Numerical simulation of self-dual U(1) lattice field theory with electric and magnetic matter

Maria Anosova [a], Christof Gattringer [a,b], Nabil Iqbal [c], Tin Sulejmanpasic [c]

[a] University of Graz
[b] FWF Austrian Science Fund
[c] Durham University

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Motivation

- Duality is a powerful tool relating weak and strong coupling regimes, often allowing for non-perturbative insights.

- We study a fully self-dual lattice model with U(1) gauge fields and electric matter as well as magnetic matter (coupled in a local way).

- Spontaneous breaking of self-duality as a function of the matter coupling parameter has been conjectured.

- ***We find a first order transition line with two end points.***

- In this project we numerically explore possible self-duality breaking, using Monte Carlo simulations based on a worldline formulation.

Setting for ***U(1) pure lattice gauge theory:*** ***Villain formulation***

$$Z = \int D[A] \sum_{\{n\}} e^{-\frac{\beta}{2}\sum_{x,\mu<\nu} F_{x,\mu\nu}F_{x,\mu\nu}}$$

Villain variables
(assigned to the plaquettes)

$n_{x,\mu\nu} \in \mathbb{Z}$

- Strength field tensor $F_{x,\mu\nu} = A^e_{x+\hat{\mu},\nu} - A^e_{x,\nu} - A^e_{x+\hat{\nu},\mu} + A^e_{x,\mu} + 2\pi n_{x,\mu\nu}$
- The theory has a shift symmetry $A^e_{x,\mu} \to A^e_{x,\mu} + 2\pi k_{x,\mu}$

- One can gauge the center symmetry imposing the 'closedness' constraint to *eliminate monopoles* :

* *C.Gattringer, T.Sulejmanpasic arXiv:1901.02637*

$$(dn)_{x,\mu\nu\rho} = 0 \ \forall\, (x, \mu < \nu < \rho).$$

The constraints are:

- implemented on the cubes
- necessary for **self-duality**
- correspond to **absence of monopoles** in the U(1) gauge theory with Villain action

Setting for ***U(1) pure lattice gauge theory:*** ***Villain formulation***

$$Z = \int D[A] \sum_{\{n\}} e^{-\frac{\beta}{2}\sum_{x,\mu<\nu} F_{x,\mu\nu}F_{x,\mu\nu}} \prod_{x} \prod_{\mu<\nu<\rho} \delta\Big((dn)_{x,\mu\nu\rho}\Big)$$

Constraints on Villain variables in the partition sum are introduced with the Kronecker deltas.

$\Longrightarrow$ In integral representation:

$$Z(\beta) = \int D[A^e] \int D[A^m] \sum_{\{n\}} e^{-\frac{\beta}{2}\sum_x \sum_{\mu<\nu}(F^e_{x,\mu\nu})^2} e^{i\sum_x \sum_{\mu<\nu<\rho} A^m_{x,\mu\nu\rho}(dn)_{x,\mu\nu\rho}}$$

Naturally introducing a magnetic gauge field

- Electric gauge field $A^e_{x,\mu}$ describes the photon dynamics.
- Magnetic gauge field $\widetilde{A}^m_{\widetilde{x},\mu} \in R$ lives on the dual lattice and removes monopoles.

U(1) pure lattice gauge theory: self-duality

$$Z(\beta) \;=\; \int D[A^e] \int D[A^m] \sum_{\{n\}} \; e^{-\frac{\beta}{2}\sum_x \sum_{\mu<\nu}((dA^e)_{x,\mu\nu}+2\pi n_{x,\mu\nu})^2} \, e^{i\sum_x \sum_{\mu<\nu<\rho} A^m_{x,\mu\nu\rho}(dn)_{x,\mu\nu\rho}}$$

Switching to the dual lattice one can show that theory is *self-dual* with the relation:

$$Z(\beta) \;=\; c\, Z(\widetilde{\beta}) \qquad \text{with} \qquad \widetilde{\beta} \;=\; \frac{1}{4\pi^2\beta}$$

The self-duality relation obviously ***maps the weak- and strong-coupling*** regions of $Z(\beta)$ onto each other.

Coupling electric and magnetic matter

$$Z(\beta, J_e, J_m) \;\equiv\; \int D[A^e] \int D[A^m]\; B_\beta[A^e, A^m]\; Z[A^e, J_e]\; \widetilde{Z}[\widetilde{A}^m, J_m]$$

We have coupled electric and magnetic matter using U(1)-valued matter fields (also possible to couple complex-valued bosonic fields and fermions).

$$\phi^e_x = e^{\,i\varphi^e_x} \text{ with } \varphi^e_x \in [-\pi, \pi]$$

$$Z[A^e, J_e] \equiv \int D[\phi^e]\, e^{\,J_e S_e[\phi^e,\, A^e]} \qquad \Longrightarrow$$

$$\Longrightarrow \qquad \mathrm{S}_e[\phi^e, A^e] \equiv \frac{1}{2} \sum_{x,\mu} \left[\phi^{e\,*}_x\, e^{iA^e_{x,\mu}}\, \phi^e_{x+\hat{\mu}} + c.c.\right] = \sum_{x,\mu} \cos\left(\varphi^e_{x+\hat{\mu}} - \varphi^e_x + A^e_{x,\mu}\right).$$

