Quantum Counter Terms for Lattice Field Theory on Curved Manifolds (Lattice 2021)

Evan Owen (speaker, Boston University)
Casey Berger (Boston University)
Richard Brower (Boston University)
George Fleming (Yale University)
Andrew David Gasbarro (Universität Bern)
Timothy Raben (University of Kansas)

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Background

We seek to probe the non-perturbative regime of ϕ^4 theory on a discretized $\mathbb{S}^2$ lattice close to the strong coupling Wilson-Fisher IR fixed point.

Expanding on the results of Brower et al. 1803.08512 / PRD 98, 014502 (2018) which studies the same model with perturbative counterterms [1].

Background

Start with continuum ϕ^4 action defined on a smooth Riemann manifold $\mathcal{M}$.

$$S = \int_{\mathcal{M}} d^d x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi(x) \partial_\nu \phi(x) + \frac{1}{2} m^2 \phi^2(x) + \lambda \phi^4(x) \right)$$

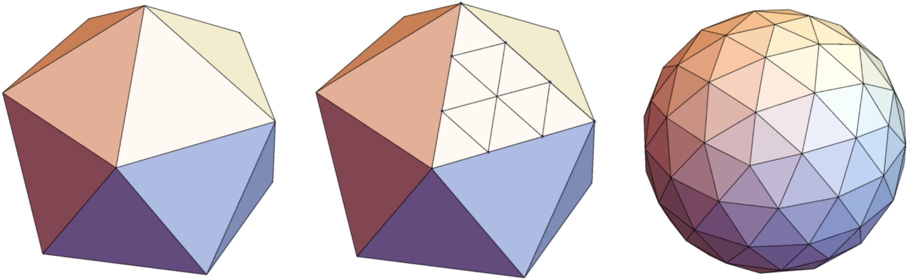
Introduce a simplicial complex to discretize the manifold. The lattice action is

$$S = \frac{1}{2} \sum_{\langle xy \rangle} \frac{V_{xy}}{\ell_{xy}^2} (\phi_x - \phi_y)^2 + \sum_x \sqrt{g_x} \left(\frac{1}{2} m^2 \phi_x^2 + \lambda \phi_x^4 \right)$$

Site/link prefactors are determined via finite element method [1]. $\sqrt{g_x}$ is the normalized area associated with site x.

Background

Discretize $\mathbb{S}^2$ using a refined icosahedron.

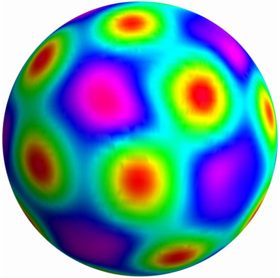


Approach continuum limit by increasing refinement (refinement of 3 is shown).

Spherical Symmetry Breaking

Quantum theory has UV divergences due to quantum fluctuations.

Spherical symmetry of UV and IR operators is lost in the continuum limit.

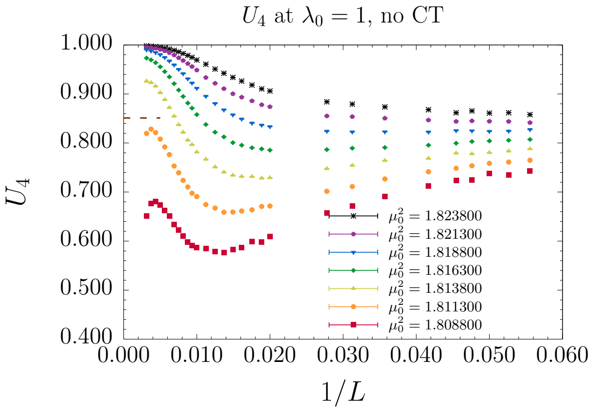


$\langle \phi_x^2 \rangle$ from Monte Carlo simulation showing breaking of spherical symmetry near the critical surface.

Unstable Critical Surface Without Counterterms

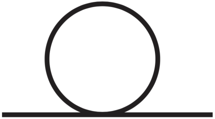
Critical surface is not well-defined as the continuum limit is approached.

We measure $U_4 = \frac{3}{2}\left(1 - \frac{\langle m^4\rangle}{3\langle m^2\rangle^2}\right)$ where $m = \sum_x \sqrt{g_x}\phi_x$



Perturbative Counter Terms

In 2d, 1-loop diagram is logarithmically UV divergent. Higher order diagrams are finite.



