Relationship between the Euclidean and Lorentzian versions of type IIB matrix model

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1. Introduction

◆The type IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)]

Space-time does not exist a priori but emerges dynamically from the degrees of freedom of matrices.

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

A_μ, Ψ : $N \times N$ Hermitian matrices $(\mu = 0, \ldots, D-1)$

$$= S_\mathrm{b} + S_\mathrm{f} \ , \quad S_\mathrm{b} = -\frac{1}{4g^2}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right), \quad S_\mathrm{f} = -\frac{1}{2g^2}\mathrm{Tr}\left(\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

- IR cutoffs to make this model well-defined: $\frac{1}{N}\mathrm{Tr}(A_0)^2 = \kappa \ , \ \frac{1}{N}\mathrm{Tr}(A_i)^2 = 1 \ \ (i = 1, \ldots, 9)$
- Partition function: $Z = \int dA d\Psi e^{iS} = \int dA \ e^{iS_\mathrm{b}} \ \mathrm{Pf}\mathcal{M}(A)$

  Phase factor: cause of the sign problem
- Wick rotation

$$S_\mathrm{b} = -\frac{1}{4g^2}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right) = N\beta\left[-\frac{1}{2}\mathrm{Tr}(F_{0i})^2 + \frac{1}{4}(F_{ij})^2\right] \quad \beta = 1/(g^2 N), \ F_{\mu\nu} = i[A_\mu, A_\nu]$$

$$\Downarrow$$

$$\tilde{S}_\mathrm{b} = -iN\beta e^{i\frac{\pi}{2}s}\left[-\frac{1}{2}e^{-ik\pi}\mathrm{Tr}(F_{0i})^2 + \frac{1}{4}(F_{ij})^2\right]$$

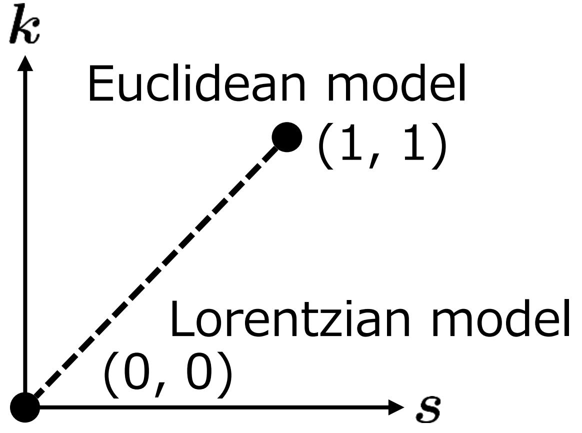
$$Z = \int dA \ e^{-\tilde{S}_\mathrm{b}} \ \mathrm{Pf}\mathcal{M}(A)$$

s : parameter of the Wick rotation on the world-sheet

k : parameter of the Wick rotation for the target space

$$A_0 \to e^{-ik\pi/2}A_0$$

Lorentzian and Euclidean models



◆ Lorentzian model [Hirasawa's talk]

- The complex Langevin method (CLM) works well.
- Continuous space structure appears, while the SSB of SO(9) symmetry doesn't occur yet.

➡ How is the SSB realized at early universe?

◆ Euclidean model well-defined without IR constraints and corresponding to $(s, k) = (1, 1)$

- Bosonic model: no sign problem, no SSB of SO(10)
- SUSY model: sign problem due to Pfaffian, SSB of SO(10) $\rightarrow$ SO(3)

Complex phase of Pfaffian causes the SSB of SO(10), which was found by using the CLM.

[Anagnostopoulos-Azuma-Ito-Nishimura-Okubo-Papadoudis ('20)]

We will show that the Lorentzian and Euclidean models are connected, and a possible scenario to describe our 4D space-time structure.

Euclidean type IIB matrix model

- 10D bosonic model

$$A_0 = \begin{pmatrix} \alpha_1 & & & & & & \\ & \alpha_2 & & & & & \\ & & \ddots & & & & \\ & & & \boxed{\begin{matrix} \alpha_{k+1} & & \\ & \ddots & \\ & & \alpha_{k+n} \end{matrix}} & & & \\ & & & & \ddots & \\ & & & & & \alpha_N \end{pmatrix}$$

n : block size

Euclidean: $(s,k)=(1,1)$

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[\tilde{A}^{\mu},\tilde{A}^{\nu}][\tilde{A}_{\mu},\tilde{A}_{\nu}] + \cancel{\frac{1}{2}\bar{\Psi}\Gamma^{\mu}[\tilde{A}_{\mu},\Psi]}\right)$$

$$\beta = 1/(g^2N),\ F_{\mu\nu} = i\left[\tilde{A}_{\mu},\tilde{A}_{\nu}\right]$$

$$S = -iN\beta\left[-\frac{1}{2}\mathrm{Tr}(F_{0i})^2 + \frac{1}{4}(F_{ij})^2\right] \xrightarrow{\text{Wick rotation}} N\beta\left[\frac{1}{2}\mathrm{Tr}(F_{0i})^2 + \frac{1}{4}(F_{ij})^2\right]$$