Coupling electric and magnetic matter

$$Z(\beta, J_e, J_m) \;\equiv\; \int D[A^e] \int D[A^m]\; B_\beta[A^e, A^m]\; Z[A^e, J_e]\; \widetilde{Z}[\widetilde{A}^m, J_m]$$

As before ***self-duality*** with the corresponding relations :

$$Z(\beta, J_e, J_m) \;=\; c\, Z\big(\widetilde{\beta}, \widetilde{J}_e, \widetilde{J}_m\big)$$

$$\text{with} \quad \widetilde{\beta} \;=\; \tfrac{1}{4\pi^2\beta}\;, \widetilde{J}_e \;=\; J_m\;, \widetilde{J}_m \;=\; J_e\;.$$

Derivatives of $\ln Z$ relate observables in *strong and weak coupling region*

(1) $\langle F^2 \rangle_\beta \equiv -\dfrac{1}{3V}\dfrac{\partial}{\partial\beta} \ln Z(\beta)$

$$\Longrightarrow \quad \beta\, \langle F^2 \rangle_{\beta, J_e, J_m} \;+\; \widetilde{\beta}\, \langle F^2 \rangle_{\widetilde{\beta}, \widetilde{J}_e, \widetilde{J}_m} \;=\; 1$$

(2) $$\langle s_e \rangle_{\beta, J_e, J_m} \;=\; \langle \widetilde{s}_m \rangle_{\widetilde{\beta}, \widetilde{J}_e, \widetilde{J}_m}$$

We simulate theory in the ***self-dual point***

- $\beta = \widetilde{\beta} = \beta^* = \dfrac{1}{2\pi}$
- $J_e = J_m = \widetilde{J}_e = \widetilde{J}_m = J$ $\Longrightarrow$ The only remaining parameter is coupling J

(1) $\beta^* \langle F^2 \rangle_{\beta^*,\, J} + \beta^* \langle F^2 \rangle_{\beta^*,\, J} = 1 \quad \Longrightarrow \quad \langle F^2 \rangle_{\beta^*,\, J} = \pi \quad \forall\, J$

(2) $\langle s_e \rangle_{\beta^*,\, J} = \langle \widetilde{s}_m \rangle_{\beta^*,\, J} \quad \forall\, J$

Can self-duality be broken spontaneously as a function of J ?

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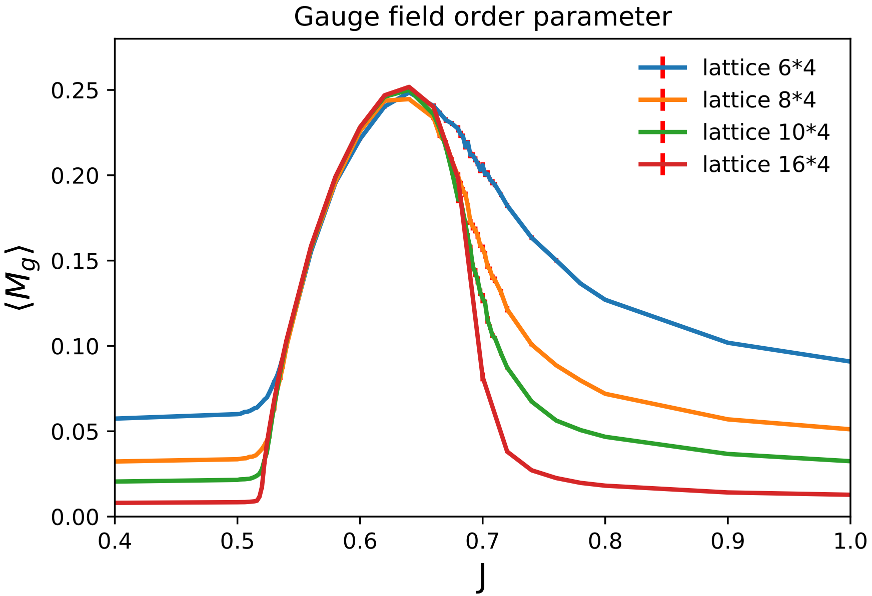
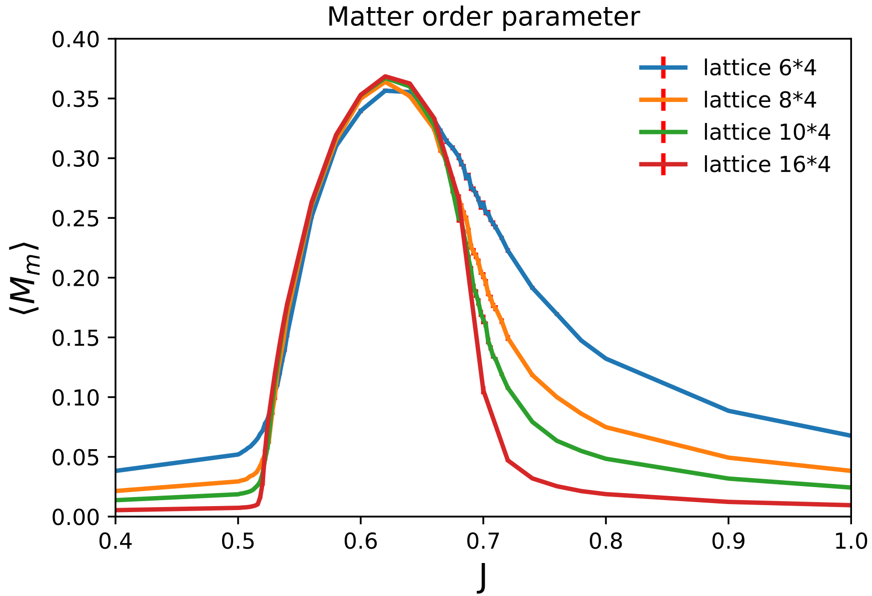
Can self-duality be broken spontaneously as a function of J ? $\Longrightarrow$ *Study behaviour of the observables*

