Global symmetry breaking in gauge theories: the case of multiflavor scalar chromodynamics

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mainly based on C. Bonati, A. Pelissetto, E. Vicari
Phys. Rev. Lett. **123**, 232002 (2019) arXiv:1910.03965
Phys. Rev. D **101**, 034505 (2020) arXiv:2001.01132
with some glimpses from subsequent works

LATTICE 2021
The 38th International Symposium on Lattice Field Theory
July 26-30 2021, Zoom/Gather@MIT

Outline

Finite-T chiral transition

Standard approach to investigate the universal properties of the finite-T chiral transition Pisarski, Wilczek PRD, **29**, 338 (1984)

1. Assume the presence of a continuous phase transition (otherwise the following points are meaningless).
2. Model the 4D finite-T transition by a 3D effective field theory.
3. Assume that the chiral transition is governed by a gauge-invariant order parameter (the $N_f \times N_f$ chiral condensate $\langle \bar{\Psi}_i (1 + \gamma_5) \Psi_j \rangle$) i.e. assume that gauge modes are irrelevant at the transition.
4. Write down the most general Φ^4 effective Lagrangian for the order parameter compatible with chiral symmetry (maybe also $U(1)_A$).
5. If IR-stable FPs of the RG-flow exist, the chiral transition can be a continuous one of the given FP universality classes or discontinuos. If no IR-stable FPs exist, discontinuous transition.

Why 3D scalar models

Main predictions of the Pisarski-Wilczek approach for the chiral transition:

- O(4) or discontinuous for $N_f = 2$
  (or $\mathrm{U}(2)_L \otimes \mathrm{U}(2)_R \to \mathrm{U}(2)_V$ Pelissetto, Vicari 1309.5446)
- discontinuous for $N_f > 2$

On the lattice the $N_f = 2$ case has a long history of contradictory results; for $N_f > 2$ a clear 1^{st} order is observed for small N_t but the 1^{st} order region seems to disappear by increasing N_t
(e.g. de Forcrand, D'Elia 1702.00330, Cuteri, Philipsen, Sciarra 2107.12739)

Our aim is to investigate the validity of the PW approach and, in particular, of its main assumption: the irrelevance of the gauge modes.

3D scalar models with global and gauge symmetries are ideal models to test the predictions of the PW approach in a setup where accurate numerical results can be obtained.

The scalar model

The basic variables of our model are $N_c \times N_f$ complex matrices $Z_{\boldsymbol{x}}^{cf}$ and the action before gauging is

$$S_{\rm inv} = -J \sum_{\boldsymbol{x},\mu} \mathrm{Re}\,\mathrm{Tr}\, Z_{\boldsymbol{x}}^{\dagger} Z_{\boldsymbol{x}+\hat{\mu}}\,, \quad \mathrm{Tr}\, Z_{\boldsymbol{x}}^{\dagger} Z_{\boldsymbol{x}} = 1\,,$$

where $\boldsymbol{x}$ stands for the site of a 3D cubic lattices. This action is $O(2N_cN_f)$ symmetric, as seen by explicitly rewriting it in term of Re and Im parts of $Z_{\boldsymbol{x}}^{cf}$.

Next we gauge the N_c color degrees of freedom, by coupling them to a $SU(N_c)$ gauge field $U_{\boldsymbol{x},\hat{\mu}}$

$$S_g = -\beta N_f \sum_{\boldsymbol{x},\mu} \mathrm{Re}\,\mathrm{Tr}\left[Z_{\boldsymbol{x}}^{\dagger}\, U_{\boldsymbol{x},\hat{\mu}}\, Z_{\boldsymbol{x}+\hat{\mu}}\right] - \frac{\beta_g}{N_c} \sum_{\boldsymbol{x},\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\Box_{\boldsymbol{x},\mu\nu}$$

The symmetries of the scalar models

$$S_g = -\beta N_f \sum_{\boldsymbol{x},\mu} \mathrm{Re}\,\mathrm{Tr}\left[Z_{\boldsymbol{x}}^\dagger\, U_{\boldsymbol{x},\hat{\mu}}\, Z_{\boldsymbol{x}+\hat{\mu}}\right] - \frac{\beta_g}{N_c} \sum_{\boldsymbol{x},\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\Box_{\boldsymbol{x},\mu\nu}$$

