Breaking the gauge symmetry in lattice gauge-invariant systems

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arXiv:2106.02503 Lattice gauge theories in the presence of a linear gauge-symmetry breaking
arXiv:2104.09892 Breaking of the gauge symmetry in lattice gauge theories

Why studying gauge theories in three dimensions

Most studies of gauge theories strictly connected with High Energy Physics and fundamental interactions.

Gauge theories are also very important in condensed matter physics.

1) U(1) gauge symmetry: charged systems...

2) Discrete gauge symmetries: $\mathbb{Z}_2$ gauge models appear in several contexts, giving rise to topological transitions (they control the dynamics of classes of topological defects)

3) Nonabelian gauge models: recently proposed to describe multicritical transitions (a model with SU(2) gauge invariance has been introduced by Sachdev and collaborators for high-T_c superconductors)

Recent experimental work: To reproduce in a lab a gauge invariant system (for instance, using cold atoms).

A common feature of many condensed-matter gauge invariant systems: The microscopic model is not invariant under the gauge symmetry group (at variance with HEP, where the gauge symmetry is exact).

The hope is that the critical (long-distance) behavior is gauge invariant: Gauge symmetry is an emergent symmetry at criticality.

OUR QUESTION in the renormalization-group language:

is gauge invariance ROBUST against gauge-symmetry breaking (GSB) perturbations ?
OR
do we need to perform tunings to obtain a gauge-invariant critical limit?

The Abelian-Higgs model (scalar electrodynamics)

We study these issues in one of the simplest models.

U(1) gauge model with scalar fields (NO FERMIONS) that transform under the fundamental rep of SU(N). Continuum Lagrangian:

$$L = |D_\mu \phi(x)|^2 + m^2 |\phi(x)|^2 + V(|\phi(x)|) + \frac{1}{g^2} F^2$$

where $F_{\mu\nu}$ is the U(1) field strength, D_μ the covariant derivative, and ϕ is an N-dimensional complex vector.

Two possible lattice formulations:

1) Compact formulation à la Wilson ($|\phi_x|^2 = 1$):

$$S = J \,\mathrm{Re} \sum_{x\mu} \phi_x^* U_{x\mu} \phi_{x+\mu} + \kappa \sum_{x,\mu<\nu} \mathrm{plaquette}_{\mu\nu}$$

2) Noncompact formulation with unbounded electromagnetic potential (NCAH) [On the lattice we use $F_{\mu\nu} = \nabla_\mu A_\nu(x) - \nabla_\nu A_\mu(x)$, with $\nabla_\mu f(x) = f(x + a\mu) - f(x)$]. U is not fundamental: $U = \exp(iA)$.

The two models are not equivalent.

Compact model: gauge fields play no role. The critical behavior for $\kappa \neq 0$ is the same as that for $\kappa = 0$.
No continuous transition is observed for any $N \geq 3$ (including $N = \infty$).
For $N = 3$ it undergoes an $O(3)$ transition where gauge fields are not dynamical as in the CP^1 model ($|z| = 1$):

$$S = \sum_{\langle xy \rangle} |z_x^* \cdot z_y|^2.$$

NCAH model: for $N < N_c \approx 7$ it behaves as the compact model: first-order transitions, unless $N = 2$.
For $N > N_c$ a charged fixed point exists for $\kappa > \kappa_c(N)$.

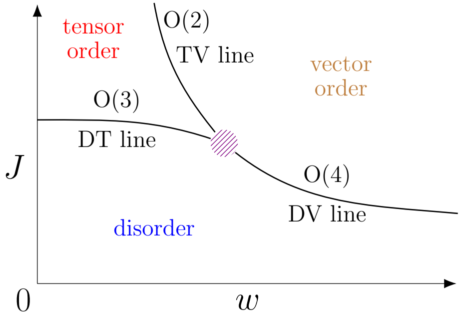
We add a gauge symmetry breaking term (photon mass):

a) compact case: $w \sum_{\langle x\mu \rangle} \mathrm{Re}\,\mathrm{Tr}\, U_{x\mu}$

b) ncAH: $w \sum_{\langle x\mu \rangle} A^2_{x\mu}$

The same term in the formal continuum limit.

Compact model for $N = 2$



For small w, gauge invariance is still present.

General statement: at phase transitions where gauge fields are not dynamical (phase behavior independent of κ), gauge invariance is not broken by small GSB terms.
Only gauge-invariant observables are considered.

An easy proof

1) We set $\kappa = 0$. Gauge fields can then be integrated out.

The action will be of the form

$$A = A_{GI}(\phi) + A_{GSB}(\phi)$$

where ϕ are the matter fields. $A_{GI}(\phi_x)$ is gauge invariant, while $A_{GSB}(\phi)$ is not gauge invariant, but is small for $w \to 0$.

