Tensor renormalization group approach to (1+1)-dimensional Hubbard model

Shinichiro Akiyama (Univ. of Tsukuba)
in collaboration with
Yoshinobu Kuramashi

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Introduction

- ✔ The Hubbard model at finite density is an important toy model to understand the physics of strongly-correlated electrons, but it is very challenging to investigate numerically the model away from the half-hilling due to the sign problem

- ✔ The TRG approach is expected to be useful to investigate the Hubbard model because **the TRG provides us a deterministic (not a stochastic) methodology, which is free from the sign problem**

- ✔ TRG is very good at dealing with interacting fermions in the path-integral formalism because **the TRG can directly evaluate the Grassmann path integral itself without introducing any auxiliary scalar field**

Hubbard model in the path-integral formalism

✔ Action of the $(d+1)$-dimensional Hubbard model

$$S = \sum_n \epsilon \left[\bar{\psi}(n) \left(\frac{\psi(n+\hat{\tau}) - \psi(n)}{\epsilon} \right) - t \sum_{\sigma=1,\cdots,d} \left(\bar{\psi}(n+\hat{\sigma})\psi(n) + \bar{\psi}(n)\psi(n+\hat{\sigma}) \right) \right]$$
$$+ \sum_n \epsilon \left[\frac{U}{2} \left(\bar{\psi}(n)\psi(n) \right)^2 - \mu \bar{\psi}(n)\psi(n) \right]$$

✔ fermions are described by two-component Grassmann numbers

$$\psi(n) = \begin{pmatrix} \psi_\uparrow(n) \\ \psi_\downarrow(n) \end{pmatrix} \qquad \bar{\psi}(n) = \left(\bar{\psi}_\uparrow(n), \bar{\psi}_\downarrow(n) \right)$$

✔ **The action of the Hubbard model is quite similar with that of the NJL model**

S. A. et al., JHEP01(2021)121

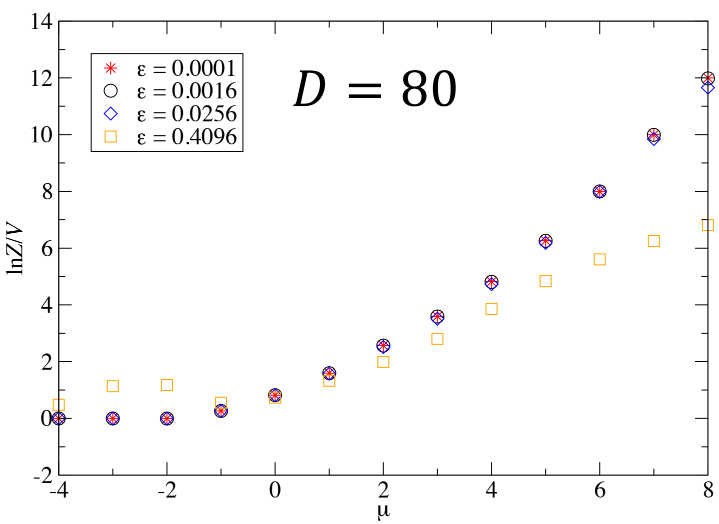
✔ We start from the (1+1)D case, exactly solved by the Bethe ansatz, to discuss the efficiency of the TRG approach to the (d+1)D Hubbard model w/ d>1