S_g is invariant under the local transformation $G_{\boldsymbol{x}} \in \mathrm{SU}(N_c)$

$$Z_{\boldsymbol{x}} \to G_{\boldsymbol{x}} Z_{\boldsymbol{x}}\ , \quad U_{\boldsymbol{x},\hat{\mu}} \to G_{\boldsymbol{x}} U_{\boldsymbol{x},\hat{\mu}} G^\dagger_{\boldsymbol{x}+\hat{\mu}}\ ,$$

and under the global transformation $M \in \mathrm{U}(N_f)$

$$Z_{\boldsymbol{x}} \to Z_{\boldsymbol{x}} M\ , \quad U_{\boldsymbol{x},\hat{\mu}} \to U_{\boldsymbol{x},\hat{\mu}}\ .$$

The $N_c = 2$ case is somehow peculiar: since SU(2) is pseudoreal the largest global symmetry is in fact $\mathrm{Sp}(N_f)$ (the subgroup of $M \in \mathrm{U}(2N_f)$ such that $MJM^T = J$). Moreover we will need $\mathrm{SO}(5)=\mathrm{Sp}(2)/\mathbb{Z}_2$.

PW approach: gauge degrees of freedom are irrelevant

We use a LGW eff. Lagrangian with the gauge invariant order parameter

$$Q_{\boldsymbol{x}}^{fg} = \sum_a \bar{Z}_{\boldsymbol{x}}^{af} Z_{\boldsymbol{x}}^{ag} - \frac{1}{N_f}\delta^{fg} \quad (Z_{\boldsymbol{x}} \to G_{\boldsymbol{x}} Z_{\boldsymbol{x}}\ ;\ Z_{\boldsymbol{x}} \to Z_{\boldsymbol{x}} M)\ .$$

$Q_{\boldsymbol{x}}$ is hermitian, traceless and $Q_{\boldsymbol{x}} \to M^\dagger Q_{\boldsymbol{x}} M$ under the global symmetry.

Most general 4^{th}-order polynomial consistent with the global symmetry:

$$\mathrm{Tr}(\partial_\mu Q)^2 + r\,\mathrm{Tr}\,Q^2 + w\,\mathrm{tr}\,Q^3 + u\,(\mathrm{Tr}\,Q^2)^2 + v\,\mathrm{Tr}\,Q^4$$

For $N_f > 2$, $\mathrm{Tr}\,Q^3 \neq 0$ and a first order phase transition is expected.

For $N_f = 2$ and $N_c > 3$ we obtain the standard LGW for the 3D O(3) universality class (more explicit if rewritten by using $\varphi_{\boldsymbol{x}}^k = \bar{Z}_{\boldsymbol{x}}^{af}\sigma_{fg}^k Z_{\boldsymbol{x}}^{ag}$)

For $N_f = 2$ and $N_c = 2$ we obtain Sp(2)≈O(5) universality class.
Beyond $\varphi_{\boldsymbol{x}}$ we also have the complex scalar $\phi_{\boldsymbol{x}} = \epsilon_{ab}\epsilon_{fg} Z_{\boldsymbol{x}}^{af} Z_{\boldsymbol{x}}^{bg}$.

Lattice observables and Finite Size Scaling

On a L^3 lattice with periodic b.c. we can define

$$Q_{\boldsymbol{x}}^{fg} \equiv \sum_a \bar{Z}_{\boldsymbol{x}}^{af} Z_{\boldsymbol{x}}^{ag} - \frac{1}{N_f}\delta^{fg}\,, \quad G(\boldsymbol{x}-\boldsymbol{y}) = \langle \mathrm{Tr}\,(Q_{\boldsymbol{x}} Q_{\boldsymbol{y}})\rangle,$$

from which we get the susceptibility χ, the (2^{nd} momentum, finite volume) correlation lenght ξ and the Binder cumulant U

