Monopole-like configurations in the O(3) spin model at the upper critical dimension

based on: *Panero, Smecca, J. High Energ. Phys. 2021, 231 (2021), [2012.12221]*

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The O(3) Model

Lattice model of interacting vector spin with global O(N) symmetry can be described by the following Hamiltonian :

$$\mathcal{H} = -J \sum_{\langle x,y \rangle} \mathbf{s}(x) \cdot \mathbf{s}(y), \tag{1}$$

Among the particular cases for this Hamiltonian, there are the Ising model, for $N = 1$, the XY model for $N = 2$ and the classical Heisenberg model, for $N = 3$, which is the one presented in this talk.

The classical Heisenberg model experience a phase transition as temperature increases, consequently we can define two phases:

- low-temperature, ordered, broken symmetry phase
- high-temperature, disordered, symmetric phase

Our work concerns the study of the Heisenberg model on the lattice at the upper critical dimension $D = 4$.

In this regime, the critical exponents at the phase transition that separates the low-temperature phase from the disordered high temperature one, are equal to their mean-field values ($\langle \mathbf{s}_i(x_i) \rangle$), up to logarithmic corrections.

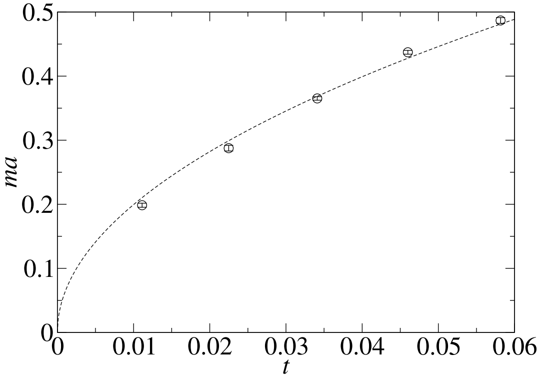
Computation Setup

- We work in a four-dimensional, isotropic, hypercubic lattice of spacing a. We denote the spatial extent of the system with L and the temporal extent as R.
- We denote T as the system temperature, and introduce the reduced temperature: $t = (T - T_c)/T_c$. T_c is the critical temperature. We use the critical temperature determined by J.-P. Lv *et al.* [J.-P. Lv, et al., National Science Review(08, 2020), [1909.10347]]
- In the high temperature phase, periodic boundary conditions are assumed. Conversely in the low-temperature phase we impose boundary conditions enforcing the existence of a "monopole-like" spin configuration
- In our Monte Carlo simulations, Markov chains of vector-field configurations are generated by a combination of local heat bath and overrelaxation updates

High -Temperature Phase ($T > T_c$)

Our results in the High-Temperature symmetric phase confirm the work done by J.-P. Lv *et al.* In particular, our results close to the critical temperature T_c indicate that the the mass of the lightest physical state that propagates in the theory vanishes in the thermodynamic limit.

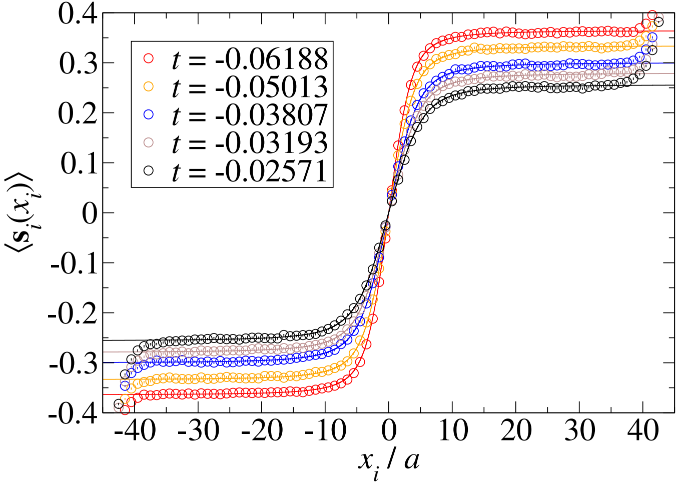
$$am = \frac{mt^{\nu}}{\Lambda_+} \qquad (2)$$



