Excitations of isolated static charges in q=2 Abelian Higgs theory

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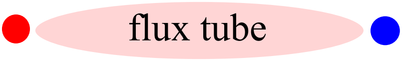
$\Psi_v =$ 

a spectrum of localized quantum excitations of the surrounding field exists for a static quark and antiquark pair in the confining phase of a pure gauge theory.

In confining phase, the color electric field associated with the pair of color charges is collimated into a **flux tube**, the flux tube exists as a number of vibrational modes.

On the other hand, in ordinary QED,

- any disturbance of the field surrounding a static charge can be viewed as the creation of some set of photons superimposed on a Coulombic background.
- then there are no stable localized excitations.
- But, could such a spectrum of localized excitations exist in **Higgs phase in gauge Higgs theories**? (interacting, yet nonconfining)

q=2 Abelian Gauge-Higgs theory

q=2 Abelian Gauge-Higgs theory,

$$S = -\beta \sum_{plaq} \mathrm{Re}[U_\mu(x) U_\nu(x+\hat{\mu}) U^*_\mu(x+\hat{\nu}) U^*_\nu(x)] - \gamma \sum_{x,\mu} \mathrm{Re}[\phi^*(x) U^2_\mu(x) \phi(x+\widehat{\mu})] \ .$$

- the scalar field has charge $q = 2$ (as do Cooper pairs)
- impose a unimodular constraint $\phi^*(x)\phi(x) = 1$ (for simplicity)
- a relativistic generalization of the Landau-Ginzburg effective model of superconductivity

q=2 Abelian Gauge-Higgs theory

What do we consider:

1. Physical states containing a static fermion and anti-fermion at sites **x**, **y**
2. **Stable localized excitations** of the U(1) gauge field and Higgs field surrounding a static charge
3. Excitations have been reported in SU(3) gauge Higgs theory (Phys. Rev. D 102, 054504 (2020)) and in chiral U(1) gauge Higgs theory (arXiv:2104.12237) by Jeff Greensite

What do we show in our work?:

- We compute the energy excitations above the ground state of Higgs + U(1) gauge field via lattice Monte Carlo simulations.

- If the excited state above the ground state is less than the photon mass, the excited state is stable!

How do we identify such localized excitations in MC simulations?

What do we exactly measure in our calculations?

Let us first consider a static fermion and anti-fermion pair at sites **x** **y** each of ± 2 electric charge

$$|\Phi_\alpha(R)\rangle = Q_\alpha(R)|\Psi_0\rangle \,,$$

where Ψ_0 is the vacuum state, and

$$Q_\alpha(R) = [\overline{\psi}(\mathbf{x})\zeta_\alpha(\mathbf{x})] \times [\zeta^*_\alpha(\mathbf{y})\psi(\mathbf{y})] \,.$$

Here the $\overline{\psi}, \psi$ are operators creating double-charged static fermions of opposite charge transforming as $\psi(x) \to e^{2i\theta(x)}\psi(x)$, and the $\{\zeta_\alpha(x)\}$ are a set of operators, which may depend on some (possibly non-local) combination of the Higgs and gauge fields, also transforming as $\zeta(x) \to e^{2i\theta(x)}\zeta(x)$, under a gauge transformation

$$U_\mu(x) \to \exp(i\theta(x))U_\mu(x)\exp(-\theta(x+\hat{\mu})).$$

Possible choice for ζ ?

One possible choice for ζ is the Higgs field $\phi(x)$.
Another set is provided by eigenstates $\zeta = \xi_\alpha$ of the covariant Laplacian, where

$$(-D_i D_i)_{xy} \xi_\alpha(\mathbf{y}; U) = \lambda_\alpha \xi_\alpha(\mathbf{x}; U)$$

and

$$(-D_i D_i)_{xy} = \sum_{k=1}^{3} [2\delta_{\mathbf{xy}} - U_k^2(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U_k^{*2}(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}}] \ .$$

Because the covariant Laplacian depends only on the squared link variable, the $\xi_\alpha(x; U)$, which we have elsewhere referred to as "pseudomatter" fields transforming like $q = 2$ charged matter fields, with the one difference that, unlike matter fields, they do not transform under a global (constant) gauge transformation. Pseudomatter fields depend nonlocally on the gauge fields, and the low-lying eigenstates and eigenvalues of the covariant Laplacian, which is a sparse matrix, can be computed numerically via the Arnoldi algorithm.

