Universal Properties of the Vortex in the (2+1)-d $O(2)$ through dualization

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Lattice 2021

Outline

- The (2+1)-d $O(2)$ Scalar Field Theory;
    - broken phase;
    - vortex excitation;

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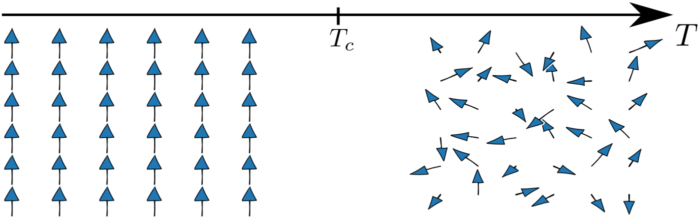
    - Dualization: vortex as a charged particle;
    - Non-perturbative quantization;
- Results and perspectives.

Broken phase | Symmetric Phase



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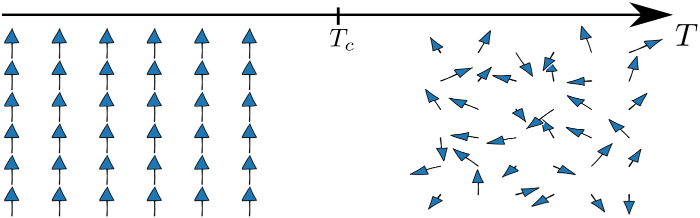

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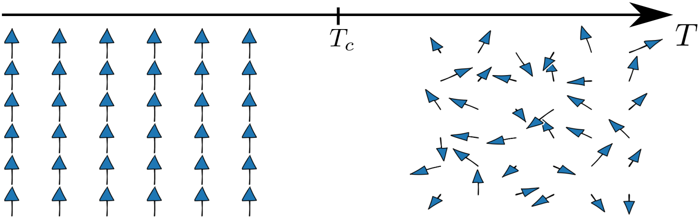

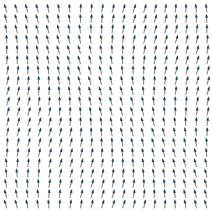

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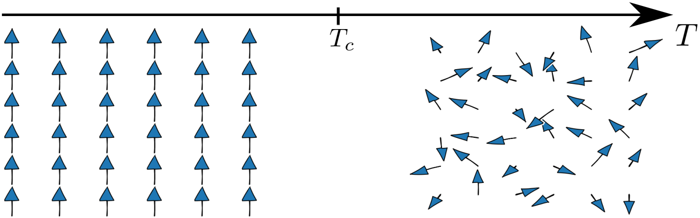

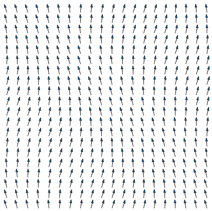

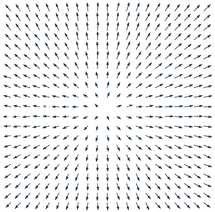

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    J.-M. Duan, Phys. Rev. B49 (1994) 12381.
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Quantizing the vortex

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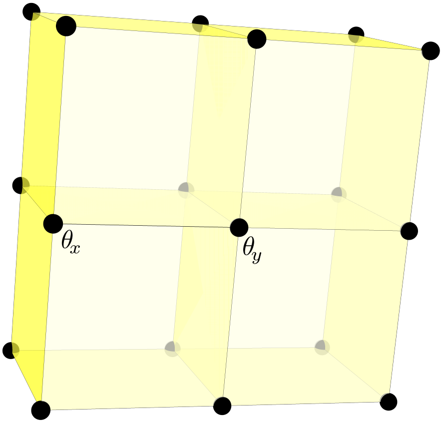
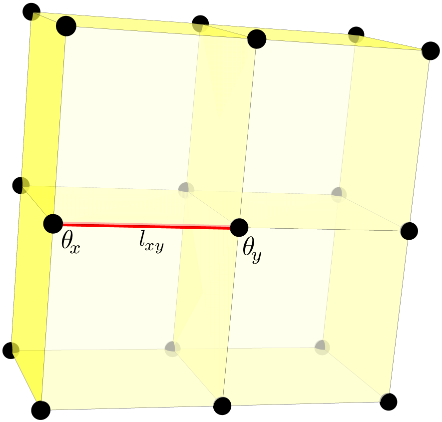
- Fully non-perturbative approach is non-trivial:
  - vortex correlation not readily amenable to numerical simulations;
  - single vortex never occurs at finite periodic volume.

