Study of a lattice 2-group gauge model

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LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

38th International Symposium on Lattice Field Theory

based on arxiv:2105.05671 [hep-lat]

TQFT and crossed-modules

- In topological field theories, where the effective description with no local dynamical degrees of freedom exists, higher form symmetries may be present
- p-form symmetry is a symmetry which acts on observables $\mathcal{O}(\Sigma_p)$ on closed p-manifolds
- A general description of a dynamical model with different p symmetries intertwined was presented by B. Ruba in the previous talk. Here I discuss in more detail results from a particular realization of such a system
- Crossed-module: { groups $\mathcal{E}, \Phi$, action $\triangleright$ of $\mathcal{E}$ on Φ, homomorphism $\Delta : \Phi \to \mathcal{E}$ }
  Consistency conditions: 1^{st},2^{nd} Peiffer identities
  This talk: $\mathcal{E} = \Phi = \mathbb{Z}_4$,

$$\Delta(n) = 2n, \qquad m \triangleright n = (-1)^m n.$$

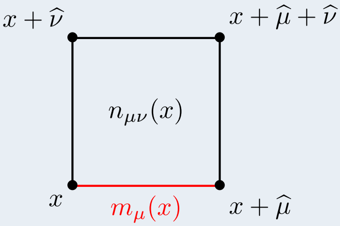
- We work on a cubic lattice with periodic boundary conditions.

Description of the model

Degrees of freedom

- $m_\mu(x)$, associated with links,
- $n_{\mu\nu}(x)$, associated with faces.

$n_{\mu\nu}(x)$ is an additional degree of freedom, not the plaquette observable $f_{\mu\nu}(x)$ constructed from $m_\mu(x)$!



Fake flatness constraint

$$2n_{\mu\nu}(x) = m_\mu(x) + m_\nu(x+\widehat{\mu}) - m_\mu(x+\widehat{\nu}) - m_\nu(x) =: f_{\mu\nu}(x)$$

Description of the model

Cube observables

$$g_{\mu\nu\rho}(x) = -n_{\mu\nu}(x) + n_{\mu\rho}(x) - n_{\nu\rho}(x)$$
$$+ (-1)^{m_\rho(x)} n_{\mu\nu}(x+\widehat{\rho}) - (-1)^{m_\nu(x)} n_{\mu\rho}(x+\widehat{\nu}) + (-1)^{m_\mu(x)} n_{\nu\rho}(x+\widehat{\mu}).$$

Six terms correspond to six sides of a cube.

Generalized Wilson action

$$S = -J_1 \sum_x \sum_{\mu<\nu} (-1)^{\frac{f_{\mu\nu}(x)}{2}} - J_2 \sum_x \sum_{\mu<\nu<\rho} (-1)^{\frac{g_{\mu\nu\rho}(x)}{2}}$$

is minimal when all plaquettes and all cubes are zero.

Nonlocal order parameters

Polyakov loop

Polyakov loop, valued in $\{\pm 1, \pm \mathrm{i}\}$:

$$P_\mu(x) = \exp\left(\frac{\mathrm{i}\pi}{2} \sum_{j=0}^{L_\mu - 1} m_\mu(x + j\widehat{\mu})\right).$$

Fake flatness $\implies$ $P_\mu(x)^2 = (-1)^{Q_\mu}$ independent of x.
Topological charges Q_μ split configurations into sectors.

Polyakov plane

"Polyakov torus" $P_{\mu\nu}(x)$. It is given by the sum of all $n_{\mu\nu}$ in a given plane weighted by $(-1)^m$ factors needed for gauge invariance.

Higher symmetries in our model

- 1-form $\mathbb{Z}_2^{(1)}$ flips sign of $P_\mu(x)$ (so Q_μ is invariant),
- 2-form $\mathbb{Z}_2^{(2)}$ flips sign of $P_{\mu\nu}(x)$.

