Dirac Spectra in the 2+1D Thirring Model

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In a Nutshell

Motivation

- Thirring model good laboratory for strong interactions.
- DW and OL operators allow chiral fermions.
- Casher-Banks relates spectral density to condensate/chiral symmetry breaking

Calculations

- Dynamically generate auxiliary Thirring fields using RHMC enabling $N_f = 1$.
- Calculate eigenranges of OL/DW Dirac operators and their kernels.
- Analyse any relation between formulations.

Findings

- Sea fermions require lower L_s than valence fermions.
- Spectral range smaller for Wilson kernels close to critical point.
- Smallest evs of Wilson and Shamir kernels related by $\lambda_S \approx \frac{\lambda_W}{2+\lambda_W}$.

Thirring Model

Eulidean Continuum Formulation

$$S = S_F + S_G$$

QED3:

$$S_F[\psi, \bar{\psi}, A] = \int d^3x \bar{\psi}(\gamma_\mu(\partial_\mu + iA_\mu) + m)\psi$$

$$S_G[A] = \frac{1}{4g^2} \int d^3x F_{\mu\nu} F_{\mu\nu}$$

Thirring:

$$S[\psi, \bar{\psi}] = \int d^3x \bar{\psi}(\gamma_\mu \partial_\mu + m)\psi + \frac{g^2}{2}(\bar{\psi}\gamma_\mu\psi)^2$$

$$S_G[A] = \frac{1}{g^2} \int d^3x A_\mu^2$$

Domain Wall Fermions

Given the massive Wilson Dirac operator, $D_W \equiv D_W(-M, x)$ and the projectors $P_\pm = \frac{1\pm\gamma_3}{2}$ a standard version of the domain wall Dirac operator is given by

$$D_{DW} = \begin{pmatrix} D_W + I & -P_- & 0 & \cdots & mP_+ \\ -P_+ & D_W + I & -P_- & 0 & \cdots \\ \vdots & & \ddots & & \vdots \\ \cdots & 0 & -P_+ & D_W + I & -P_- \\ mP_- & \cdots & 0 & -P_+ & D_W + I \end{pmatrix}$$

M is the domain wall height. m is the bare mass of a fermion defined on the walls

$$q(x) = P_+\Psi(x, L_S) + P_-\Psi(x, 1)$$
$$\bar{q}(x) = P_-\bar{\Psi}(x, L_S) + P_+\bar{\Psi}(x, 1)$$

with condensate given by

$$C = \frac{1}{V}\langle \bar{q}q \rangle$$

Overlap Fermions

$$D_{OL} = \frac{1+m}{2} + \frac{1-m}{2}V$$

$$V = \gamma_5 \mathrm{sgn}(H)$$

$$\mathrm{sgn}(H) = cH(a_0 + \sum_j \frac{a_j}{H^2 - d_j})$$

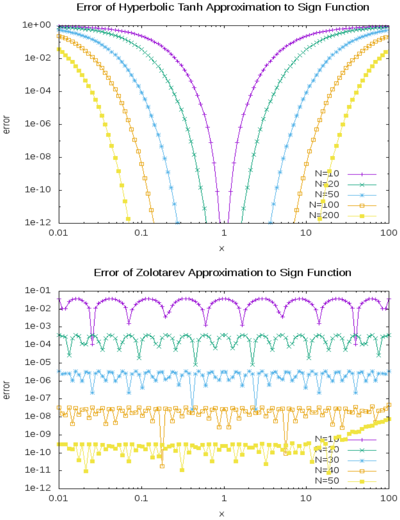
Wilson kernel:

$$H_W = \gamma_5 D_W$$

Shamir kernel:

$$H_S = \gamma_5 \frac{D_W}{2 + D_W}$$

Require spectral range of kernel to use Zolotarev



Equivalence of Methods

$$K_{DW} = \begin{pmatrix} D_{OL}(m) & 0 & 0 & \cdots \\ -(1-m)\triangle_2^R & 1 & 0 & \\ -(1-m)\triangle_3^R & 0 & \ddots & \end{pmatrix} = C^\dagger D_{DW}^{-1}(1) D_{DW}(m) C$$