Four lowest-lying Laplacian eigenstates + Higgs field

In our calculation we make use of the four lowest-lying Laplacian eigenstates and the Higgs field to construct the Φ_α,

the four lowest lying Laplacian eigenstates,

$$\zeta_i(x) = \begin{cases} \xi_i(x) & i = 1, 2, 3, 4 \\ \phi(x) & i = 5 \end{cases}$$

In general the five states $\Phi_\alpha(R)$ are non-orthogonal at finite R. Of course $\phi(x)$ is a $q = 2$ matter field, rather than pseudomatter field. We express the operator Q_α in terms of a non-local operator $V_\alpha(\mathbf{x}, \mathbf{y}; U)$

$$\begin{aligned} Q_\alpha(R) &= \overline{\psi}(\mathbf{x}) V_\alpha(\mathbf{x}, \mathbf{y}; U) \psi(\mathbf{y}) \\ V_\alpha(\mathbf{x}, \mathbf{y}; U) &= \zeta_\alpha(\mathbf{x}; U) \zeta^*_\alpha(\mathbf{y}; U) \ , \end{aligned}$$

and define the Euclidean time evolution operator of the lattice abelian Higgs model, $\mathcal{T} = e^{-(H - \mathcal{E}_0)}$, which is the transfer matrix multiplied by a constant $e^{\mathcal{E}_0}$ where $\mathcal{E}_0$ is the vacuum energy, evolving states for one unit of discretized time.

Calculations of the Transfer Matrix

$[\mathcal{T}]$ is the matrix element in the five non-orthogonal states Φ_α, with the matrix of overlaps, $[\mathcal{O}]$, of such states.

$$
\begin{aligned}
[\mathcal{T}]_{\alpha\beta} &= \langle \Phi_\alpha | e^{-(H-\mathcal{E}_0)} | \Phi_\beta \rangle = \langle Q^\dagger_\alpha(R,1) Q_\beta(R,0) \rangle \\
[O]_{\alpha\beta} &= \langle \Phi_\alpha | \Phi_\beta \rangle = \langle Q^\dagger_\alpha(R,0) Q_\beta(R,0) \rangle
\end{aligned}
$$

We obtain the five orthogonal eigenstates of $[\mathcal{T}]_{\alpha\beta}$ in the subspace of Hilbert space spanned by the Φ_α by solving the generalized eigenvalue problem.

$$
[T]_{\alpha\beta} v^{(n)}_\beta = \lambda_n [O]_{\alpha\beta} v^{(n)}_\beta \ ,
$$

with eigenstates,

$$
\Psi_n(R) = \sum_{\alpha=1}^{5} v^{(n)}_\alpha \Phi_\alpha(R)
$$

and ordered such that λ_n decreases with n.

Euclidean time evolution

Consider evolving the states Ψ_n in Euclidean time,

$$
\begin{aligned}
\mathcal{T}_{nn}(R,T) &= \langle \Psi_n | e^{-(H-\mathcal{E}_0)T} | \Psi_n \rangle \\
&= v_\alpha^{*(n)} \langle \Phi_\alpha | e^{-(H-\mathcal{E}_0)T} | \Phi_\beta \rangle v_\beta^{(n)} \\
&= v_\alpha^{*(n)} \langle Q_\alpha^\dagger(R,T) Q_\beta(R,0) \rangle v_\beta^{(n)} \ ,
\end{aligned}
$$

where Latin indices indicate matrix elements with respect to the Ψ_n rather than the Φ_α, and there is a sum over repeated Greek indices.

To calculate this expression, we define timelike $q = 2$ Wilson lines of length T,

$$
P(\mathbf{x},t,T) = U_0^2(\mathbf{x},t) U_0^2(\mathbf{x},t+1) \dots U_0^2(\mathbf{x},t+T-1) \ .
$$

After integrating out the massive fermions, whose worldlines lie along timelike Wilson lines, we have

$$
\langle Q_\alpha^\dagger(R,T) Q_\beta(R,0) \rangle = \langle \mathrm{Tr}[V_\alpha^\dagger(\mathbf{x},\mathbf{y};U(t+T)) P^\dagger(\mathbf{x},t,T) V_\beta(\mathbf{x},\mathbf{y};U(t)) P(\mathbf{y},t,T)] \rangle \ .
$$