If there are no IR constraints, the Lorentzian model can be connected to the Euclidean one by the analytic continuation.

$$\begin{cases} A_0 = e^{i\frac{\pi}{8}s - i\frac{\pi}{2}k}\tilde{A}_0 = e^{-i\frac{3\pi}{8}u}\tilde{A}_0 \\ A_i = e^{i\frac{\pi}{8}s}\tilde{A}_i \end{cases}$$

$$s = k = u$$

$$u = 0 \mapsto u = 1$$

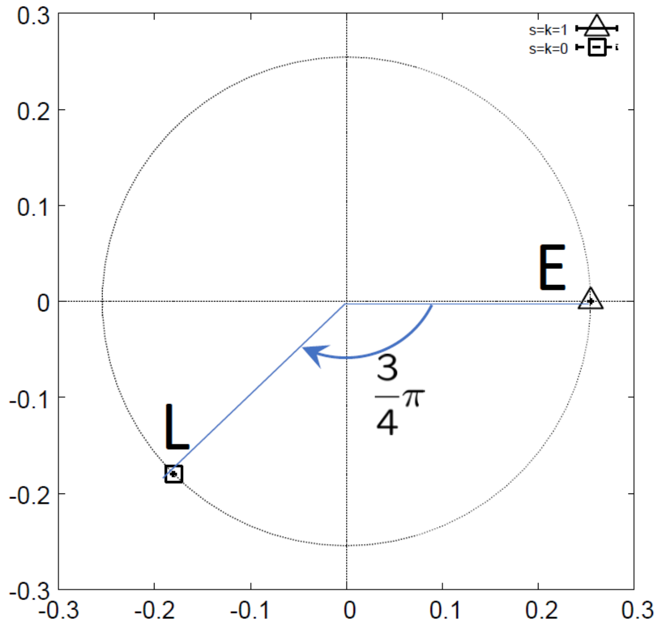
Lorentzian Euclidean

Contents

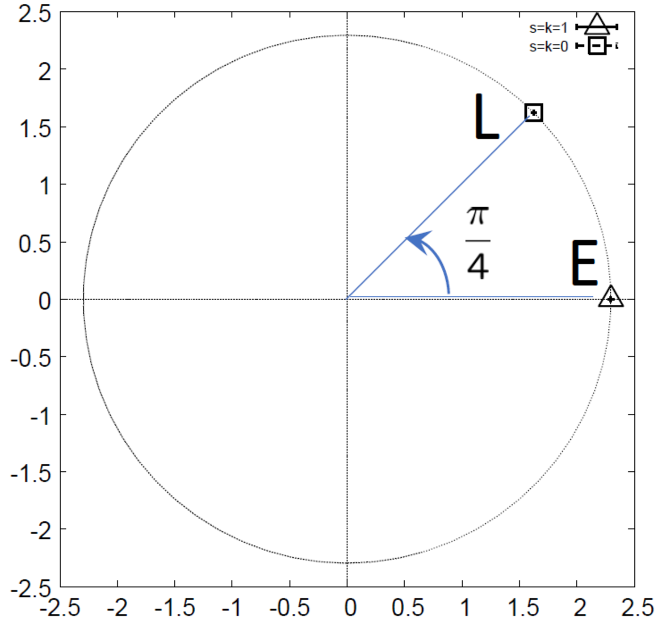
2. Relationship between the Euclidean and Lorentzian versions of type IIB matrix model

Equivalence for the case without IR constraints

$$\left\langle \frac{1}{N}\,\mathrm{Tr}\,A_0^2 \right\rangle_{\mathrm{L}} = e^{-i\frac{3\pi}{4}} \left\langle \frac{1}{N}\,\mathrm{Tr}\,\tilde{A}_0^2 \right\rangle_{\mathrm{E}}$$



$$\left\langle \frac{1}{N}\,\mathrm{Tr}\,A_i^2 \right\rangle_{\mathrm{L}} = e^{i\frac{\pi}{4}} \left\langle \frac{1}{N}\,\mathrm{Tr}\,\tilde{A}_i^2 \right\rangle_{\mathrm{E}}$$



$$\begin{cases} A_0 = e^{-i\frac{3\pi}{8}u}\tilde{A}_0 \\ A_i = e^{i\frac{\pi}{8}u}\tilde{A}_i \end{cases}$$

$$u = 0 \mapsto u = 1$$

Lorentzian Euclidean

Without IR constraints, one can find that the Lorentzian model (L) is equivalent to the Euclidean one (E) if A_μ are rotated with an appropriate phase.
Both quantities are complex (not real) in the Lorentzian model.