Numerical setup

Adapted Metropolis algorithm

We define two types of local moves which satisfy the fake-flatness constraint:

- *link moves*

$$
\begin{aligned}
m_\mu(x) &\to m_\mu + 2\psi_\mu(x) \\
n_{\mu\nu}(x) &\to n_{\mu\nu}(x) + (-1)^{m_\mu(x)}\psi_\mu(x) + (-1)^{m_\mu(x+\hat{\nu})+m_\nu(x)}\psi_\nu(x+\hat{\mu}) \\
&\quad - (-1)^{m_\mu(x+\hat{\nu})+m_\nu(x)}\psi_\mu(x+\hat{\nu}) + (-1)^{m_\nu(x)}\psi_\nu(x)
\end{aligned}
$$

- *face moves*

$$
\begin{aligned}
m_\mu(x) &\to m_\mu(x) \\
n_{\mu\nu}(x) &\to n_{\mu\nu}(x) + \chi_{\mu\nu}(x)
\end{aligned}
$$

- *overrelaxation moves* do not change the value of the action but flip the signs of the Polyakov loop and surface

Numerical setup

Adapted Metropolis algorithm

- *link moves*

$$\psi_\mu \in \mathbb{Z}_4$$

  For $\psi_\mu(x)$ even, the move reduces to a gauge transformation.

- *face moves*

$$\chi_{\mu\nu} \in \{0, 2\}$$

With a finite sequence of such moves any two configurations with equal topological charge can be related. We will perform simulations at fixed topological charge.

Starting configuration

The simulation is started from one of the configurations listed below

- *cold*: all links and faces are set to 0, the constraint is automatically satisfied
- *hot*: all links are random, faces are set such that the constraint is satisfied $\pm$ random choice of the remaining freedom

Analytic results

Factorization theorem

$$\langle \mathcal{O}_1[m]\mathcal{O}_2[g]\rangle(J_1, J_2) = \langle \mathcal{O}_1[m]\rangle(J_1)\langle \mathcal{O}_2[g]\rangle(J_2).$$

Dynamics

Dynamics of $f_{\mu\nu}, P_\mu \equiv$ Wegner's model. First order critical point for $D = 4$ known exactly from Kramers-Wannier (KW) duality.
Dynamics of $g_{\mu\nu}$ dual to Ising. Expected continuous transition.

Expected phases

$J_1 \in \{0, \infty\}$, $J_2 \in \{0, \infty\}$, corresponding to four distinct TQFTs.

- $J_2 < J_2^{\mathrm{crit}}$: $n_{\mu\nu}$ d.o.f.s are disordered, expect unbroken $\mathbb{Z}_2^{(2)}$.
- $J_2 > J_2^{\mathrm{crit}}, J_1 > J_1^{\mathrm{crit}}$ – we classify minima of the action. Results suggest that $\mathbb{Z}_2^{(2)}$ is broken.
- $J_2 > J_2^{\mathrm{crit}}$, $J_1 < J_1^{\mathrm{crit}}$ – phase related to Yetter's model. Monte Carlo: $\mathbb{Z}_2^{(2)}$ breaking depends on topological charge.

Local observables

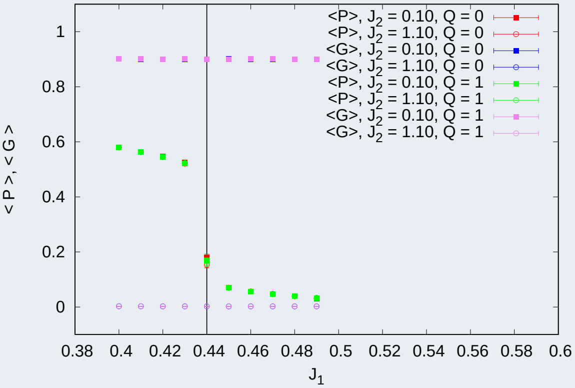


Figure: Mean value of plaquettes $\langle P \rangle$ and cubes $\langle G \rangle$ as a function of J_1 simulated on a 4^4 lattice. $Q = 0$ and $Q = 1$ overlap, so no sensitivity.