Transfer matrix, a sum of exponentials

On general grounds, $\mathcal{T}_{nn}(R,T)$ is a sum of exponentials

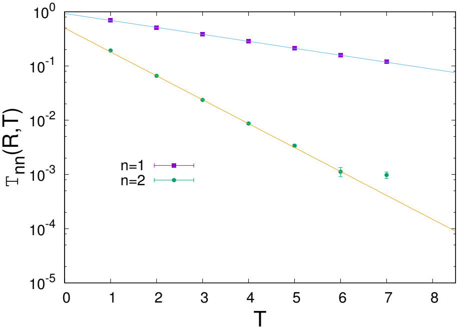
$$
\begin{aligned}
\mathcal{T}_{nn}(R,T) &= \langle \Psi_n(R) | e^{-(H-\mathcal{E}_0)T} | \Psi_n(R) \rangle \\
&= \sum_j |c_j^{(n)}(R)|^2 e^{-E_j(R)T} ,
\end{aligned}
$$

where $c_j^{(n)}(R)$ is the overlap of state $\Psi_n(R)$ with the j-th energy eigenstate of the abelian Higgs theory containing a static fermion-antifermion pair at separation R, and $E_j(R)$ is the corresponding energy eigenvalue minus the vacuum energy.

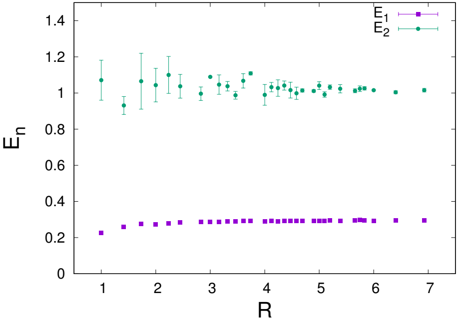
Numerics

I work in the Higgs region at β=3 and γ=0.5, the photon mass is determined from the plaquette-plaquette correlator to be 1.57 in lattice units.

The energies $E_n(R)$ for $n = 1, 2$ are also obtained by fitting the data for $\mathcal{T}_{nn}(R, T)$ vs. T, at each R, to an exponential falloff.



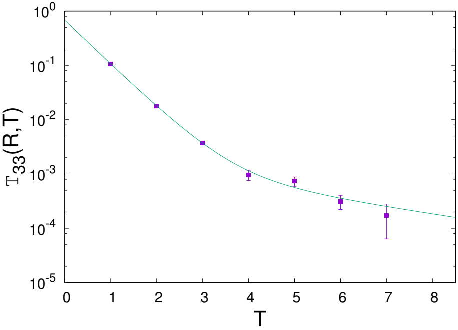
An example of these fits at $R = 6.93$ on a 16^4 lattice with couplings $\beta = 3, \gamma = 0.5$. Fitting through the points at $T = 2 - 5$, with $E_1 = 0.2929(6)$ and $E_2(R) = 1.01(1)$



Energy expectation values $E_n(R)$ vs. R for $n = 1$ and $n = 2$, obtained from a fit to a single exponential. The data and errors were obtained from ten independent runs, each of 77,000 sweeps after thermalization, with data taken every 100 sweeps, computing $\mathcal{T}_{nn}$ from each independent run.

Second stable excited state

...see if there is any indication of a second stable excited state

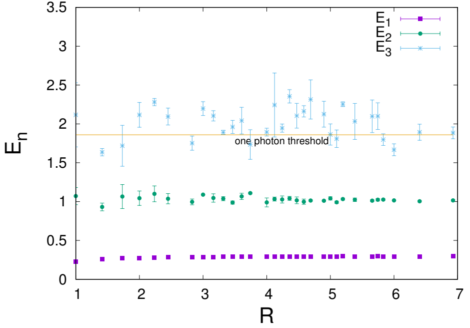


$\mathcal{T}_{33}(R,T)$ vs. T at fixed $R = 6.93$. The fit shown is to the sum of exponentials

$$\mathcal{T}_{33}(R,T) \approx a_1(R)e^{-E_1 T} + a_2(R)e^{-E_2 T} + a_3(R)e^{-E_3 T} \ ,$$

where $E_1 = 0.29, E_2 = 1.02$ are taken from the previous fits. A sample fit, again at $R = 6.93$, is shown. Obviously one cannot be very impressed by a four parameter fit through a handful of data points. A sample fit, again at $R = 6.93$ is shown.

the values of E_1, E_2, E_3, together with the one photon threshold



The one photon threshold is simply $E_1 + m_{photon} = 0.29 + 1.57(1) = 1.86(1)$ in lattice units. The important observation is that $E_2(R)$ lies well below this threshold, which implies that the first excited state of the static fermion-antifermion pair is stable. The second point to note is that $E_3(R)$ seems to lie above or near the one photon threshold. The indications are that there is no second stable excited state. States above the first excited state most likely lie above the threshold, and are probably combinations of the ground state plus a massive photon.