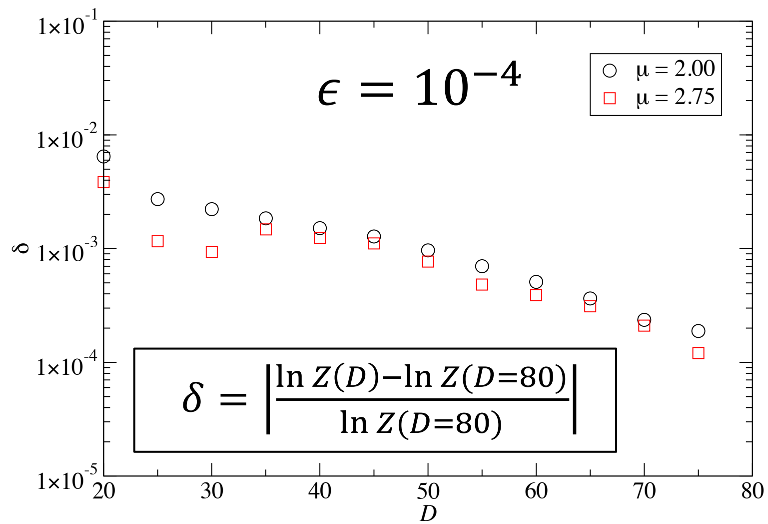
Algorithmic parameter dependence

with $U/t = 4, N_\tau = 2^{24}, N_\sigma = 2^{12}, D \leq 80$

✓ Calculation by the Grassmann HOTRG
(whose formulation is based on SA-Kadoh, arXiv:200507570 [hep-lat])

✓ Imaginary time evolution + Space-time coarse graining
(a similar treatment to previous HOTRG study of (2+1)D quantum Ising model in Xie et al, PRB86(2012)045139)





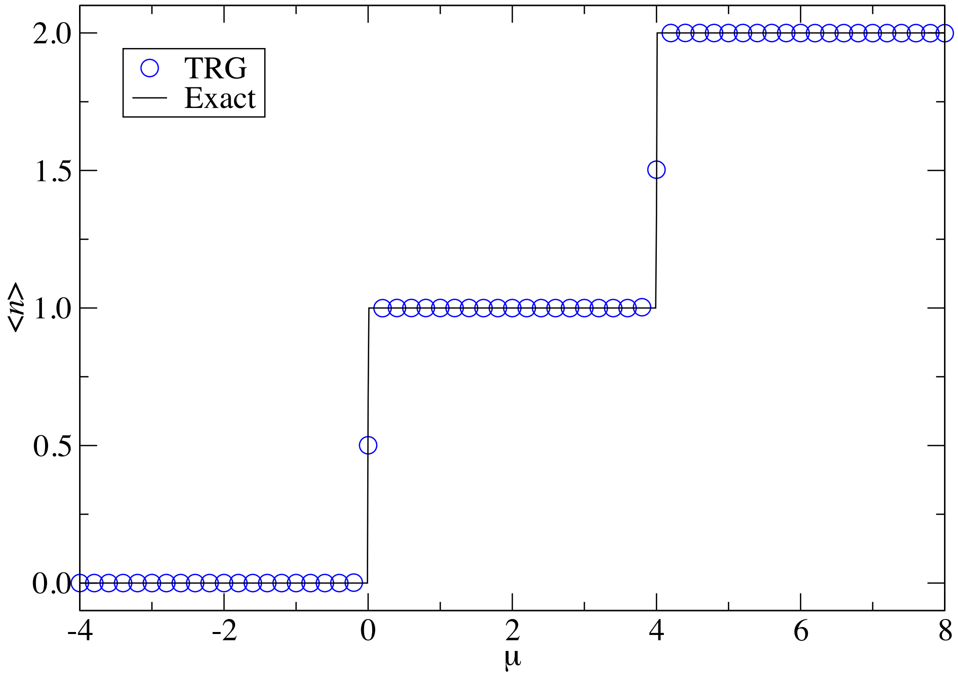
$\epsilon = 10^{-4}$ is sufficiently small

Scaling w. r. t. D is discussed later

The model with $t = 0$ (one-site model)

