THE SU(N) RUNNING COUPLING IN THE TWISTED GRADIENT FLOW SCHEME AND VOLUME INDEPENDENCE

J.L, DASILVA GOLÁN, M. GARCÍA PÉREZ, A. RAMOS

Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid

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THE GRADIENT FLOW

Renormalization procedure where the gauge field $A_\mu(x)$ is replaced by a set of smooth, flow time dependent, fields $B_\mu(x)$ driven by the so-called "Flow equations":

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t) \quad B_\mu(x, t=0) = A_\mu(x)$$

SMOOTHING THE GAUGE FIELDS IN A RANGE $\sqrt{8t}$

Gauge invariant composite observables are automatically renormalized quantities for $t > 0$

[arXiv:1101.0963]

Renormalized couplings can be trivially introduced, as for example, with the energy density:

$$E(t) = \frac{1}{2} Tr\left(G_{\mu\nu}(x,t) G_{\mu\nu}(x,t)\right)$$

THE COUPLING:

$$\lambda(\mu) = \mathcal{N} \left\langle t^2 E(t) \right\rangle \Big|_{\sqrt{8t}=\mu^{-1}}$$

GEOMETRY AND BOUNDARY CONDITIONS

The twisted gradient flow (TGF) scheme is defined by introducing $SU(N)$ YM theories on an asymmetrical hyper-box of size $l^2 \times (Nl)^2$. [arXiv:1903.08029 & arXiv:2001.03735]

SHORT DIRECTIONS: TWISTED BOUNDARY CONDITIONS

$$A_\mu(x + l\hat{\nu}) = \Gamma_\nu A_\mu(x) \Gamma_\nu^\dagger, \text{ for } \nu = 1{,}2$$

$$\Gamma_1 \Gamma_2 = Z_{12} \Gamma_2 \Gamma_1$$

LONG DIRECTIONS: PERIODIC BOUNDARY CONDITIONS

$$\tilde{l} \equiv N \times l$$

Projecting within the sector of zero topological charge, the coupling definition adopted is:

$$\lambda_{TGF}(\mu) = \frac{128\pi^2 t^2}{3N\mathscr{A}(\pi c^2)} \frac{< E(t)\, \delta_Q >}{< \delta_Q >} \Bigg|_{\sqrt{8t} = c\tilde{l} = \mu^{-1}}$$

Characterising the scheme (we use $c = 0.3$)

THE Λ PARAMETER

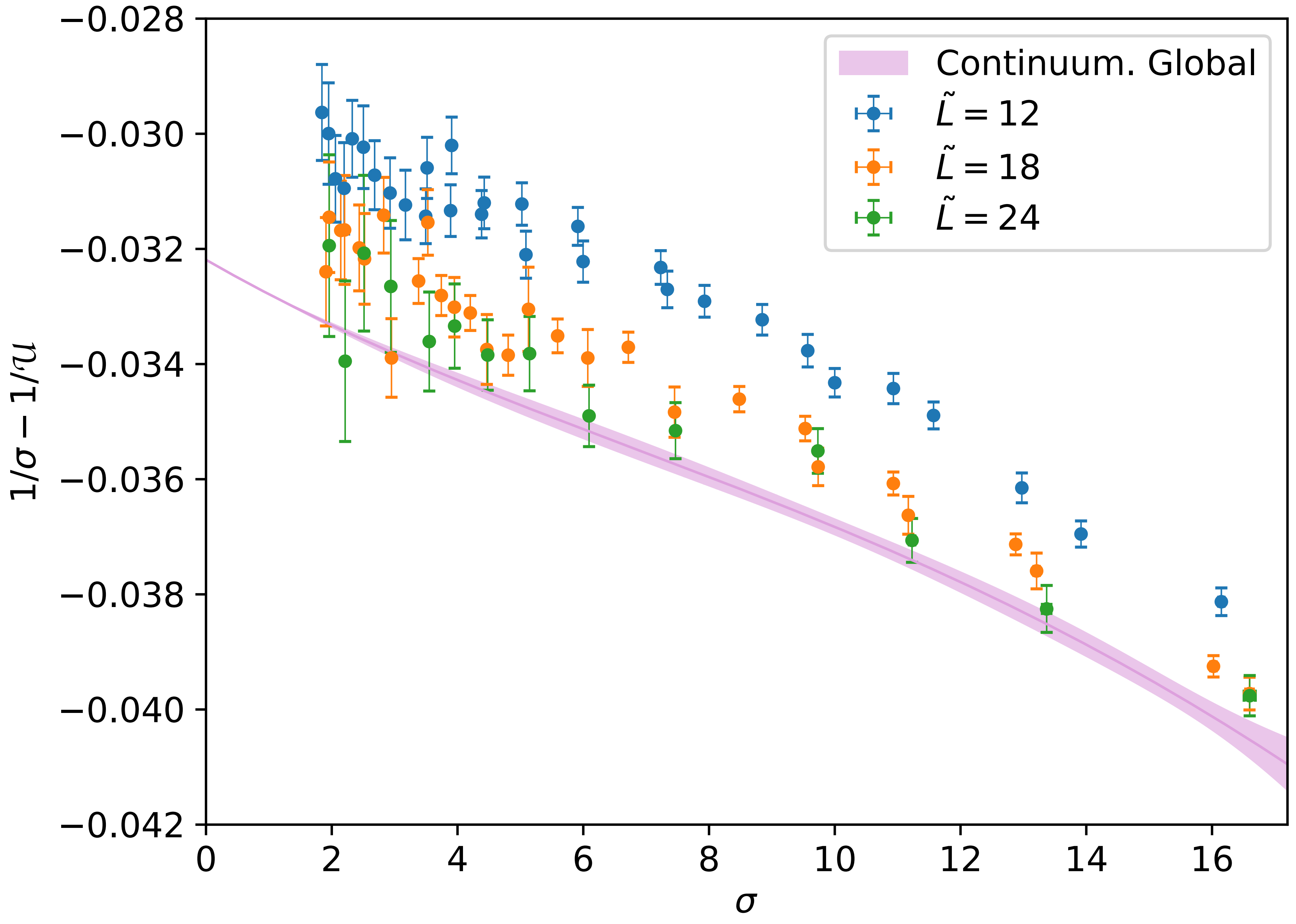
$$\frac{\Lambda_{TGF}}{\mu_{ref}} = \underbrace{\lim_{\lambda_{TGF}(\mu_{pt})\to 0} \left(b_0 \lambda_{TGF}(\mu_{pt})\right)^{\frac{-b_1}{2b_0^2}} \exp\left(\frac{-1}{2b_0\lambda_{TGF}(\mu_{pt})}\right) I^{(n)}_{TGF}\left(\lambda_{TGF}(\mu_{pt})\right)}_{\Lambda_{TGF}/\mu_{pt}} \times \underbrace{\exp\left[-\int_{\lambda_{TGF}(\mu_{pt})}^{\lambda_{TGF}(\mu_{ref})} \frac{dx}{2\beta_{TGF}(x)}\right]}_{\mu_{pt}/\mu_{ref}}$$

Finite size scaling allows to simulate large energy scales by linking the renormalization scale with the physical size of the box; consequently, we use the so-called step scaling function:

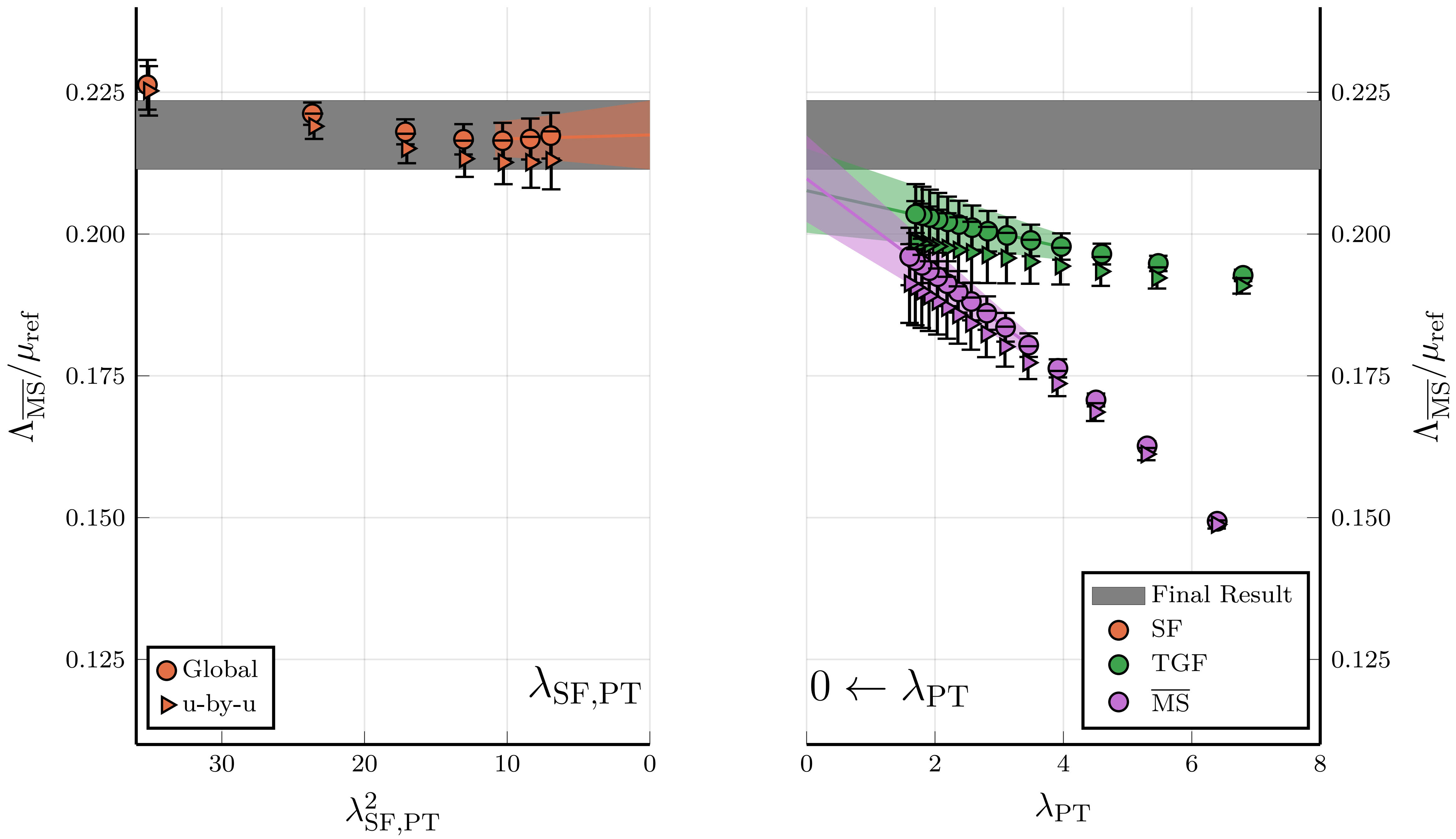
$$\sigma(u) = \lambda_{TGF}(\mu/2)\Big|_{\lambda_{TGF}(\mu)=u}$$

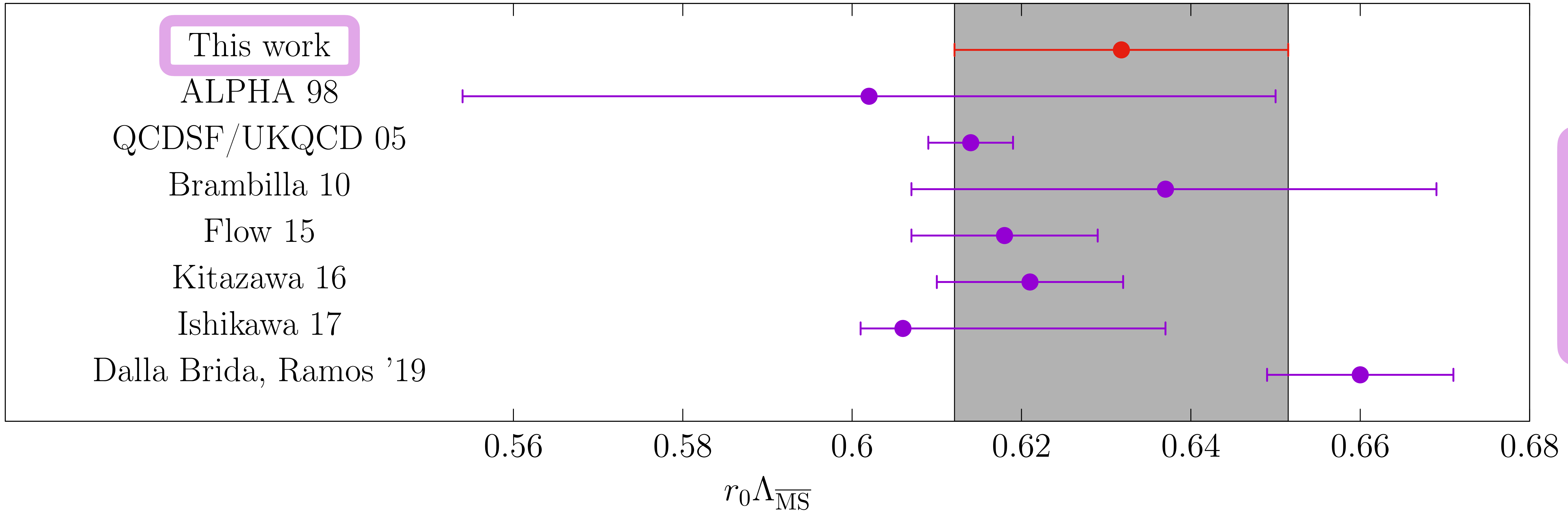
EACH APPLICATION:

$$\int_u^{\sigma(u)} \frac{dx}{\beta_s(x)} = -2\log 2$$



We have determined $\frac{\Lambda_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{ref}}}$ by running the TGF coupling, by doing the matching with SF data, and by matching with the 5-loop expansion of $\overline{\mathrm{MS}}$ scheme [arXiv:2107.03747]

