Large N simulation of the twisted reduced matrix model with an adjoint Majorana fermion

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in collaboration with

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Related Talk by Butti @7/27,6:30(EDT)

Contents

1. Introduction

- 4D $\mathcal{N}=1$ SUSY YM theory
  - common features with YM theory:
    confinment, chiral condensate etc.
- Large N limit of $\mathcal{N}=1$ SUSY YM theory
  - simplification by a planer limit, factorization ..
  - has a relation to AdS/CFT ($\mathcal{N}$=4 SYM)?
  - lattice version: volume reduction to a matrix model

't Hooft (1974), Witten (1980)

Eguchi-Kawai (1982)
Bhanot-Heller-Neuberger (1982)
Gross-Kitazawa (1982)
Gonzalez-Arroyo-Okawa(1983)

We focus on

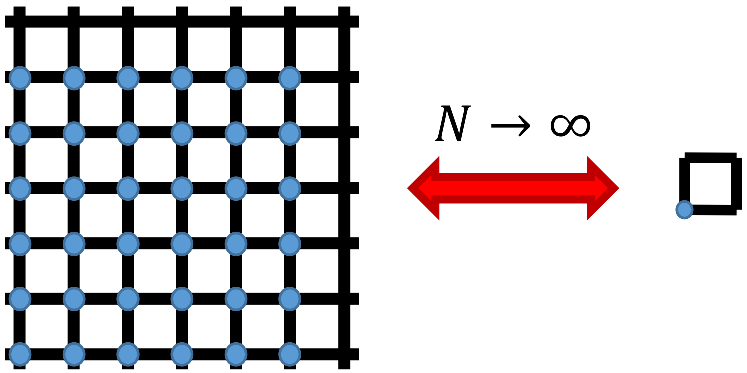
Twisted reduced matrix model with one adjoint Majorana fermion

toward the 1st step: check the SUSY spectrum, SUSY W.-T. identity ....

In this talk: Simulation algorithm, status
and low eigenvalues of D_W

Works on SU(3),SU(2) lattice SYM:
Steinhauser et al, JHEP(2021)
Ali et al., EPJC80(2020),Latt19,PRL12(2019)...
Bergner et al.,Latt18,...
Demmouche et al., EPJC69(2010), Latt09...
Farchioni et al, EPJC(2002),...
Fleming et al. PRD64(2001), Giedt et al, PRD79(2009)
Campos et al. EPJC(1999)

2. Twisted reduced matrix model with an adjoint Majorana fermion



- Based on the SU(N) Lattice Wilson gauge action
  $+\ N_f = 1/2$ Wilson-Dirac fermion in adjoint rep.

  Lattice Wilson action : Montvay (1999)

- Twisted gauge boundary condition

- To take the large N limit, we utilize the volume independence property of the theory. We also expect the automatic SUSY recovery in the chiral limit.

- Partition Function :
$$Z = \int \prod_{\mu=1}^{4} dU_\mu \, \mathrm{Pf}(CD_W) \mathrm{e}^{-S_G[U]}$$
U_μ: SU(N) matrix. four matrices.

$$S_G[U] = bN \sum_{\mu,\nu=1,\mu\neq\nu}^{4} \mathrm{Tr}\left[I - z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right]$$

$$z_{\mu\nu} = \exp\left[\frac{2\pi i k}{\sqrt{N}} \epsilon_{\mu\nu}\right]$$: Twist phase

Twisted E.-K. Wilson action

$$D_W = I - \kappa_{\mathrm{adj}} \sum_{\mu=1,2,3,4} \left[(1-\gamma_\mu)V_\mu + (1+\gamma_\mu)V_\mu^T\right]$$

$C = \gamma_4 \gamma_2$: Charge Conjug.

Wilson-Dirac action with adjoint link

$\mathrm{Pf}(M)$: Pfaffian for skew symmetric matrices

$$M \equiv CD_W, \qquad M^T = -M,$$

3. Simulation method

RHMC: Horvath-Kennedy-Sint-Clark

- In order to incorporate the Pfaffian, we apply the RHMC algorithm to the following partition function.