with $U = 4, \epsilon = 10^{-4}, N_\tau = 2^{24}$

-> No hopping structure in the spatial direction

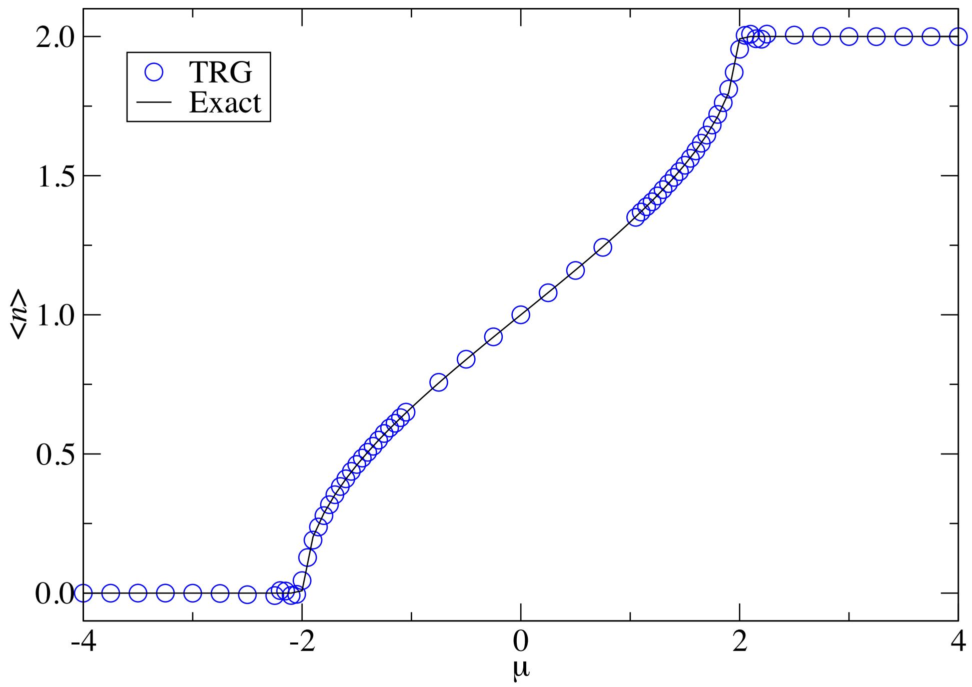


Imaginary time evolution restores the exact results

The model with $U = 0$ (free fermion model)

with $t = 1, \epsilon = 10^{-4}, N_\tau = 2^{24}, N_\sigma = 2^{12}, D = 80$

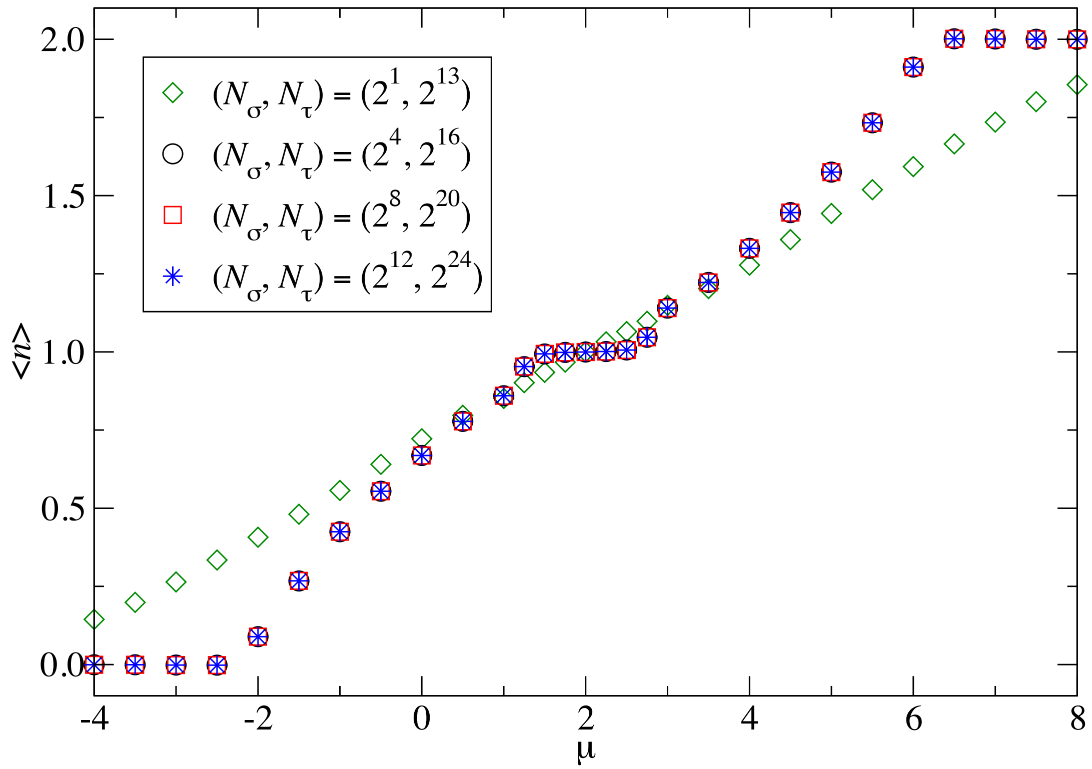
-> TRG approximately evaluates the tensor network contraction