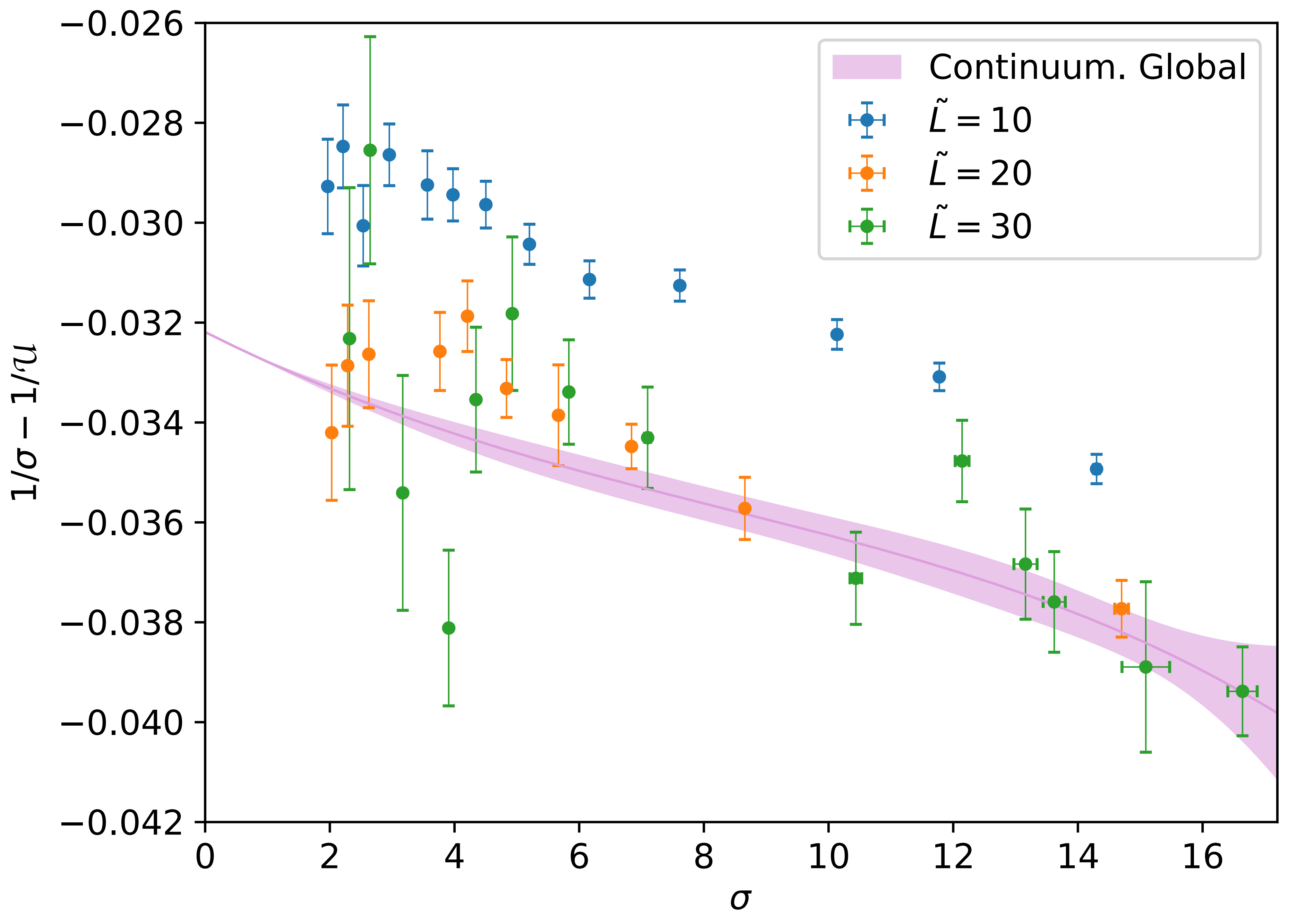




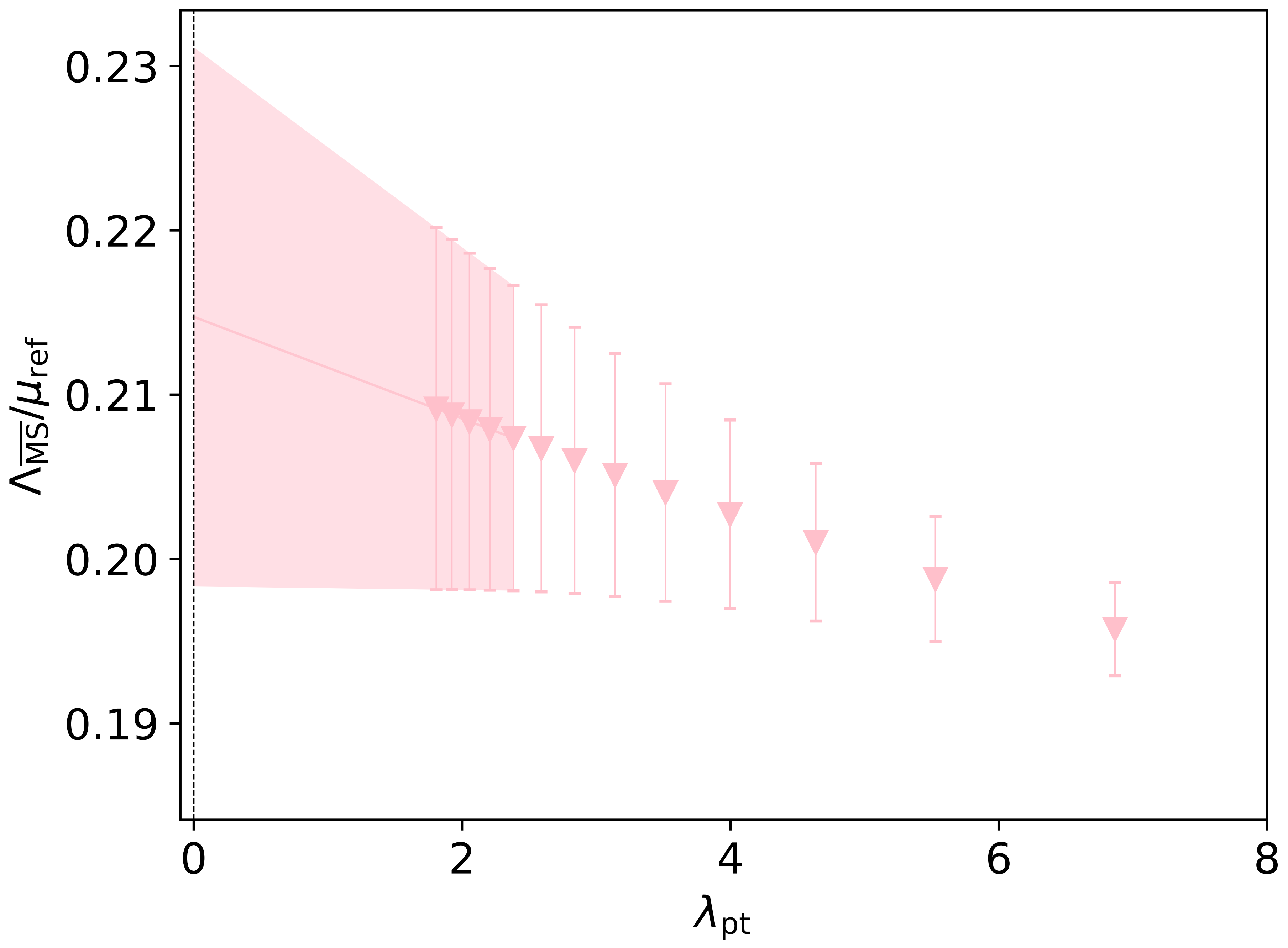
$$r_0 \times \Lambda_{\overline{\mathrm{MS}}} = 0.632(20)$$
$$\sqrt{8t_0} \times \Lambda_{\overline{\mathrm{MS}}} = 0.603(17)$$

THE Λ PARAMETER

Extracted with the same procedure as SU(3), using the step scaling function to relate the high energy scale μ_{ref} and the perturbative one μ_{pt}.