Ref: Montvay, IJMP A 17(2002)2377

$$Z_{RHMC} = \int \prod_{\mu=1}^{4} dU_\mu \, |\det(Q_W Q_W)|^{1/4} e^{-S_G[U]} \qquad Q_W \equiv D_W \gamma_5$$

$$Z = \int \prod_{\mu=1}^{4} dU_\mu \, \mathrm{sign}(\mathrm{Pf}(CD_W)) |\det(Q_W Q_W)|^{1/4} e^{-S_G[U]}$$

$$\langle O \rangle = \frac{\langle O \, \mathrm{sign}(\mathrm{Pf}(CD_W)) \rangle_{RHMC}}{\langle \mathrm{sign}(\mathrm{Pf}(CD_W)) \rangle_{RHMC}}$$

$$\mathrm{Pf}(CD_W) = \mathrm{sign}(\mathrm{Pf}(CD_W)) \, |\mathrm{Pf}(CD_W)| = \mathrm{sign}(\mathrm{Pf}(CD_W)) |\det(Q_W Q_W)|^{1/4}$$

- The sign of Pfaffian is incorporated as observables.
  - by counting the number of negative real eigenvalues of D_W

Bergner et al., arXiv:1111.3012 (Latt 2011)
Bergner-Wuilloud, CPC 183(2012)299

- The weight $|\det(Q_W Q_W)|^{1/4}$ is evaluated by the following action:

$$|\det(Q_W Q_W)|^{1/4} = \int d\phi d\phi^\dagger \, e^{-S_Q} \qquad S_Q = \mathrm{Tr}\left[\phi^\dagger R_N^{(-1/4)}(Q_W^2)\phi\right] = \mathrm{Tr}\left[\left|R_M^{(-1/8)}(Q_W^2)\phi\right|^2\right]$$

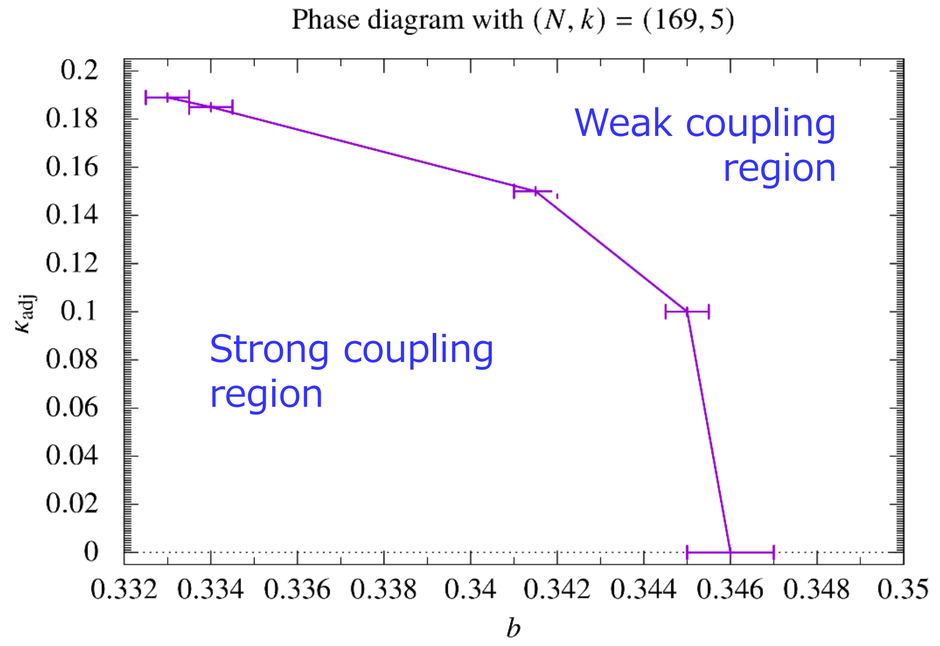
To control the accuracy of the approximation, we monitor the lowest and highest eigenvalues of Q_W^2 in every HMC steps.