The emergent space-time should be interpreted as being Euclidean.

IR constraints

We introduce IR constraints to realize the real time in the Lorentzian model.

- Order of eigenvalues of A_0 [Nishimura-Tsuchiya ('19)]

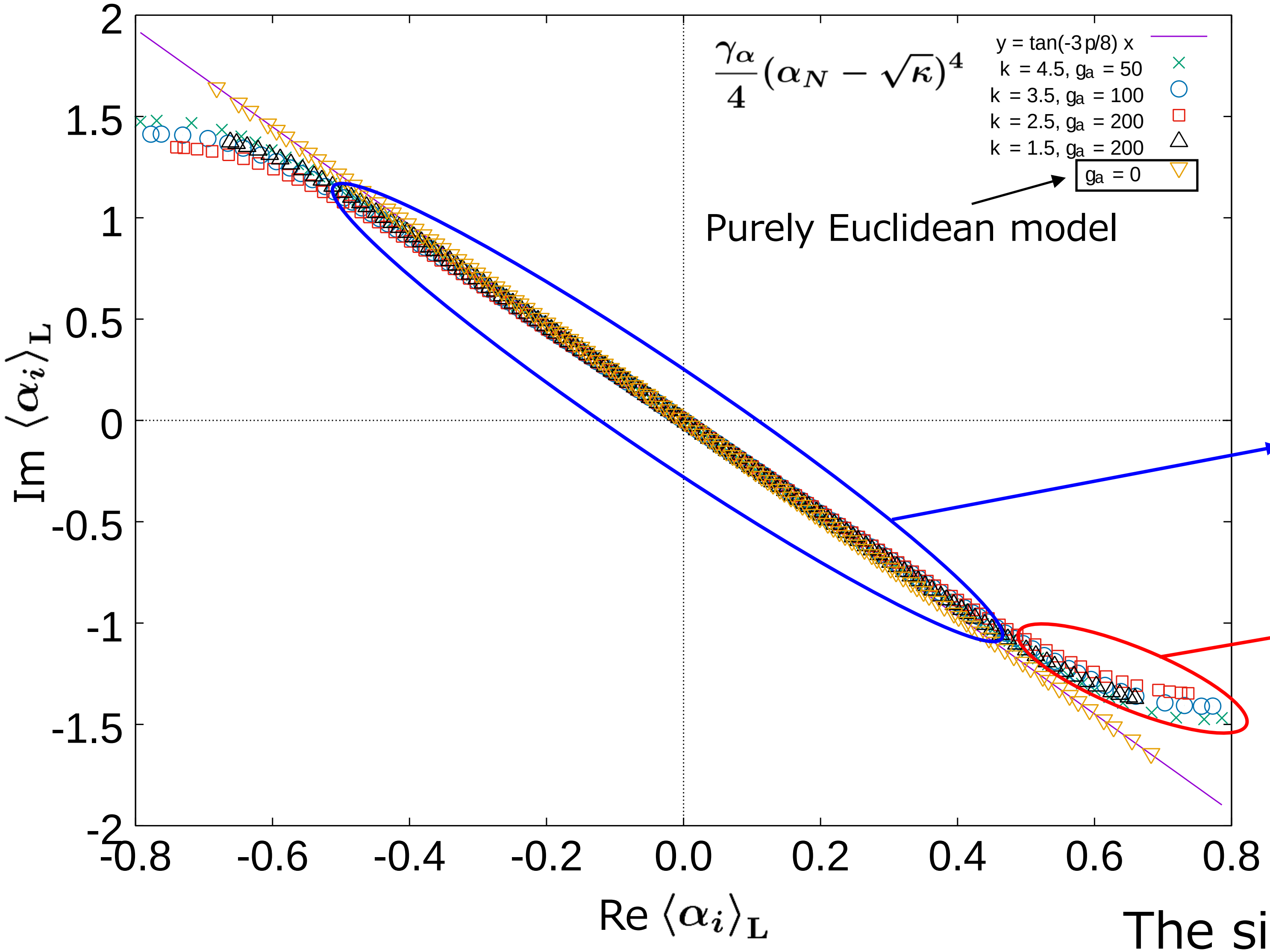
$$\tilde{A}_0 = \mathrm{diag}(\tilde{\alpha}_1, \ldots, \tilde{\alpha}_N)\ , \quad \tilde{\alpha}_1 < \tilde{\alpha}_2 < \cdots < \tilde{\alpha}_N$$