$$\det[D_{OL}] = \det[K_{DW}]$$

Model	Expansion	Kernel	Mass	Gamma
DW	HT	Shamir	I	γ_3
OL	ZOLO	Wilson	M_3	γ_5

$$DW/Zolo/Shamir \neq OL/Zolo/Shamir$$

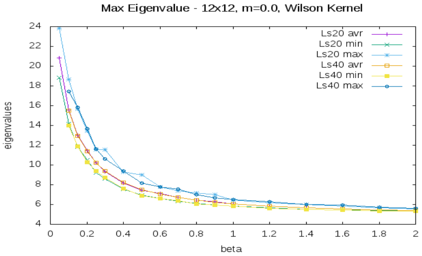
Q: Is partially quenched swapping Shamir and Wilson kernels acceptable?

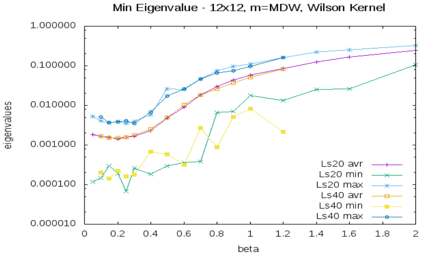
Generation of Auxiliary Fields

- Lattice used is a 12x12x12 linked mesh with fermions on the nodes and Thirring field cast on the links.
- Rational Hybrid Monte Carlo so can use $N_f = 1$.
- Dynamically generated - sea fermions are calculated.
- Compact (unitary) link fields: $U_\mu = e^{iA_\mu}$
- Non-compact (non-unitary) link fields: $U_\mu = 1 + iA_\mu$
- All auxiliary fields used here are generated with domain wall, Shamir kernel, and hyperbolic tanh approximation with order given by L_s.

Wilson Kernel

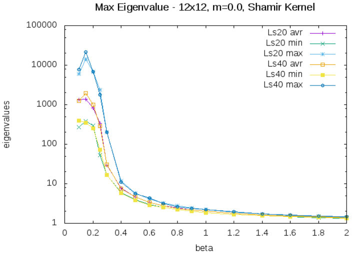
- $H_W = \gamma_3 D_W$
- minimum/maxium eigenvalues
- average, minimum, and maximum, over 100 auxiliary fields
- independent of L_s
- suggests less stringent L_s requirements in generation of auxiliary fields than in measurements
- spectral range $[1 \times 10^{-4}, 20]$

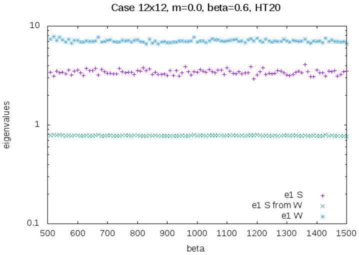


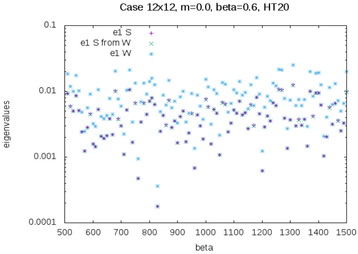


Shamir Kernel

- $H_S = \frac{\gamma_3 D_W}{D_W + 2}$
- independent of L_s again
- no formal equivalence
  $\text{eig}[H_S] \neq \frac{\text{eig}[H_W]}{\text{eig}[H_W]+2}$
- holds for small eigenvalues
- spectral range $[5 \times 10^{-5}, 10000]$

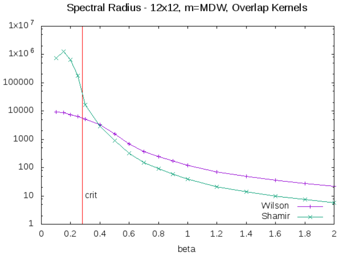




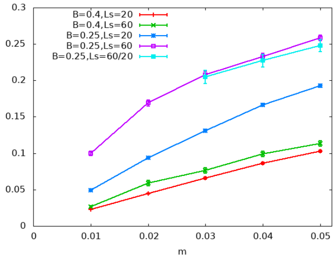


Kernels and Condensates

- $\kappa(H) = \frac{eig[H]_{max}}{eig[H]_{min}}$
- H_W is better conditioned close to critical point