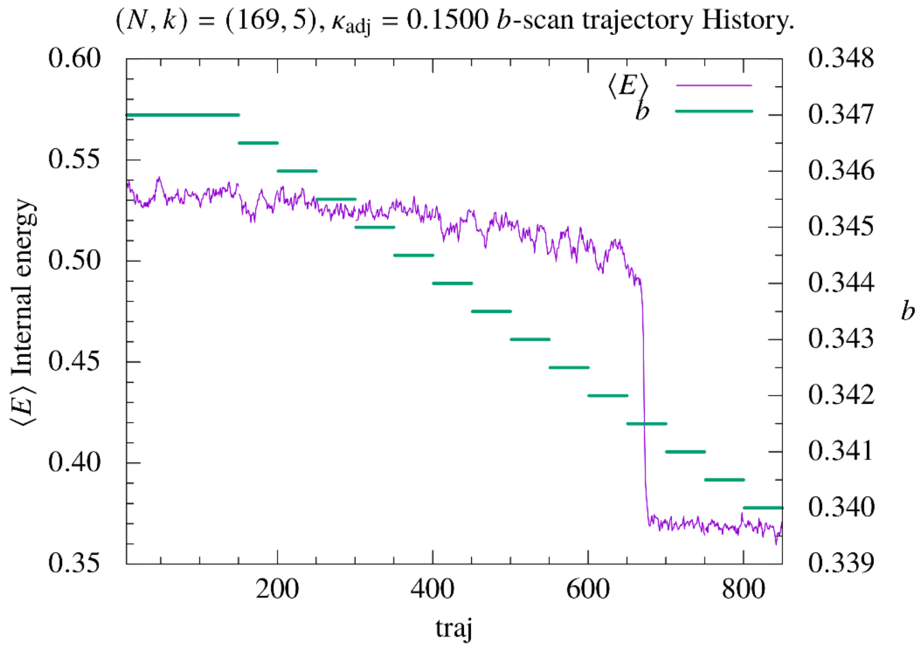
$$x^p \simeq R_N^{(p)}(x) \equiv \alpha_0^{(p)} + \sum_{j=1}^{N} \frac{\alpha_j^{(p)}}{x - \beta_j^{(p)}}$$

Rational approximation coefficients are determined with Remez alg.

4. Sign of Pfaffian and lowest eigenvalues of D_W and Q_W

- Parameter survey to avoid the 1st order phase transition
  - The model without fermions has a 1st order phase transition in a mid coupling region separating strong-weak coupling region. With dynamical fermions, this phase transition still remain. To make valid continuum limit, we have to simulate in weak coupling region. We first roughly survey the parameter region in (b, κ_{adj}) space.





We generated configurations at several parameters in the weak coupling region.
We also maintain parameters suitable for large N limit, chiral limit, and continuum limit.