Change of variables: $\tilde{\alpha}_1 = 0,\ \tilde{\alpha}_2 = e^{\tau_1},\ \tilde{\alpha}_3 = e^{\tau_1} + e^{\tau_2},\ \cdots,\ \tilde{\alpha}_N = \sum_{k=1}^{N-1} e^{\tau_k}$

$$A_0 = e^{-i\frac{3\pi}{8}u}\tilde{A}_0 \Longrightarrow \langle \alpha_i \rangle_{\mathrm{L}} = e^{-i\frac{3\pi}{8}u} \langle \tilde{\alpha}_i \rangle_{\mathrm{E}}$$

We introduce a new constraint: $\alpha_N = \sqrt{\kappa} \in \mathbb{C}$ instead of $\frac{1}{N}\mathrm{Tr}(A_0)^2 = \kappa\ ,\ \frac{1}{N}\mathrm{Tr}(A_i)^2 = 1$, and add a new term $\frac{\gamma_\alpha}{4}(\alpha_N - \sqrt{\kappa})^4$ to the effective action.

The expectation value of the time coordinates



$\beta = 1.0$ for all case

$$\langle \alpha_i \rangle_{\mathbf{L}} = e^{-i\frac{3\pi}{8}u} \langle \tilde{\alpha}_i \rangle_{\mathbf{E}}$$

$$(\Delta \alpha_i)_{\mathbf{L}} = (\alpha_{i+1})_{\mathbf{L}} - (\alpha_i)_{\mathbf{L}}$$

$(\Delta \alpha_i)_{\mathbf{L}} \propto e^{-i\frac{3\pi}{8}}$ (Euclidean regime)

$(\Delta \alpha_i)_{\mathbf{L}} > 0$ (Lorentzian regime)

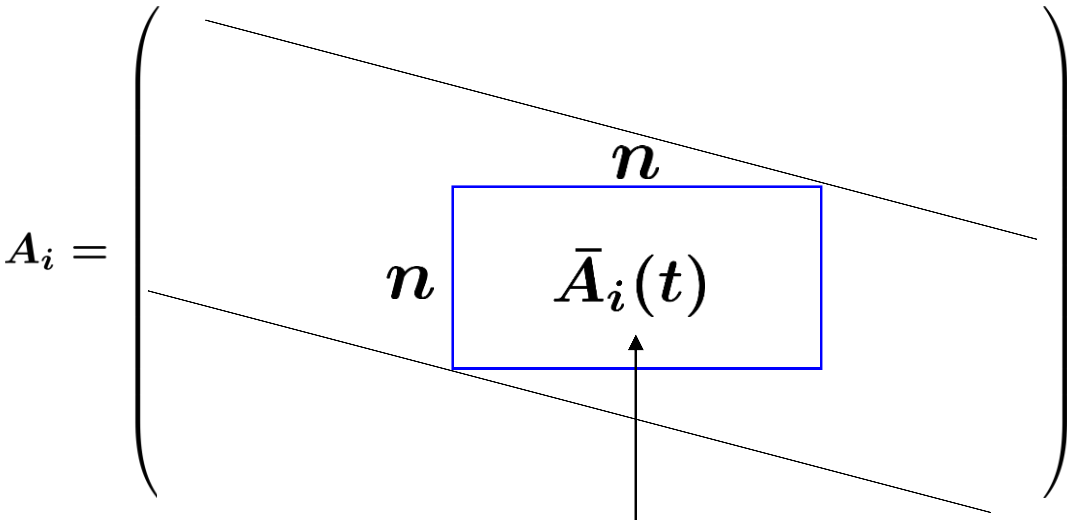
The signature change occurs dynamically.
How about space?