Conclusions

We have presented lattice Monte Carlo evidence for the existence of a stable excitation of the quantized fields surrounding isolated static charges, in the Higgs phase of the $q = 2$ abelian Higgs model in $D = 4$ spacetime dimensions:

Some obvious next questions,

1. excitations of the type seen in the abelian Higgs model would also be found in non-relativistic models of that kind (application to SC theory)
2. how they might be observed experimentally in a real superconductor?
3. whether heavy fermions (or even light fermions) have a spectrum of excitations in the electroweak sector of the Standard Model? (a further question)

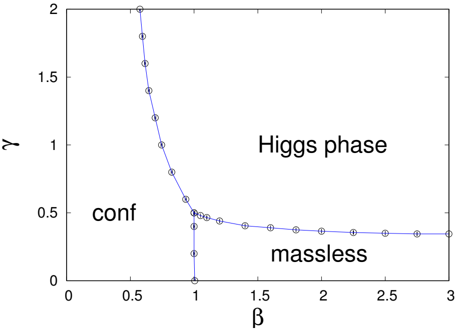
EXTRA

SLIDES

Numerical Results:

Where do we start looking for such excitation spectrum in the phase diagram of $q = 2$ Abelian Higgs theory?

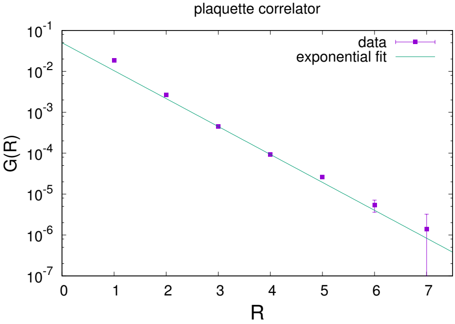
We are interested in determining $E_n(R)$ in the Higgs phase and, because the calculation involves fitting exponential decay, we would like both the mass of the photon and the energies $E_n(R)$ to be not much larger than unity in lattice units. For this reason we choose to work at the edge of the phase diagram shown just above the massless-to-Higgs transition line at $\beta = 3, \gamma = 0.5$.



We compute the photon mass from the gauge invariant on-axis plaquette-plaquette correlator with the same $\mu\ \nu$ orientation

$$G(R) = \left\langle \mathrm{Im}[U_\mu(x)U_\nu(x+\hat{\mu})U^*_\mu(x+\hat{\nu})U^*_\nu(x)] \times \mathrm{Im}[U_\mu(y)U_\nu(y+\hat{\mu})U^*_\mu(y+\hat{\nu})U^*_\nu(y)] \right\rangle ,$$

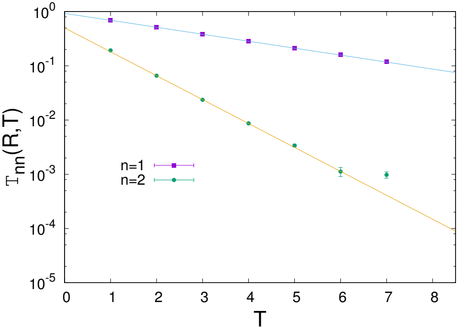
where $y = x + R\hat{k}$, and $\hat{k}$ is a unit vector orthogonal to the $\hat{\mu}, \hat{\nu}$ directions.
The result for the $\beta = 3, \gamma = 0.5$ parameters is

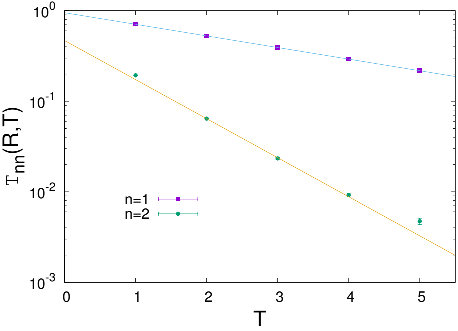


From an exponential fit, disregarding the initial points, we find a photon mass of $m_\gamma = 1.57(1)$ in lattice units. Data was obtained on a 16^4 lattice with 1,600,000 sweeps and data taken every 100 sweeps. We have checked that if the calculation is done just below the transition, in the massless phase, then $G(R)$ is fit quite well by a $1/R^4$ falloff, as expected.

Any finite size effects?

To check a finite size effect we can make the same computation, with the same number of sweeps, only on a 12^4 lattice.