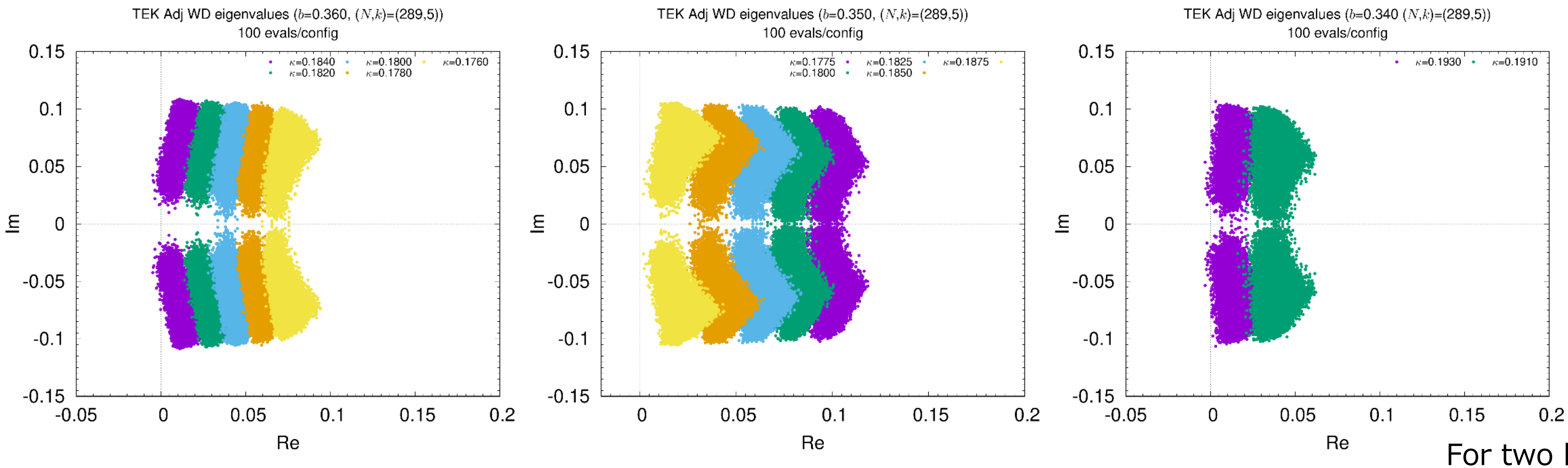
- Parameters and statistics
  - Three N's, three b's, and several κ_{adj}'s

b	(N, k)	κ_{adj}	Statistics
0.360	(289,5)	0.1500-0.1750, 0.1760,0.1780,0.1800,0.1820,0.1840	500, ≥600
0.350	(169,5)	0.1775,0.1800,0.1825,0.1850,0.1875	≥600
	(289,5)	0.1500-0.1750, 0.1775,0.1800,0.1825,0.1850,0.1875	500, ≥600
	(361,7)	0.1775,0.1800,0.1825,0.1850,0.1875	≥600
0.340	(169,5)	0.1850,0.1875,0.1890,0.1910,0.1930	≥600
	(289,5)	0.1850,0.1875,0.1890,0.1910,0.1930	≥600
	(361,7)	0.1850,0.1875,0.1890,0.1910,0.1930	≥600

Resources:
- Osaka U. : SX-ACE (NEC vector machine)
- U.Tokyo : Oakbridge-CX (Intel Xeon cluster)
- Kyushu U.: Ito system-B (GPU Nvidia V100 cluster)

- Sign of Pfaffian
  - We compute low eigenvalues of D_W on each configuration using ARPACK.
  - There are no negative real eigenvalues. The sign is positive for all simulation parameters.