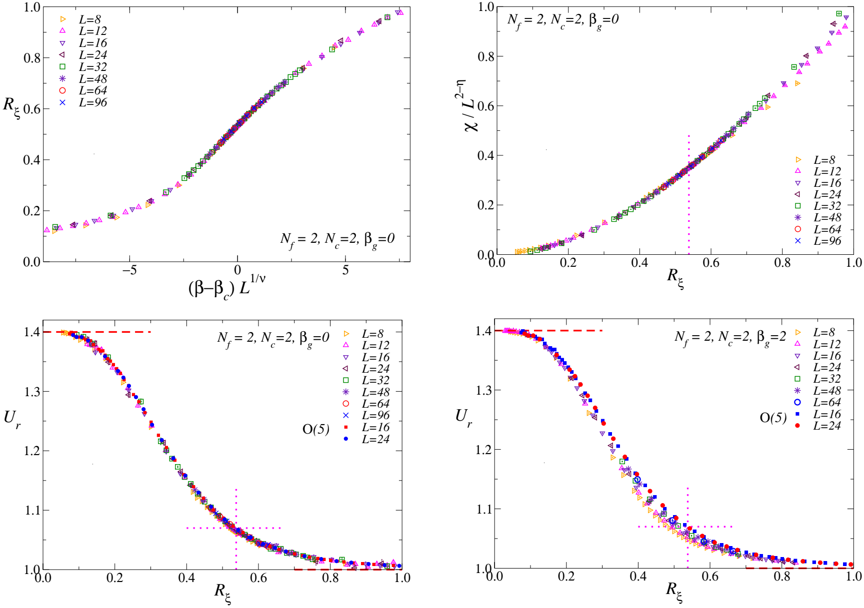
$$\chi = \sum_{\boldsymbol{x}} G(\boldsymbol{x})\,, \quad \xi^2 = \frac{1}{4\sin^2(\pi/L)} \frac{\widetilde{G}(0) - \widetilde{G}(\boldsymbol{p}_m)}{\widetilde{G}(\boldsymbol{p}_m)}$$

$$U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \frac{1}{V^2} \sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Tr}\, Q_{\boldsymbol{x}} Q_{\boldsymbol{y}}\,.$$

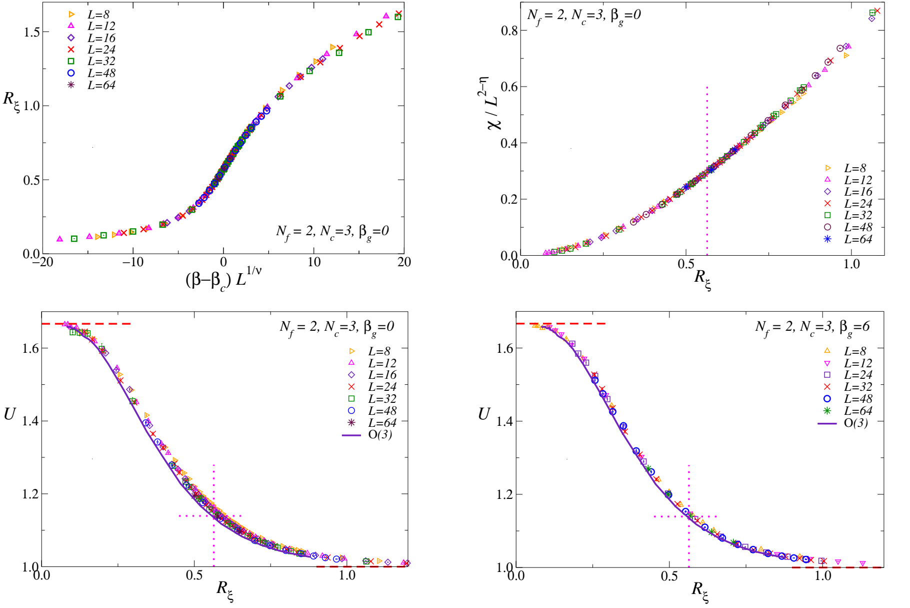
U and $R_\xi = \xi/L$ are RG invariants, hence close to the transition they satisfy $R(\beta, L) = f_R(X) + L^{-\omega} g_R(X) + \cdots$ with $X = (\beta - \beta_c)L^{1/\nu}$ in particular