Performing the perturbative expansion in λ, renormalization requires the addition of *local* mass counterterms. To first order,

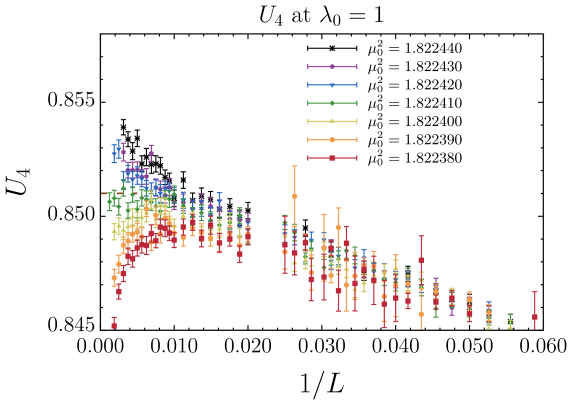
$$\delta m_x^2 = 6Q\lambda \log\left(\sqrt{g_x}\right)$$

where $Q = \sqrt{3}/8\pi$ and $\sqrt{g_x}$ is the normalized area associated with site x.

$$S \rightarrow S + \frac{1}{2}\sum_x \sqrt{g_x}\,\delta m_x^2 \phi_x^2$$

Restoration of Critical Surface

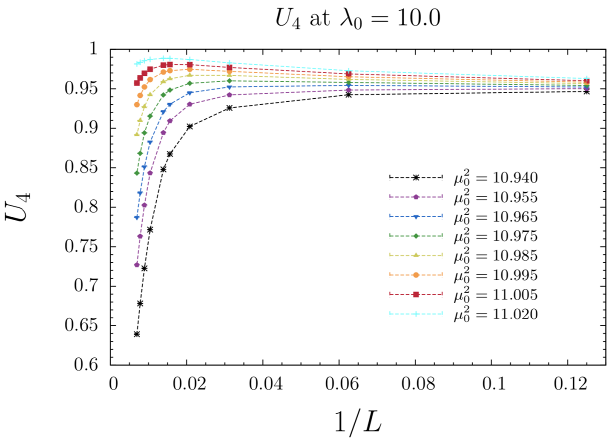
Spherical symmetry and critical surface are restored at $\lambda \leq 1$ after adding local perturbative counterterms.



Binder cumulant approaches the correct continuum value of $U_4^* \simeq 0.851$

Critical Surface Near Wilson-Fisher Fixed Point

Spherical symmetry remains broken at $\lambda > 1$. (Gasbarro, 2019 [2])

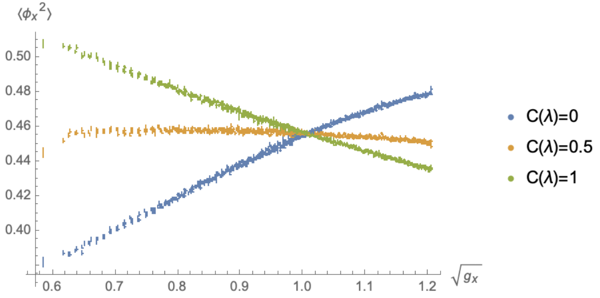


Unable to approach Wilson-Fisher fixed point ($10 \lesssim \lambda_{WF} \lesssim 20$)

Non-Perturbative Counter Terms

Conjecture that symmetry can be restored at strong coupling by adjusting strength of counterterms as a function of λ.

$$\delta m_x^2 \to C(\lambda)\delta m_x^2$$

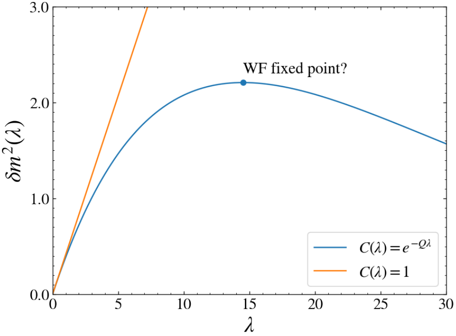


Plotted at $\lambda = 10$, refinement of 96.

Non-Perturbative Counter Terms

We obtain good results with $C(\lambda) = e^{-Q\lambda}$. Counterterms become

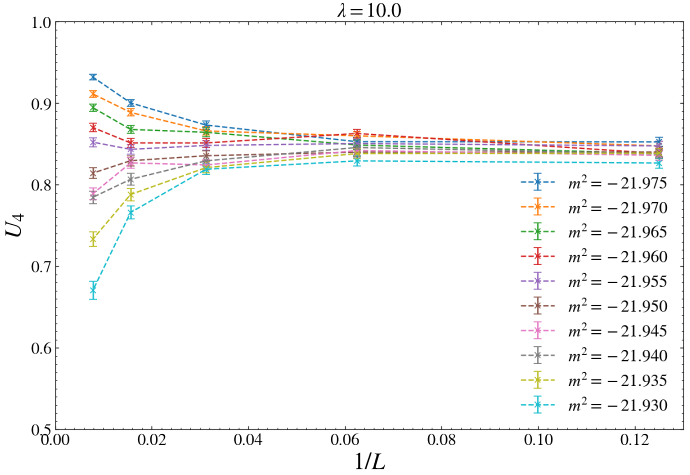
$$\delta m_x^2(\lambda) = 6Q\lambda e^{-Q\lambda} \log\left(\sqrt{g_x}\right)$$



Maximum at $\lambda_{WF} \simeq 14.5$, possibly corresponding to the Wilson-Fisher fixed point.