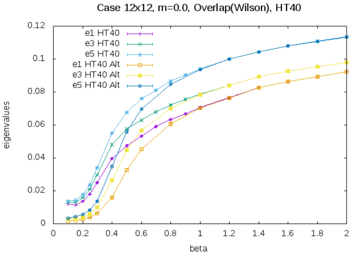


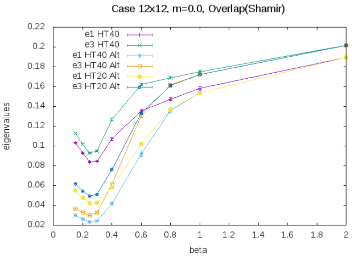
- Measure condensate with valence fermions
- Can capture $L_s = 60$ measurements using $L_s = 20$ sea fermions.

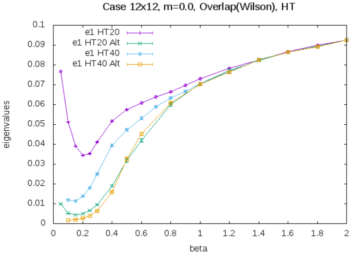


Fermion Spectra - matching L_s

- preliminary results
- smallest eig$[D_{OL}^{\dagger} D_{OL}]^{1/2}$
- eigenvalues doubled - look at 1st, 3rd, 5th eigenvalues
- $D_{OL}^{\dagger} D_{OL} = 2 + V + V^{\dagger}$
- $D_{OL}^{\dagger} D_{OL} = 1 + V + V^{\dagger} + V^{\dagger} V$

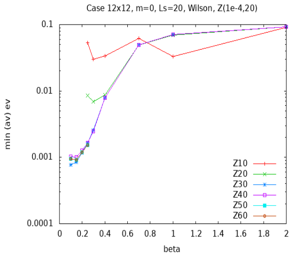




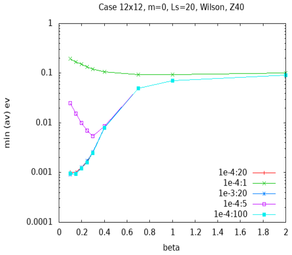


Fermion Spectra - Zolotarev requirements

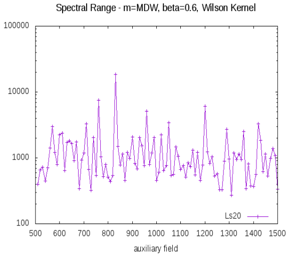
- Z40 sufficient



- 1×10^{-4} bound too stringent
- Upper bound necessary



- Is consistent range required for each auxiliary field?



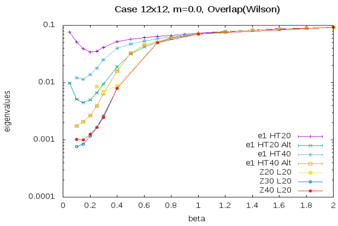
References

- S Hands (2015), JHEP (09), hep-lat/1507.07717
- S Hands (2016), Phys. Lett. B(754), hep-lat/1512.05885
- R Narayanan, H Neuberger (1995), Nuc. Phys. B (443), hep-lat/9411108
- R Narayanan (2021), Phys. Rev. D (103), hep-lat/2102.11367
- A Borici (2000), Nucl. Phys. Proc. Suppl. (83), hep-lat/9909057
- R Brower, H Neff, K Originos (2017), Comput. Phys. Commun.
- T-W Chiu (2003), Phys. Rev. Lett. (90), hep-lat/0209153 (220), hep-lat/1206.5214
- D Kaplan (1992), Phys. Lett. B (288), hep-lat/9206013
- A Kennedy (2006), Nucl. Phys. Proc. Suppl. (128C), hep-lat/0607038
- H Neuberger (1998), Phys. Lett. B(417), hep-lat/9707022

Outlook

Summary

- Looked at Dirac spectra in the non-compact Thirring model
- Sea fermions require lower L_s than valence fermions.
- Spectral range smaller for Wilson kernels close to critical point.
- Smallest evs of Wilson and Shamir kernels related by $\lambda_S \approx \frac{\lambda_W}{2+\lambda_W}$.



Next Steps

- Explore overlap with Shamir kernel
- Explore Banks-Casher relation
- Explore compact formulation