Self-duality breaking parameter, susceptibility, Binder cummulant

Defining the **order parameters** of self-duality breaking

- $M_g \equiv |F^2 - \pi|$
- $M_m \equiv |s_e - s_g|$





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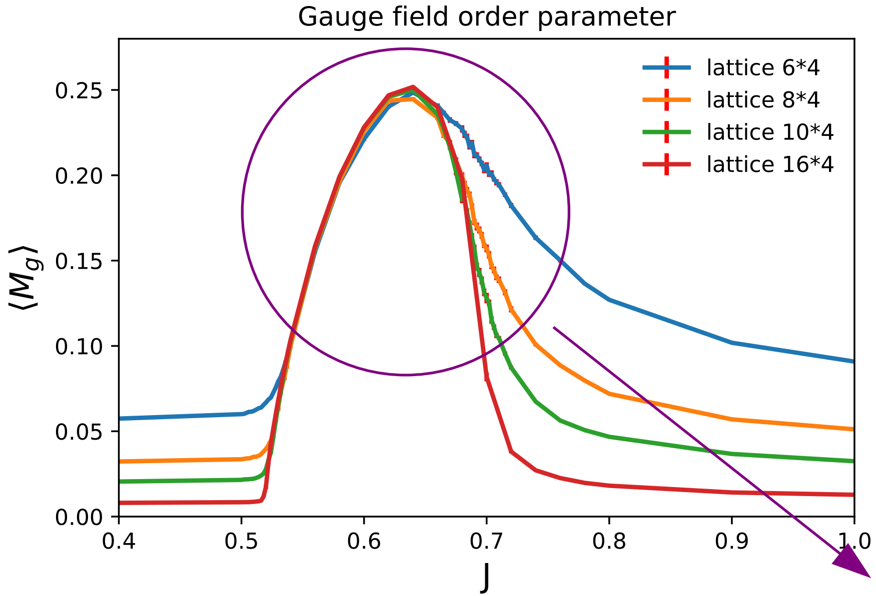
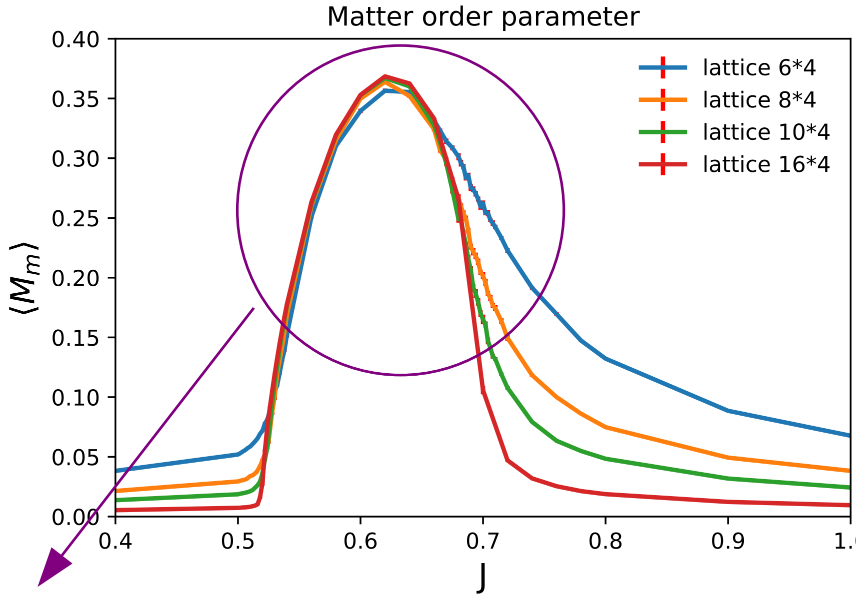