The quantum vortex as a quantum particle



$$s\,(\theta_x - \theta_y)$$

The quantum vortex as a quantum particle



$$s\left(\theta_x - \theta_y\right) \rightarrow s\left(l_{xy}\right)$$

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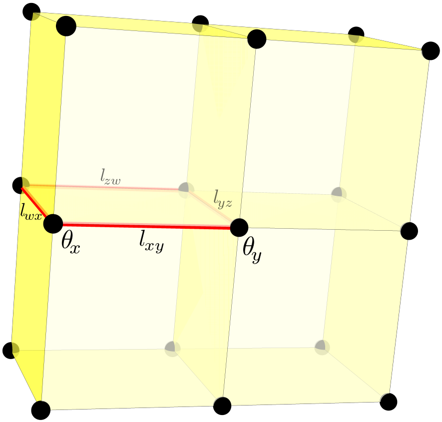
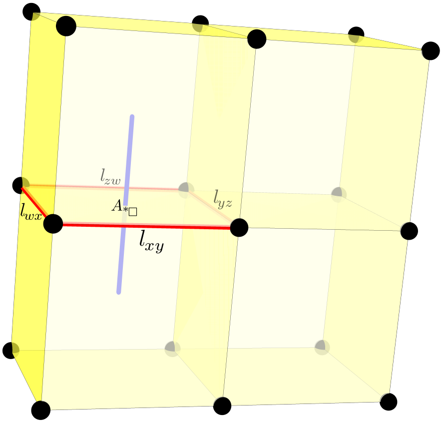
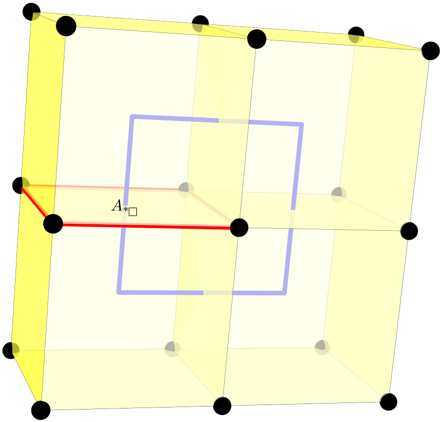


$$s\,(l_{xy}) \quad \text{with constraints} \quad l_{xy} + l_{yz} + l_{zw} + l_{wx} = 0$$

The quantum vortex as a quantum particle

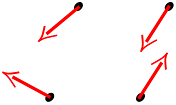


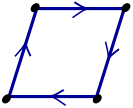
$$s\left(l_{xy}, A_{*\square}\right) \quad \text{with} \quad \delta\left(l_{xy} + l_{yz} + l_{zw} + l_{wx}\right) = \sum_{A_{*\square} \in \mathbb{Z}} e^{iA_{*\square}(l_{xy}+l_{yz}+l_{zw}+l_{wx})}$$



Integrate out l_{xy} $\Rightarrow$ Gauge Theory : $\tilde{s}\,(A_{*\square})$

The dualization picture





Continuous spin model	DUAL	**Scalar QED**
$O(2)$ global symmetry	$\rightleftarrows$	$U(1)$ local symmetry
Weak/Strong coupling	$\rightleftarrows$	Strong/Weak coupling
Symmetric phase	$\rightleftarrows$	Higgs phase
Broken phase	$\rightleftarrows$	Coulomb phase
Goldstone bosons	$\rightleftarrows$	Photons
Vortex	$\rightleftarrows$	Charged scalar