(N,k)=(289,5) b=0.360,0.350,0.340 series



For two lighter κ_{adj}

- Sign of Pfaffian
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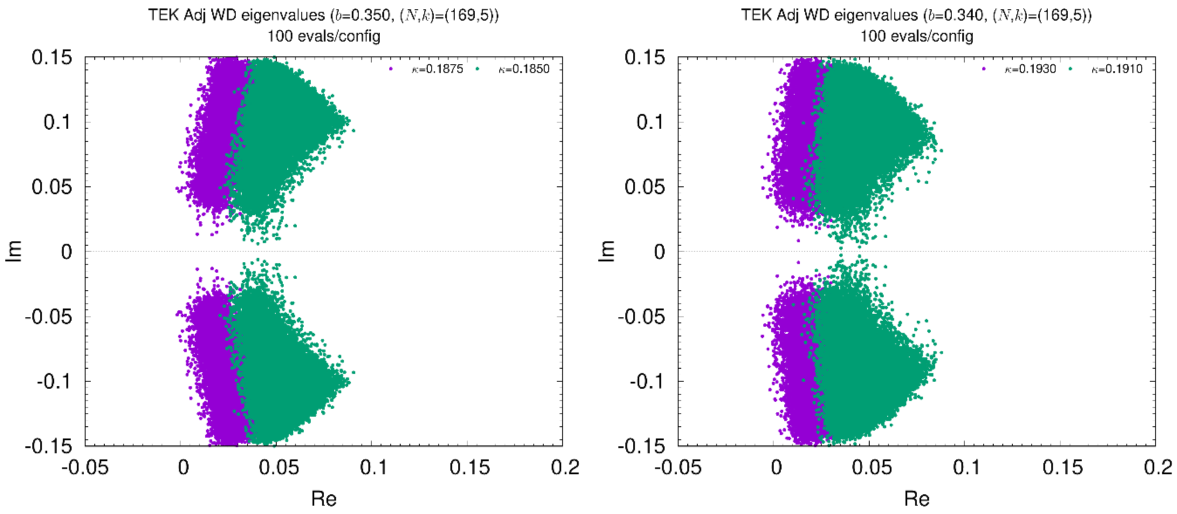
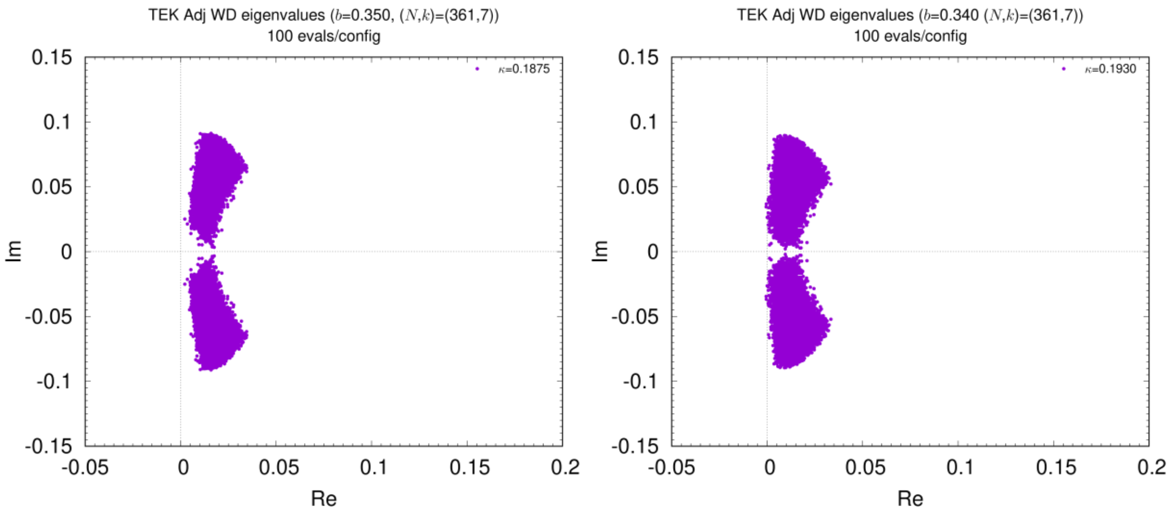
(N,k)=(169,5) b=0.350,0.340 series