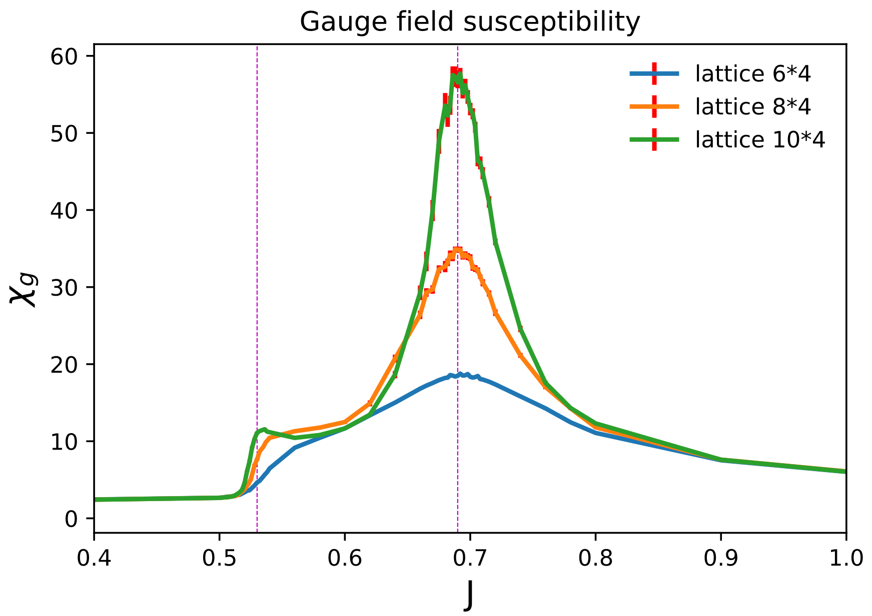


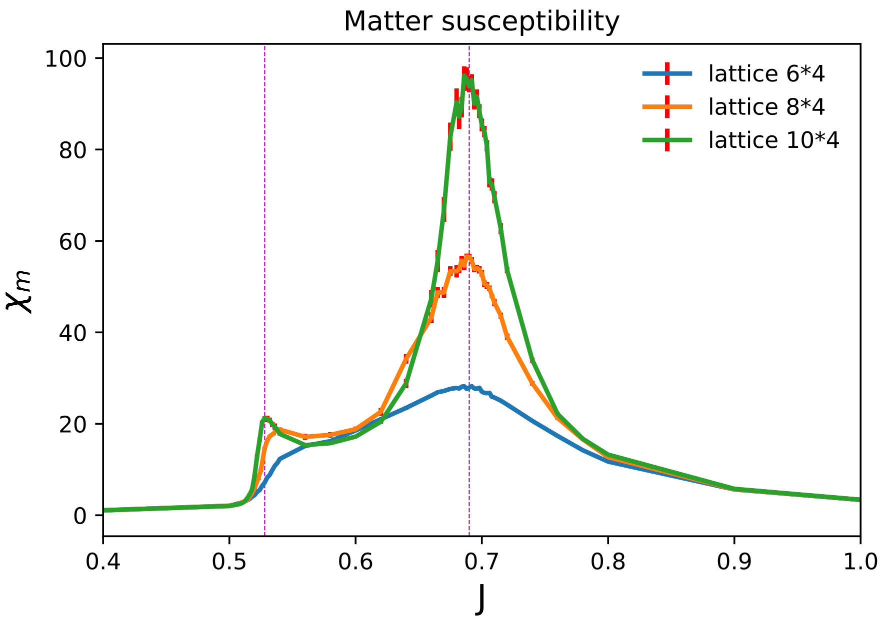
$$\langle M_g \rangle_{\beta^*,\, J} \neq 0 \quad \langle M_m \rangle_{\beta^*,\, J} \neq 0$$

Spontaneous breaking of self-duality

Defining the **susceptibilities**

- $\chi_g \equiv V \left\langle (M_g - \langle M_g \rangle_{\beta^*,\, J})^2 \right\rangle_{\beta^*,\, J}$
- $\chi_m \equiv V \left\langle (M_m - \langle M_m \rangle_{\beta^*,\, J})^2 \right\rangle_{\beta^*,\, J}$



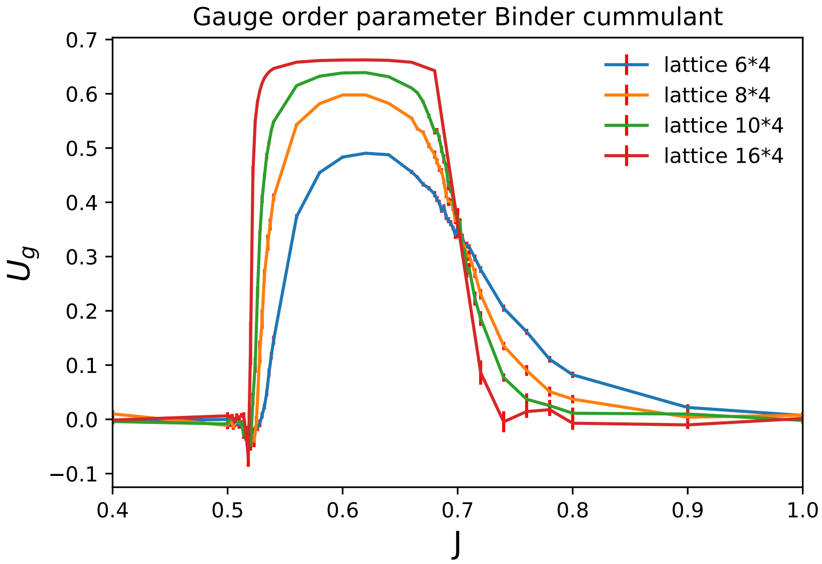


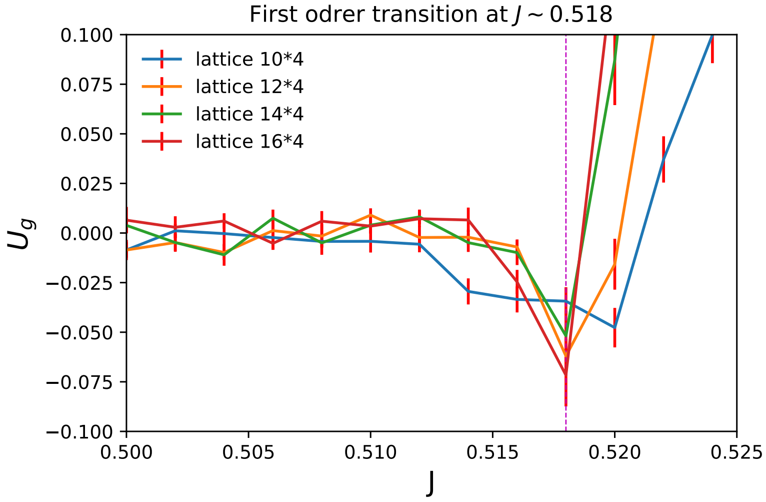
First order phase transition at J ~ 0.52