The current approach (= imaginary time evolution + space-time coarse graining) restores the free-field solution

Number density with $U/t = 4$

with $\epsilon = 10^{-4}, D = 80$

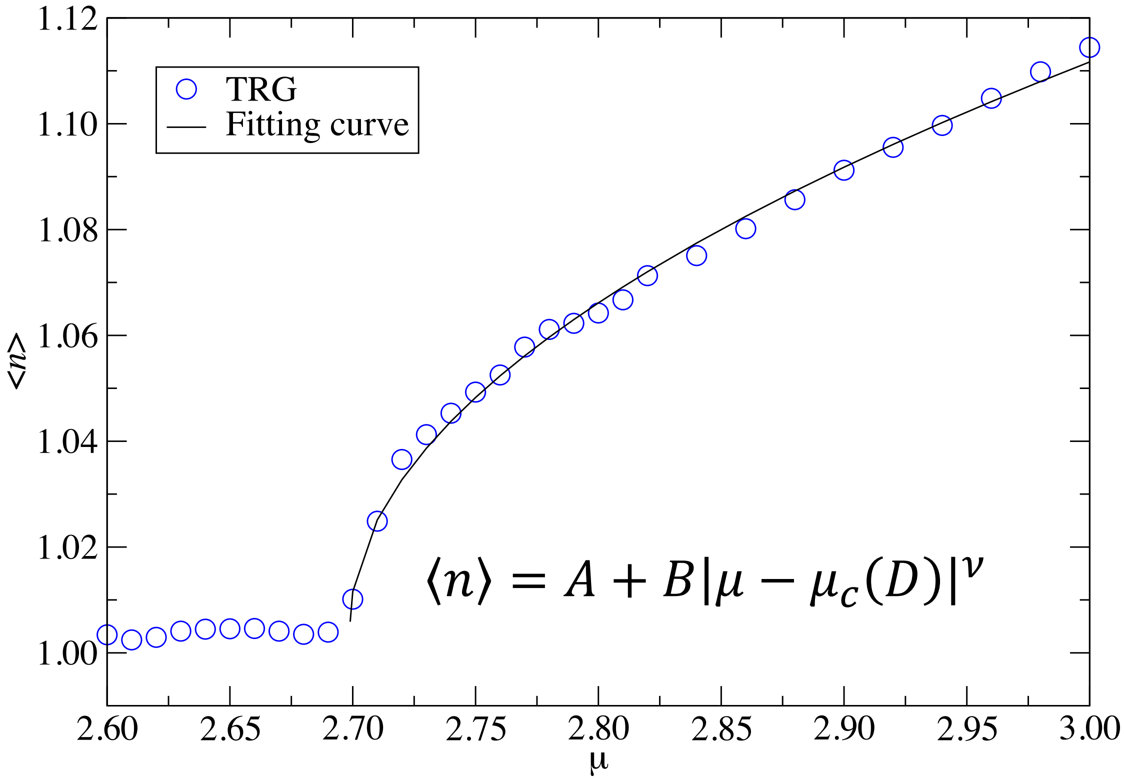


The TRG allows us to investigate the system away from the half filling ($\mu = 2$)