Broken symmetry phase ($T < T_c$)

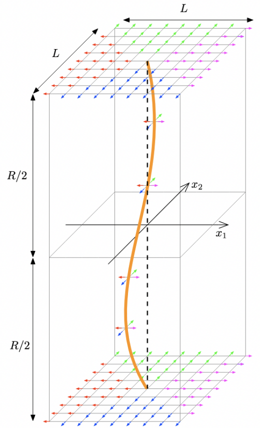
- Our result in the low-temperature phase follow closely the analytical results predicted by Delfino in his work: [G. Delfino, J. Phys. A47(2014) 132001, [1401.2041].], where he studies spin models through field theory arguments.
- For the first time, the analytical predictions worked out by Delfino are quantitatively confirmed in 4 Euclidean dimensions. The agreement between the analytic curve and the numerical results is excellent.
- All fits are done in the range $-25 < x_i/a < 25$ to avoid systematic effects due to the boundaries of the system.

$$\langle \mathbf{s}_i(x_i) \rangle = v \left[\left(1 - \frac{1}{2z^2} \right) \mathrm{erf}(z) + \frac{\exp\left(-z^2\right)}{\sqrt{\pi} z} \right] \; ; \; z = x_i \sqrt{\frac{2M}{R}} \qquad (3)$$



Derrick's Theorem

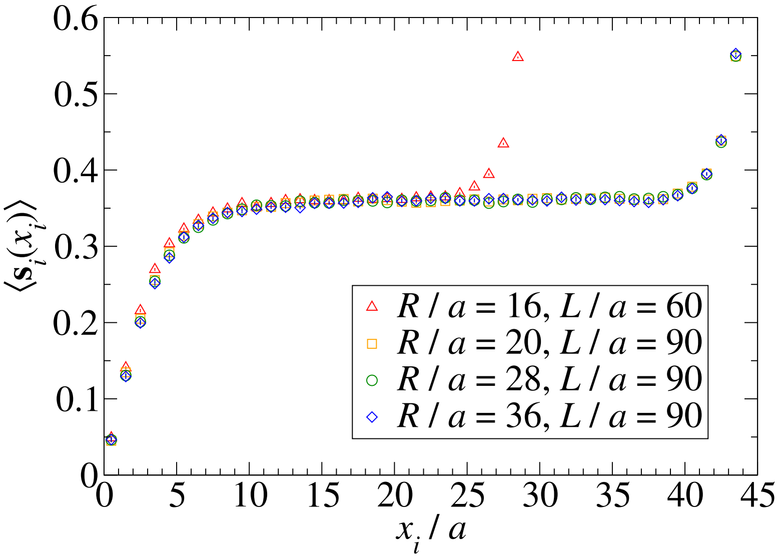
- It states that non-trivial, static, regular, soliton-like configurations in scalar theory cannot exist in more than 2 spacetime dimensions, as their energy is unstable under scale transformations
- Recent work on the XY model (O(2)) in 3-dimensions pointed out the possibility of violating Derrick's Theorem in scalar QFT as the existence of a well-defined mass for the vortex in the continuum limit is found. [Delfino, Selke, Squarcini, Phys. Rev. Lett.122(2019) 050602, [1808.09276]]



DSS, Phys. Rev. Lett.122(2019) 050602, [1808.09276]

- This possibility can be investigated in our numerical analysis in 4-dimensions by studying the scaling of the fitted values of Ma as a function of the parameters of the theory L, R and t.
- According to the interpretation of the topological field configuration as a particle excitation, M should be interpreted as the mass of the particle, hence, independent from R.
- Our fit results, however, indicate that the parameter Ma scales approximately proportionally to R. This fact questions the interpretation of the topological excitation as a physical particle, as shown by the plots in the following slides:

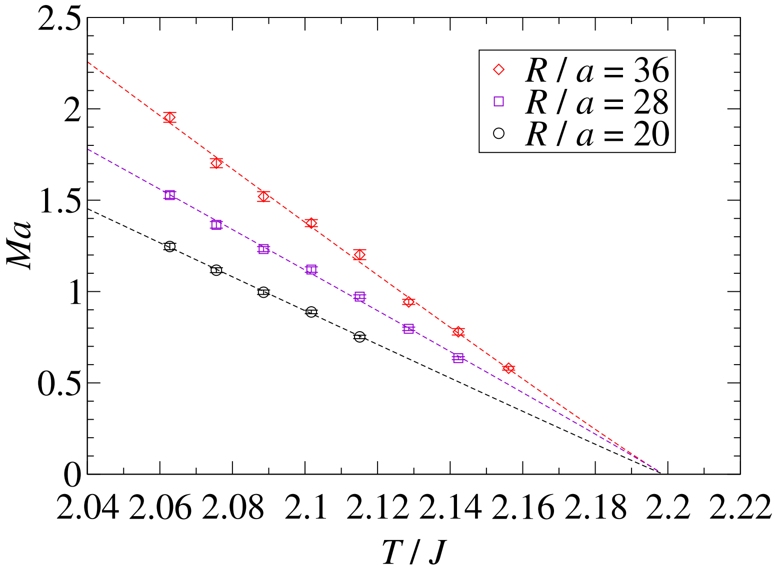
Scaling of $\langle \mathbf{s}_i(x_i) \rangle$



- All data for the spin profile collapse onto the same curve, it follows that $M \propto R$, or, equivalently, that the topological excitation is characterised by an approximately constant $\mu = M/R$
- If μ should be thought of as a physical quantity, then simulations at fixed R but different t should show aM scaling like: $a\mu R = aM \propto |t|^{2\nu}$
- Fitting our data to the function $aM = A_M \left(1 - \frac{T}{T_c}\right)^E$ we find results for E very close to 1, hence twice the value of ν predicted in the Gaussian approximation.

R/a	A_M	E	χ^2_{red}
20	21.3(1.5)	1.021(23)	0.50
28	23.4(1.8)	0.981(24)	2.34
36	34.6(2.3)	1.039(21)	1.76

Scaling of *Ma*



Potential Applications

- Alternatives that violates Derrick's Theorem do exist but they are characterised by an intrinsic cutoff scale. Some examples in condensed matter physics exist such as experimental studies on Manganese Germanide [Phys. Rev. Lett. 125 (2020) 137202.]
- Another area where our study on monopole-like objects might be useful is synthetic magnetism which is one of the possible routes to Quantum Simulation [Nature 462 (2009) 628-632, [1007.0294]]

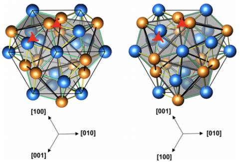


Figure: Manganese Germanide [Sci Rep 10, 4065 (2020)]

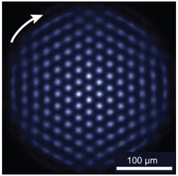


Figure: Quantum Simulator Crystal[Britton, NIST, Nature 484, 489-492 (2012)]