How to extract the time evolution

$A_\mu \to U A_\mu U^\dagger,\ U$ diagonalizes A_0.

n : block size

$$A_0 = \begin{pmatrix} \alpha_1 & & & & & \\ & \alpha_2 & & & & \\ & & \ddots & & & \\ & & & \boxed{\begin{matrix} \alpha_{k+1} & & \\ & \ddots & \\ & & \alpha_{k+n} \end{matrix}} & & \\ & & & & \ddots & \\ & & & & & \alpha_N \end{pmatrix}$$



$$\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C}$$

represent space at t

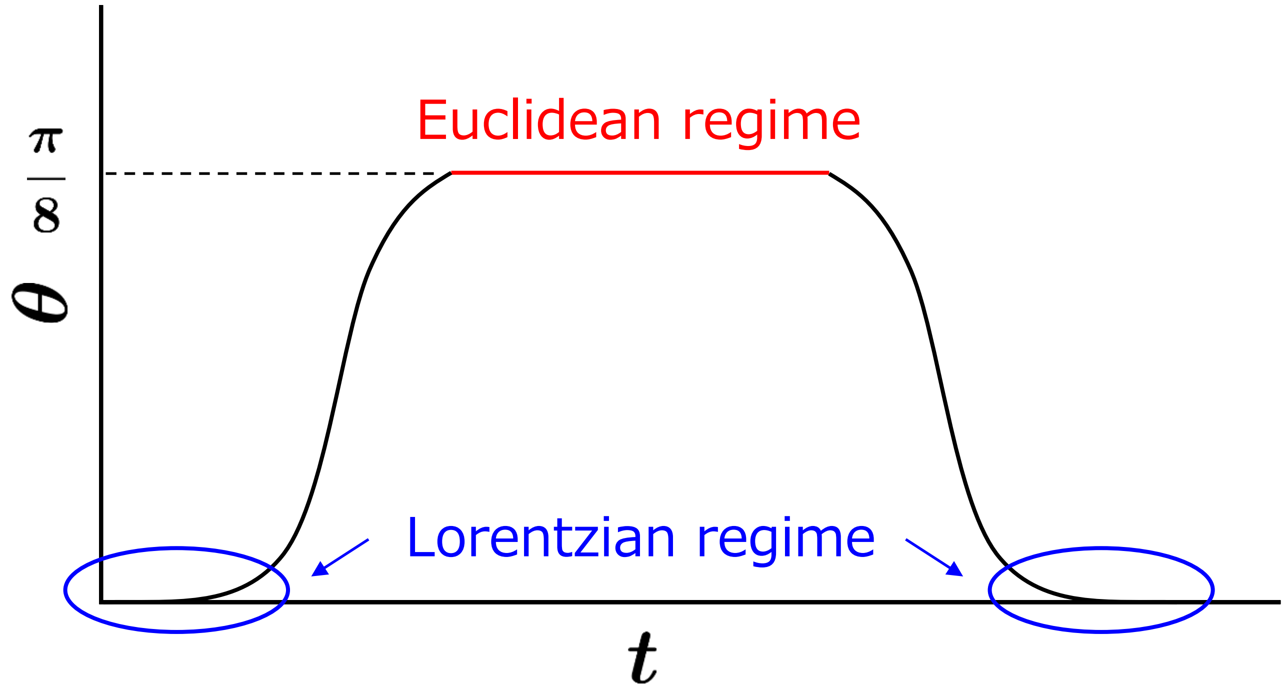
$$t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

The extent of space $R^2(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle$

Once, we introduce the constraint $\alpha_N = \sqrt{\kappa} \in \mathbb{C}$, $\left\langle \frac{1}{N} \mathrm{Tr}\, A_i^2 \right\rangle_{\mathrm{L}} = e^{i\frac{\pi}{4}} \left\langle \frac{1}{N} \mathrm{Tr}\, \tilde{A}_i^2 \right\rangle_{\mathrm{E}}$ is not true anymore.

$$R^2(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta} \left| R^2(t) \right|$$