2) Now, perform a change of variables: redefine the fields $\phi_x = e^{i\alpha_x}\psi_x$ (gauge transformations). For gauge-invariant observables, we can simply integrate over the α's ending up with a gauge-invariant model with short-range interactions [it can be proved by performing a strong-coupling expansion in powers of w].

For gauge-invariant observables and for small w, averages can be computed in an equivalent gauge-invariant model.

Charged fixed point

The argument does not apply to the charged fixed point (κ is positive and gauge fields cannot be integrated away)..
We have verified that that a photon mass is a relevant perturbation of the charged fixed point.

For any $w \neq 0$ the continuum limit is not gauge invariant. If we start from a non gauge-invariant microscopic theory, a tuning is needed to obtain a large-distance gauge-invariant behavior.

To characterize it, one needs the compute the RG dimension y_{GSB} of the perturbation at the charged fixed point. The exponent provides information on how to scale w to keep GSB effects small.

When the correlation length ξ increases, approaching the continuum limit, one should decrease w faster than $\xi^{-y_{GSB}}$ to ensure that GSB effects are negligible.

The determination of the exponent is tricky in the NCAH. The use of unbounded gauge fields makes the nongauge invariant sector ill defined for $w = 0$ and correspondingly the limit $w \to 0$ is singular.

How these problems affect the results is not yet understood.

We have studied a gauge-fixed version of the AH model at the charged fixed point.

1) We consider the AH model in the axial (temporal) gauge. The introduction of the gauge fixing does not change gauge-invariant observables for $w = 0$, but it makes the nongauge invariant sector well defined.

2) We add the symmetry breaking term to the gauge-fixed theory.

NOTE: For $w \neq 0$ this model cannot be related to the model without gauge fixing (obvious: the GF is only irrelevant for $w = 0$).

We find: $y_{GSB} = 2.55(5), 2.55(10)$ for $N = 25$ and 15, resp. (the charged fixed point exists for $N \gtrsim 7$)

We also considered a soft version of the Lorentz gauge, adding to the action

$$\frac{a}{2} \sum_x \left(\sum_\mu \Delta_\mu A_{x,\mu}\right)^2.$$

For two values of a ($a = 1$ and $a = 10$) we find $y_{GSB} = 1.4(1)$ ($N = 25$).

OPEN PROBLEM: Why does y_{GSB} depend on the gauge fixing used and/or on how it is implemented [hard GF (it corresponds to $a = \infty$) and soft GF (finite a)]?

The broken theory

QUESTION: Once gauge invariance is lost, what kind of continuum limit do we observe?

The surprising answer is that a gauge symmetry breaking gives rise to an enlargement of the global symmetry.

An argument: In a gauge invariant theory, some scalar degrees of freedom are “eaten" by the gauge bosons

Once gauge invariance is broken, some scalar degrees of freedom become again dynamical, allowing for a larger global symmetry group.

The U(N) Landau-Ginzburg-Wilson theory

Once we have broken gauge invariance, only the scalar field is dynamical. As it is a complex N dimensional vector, the corresponding effective theory is the $U(N)$ Landau-Ginzburg-Wilson:

$$\mathcal{L}_{\rm LGW} = \partial_\mu \Psi^* \cdot \partial_\mu \Psi + w\, \Psi^* \cdot \Psi + \frac{u}{4} \left(\Psi^* \cdot \Psi\right)^2 .$$

It is easy to prove that this model is $O(2N)$ invariant. The lowest dimensional operator that is $U(N)$, but not $O(2N)$ invariant, has dimension 6 and therefore it does not contribute to the critical behavior.

Once gauge invariance is lost, if the model undergoes a continuous transition, the global invariance group is $O(2N)$.

Cyclic RG flow

Our results indicate the presence of a cyclic RG flow.

We start from the $O(2N)$ invariant model

$$L = |\partial_\mu \phi(x)|^2 + m^2 |\phi(x)|^2 + V(|\phi(x)|)$$

We gauge the $U(1)$ invariance, obtaining the AH model, that has a charged fixed point: the $O(2N)$ fixed point is unstable under the addition of the gauge coupling.

We add a gauge breaking term and we end up again with an $O(2N)$ invariant model (at criticality).

Conclusions

1. The role of GSB terms depends on the nature of the transition in the gauge-invariant model.
If gauge fields are not dynamical at the transition (one can set $\kappa = 0$ and integrate them out), gauge invariance is not broken by small GSB perturbations.

2. The charged fixed point in the NCAH model is unstable with respect to the gauge perturbation $\sum_{x\mu} A_{x\mu}^2$. General result?

3. We compute the RG dimension of the perturbation for $N = 25$ obtaining $y_{GSB} = 2.55$ for the axial gauge-fixed theory and $y_{GSB} = 1.4$ for the theory with a soft Lorentz term. Why are they different? Not understood.

4. Once gauge invariance is lost, continuous transitions should belong to the $O(2N)$ universality class.