J. Fröhlich, G. Morchio, F. Strocchi, Phys. Lett. B89 (1979) 61
D. Buchholz, K. Fredenhagen, Commun. Math. Phys. 84 (1982) 1
D. Buchholz, Commun. Math. Phys. 85 (1982) 49
D. Buchholz, J. E. Roberts, Commun. Math. Phys. 330 (2014) 935

The charged particle

- Vortex correlation $\rightleftarrows$ charged particle (χ_x) correlation;

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- The proper gauge invariant operator creates the particle along with a cloud of soft photons

$$\chi_x^C = e^{i\triangle^{-1}\delta A_x}\chi_x$$

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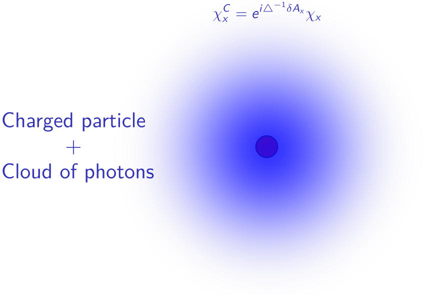
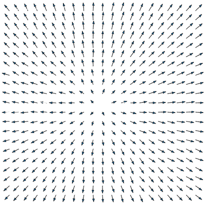


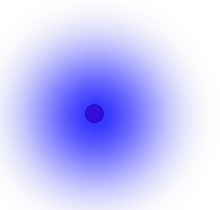
- Vortex correlation function: $\langle \chi_x^C \chi_y^{C\dagger} \rangle$

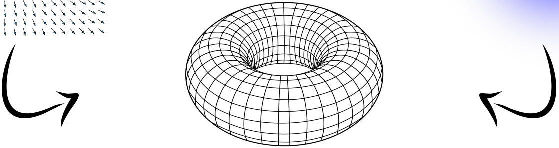
P. A. M. Dirac, Canad. J. Phys. 33 (1955) 650
J. Fröhlich, P. A. Marchetti, Euro. Phys. Lett. 2 (1986) 933

Vortex
+
Cloud of Goldstone bosons



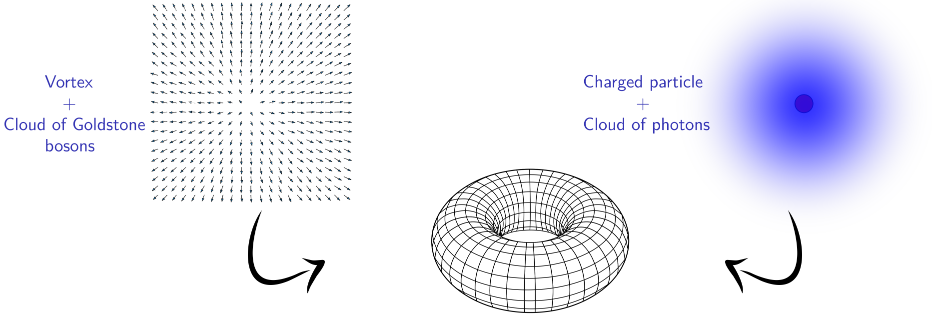
Charged particle
+
Cloud of photons





- No net charge on the torus;

The charged particle at finite volume

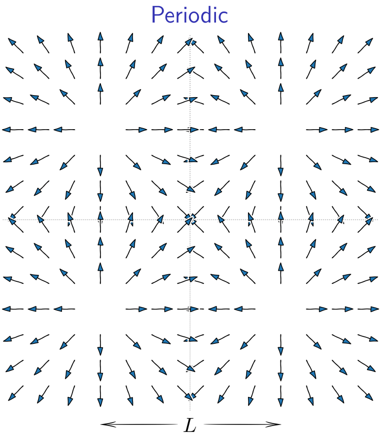


- No net charge on the torus;
- C-periodic boundary conditions (C* boundary conditions):

$$A_\mu\left(x + L\hat{i}\right) = -A_\mu\left(x\right) - \partial_\mu \varphi_i\left(x\right)$$
$$\chi\left(x + L\hat{i}\right) = \chi\left(x\right)^* e^{i\varphi_i(x)}$$