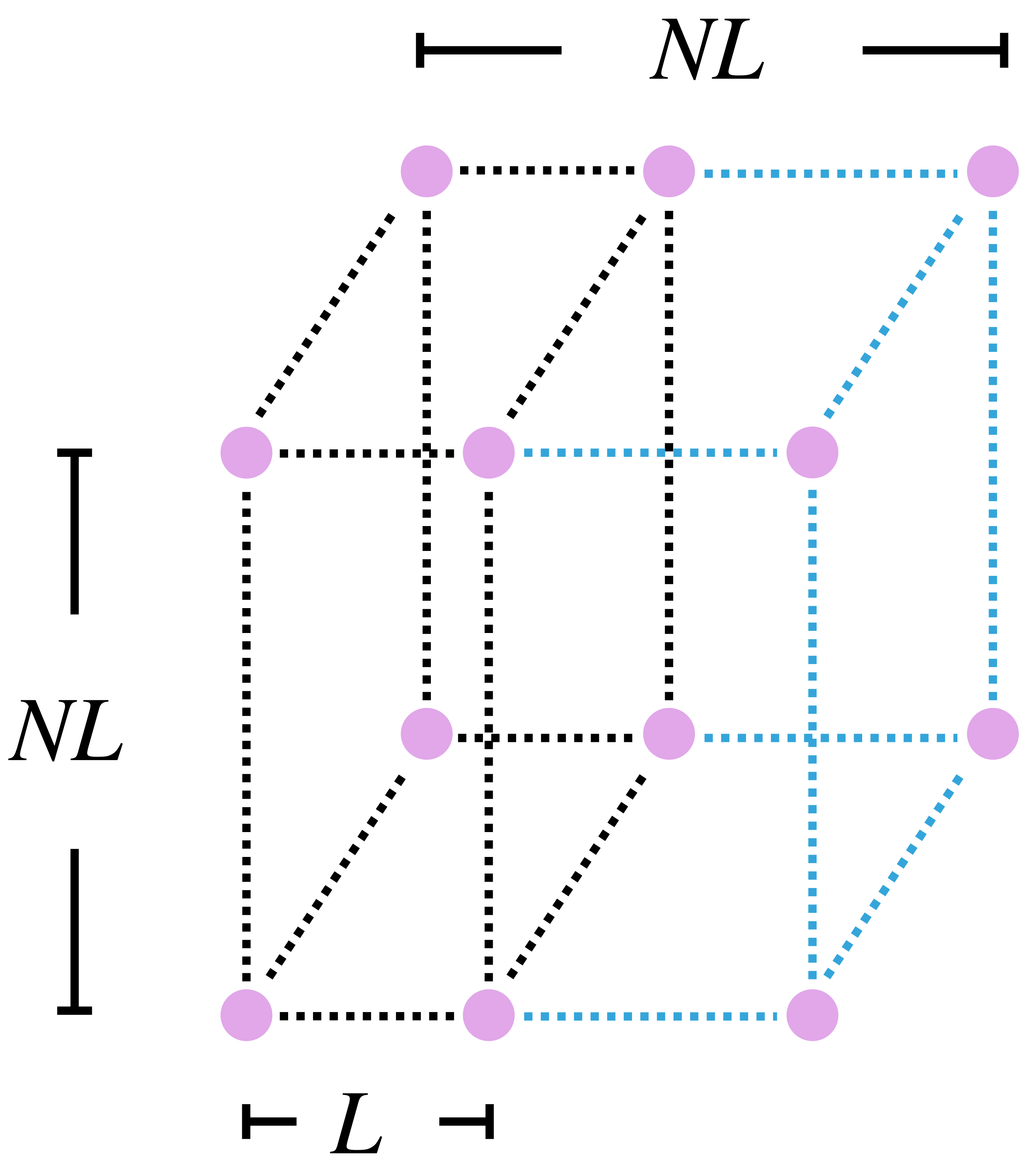


$$\frac{\Lambda_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{ref}}} = 0.215(16)$$



Same renormalization condition as SU(3), as λ_{TGF} is the quantity with "good" N scaling

TESTING FINITE VOLUME EFFECTS PER PLANES



We separate the computation per planes:

✓ *TT*: plane having non-trivial twist in both directions.

✓ *PT*: four planes sharing one direction with the twisted planes.

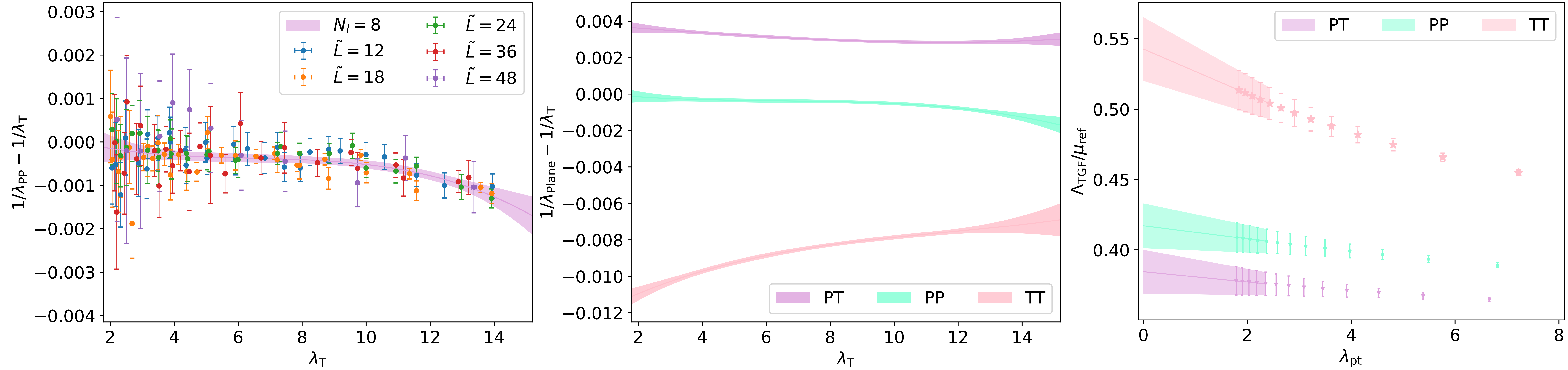
✓ *PP*: plane orthogonal to the twisted one.

μ_{ref} is fixed by doing the matching between the coupling in the total computation and the coupling in each plane. The symmetry of the box is tested with the ratio:

$$R(\Lambda_{\text{TGF}}) = \frac{\Lambda_{\text{TGF}}(\text{Plane})}{\Lambda_{\text{TGF}}(\text{All})}$$

$R \to 1$ IN THE FULLY SYMMETRIC CASE

MATCHING BETWEEN PLANES AND TOTAL COMPUTATION



$$\frac{1}{\lambda_{\mathrm{Plane}}(\mu)} - \frac{1}{u(\mu)} = \sum_{n=0}^{3} c_k u^k + \left(\frac{1}{\tilde{L}^2}\right) \times \sum_{n=0}^{N_l} \rho_k u^k$$

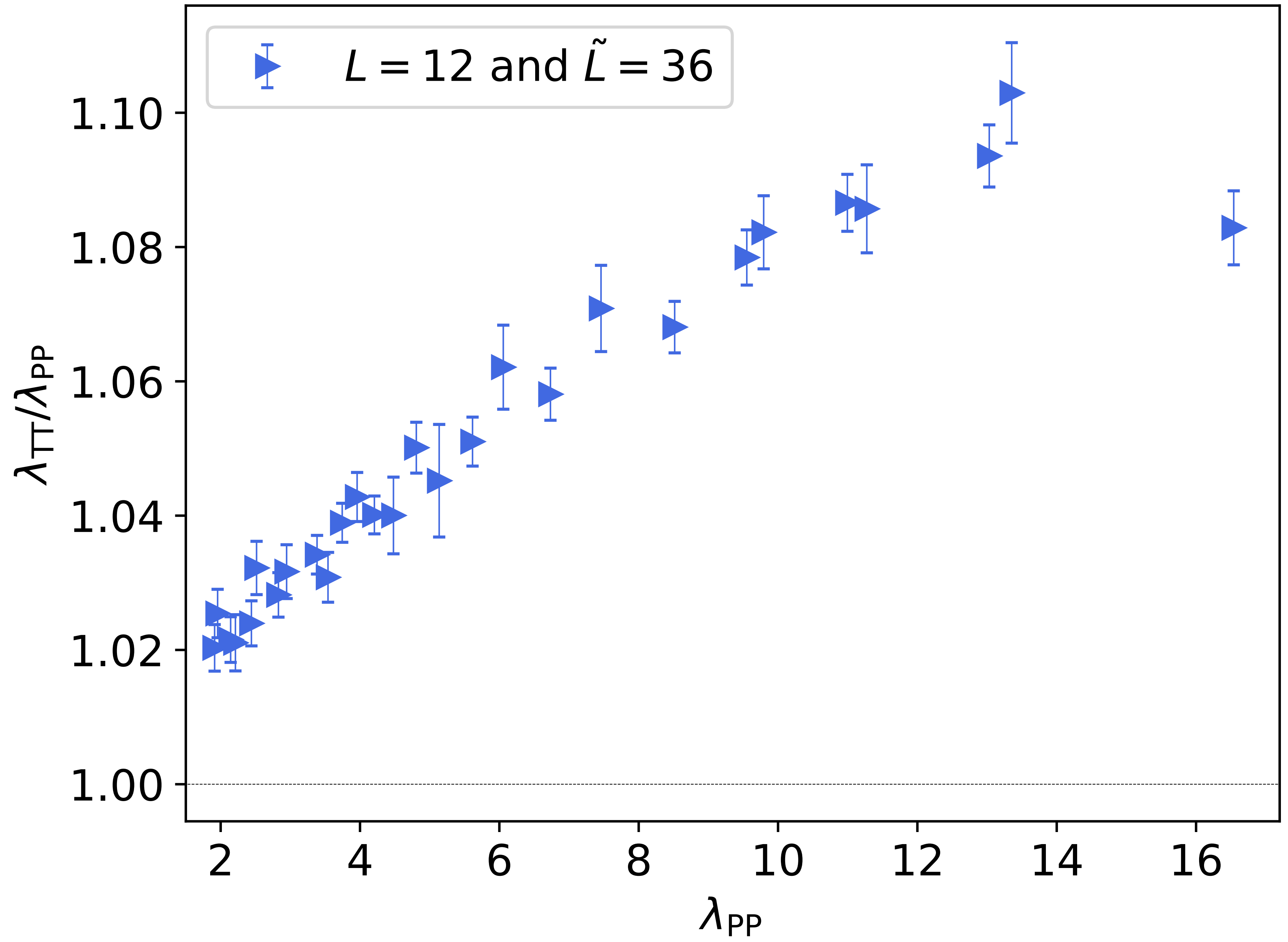
$$u(\mu) = \lambda_{\mathrm{T}}(\mu)$$

Plane	$\Lambda_{\mathrm{TGF}}/\mu_{\mathrm{ref}}$	$R(\Lambda_{\mathrm{TGF}})$
Total	0.418(17)	-------
TT	0.543(23)	1.299(74)
PT	0.385(16)	0.921(52)
PP	0.417(16)	0.999(55)