$$U(\beta, L) = F_U(R_\xi) + O(L^{-\omega})\,.$$

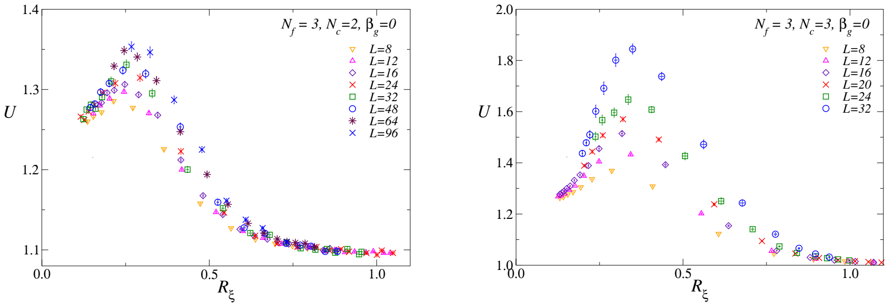
The case $N_f = 2$, $N_c = 2$: O(5)



The case $N_f = 2$, $N_c = 3$: O(3)



The case $N_f \geq 3$: (hints of) 1^{st} order



No clear signal of metastabilites, but data of U versus R_ξ do not seem to converge to a scaling curve, as they should for a second order phase transition.

The peak of U is expected to grow as $U \propto V$ for 1^{st} order transitions on large enough volume.

Conclusions

We have numerically investigated a lattice 3D model with SU(N_c) gauge symmetry and U(N_f) global symmetry, and verified that the EFT that correctly describe its critical behaviour is the LGW theory based on a gauge-invariant order parameter.

This supports the Pisarski-Wilczek approach to massless QCD but leaves several open questions:

- The PW approach sometimes fails for abelian gauge field (see Bonati, Pelissetto, Vicari 2010.06311). Do systems with non-abelian gauge symmetry exist for which the PW approach is not the correct one?
- What happens if we start from an initial symmetry that is not the maximal O($2N_cN_f$)? Can "3D continuum scalar QCD" be the correct continuum EFT in some cases? Preliminary indications from Bonati, Franchi, Pelissetto, Vicari 2106.15152

Thank you for your attention!

Backup with something more

The global symmetry for $N_c = 2$ (I)

$$S_h = \frac{1}{2}\sum_{f,a,b}\left[\bar{Z}^{af}_{\boldsymbol{x}}\, U^{ab}_{\boldsymbol{x},\hat{\mu}}\, Z^{bf}_{\boldsymbol{x}+\hat{\mu}} + Z^{af}_{\boldsymbol{x}}\, \bar{U}^{ab}_{\boldsymbol{x},\hat{\mu}}\, \bar{Z}^{bf}_{\boldsymbol{x}+\hat{\mu}}\right] =$$

$$= \frac{1}{2}\sum_{f,a,b}\left[\bar{Z}^{af}_{\boldsymbol{x}}\, U^{ab}_{\boldsymbol{x},\hat{\mu}}\, Z^{bf}_{\boldsymbol{x}+\hat{\mu}} + \bar{Y}^{af}_{\boldsymbol{x}}\, U^{ab}_{\boldsymbol{x},\hat{\mu}}\, Y^{bf}_{\boldsymbol{x}+\hat{\mu}}\right] ,$$

where

$$Y^{af}_{\boldsymbol{x}} \equiv i\sigma_2^{ab}\bar{Z}^{bf}_{\boldsymbol{x}} , \quad \bar{U} = \sigma_2 U \sigma_2 , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

We can now define

$$\Gamma^{a\gamma}_{\boldsymbol{x}} = \begin{cases} Z^{a\gamma}_{\boldsymbol{x}} & \text{if} \quad 1 \le \gamma \le N_f \\ Y^{a\,(\gamma - N_f)}_{\boldsymbol{x}} & \text{if} \quad N_f + 1 \le \gamma \le 2N_f \end{cases} ,$$

then

$$S_h = \frac{1}{2}\sum_{\gamma,a,b}\bar{\Gamma}^{a\gamma}_{\boldsymbol{x}}\, U^{ab}_{\boldsymbol{x},\hat{\mu}}\Gamma^{b\gamma}_{\boldsymbol{x}+\hat{\mu}} = \frac{1}{2}\mathrm{Tr}(\Gamma^{\dagger}_{\boldsymbol{x}} U_{\boldsymbol{x},\hat{\mu}}\Gamma) .$$