U.-J. Wiese, Nucl. Phys. B375 (1992) 45
B. Lucini, A. Patella, A. Ramos, N. Tantalo, JHEP 1602 (2016) 076C
I. Campos, P. Fritzsch, M. Hansen, M. K. Marinkovic, A. Patella, A. Ramos, N. Tantalo, Eur. Phys. J. C (2020) 80:195

The C-periodic vortex: mass and charge computation



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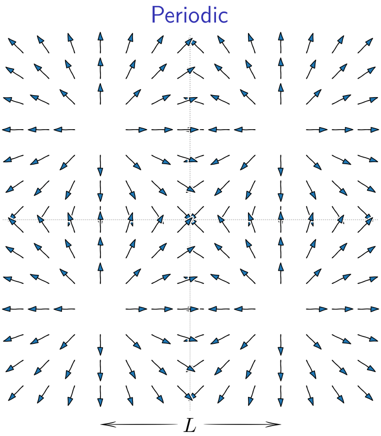
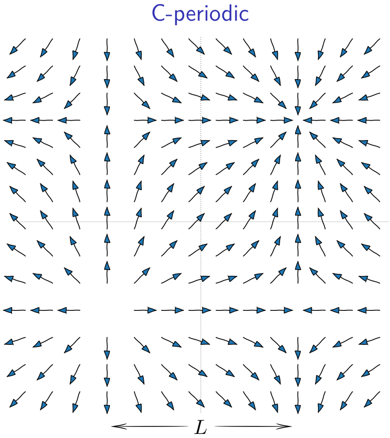
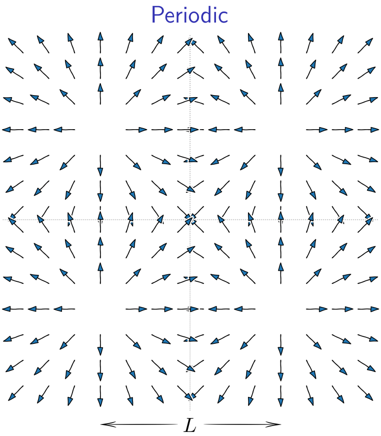
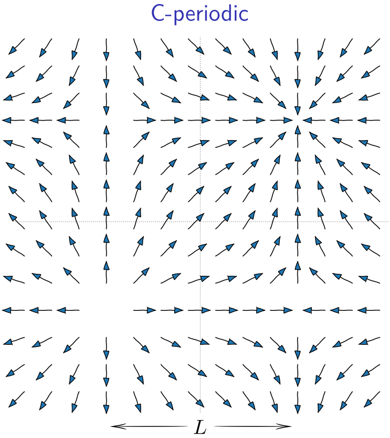


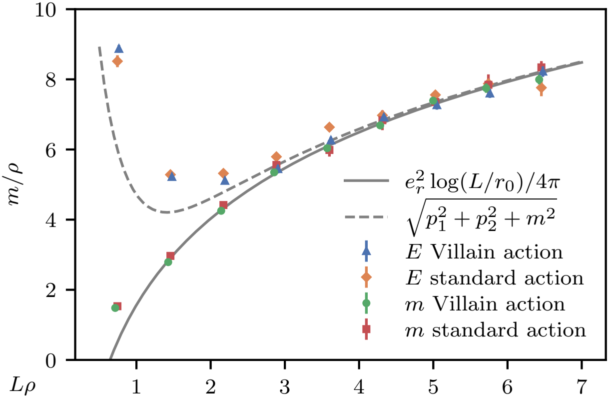






Vortices interact with their C-periodic copy

$$m = \frac{e^2}{4\pi} \log\left(\frac{L}{r_0}\right) \rightarrow \text{Determine the charge}$$

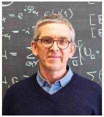


- Log divergent mass;
- Universal vortex charge: $e_r^2 = 3.58(8) \times (4\pi\rho)$;
- Breaking of Lorentz invariance: $E = m + \frac{p^2}{2m_k} \Rightarrow \frac{m_k}{m} = 0.71(3)$ (for $L\rho = 1.43(2)$).