All ratios close to 1

FINITE VOLUME EFFECTS IN THE STANDARD SET-UP

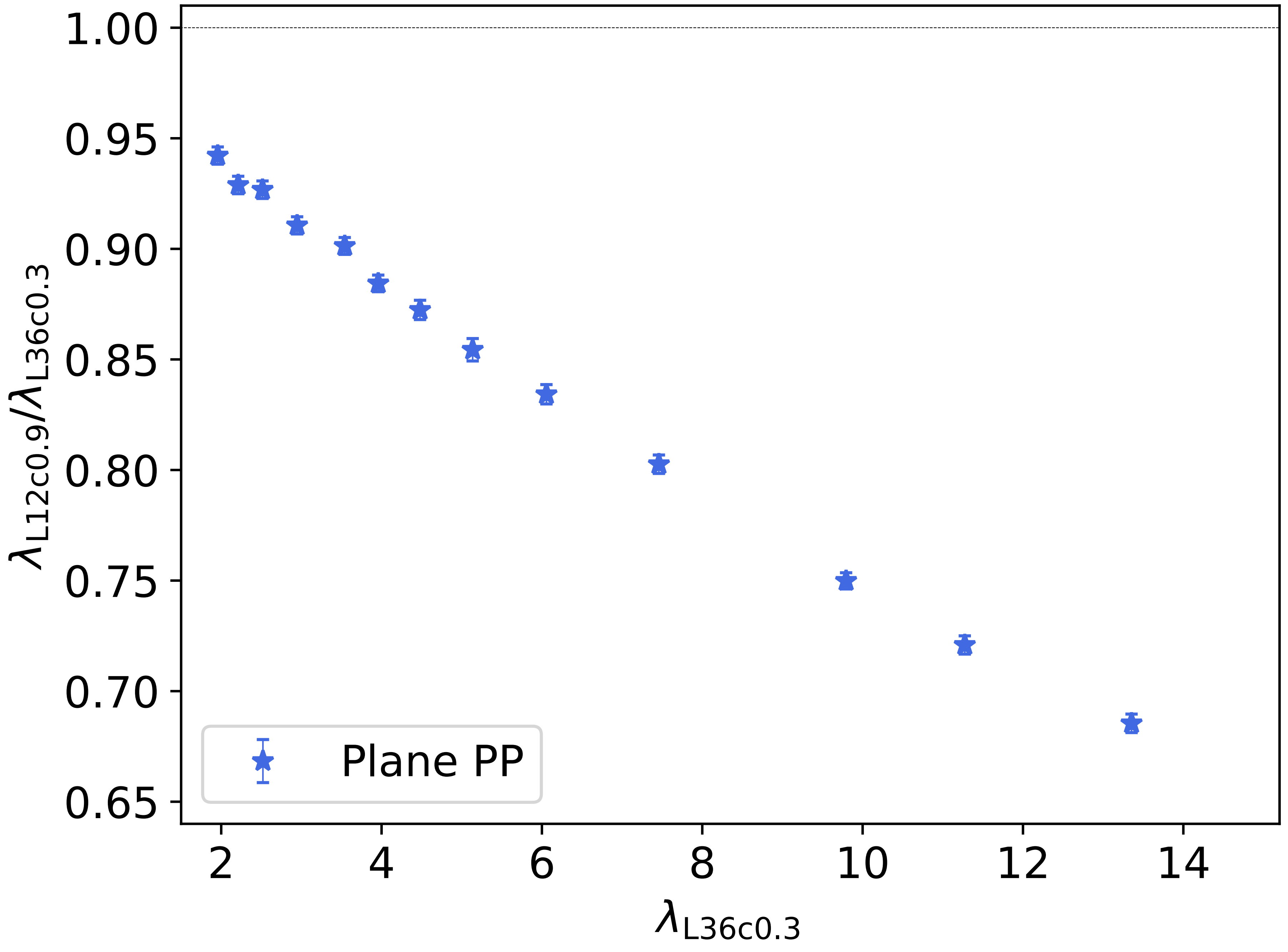
A measure of the range of finite volume effects can be obtained by comparing TT and PP calculations, and more over its differences with the standard set-up at large values of c.