<Our expectation>



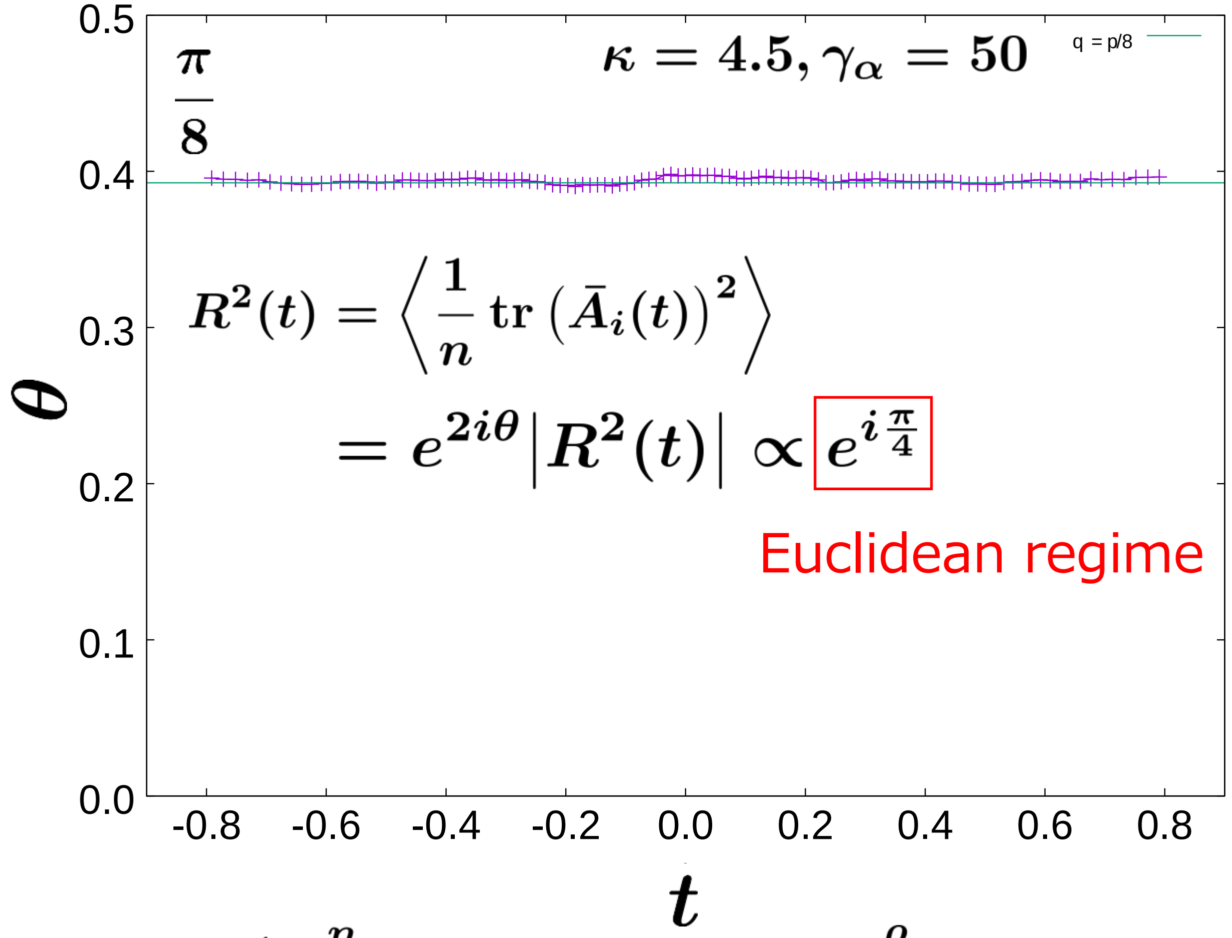
Thus, the signature of the space-time can change dynamically in this model.