Overview and Outlook

- Computation of the vortex mass and vortex charge
  - The logarithmic divergent mass survives quantization;
  - The vortex charge gets renormalized.

M. Hornung, JCPB, and U-J. Wiese arXiv:2106.16191 (2021).



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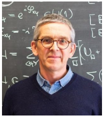

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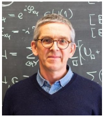

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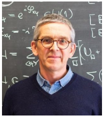

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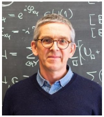

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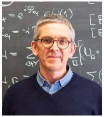
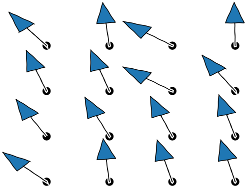


Lattice discretization and dualization



Standard: $e^{-S(\{\theta\})} = \prod_{\langle x,y \rangle} e^{\frac{1}{g^2}\cos(\theta_x - \theta_y)}$

Villain: $e^{-S(\{\theta\})} = \prod_{\langle x,y \rangle} \sum_{n_{xy} \in \mathbb{Z}} e^{-\frac{1}{2g^2}(\theta_x - \theta_y + 2\pi n_{xy})^2}$

The duallized action

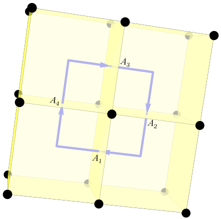
- The final gauge theory will depend on the initial action;
- We probe two actions:
  - Villain action

$$e^{-S(\{\theta\})} = \prod_{\langle x,y\rangle} \sum_{n_{xy}\in\mathbb{Z}} e^{-\frac{1}{2g^2}\left(\theta_x-\theta_y+2\pi n_{xy}\right)^2} \overset{\text{Dual}}{\rightleftarrows} e^{-\tilde{S}(\{A\})} = \prod_{\square} e^{-\frac{g^2}{2}F_\square^2}$$

  - Standard action

$$e^{-S(\{\theta\})} = \prod_{\langle x,y\rangle} e^{\frac{1}{g^2}\cos\left(\theta_x-\theta_y\right)} \overset{\text{Dual}}{\rightleftarrows} e^{-\tilde{S}(\{A\})} = \prod_{\square} I_{F_\square}\left(\frac{2}{g^2}\right)$$

$$F_\square = A_1 + A_2 - A_3 - A_4$$



- Integer gauge theory

$$\text{Dual action}: A_{x\mu} \in \mathbb{Z}$$

$$\tilde{S}(\{A\}) = \sum_{\square} \tilde{s}(F_{\square})$$

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J. Fröhlich, P. A. Marchetti, Euro. Phys. Lett. 2 (1986) 933

$$\text{Scalar QED}: \bar{A}_{x\mu} \in \mathbb{R}$$

$$S_{\mathrm{QED}}(\{\bar{A}\}, \{\chi\}) = \sum_{\Box} s(\bar{F}_{\Box}) - \frac{\kappa}{2} \sum_{x,\mu} \left(\chi^*_{x+\hat{\mu}} e^{i\bar{A}_{x\mu}} \chi_x + \chi^*_x e^{-i\bar{A}_{x\mu}} \chi_{x+\hat{\mu}} \right)$$