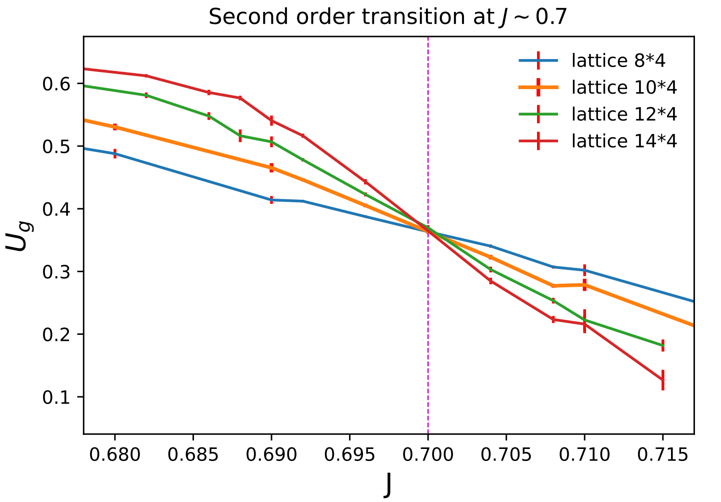
Second order phase transition at J ~ 0.69

Defining the **Binder cummulants**

$$U_g \equiv 1 - \frac{\left\langle (M_g)^4 \right\rangle_{\beta^*,\, J}}{3 \left\langle (M_g)^2 \right\rangle^2_{\beta^*,\, J}}$$