Results

Local observables

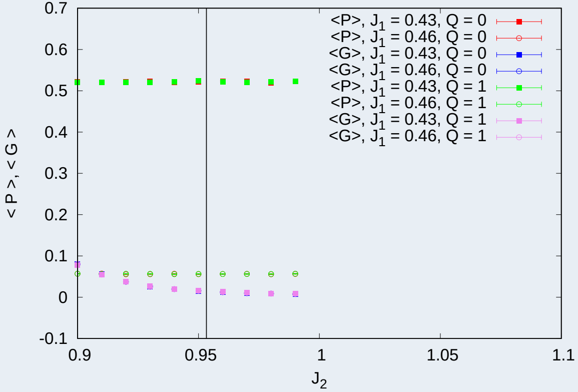


Figure: Mean value of plaquettes $\langle P \rangle$ and cubes $\langle G \rangle$ as a function of J_2 simulated on a 4^4 lattice. $Q = 0$ and $Q = 1$ overlap, so no sensitivity.

Local observables

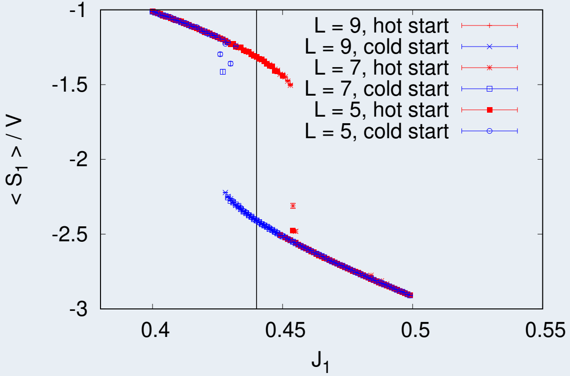


Figure: Mean action $\langle S_1 \rangle$ as a function of J_1. Data demonstrate a first order phase transition around $J_1 \approx 0.44$.

Results

Local observables

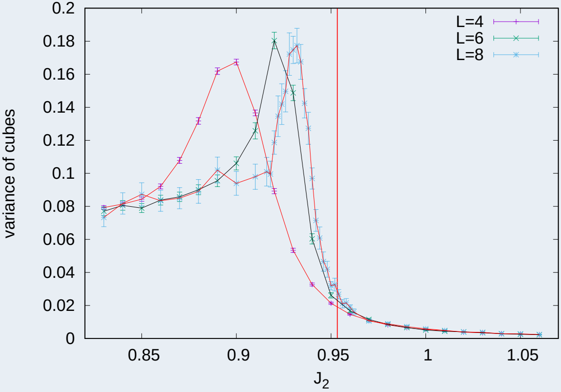


Figure: Mean fluctuations of cubes $\langle G \rangle$ (variance) as a function of J_2. Expected second order phase transition at $J_2 \approx 0.95$ is marked with solid vertical red line.

Non-local observables

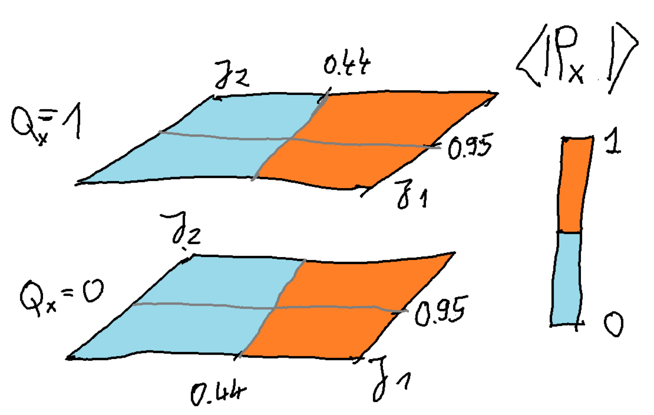


Figure: Schematic view of mean Polyakov line $\langle P_x \rangle$ as a function of the phase space.

Non-local observables

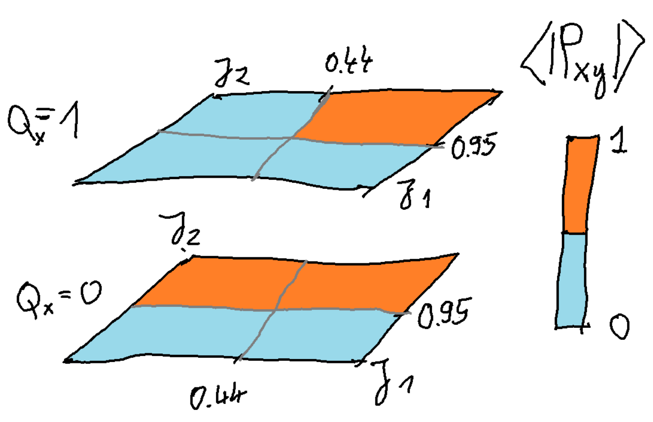


Figure: Schematic view of mean Polyakov surface $\langle P_{xy} \rangle$ as a function of the phase space.