- Integer gauge theory

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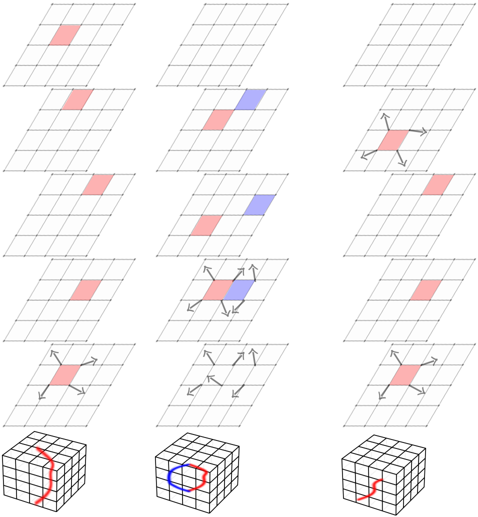
- Limit $\kappa \to +\infty$ and unitary gauge, $\chi_x = 1$:

$$-\frac{\kappa}{2} \sum_{x,\mu} \left(\chi^*_{x+\hat{\mu}} e^{i\bar{A}_{x\mu}} \chi_x + \chi^*_x e^{-i\bar{A}_{x\mu}} \chi_{x+\hat{\mu}}\right) \to -\kappa \sum_{x,\mu} \cos \bar{A}_{x\mu} \Rightarrow \bar{A}_{x\mu} \in 2\pi\mathbb{Z}$$

The vortex as a charged particle

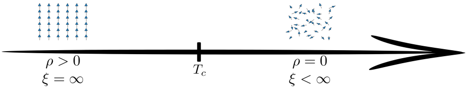
$$\left\langle \chi_x^C \chi_y^{C\dagger} \right\rangle \underset{\text{back}}{\overset{\text{Dualize}}{\longrightarrow}} \frac{Z_v}{Z}$$



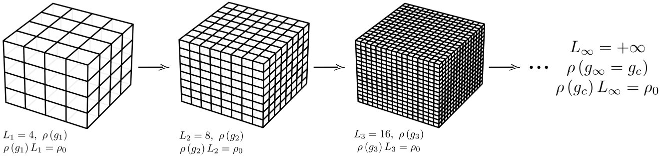
Contribution to Z Contribution to Z_v

Computational strategy

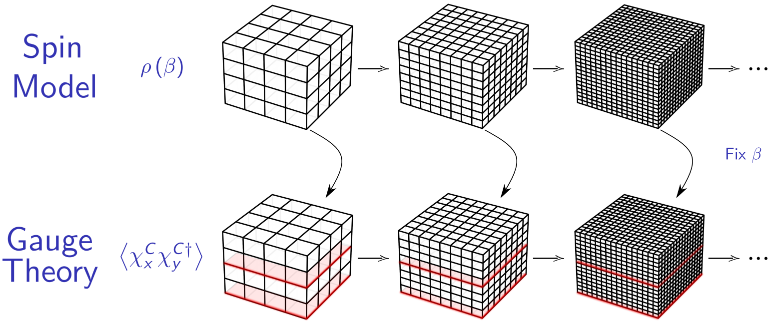
- The spin stiffness characterizes the distance to the critical point;

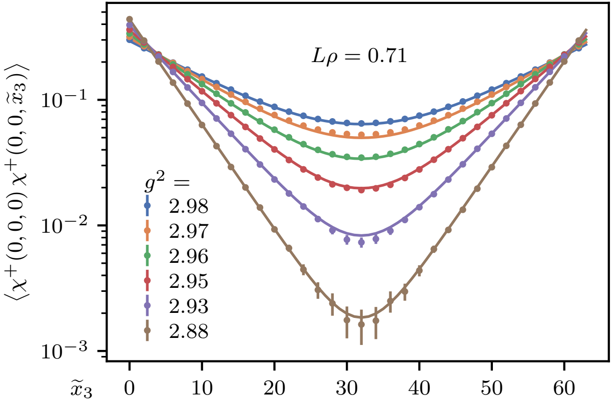


- Approaching the continuum limit



Approach the continuum limit at a finite volume characterized by $\rho_0 = \rho L$




$L\rho = 0.71$
$g^2 =$
2.98
2.97
2.96
2.95
2.93
2.88
$\langle \chi^+(0,0,0)\, \chi^+(0,0,\widetilde{x}_3) \rangle$
10^{-1}
10^{-2}
10^{-3}
$\widetilde{x}_3$
0
10
20
30
40
50
60

Results: approaching the continuum limit