The global symmetry for $N_c = 2$ (II)

$$\mathrm{Tr}(\Gamma_{\boldsymbol{x}}^\dagger U_{\boldsymbol{x},\hat{\mu}} \Gamma)$$

is invariant under the local transformation $\Gamma_{\boldsymbol{x}} \to G_{\boldsymbol{x}} \Gamma_{\boldsymbol{x}}$ with $G_{\boldsymbol{x}} \in \mathrm{SU}(N_c)$ and it is invariant under the global $\mathrm{U}(2N_f)$ transformation

$$\Gamma_{\boldsymbol{x}} \to \Gamma_{\boldsymbol{x}} M \ , \quad M \in \mathrm{U}(2N_f) \, . \tag{1}$$

However Γ variables are not generic, they have the structure

$$\Gamma = \left(Z \, , \ Y = i\sigma_2 \bar{Z} \right) \ ,$$

which is equivalent to say that they satisfy the relation

$$i\sigma_2 \bar{\Gamma} = \Gamma J \ , \quad J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \ ,$$

where I is the $N_f \times N_f$ identity matrix. The global invariance is thus the subgroup of matrices $M \in \mathrm{U}(2N_f)$ which leave invariant this relation, i.e.

$$M J M^T = J \, .$$

The complete order parameter for $N_c = 2$

Instead of $Q_{\boldsymbol{x}}^{fg}$ we can define

$$\mathcal{T}_{\boldsymbol{x}}^{\alpha\beta} = \sum_a \overline{\Gamma}_{\boldsymbol{x}}^{a\alpha} \Gamma_{\boldsymbol{x}}^{a\beta} - \frac{\delta^{\alpha\beta}}{2N_f} \sum_{a\gamma} \overline{\Gamma}_{\boldsymbol{x}}^{a\gamma} \Gamma_{\boldsymbol{x}}^{a\gamma} \,,$$

but it is not difficult to show that these quantities can be expressed in term of $Q_{\boldsymbol{x}}^{fg}$ and of $D_{\boldsymbol{x}}^{fg} = \sum_{ab} \epsilon^{ab} Z_{\boldsymbol{x}}^{af} Z_{\boldsymbol{x}}^{bg}$.

Since $\mathcal{T}_{\boldsymbol{x}}^{\alpha\beta}$ is not independent of $Q_{\boldsymbol{x}}^{fg}$ (in particular $\mathcal{T}_{\boldsymbol{x}}^{ab} = Q_{\boldsymbol{x}}^{ab}$ for $a, b = 1, \ldots, N_f$), its critical behaviour can be investigated by studying just $Q_{\boldsymbol{x}}^{fg}$.

We only have to pay attention to the fact that

$$\frac{\langle (\sum_{k=1}^{3} M^k M^k)^2 \rangle}{\langle \sum_{k=1}^{3} M^k M^k \rangle^2} = \frac{21}{25}\, U_{\mathrm{O}(5)} \,.$$

therefore the correct quantity to be studied to achieve matching with the universal Binder parameter of the O(5) vector model is $U_r \equiv \frac{21}{25}\, U$.

Critical exponents and RG-invariant quantities

- 3D O(5) universality class

$$\nu = 0.779(3)\,, \quad \eta = 0.034(1)\,, \quad \omega = 0.79(2)$$
$$R_\xi^* = 0.538(1)\,, \quad U^* = 1.069(1)\,.$$

- 3D O(3) universality class

$$\nu = 0.7117(5)\,, \quad \eta = 0.0378(3)\,, \quad \omega = 0.782(13)\,,$$
$$R_\xi^* = 0.5639(2)\,, \quad U^* = 1.1394(3)\,.$$