Non-local observables

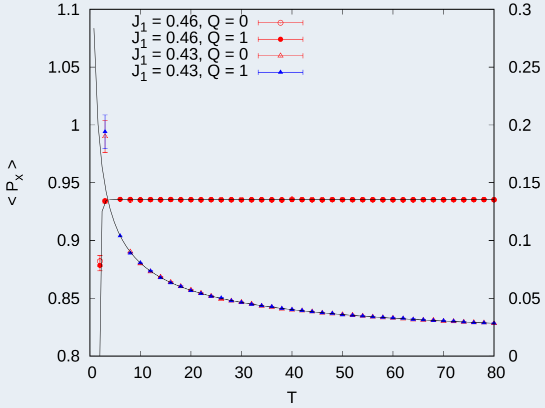


Figure: Mean Polyakov line $\langle P_x \rangle$ as a function of time extent T at small J_2.

Non-local observables

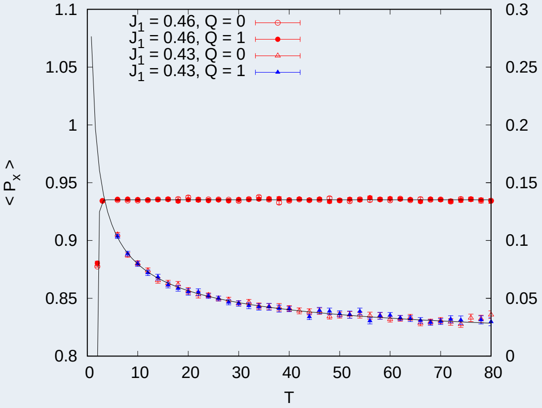
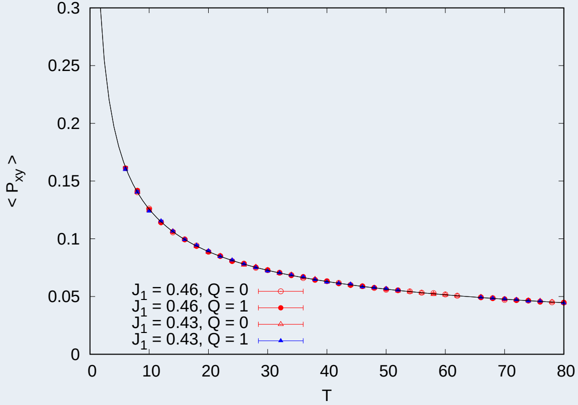


Figure: Mean Polyakov line $\langle P_x \rangle$ as a function of time extent T at large J_2.

Non-local observables



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Non-local observables

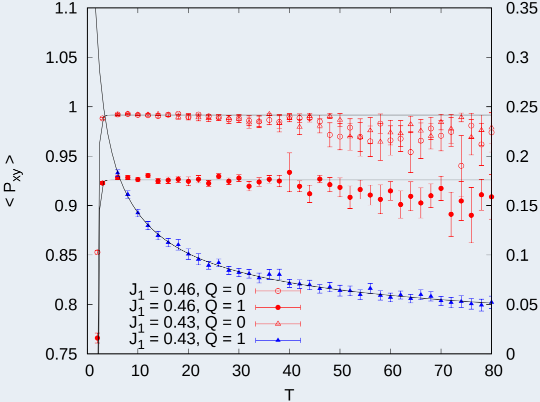

Figure: Mean Polyakov surface $\langle P_{xy} \rangle$ as a function of time extent T at large J_2.

Summary and outlook

Simplest 2-group model

- we have introduced a new model based on 2-group local symmetries
- we have analyzed its phase space and identified possible phases
- we have introduced new non-local order parameter - the Polyakov surface
- we have performed numerical simulations of the model which confirmed the proposed phase diagram of the model

Outlook

- simulations were performed at fixed topological charge $\rightarrow$ new algorithm needed
- can we construct a model where the factorization breaks?