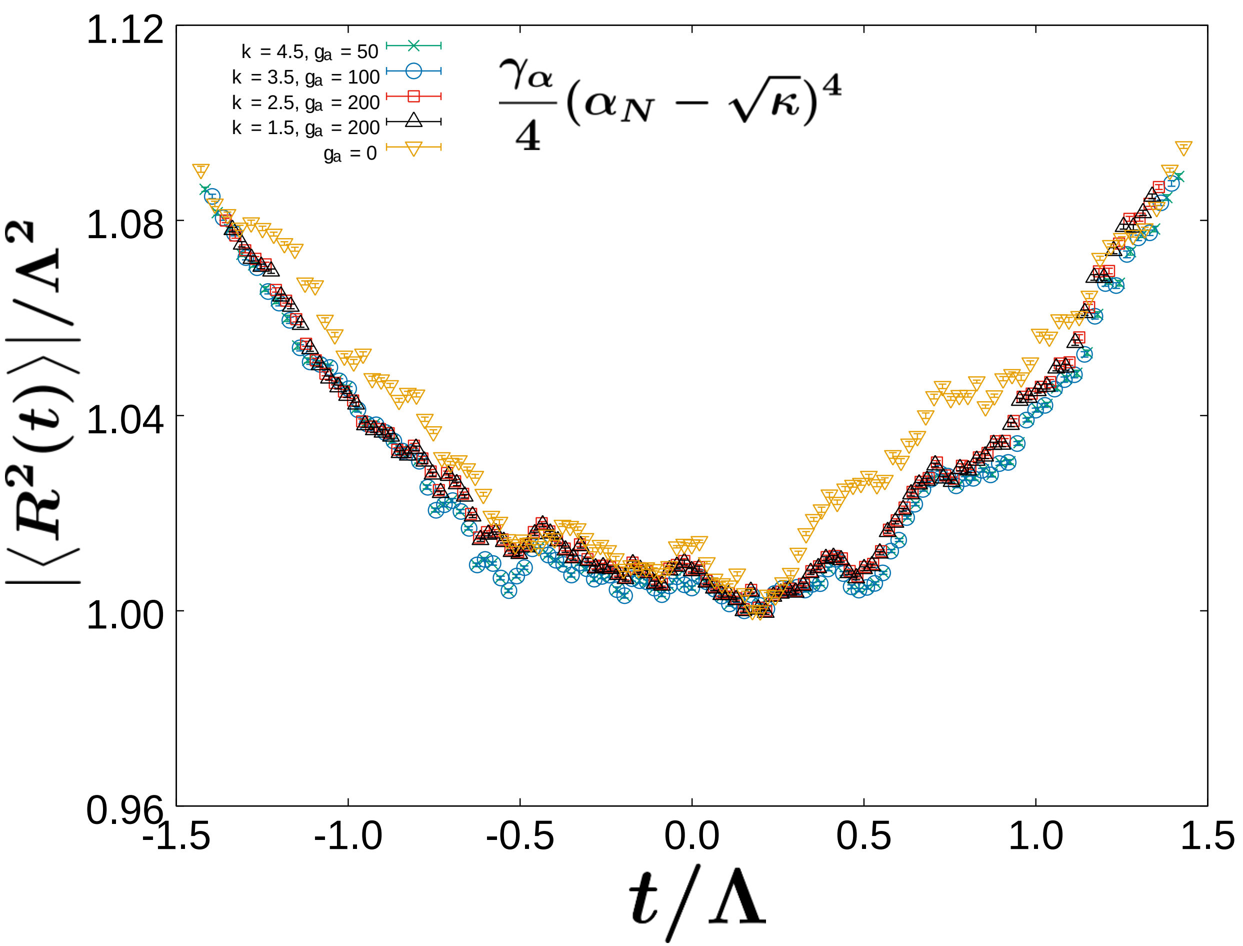
3. Results

The time evolution of space

$$N = 128,\ n = 16,\ \beta = 1.0$$

Λ : minimum value of $\sqrt{|\langle R^2(t)\rangle|}$





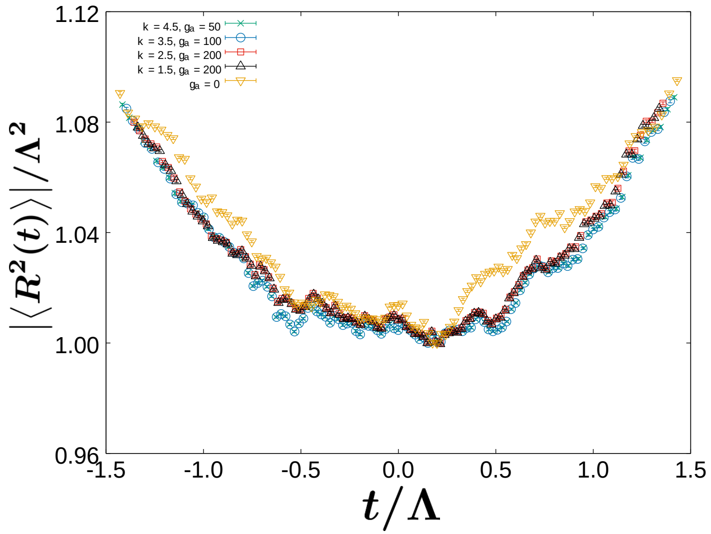
$$\bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C}\ ,\ t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

➡ Scaling behavior observed.