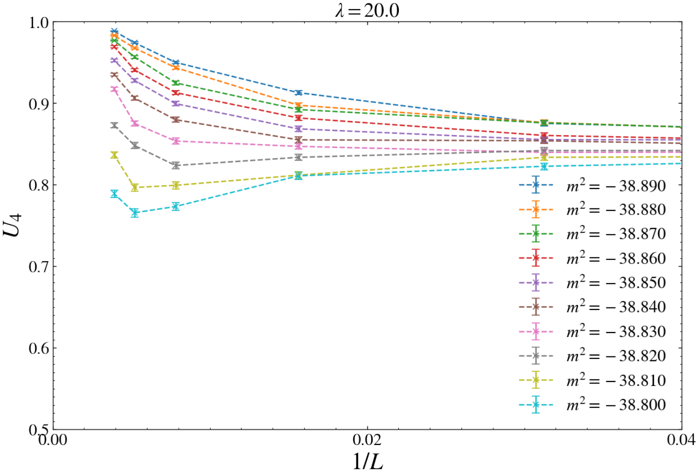
Critical Surface with Non-Perturbative Counter Terms

Restoration of critical surface is greatly improved at $\lambda = 10$.



Critical Surface with Non-Perturbative Counter Terms

However, obstruction still exists at larger λ



Conclusion

- Stability of critical surface near the strong coupling Wilson-Fisher fixed point is improved by tuning the strength of the perturbative counterterms.
- Further tuning is required to fully restore symmetry in the continuum limit.
- Exact form of non-perturbative counterterms is not well understood, but may provide direct access to the Wilson-Fisher fixed point.

Future Directions

- Tune non-perturbative counterterms to improve restoration of spherical symmetry at larger λ.
- Improve statistics, larger lattices (USQCD allocation on Fermilab lattice cluster).
- Measure projection of operators onto spherical harmonics to improve quantitative measurement of spherical symmetry breaking.
- Explore other manifolds (boundary AdS theory w/ Cameron Cogburn et al.)
- Explore other theories, add fermions (previous talk by Rich Brower).

References

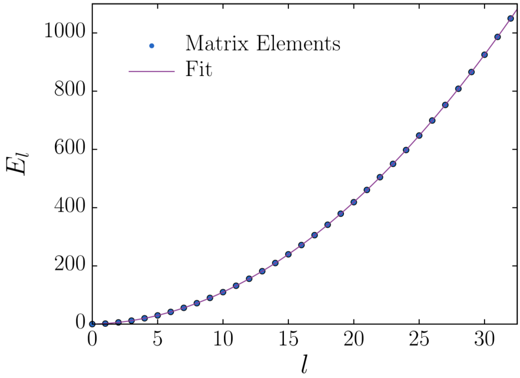
[1] Richard C. Brower, Michael Cheng, Evan S. Weinberg, George T. Fleming, Andrew D. Gasbarro, Timothy G. Raben, and Chung-I Tan.
Lattice ϕ^4 field theory on Riemann manifolds: Numerical tests for the 2-d Ising CFT on $\mathbb{S}^2$.
Phys. Rev. D, 98(1):014502, 2018.

[2] Andrew David Gasbarro.
Studies of conformal behavior in strongly interacting quantum field theories, 2019.

Classical Theory

The free action ($\lambda = 0$) in terms of a quadratic interaction matrix M_{xy} is

$$S_{\text{free}} = \sum_{x,y} \phi_x M_{xy} \phi_y$$



Eigenvalues of M_{xy} projected onto spherical harmonics approach the appropriate values $\ell(\ell+1)$ and degeneracy $2\ell+1$ as the continuum limit is approached (shown for refinement of 128).

Perturbative Counter Terms

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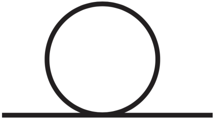


Performing the perturbative expansion in λ, renormalization requires the addition of *local* mass counterterms $\delta m_x^2 = 6\lambda\delta G_{xx}$ where δG_{xx} is the diagonal of $[M_{xy}]^{-1}$ after subtracting out the position-independent contribution to remove the logarithmic divergence.

$$\delta G_{xx} = [M_{xx}]^{-1} - \sum_y \sqrt{g_y}[M_{yy}]^{-1}$$

Perturbative Counter Terms

Continuum form of δG_{xx} is

$$\delta G_{xx} = Q \log \left(\frac{a^2}{a_x^2} \right) = -Q \log \left(\sqrt{g_x} \right)$$

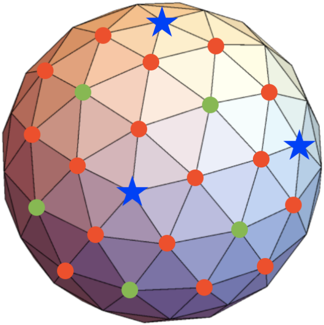
where $Q = \sqrt{3}/8\pi$ and $\sqrt{g_x}$ is the normalized area associated with site x. Continuum counterterm is then

$$\delta m_x^2 = 6Q\lambda \log \left(\sqrt{g_x} \right)$$

In most cases, calculating $[M_{xy}]^{-1}$ numerically results in a more uniform handling of the 12 exceptional points than this continuum form.

Icosahedrally Distinct Points

We take advantage of icosahedral symmetry by binning in groups of "distinct" icosahedral points.



Shown for a refinement of 3. The stars are the 12 "exceptional" points.