Conclusions

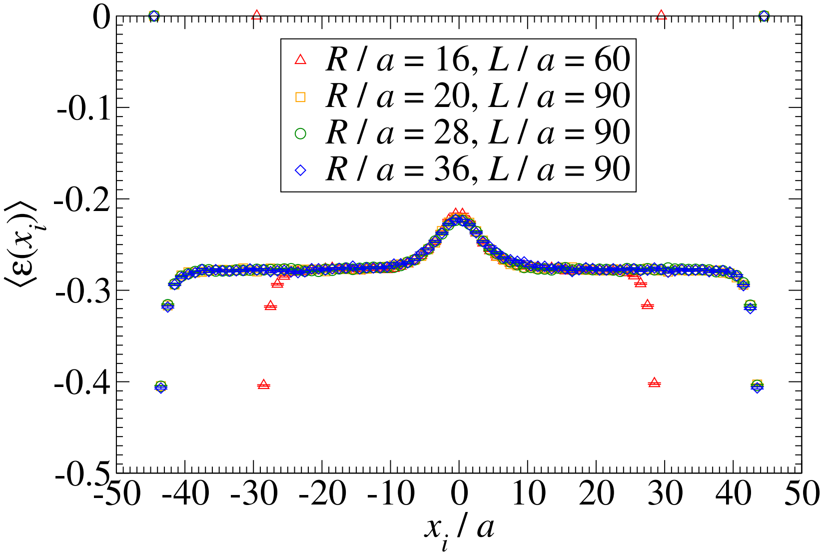
- We studied the Heisenberg model in four spacetime dimensions, confirming numerical results in the high-temperature phase
- In this work, for the first time, we have a numerical confirmation of the Field Theoretical analytic prediction worked out by Delfino in his paper
- Our findings might see practical applications in the areas of condensed matter physics and quantum simulations
- Future studies might include studying the system in an out of equilibrium setting, in order to study the non-equilibrium dynamics of the monopole-like excitation

Thank You for your Attention

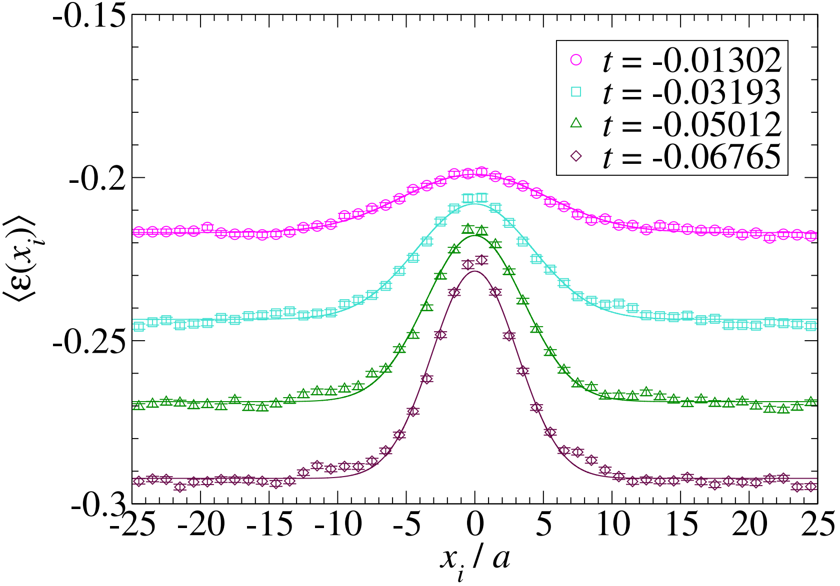
Any Questions?

Backup Slides I

- At any fixed temperature $T < T_c$, in the infinite volume limit, $\varepsilon(x_i)$ is expected to tend to to a spatially uniform value C at long distances.
- Like for the spin profile, we observe that our numerical results at the same T but different R collapse on the same curve.
- Also, the numerical results exhibit the expected qualitative features, for instance the peak at the centre of the systems becomes larger as $T \to T_C^-$, and are in excellent agreement with the theoretical model.



0
-0.1
-0.2
-0.3
-0.4
-0.5
-50 -40 -30 -20 -10 0 10 20 30 40 50
$\langle \varepsilon(x_i) \rangle$
x_i / a
$R/a = 16, L/a = 60$
$R/a = 20, L/a = 90$
$R/a = 28, L/a = 90$
$R/a = 36, L/a = 90$



-0.15
-0.2
-0.25
-0.3
-25 -20 -15 -10 -5 0 5 10 15 20 25
$\langle \varepsilon(x_i) \rangle$
x_i / a
t = -0.01302
t = -0.03193
t = -0.05012
t = -0.06765