For two lighter κ_{adj}

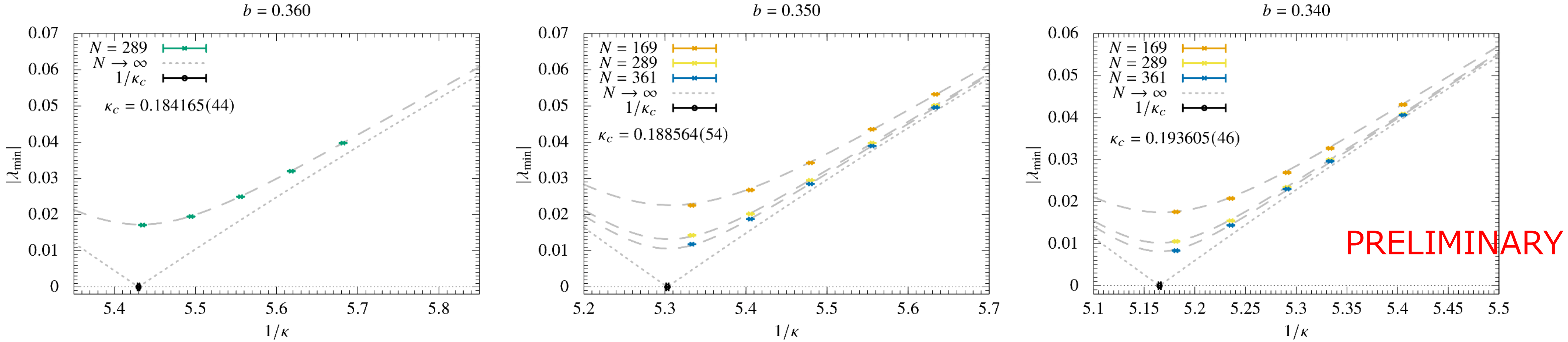
(N,k)=(361,7) b=0.350,0.340 series



For lightest κ_{adj}

- Lowest eigenvalue of $Q_W = D_W\gamma_5$ and the chiral limit ($\kappa_{adj,critical} = \kappa_c$)
  - The lowest eigenvalue $|\lambda_{min}|$ of Q_W is computed every trajectory. We can estimate the chiral limit where the lowest eigenvalue vanishes.
  - We fit $|\lambda_{min}|^2$ in the following form as a function of N, κ_{adj}.

$$\left|\frac{\lambda_{min}}{\kappa_{adj}}\right|^2 \leftarrow f(\kappa_{adj}, N) = A\left(\frac{1}{\kappa_{adj}} - \frac{1}{\kappa_c}\right)^2 + \frac{\delta}{N^2}$$



- Finite N correction term can describe the data well, but chi^2 is not so good as k dependent terms and higher $O(1/N^4)$ corrections could exist.
- b=0.360 has a large N correction meaning a fine lattice spacing.
- κ_c can be compared to those determined from PCAC mass and meson spectrum.
- We can estimate the lattice spacing a scaling from the finite N correction term as δ scales with $\delta \propto 1/a^2$, $N = \hat{L}^2$.

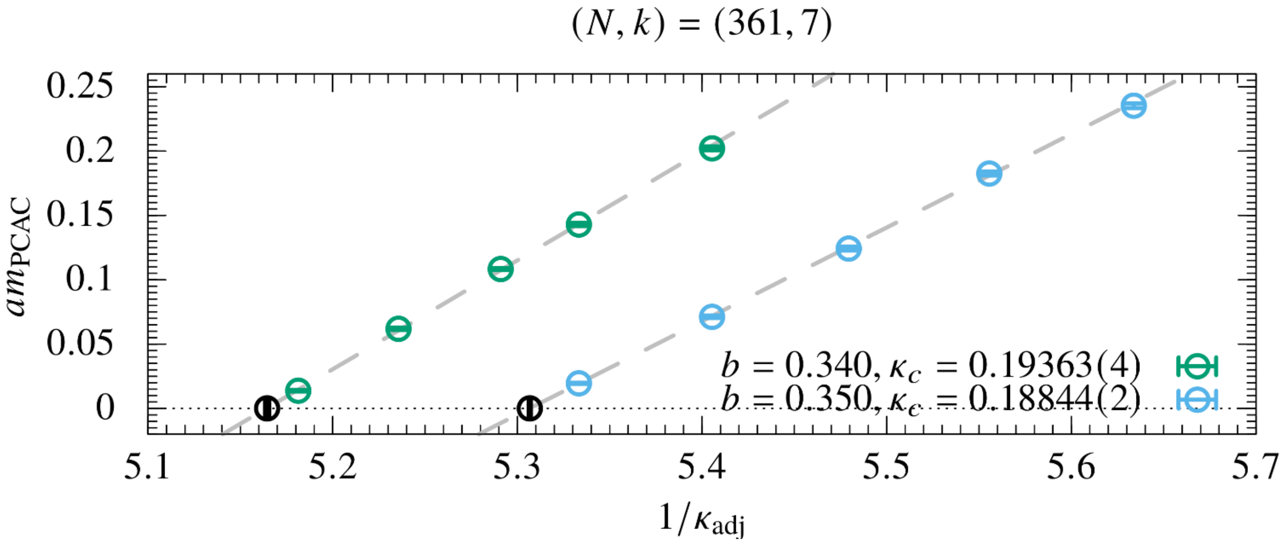
Scale setting and fundamental meson spectrum
Talk by Butti @7/27,6:30(EDT)