Characteristic feature of the metal-insulator transition is captured by the TRG

Fitting of the number density

with $U/t = 4, \epsilon = 10^{-4}, D = 80$



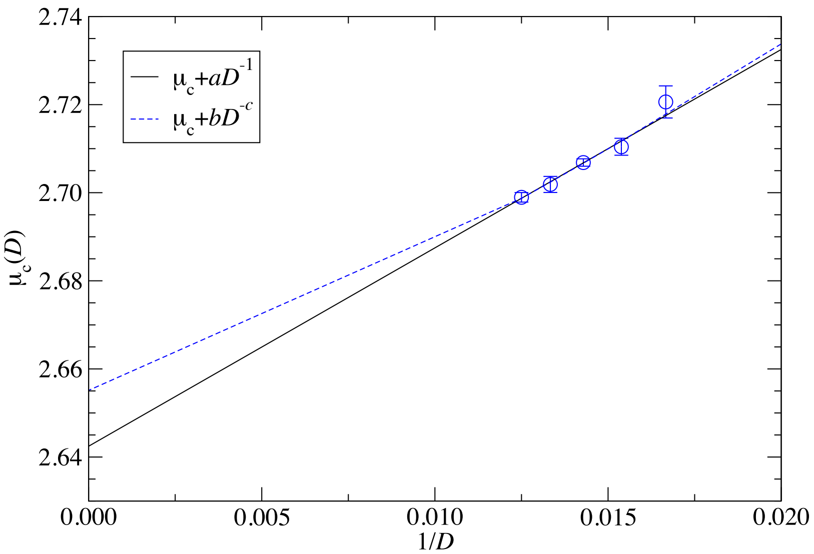
At $D = 80$, we obtain $\mu_c(D) = 2.698(1)$ and $\nu = 0.51(2)$

Exact solutions -> $\mu_c = 2.643 \ldots$ and $\nu = 0.5$

$D \to \infty$ limit

with $U/t = 4, \epsilon = 10^{-4}, D \leq 80$

D	60	65	70	75	80	∞
fit range	[2.72,3.00]	[2.70,3.00]	[2.70,3.00]	[2.69,3.00]	[2.68,3.00]	–
$\mu_c(D)$	2.720(3)	2.710(1)	2.7068(8)	2.701(1)	2.698(1)	2.642(05)(13)
ν	0.49(3)	0.52(1)	0.50(2)	0.51(2)	0.51(2)	–



$\mu_c(D \to \infty) = 2.642(05)(13)$ and ν are consistent with the exact values, $\mu_c = 2.643\ldots$ and $\nu = 0.5$

Summary

- No difficulty to apply the TRG approach to fermions on a lattice

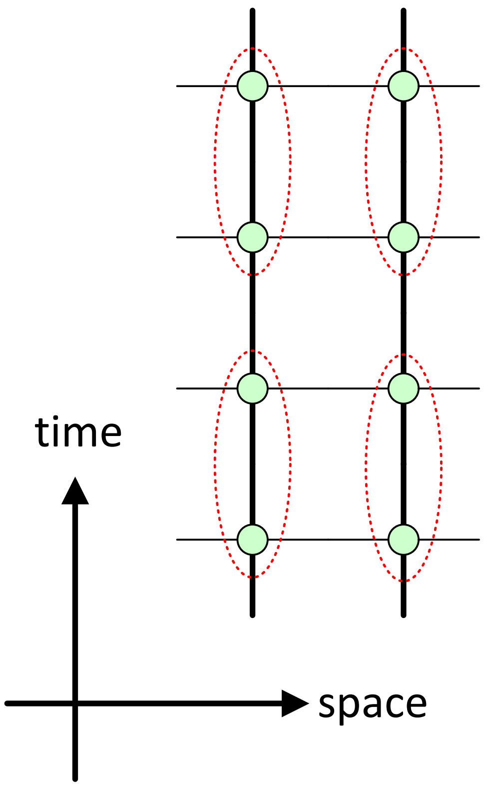
- (1+1)D Hubbard model is investigated within the path-integral formalism and the numerical results are consistent with the exact values **even away from half filling**