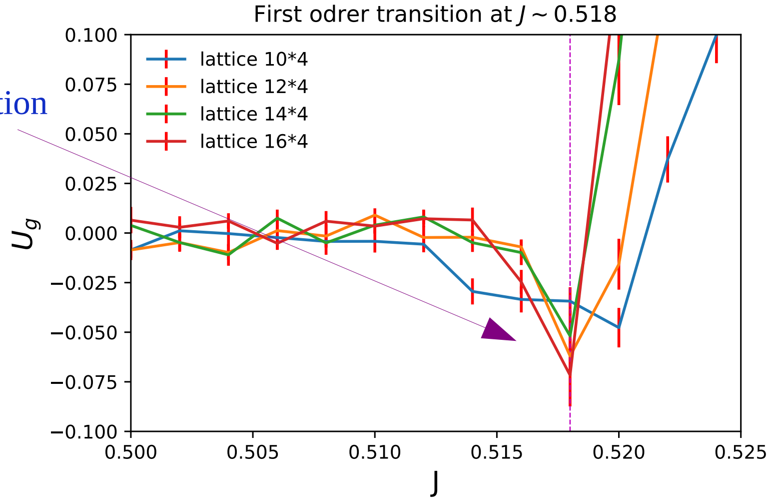


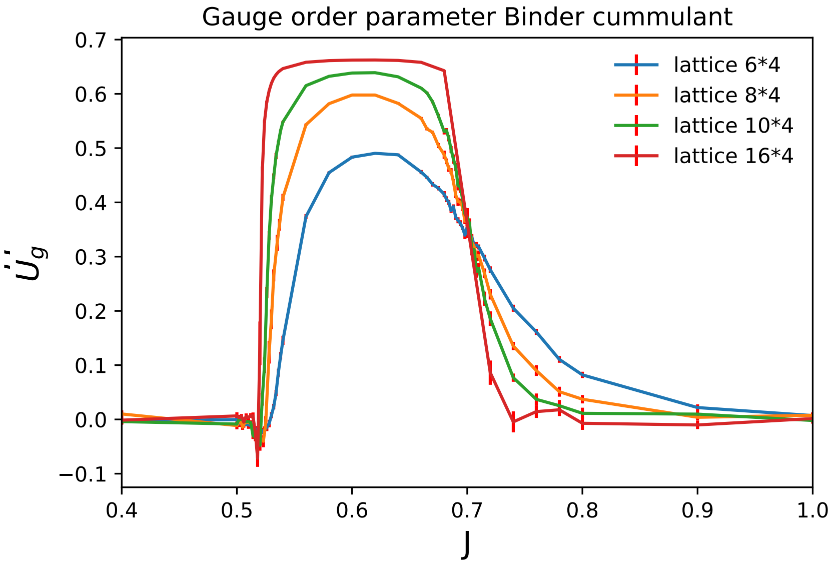


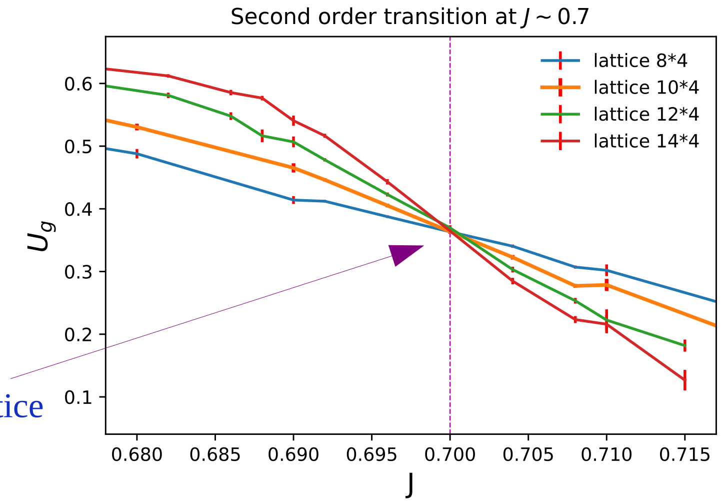


Binder cummulant of *gauge order parameter*

Develops a minimum at a first order transition

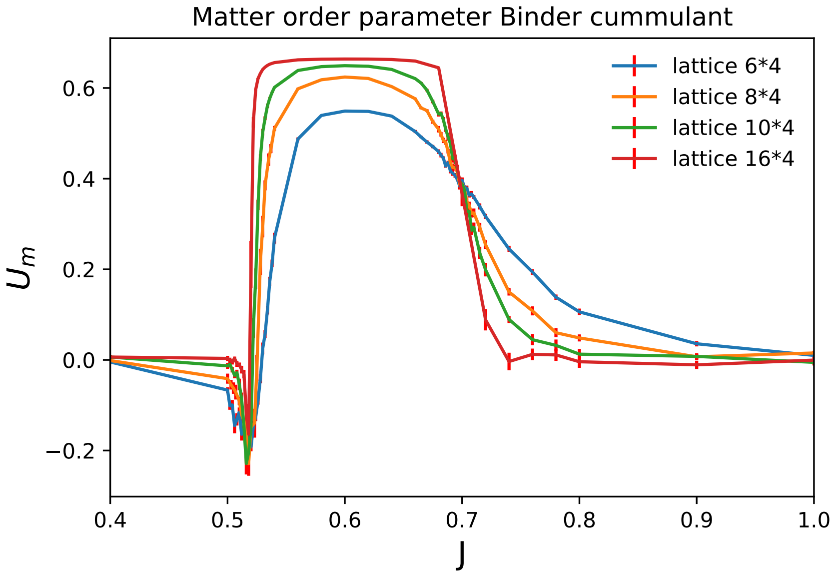


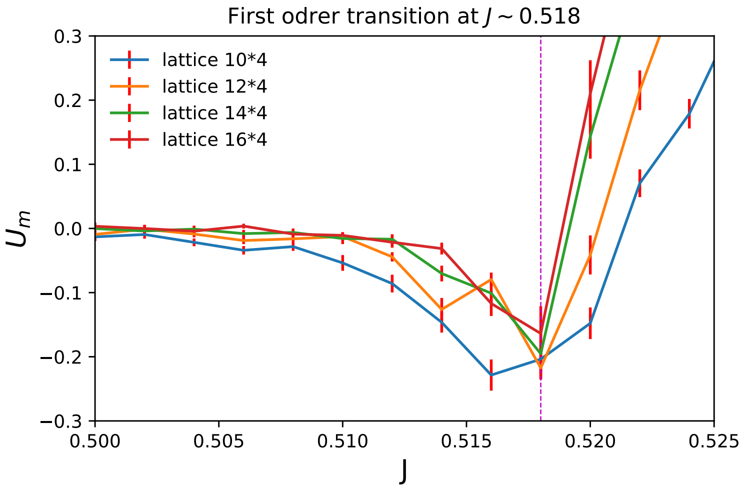


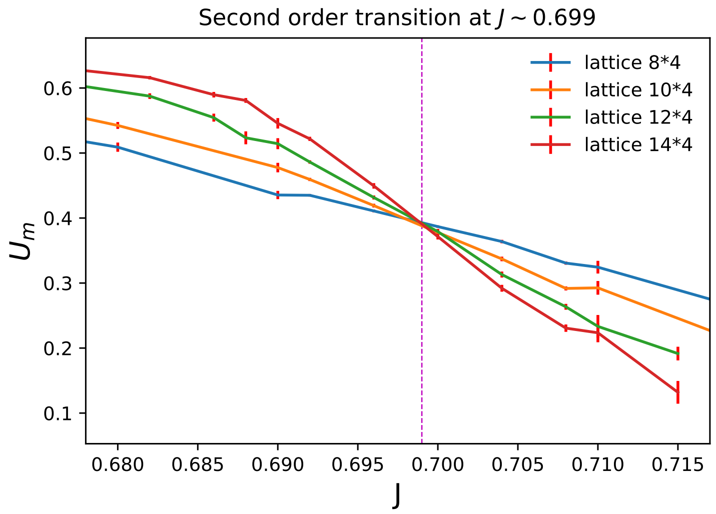


The intersection point converges to a critical point in the infinite lattice