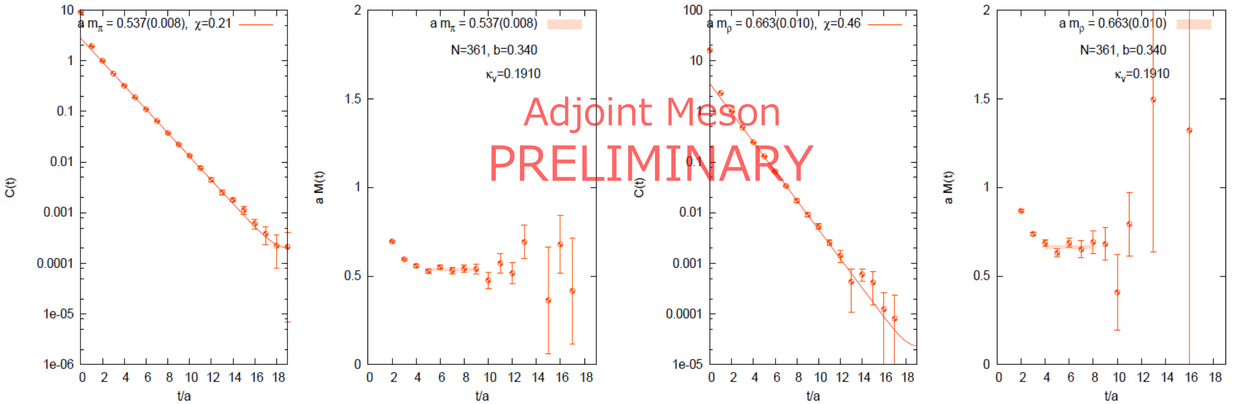
Meson in reduced model:
Garcia Perez Gonzalez-Arroyo Okawa
JHEP 04(2021)

- Meson spectrum with Adjoint Dirac fermions [PRELIMINARY]
  - We started the meson spectrum computation with both fundamental and adjoint fermions.
  - The lattice spacings are also investigated via the gradient flow scale.

- Consistency check on κ_c with the PCAC mass

Scale setting and fundamental meson spectrum
Talk by Butti @7/27,6:30(EDT)





- Uses Wuppertal smearing. Wave function tuning is not yet completed.

$$b = 0.340, \quad \kappa_c = 0.19363(4) \ \ [PCAC], \quad 0.19361(5) \ \ [|\lambda_{min}|]$$
$$b = 0.350, \quad \kappa_c = 0.18844(2) \ \ [PCAC], \quad 0.18856(5) \ \ [|\lambda_{min}|]$$

PRELIMINARY

- Consistent results from two-point meson functions with adjoint Dirac fermions

5. Summary

- We have started configuration generation to study the large N limit of SUSY YM via the twisted reduced matrix model.
- Dynamical one adjoint Majorana fermion is included via the RHMC algorithm and reweighting method.
- The sign of Pfaffian is all positive in the current parameter set.
- The critical kappa κ_c is determined via the lowest eigenvalues of Q_W^2 and the PCAC mass from adjoint meson two-point functions.
- We found consistent results for κ_c both at $b = 0.350, 0.340$.

TO DO:

- Continuum limit: Scale setting via spectrum, gradient flow, Wilson loops ...(ongoing)
- Meson spectrum in $N \to \infty$: Signal tuning via variational method ...(ongoing)
- SUSY : Extract Majorana component for SUSY spectrum, SUSY W.-T. identity....