- The Grassmann ATRG algorithm is expected to be useful to investigate the higher-dimensional Hubbard model (in progress)

Appendices

Imaginary time evolution + space-time coarse-graining

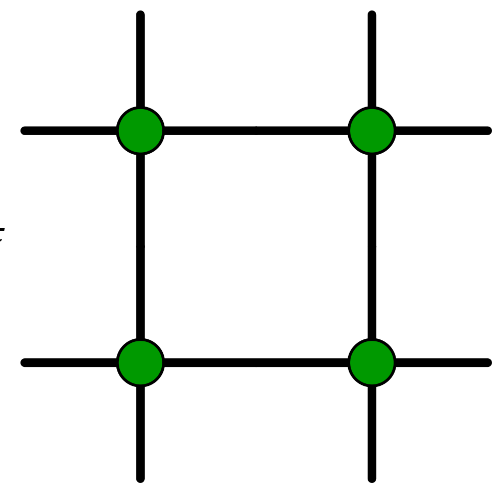
✔ We have different hopping structures in temporal and spatial directions
-> 2D tensor network is constructed by anisotropic local Grassmann tensors



① Imaginary time evolution
by the HOTRG in advance

$\beta \sim \epsilon$ ⇒ $\beta \sim \epsilon 2^{m_\tau}$

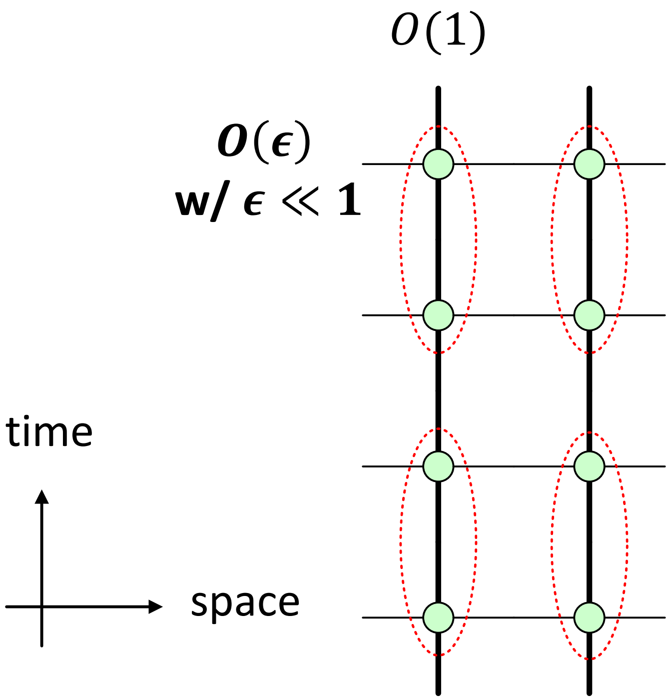
We choose m_τ and ϵ
via the condition $\epsilon 2^{m_\tau} \sim O(10^{-1})$



② Space-time coarse-graining
by the HOTRG

Imaginary time evolution + space-time coarse-graining

$$S = \sum_n \left[\bar{\psi}(n)\big(\psi(n+\hat{\tau}) - \psi(n)\big) - \boldsymbol{t\epsilon} \sum_{\boldsymbol{\sigma=1,\cdots,d}} \left(\bar{\boldsymbol{\psi}}(\boldsymbol{n}+\hat{\boldsymbol{\sigma}})\boldsymbol{\psi}(\boldsymbol{n}) + \bar{\boldsymbol{\psi}}(\boldsymbol{n})\boldsymbol{\psi}(\boldsymbol{n}+\hat{\boldsymbol{\sigma}}) \right) \right] + \sum_n \epsilon \left[\frac{U}{2}\big(\bar{\psi}(n)\psi(n)\big)^2 - \mu\bar{\psi}(n)\psi(n) \right]$$



✔ With sufficiently small ϵ, one has almost vanishing spatial interaction compared with the temporal interaction

✔ Temporal Decimation by the HOSVD works much better than spatial decimation