Binder cummulant of *matter order parameter*

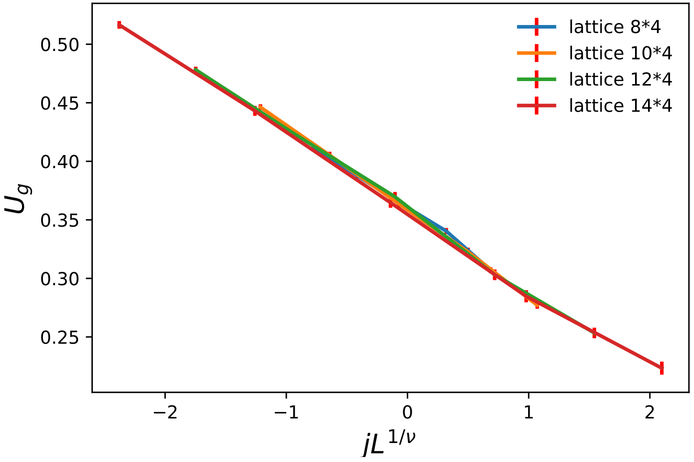




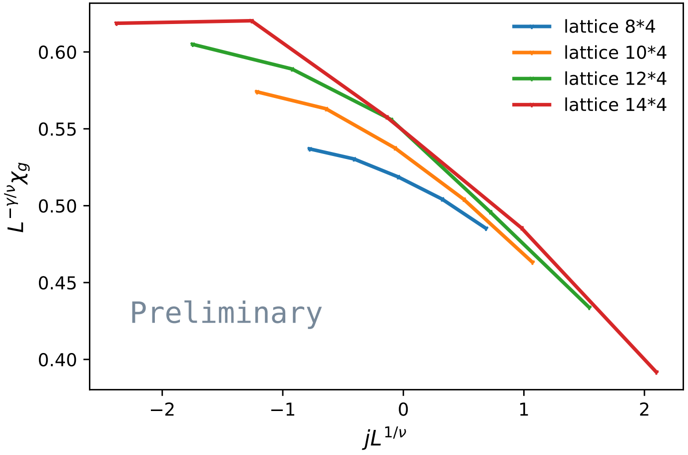


Critical exponents collapsing Binder cummulants and susceptibilities: *Mean field values*

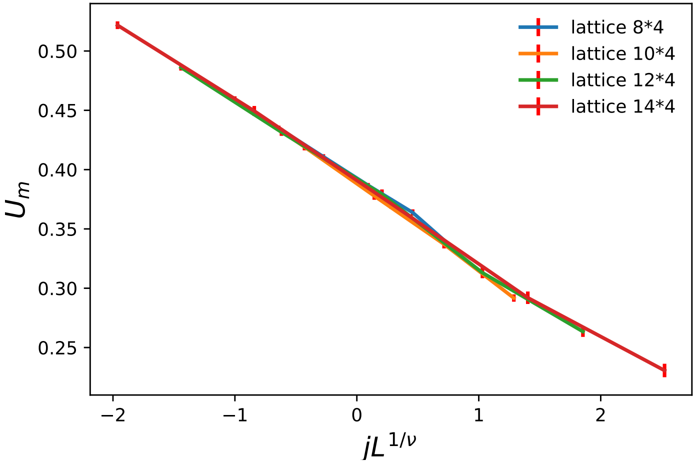
Second order transition gauge J_c = 0.7005, $\nu = 0.5$



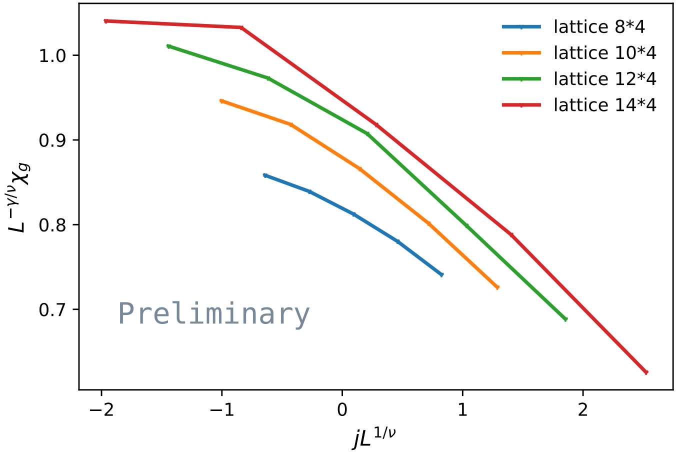
Gauge susceptibility J_c = 0.7005, $\nu = 0.5$, γ = 1