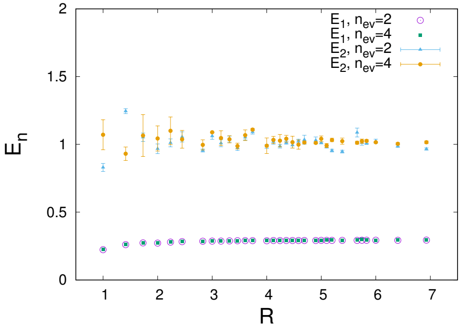


An example of these fits at $R = 6.93$ on a 16^4 lattice with couplings $\beta = 3, \gamma = 0.5$. Fitting through the points at $T = 2 - 5$, with $E_1 = 0.2929(6)$ and $E_2(R) = 1.01(1)$

Fitting result at $R = 6.93$ on a 12^4 through the points $T = 2 - 4$ yields $E_2(R) = 0.99(2)$. At $R = 5$ point lies a little above the straight line fit, and again this effect is seen at all R indicating that the deviation of the last data point from the fit to the other points is probably a finite size effect.

E_1, E_2 obtained using $n_{ev} = 2$ and $n_{ev} = 4$ Laplacian eigenstates.

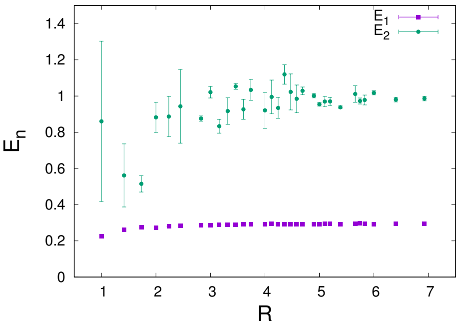
compare E_1 and E_2 values obtained from using $n_{ev} = 2$ Laplacian eigenstates with the values obtained using $n_{ev} = 4$ Laplacian eigenstates,

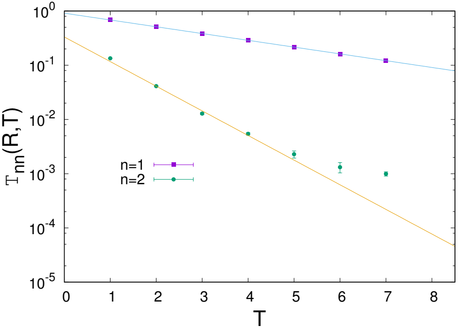


There is not much difference in the E_2 values, at least for $R \geq 3$, and the E_1 values cannot even be distinguished in the plot.

some more details of a finite size effect

the numerical solution by the standard Matlab **eig** routine (ultimately derived from LAPACK) shows a small $O(10^{-3})$ but non-negligible deviation from this orthogonality condition, $\langle \Psi_i | \Psi_j \rangle = \delta_{ij}$.



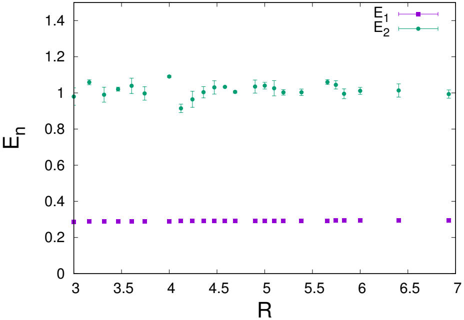


Energy expectation values of 16^4 lattice. E_2 is obtained from fits to data points in the range $T = 3 - 6$, rather than $T = 2 - 5$. the values of E_2 obtained from a fit in the $T = 3 - 6$ interval. In general the E_2 values cluster around $E_2 = 1$, as in the previous fit. But there are large error bars for some of the points, especially at the lower R values, and significant deviations from $E_2 \approx 1$

$T_{22}(R)$ vs. T at $R = 3.16$. The reason for a discrepancy between a fit of data points at $T = 2 - 5$, and $T = 3 - 6$ is apparent. both the last data points for $\mathcal{T}_{22}$ at $T = 7$ *and* the next-to-last data point at $T = 6$ deviate very significantly from the fit in the $T = 2 - 5$ range. We are inclined to attribute both deviations to finite size effects, which seem especially apparent at lower R.

Energy expectation values E_1, E_2 vs. R at $R > 3$, obtained on a 12^4 lattice.

...E_1, E_2 obtained on a 12^4 lattice volume results,



As in the larger volume, the data for E_2 clusters around $E_2 \approx 1$, albeit with a few outliers. These values, however, are obtained from a fit through only three data points at $T = 2, 3, 4$, and also the χ^2 values of these fits tend to be significantly larger than unity, indicating a possible underestimate of the error bars.