λ_{TT} VS λ_{PP}

NOT EXCEEDING 10 % LEVEL

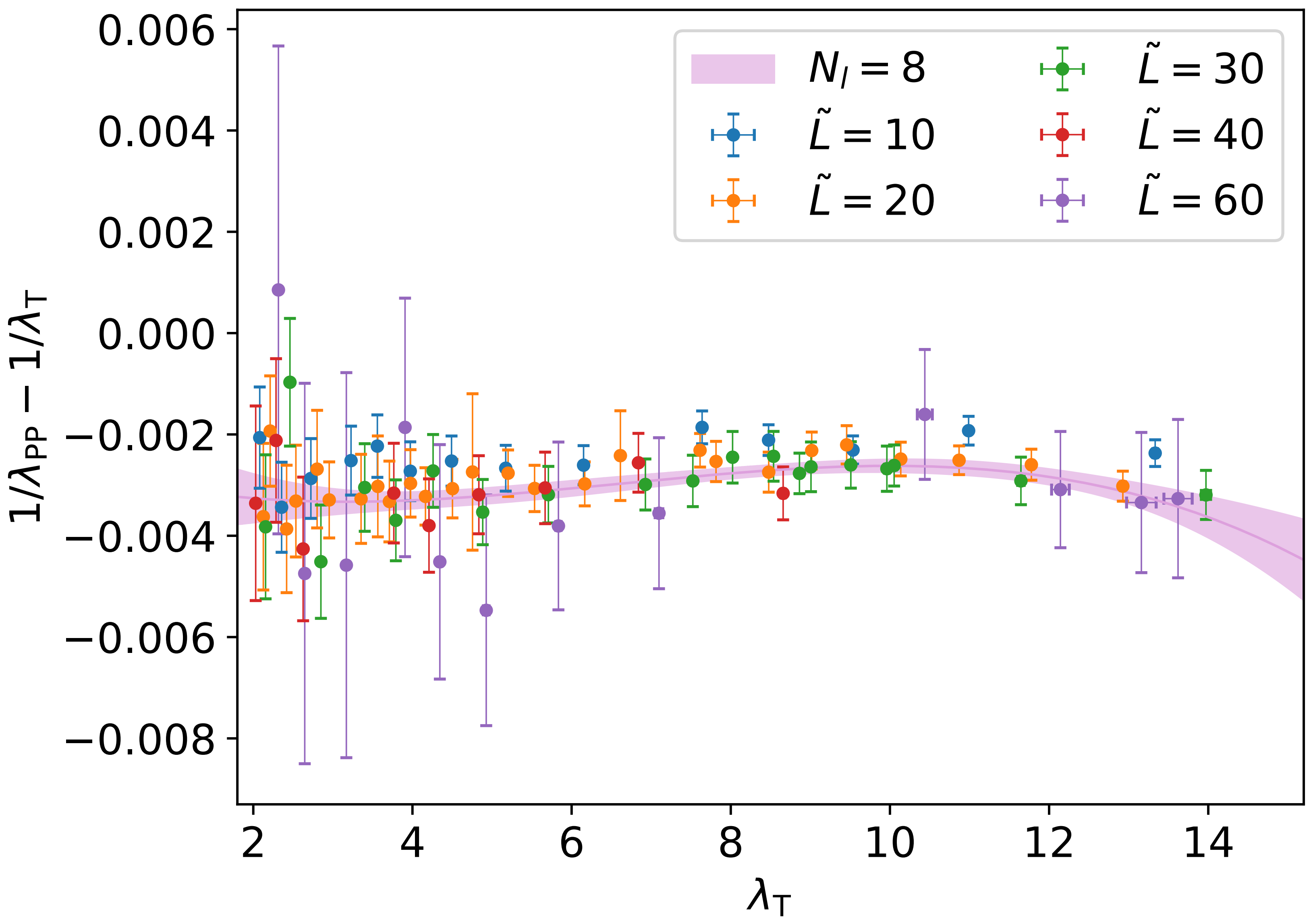
RECALL:
$\mu^{-1} = c\tilde{L}$

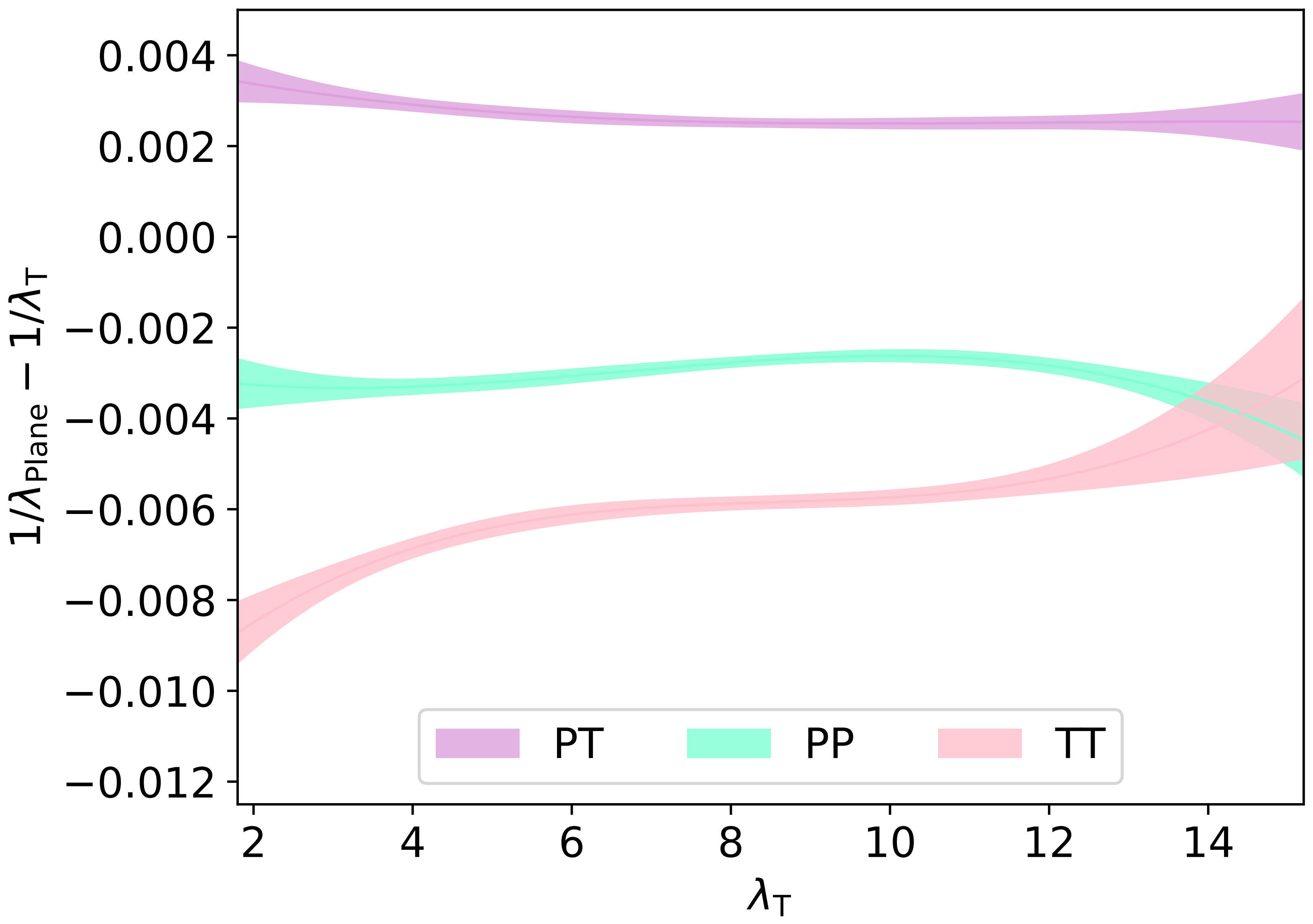


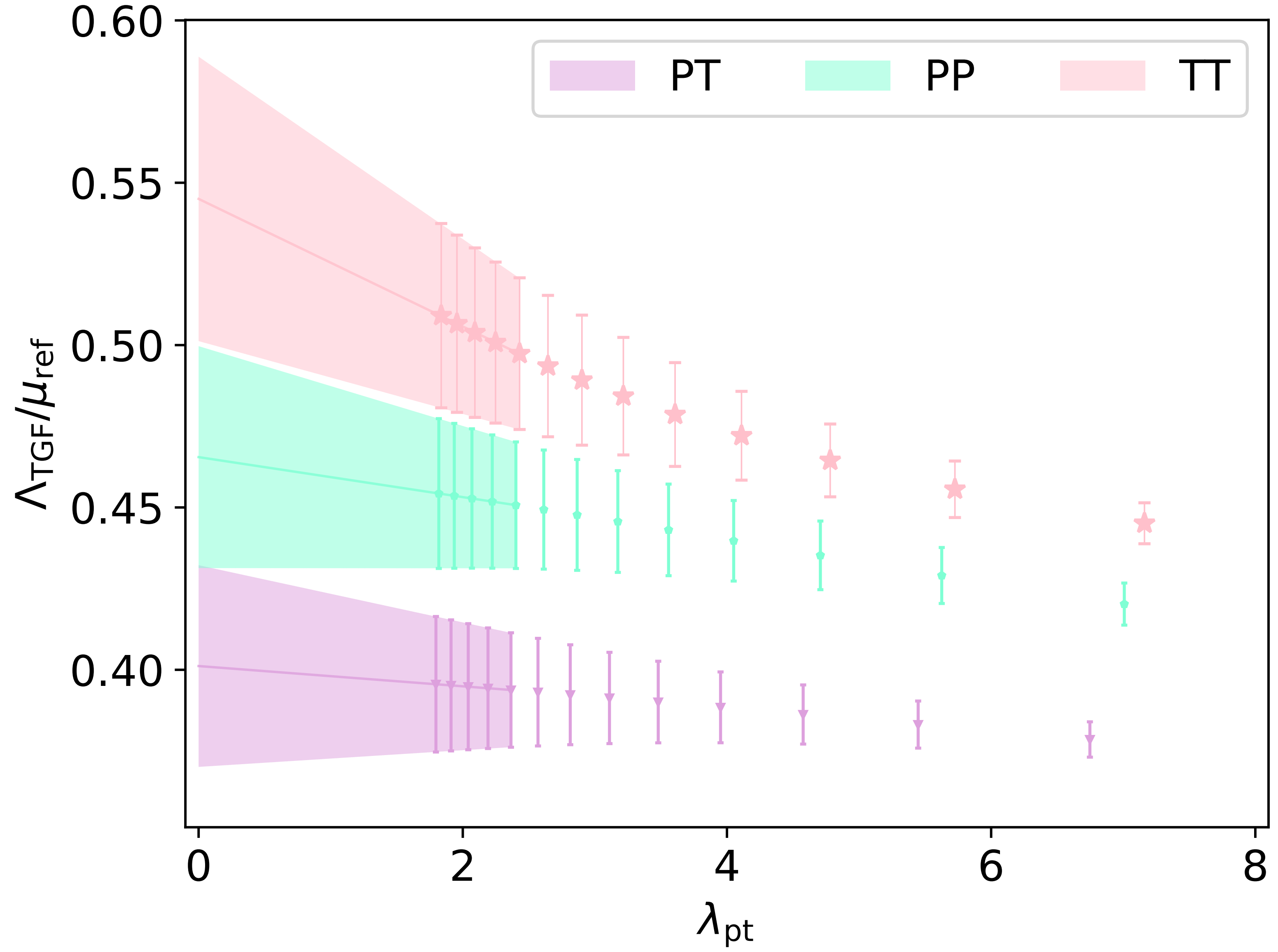
$\tilde{L} = 12\,|_{c=0.9}$ VS $\tilde{L} = 36\,|_{c=0.3}$