4. Conclusion and outlook

Conclusion

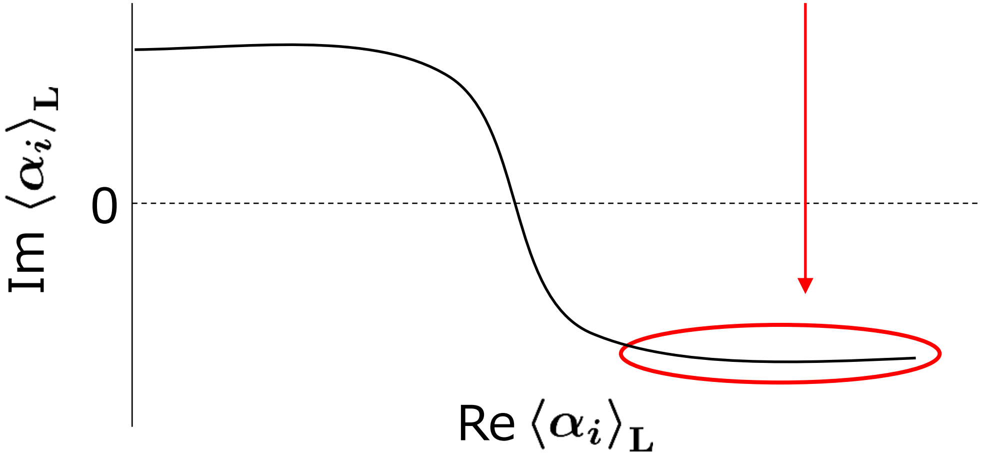
- We performed complex Langevin simulations of the type IIB matrix model.
- We showed that the Lorentzian and Euclidean versions of the type IIB matrix model are equivalent if we do not introduce IR cutoffs.
- The emergent space-time should be interpreted as being Euclidean.
- We introduce a "boundary condition" on both ends of the eigenvalue spectrum of A_0, $\alpha_1 = 0,\ \alpha_N = \sqrt{\kappa} \in \mathbb{C}$.
- Scaling behavior of the extent of space is confirmed.



Outlook

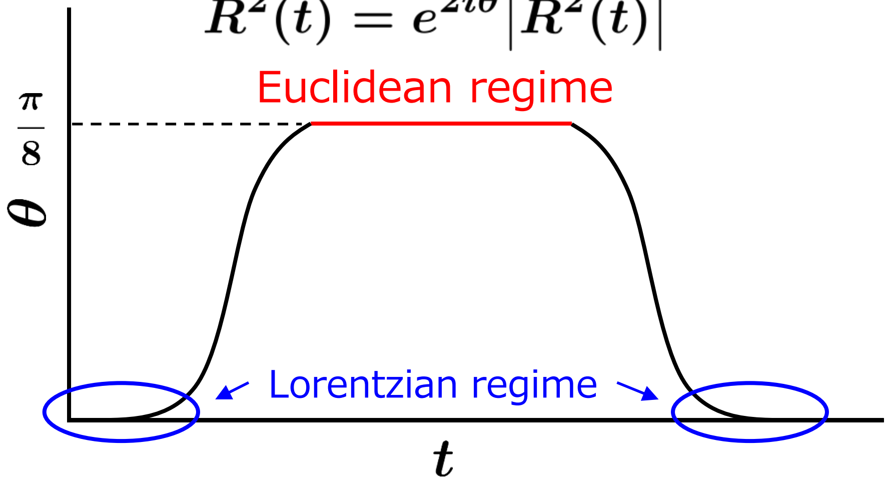
We can make the real time appear at both ends of the eigenvalue spectrum of A_0 , while it is nontrivial whether space becomes real at late times.

- Can we make the real-time regime extended by the condition at larger N ?



<Our expectation>

$$R^2(t) = e^{2i\theta}\left|R^2(t)\right|$$



- Can we observe space to be real at late times?
  So far, we have observed complex-valued $R^2(t)$.

- Include fermionic matrices and simulate SUSY model. Does the SSB in the Euclidean model imply that SO(3) is realized in the present model as well?

Back up

Complex Langevin method

Complex-valued function

$$Z = \int dx\, w(x), \quad x \in \mathbb{R} \longrightarrow z \in \mathbb{C}$$

complexify variable

[Parisi ('83), Klauder ('84)]

◆Complex Langevin equation (t_L: Langevin time)

$$\frac{dz_k}{dt_\mathrm{L}} = \frac{1}{w}\frac{\partial w}{\partial z_k} + \eta_k(t_\mathrm{L}) \qquad P(\eta_k(t_\mathrm{L})) \propto \exp\left(-\frac{1}{4}\int dt_\mathrm{L} \sum_k [\eta_k(t_\mathrm{L})]^2\right)$$

drift term

Gaussian noise, real

◆Necessary and sufficient condition to justify the CLM

[Nagata-Nishimura-Shimasaki ('16)]

The probability distribution of the drift term should be exponentially suppressed for large values.