Second order transition matter J_c = 0.699 $\nu = 0.5$



Matter susceptibility J_c = 0.699, $\nu = 0.5$, γ = 1



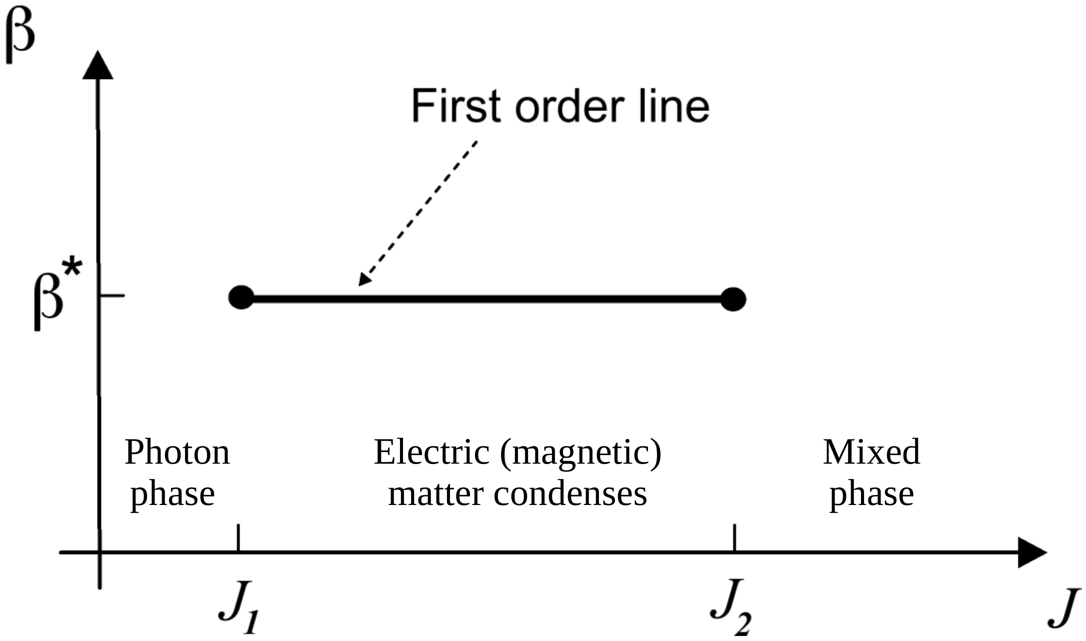
$\nu = 0.5 \pm 0.01$

Subleading logarithmic corrections are to be understood..

Phase diagram

The self-dual coupling

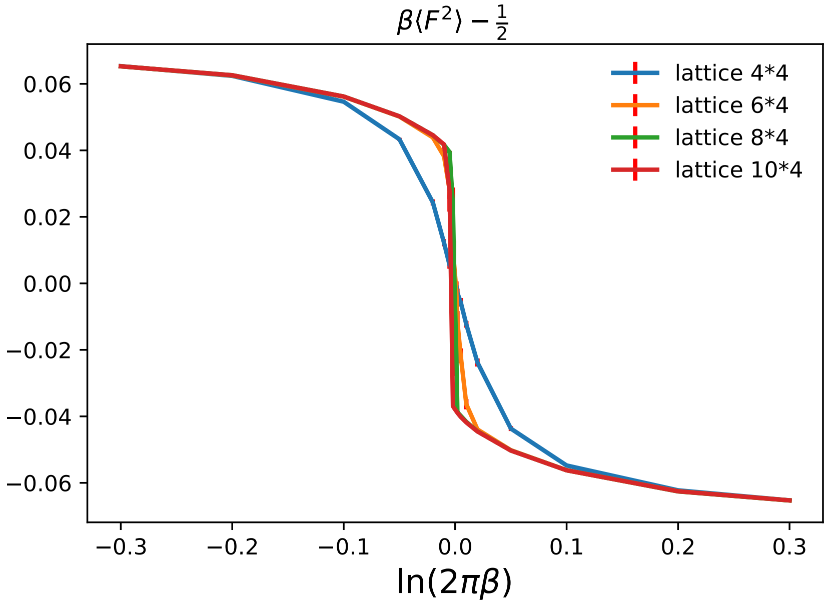
$\beta^* = \frac{1}{2\pi}$

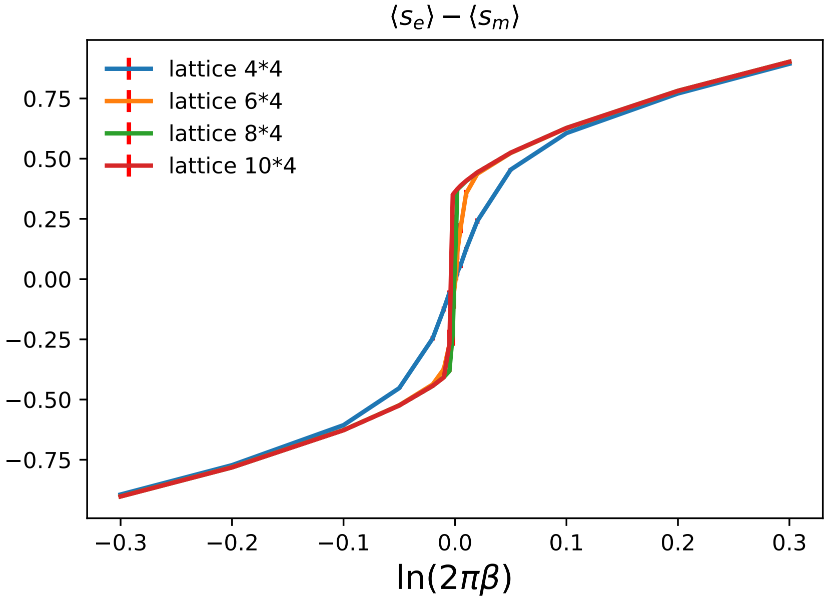


Two end points $J_1 \sim 0.518$ and $J_2 \sim 0.7$

Cross-check of the transition at J=0.61

Crossing the 1st order line in vertical direction.





Conclusions

- *We study self-dual U(1) gauge theory with electric and magnetic matter.*
- Use worldline representation for numerical simulation of self-duality relations.
- We observe *spontaneous breaking of self-duality* at $\beta^* = \frac{1}{2\pi}$ as a function of J in the region with two endpoints.
- Binder cummulants allow an assessment of the nature of the endpoints:

  the first order at $J_1 \sim 0.518$ *and the second order at* $J_2 \sim 0.7$.
- Second order transition established at $J_2 \sim 0.7$ with *mean field critical exponents*.

Outlook:

- Subleading *logarithmic corrections* need to be understood.
- Introduce a deformation to change the nature of transition at J_1