40 % DIFFERENCES AT STRONG COUPLING

MATCHING BETWEEN PLANES AND TOTAL COMPUTATION







Plane	Λ_{TGF}/μ_{ref}	$R(\Lambda_{TGF})$
Total	0.436(33)	-------
TT	0.545(44)	1.25(14)
PT	0.465(34)	1.07(11)
PP	0.401(31)	0.92(10)

$$\frac{\Lambda_{TGF}}{\mu_{ref}} = 0.418(17) \textbf{ for SU(3)}$$

MORE ANALYSIS IS NEEDED, WITH EXTRA VALUES OF N COMING SOON

- We have investigated the running coupling of $SU(N)$ YM theories for several values of N using a finite size scaling technique that combines **three main ingredients**: a coupling based on the gradient flow, twisted boundary conditions and an asymmetrical geometry.

- We have determine $\Lambda_{\overline{\mathrm{MS}}}$ in units of $\sqrt{8t_0}$ and r_0 for the case of $SU(3)$, obtaining good agreement with the literature. We want to stress that very precise determinations of $\Lambda_{\overline{\mathrm{MS}}}$ can be made within this scheme, that can be used via a non-perturbative matching between QCD and the pure gauge theory using heavy quarks, into a **very precise determination of α_s**, a key quantity for phenomenology in high energy physics. [arXiv:1912.06001]

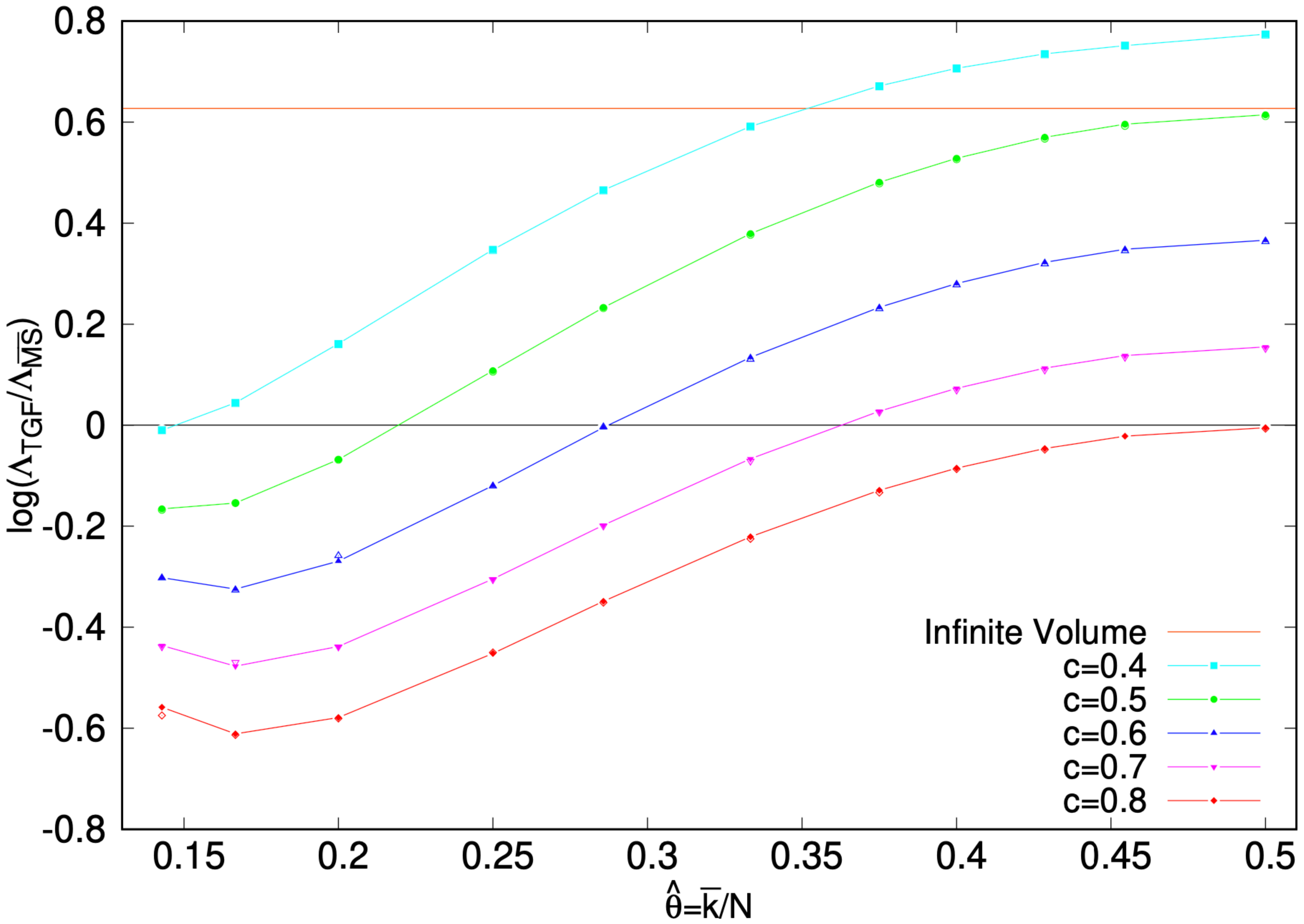
- This scheme is specially suitable for **extracting the N dependence of the Λ-parameter**, as the computation is cheaper than a brute force approach. We have presented some results for $SU(5)$ and more values of N will be coming soon.

- We have explored the finite volume effects of these calculations. We computed Λ_{TGF} per planes in both $SU(3)$ and $SU(5)$, and the extracted ratios $R(\Lambda_{\mathrm{TGF}})$ are **close to 1 in both cases**.

THANK YOU FOR YOUR ATTENTION.

QUESTIONS?

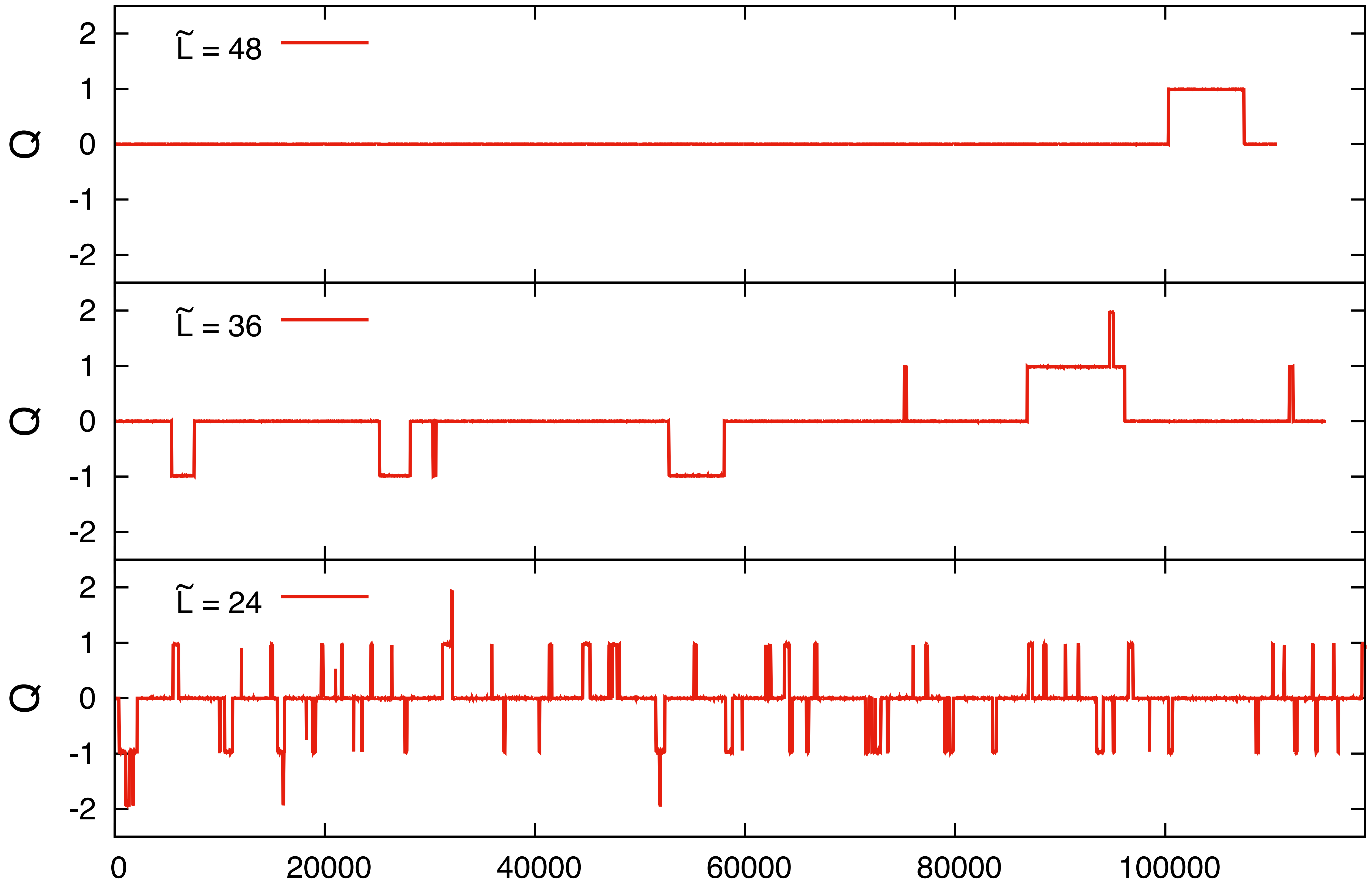
RATIO OF TGF AND MS

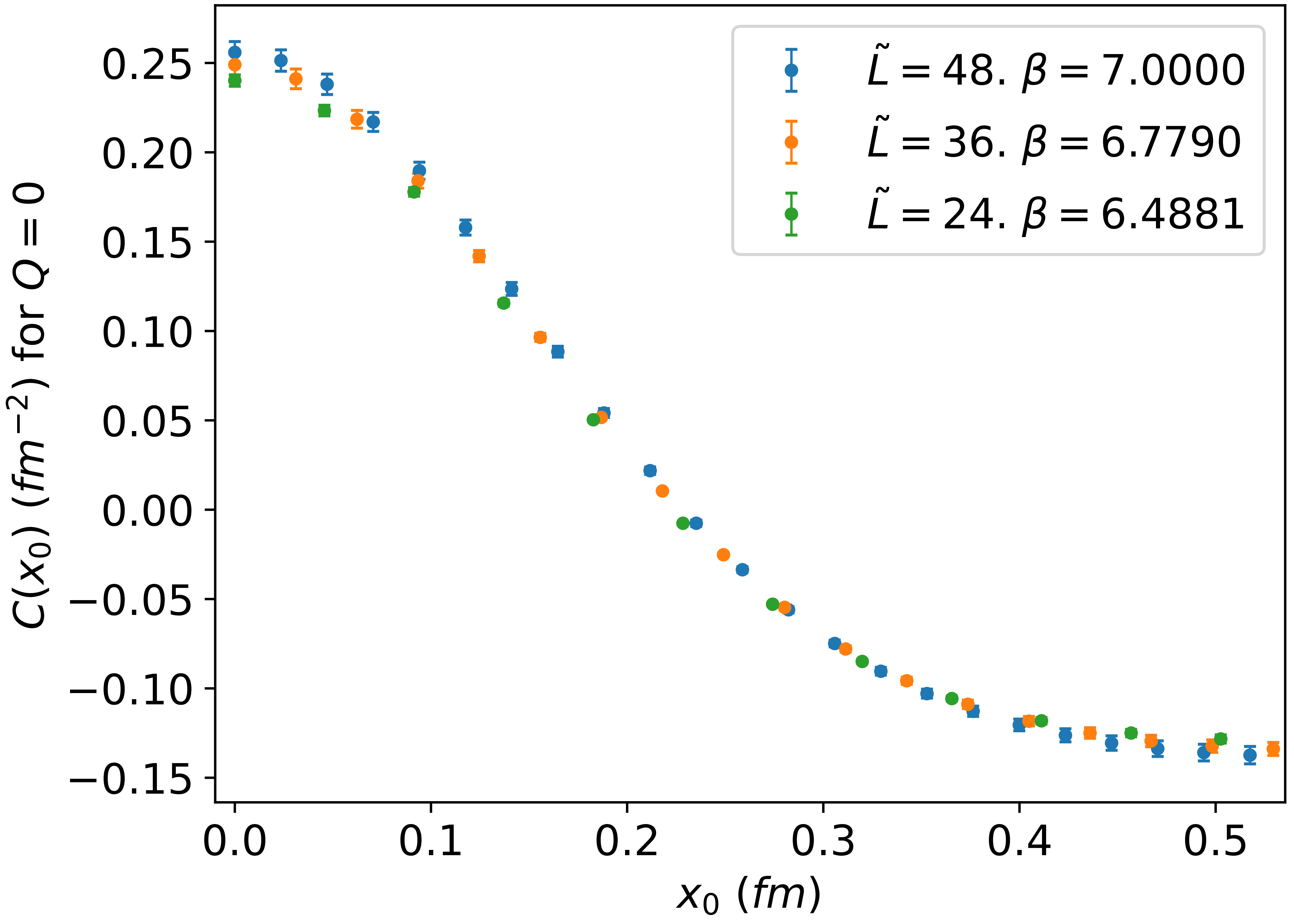


$\bar{k}$	N	$\hat{\theta}=\bar{k}/N$	c=0.4	c=0.5	c=0.6	c=0.7	c=0.8
1	7	0.1429	-4.672(15)	-5.814(20)	-6.813(26)	-7.799(39)	-8.693(62)
1	6	0.1667	-4.274(14)	-5.729(19)	-6.979(24)	-8.097(35)	-9.080(47)
1	5	0.2000	-3.417(12)	-5.098(17)	-6.573(23)	-7.811(33)	-8.843(43)
1	4	0.2500	-2.049(12)	-3.808(16)	-5.475(22)	-6.833(30)	-7.912(40)
2	7	0.2857	-1.187(13)	-2.891(15)	-4.634(20)	-6.050(29)	-7.156(39)
1	3	0.3333	-0.261(14)	-1.818(14)	-3.614(19)	-5.087(29)	-6.220(38)
3	8	0.3750	0.327(14)	-1.073(16)	-2.888(19)	-4.395(26)	-5.545(37)
2	5	0.4000	0.583(14)	-0.724(16)	-2.542(19)	-4.064(26)	-5.222(37)
3	7	0.4286	0.791(12)	-0.418(16)	-2.236(19)	-3.771(26)	-4.937(36)
5	11	0.4545	0.911(09)	-0.228(15)	-2.045(19)	-3.587(26)	-4.757(36)
1	2	0.5000	1.077(13)	-0.092(16)	-1.914(19)	-3.461(26)	-4.634(36)

RATIO $\log\left(\Lambda_{TGF}/\Lambda_{\overline{MS}}\right)$ AS A FUNCTION OF $\hat{\theta}=\bar{k}/N$

TOPOLOGY FREEZING





LATTICE SPACING OF

$a = 0.023524(96), 0.031141(11)$ AND $0.045691(92)$.

$\tilde{l} \sim 1.1$ FM