Spin-1 fields and RG flows in 4 dimensions

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Warning: not directly a lattice talk

We all are interested in gauge theory, RG flows and asymptotic freedom

The questions along these lines I was trying to answer are elementary QFT questions

Motivation

Many of us teach QFT 101 following Weinberg's (1933-2021) book

Construct interacting QFT based on unitary irreducible representations of Poincare group

Introduce fields with the right quantum numbers and write down most general $\mathscr{L}$ with renormalizable and local interactions

Motivation

We literally follow the recipe for spin 0 and spin 1/2

- spin 0: scalar field
- spin 1/2: Dirac field (reducible)
- spin 1: gauge theory
- ...

With spin 1, new ingredient: gauge invariance is introduced, which excludes lots of terms which would be allowed by locality, renormalizability

Motivation

What if we followed the original recipe with spin 1?

Write down most general renormalizable and local $\mathscr{L}$ with vector fields and quantize, i.e. forget about gauge invariance

If we get a QFT it won't be unitary for sure, gauge invariance is required for getting rid of negative norm states

Perhaps we get a well-defined Euclidean QFT (or several)?

Can we get asymptotically free QFT without gauge invariance?

Most general $\mathscr{L}$

Elementary fields A_μ^a in adjoint of (global) $SU(N)$, keep only dimensionless couplings, need all dimension 4 operators

2 different kinetic terms

$$\mathrm{Tr}\,(\partial_\mu A_\nu \partial_\mu A_\nu) \qquad \mathrm{Tr}\,(\partial_\mu A_\mu \partial_\nu A_\nu)$$

2 different cubic terms

$$\mathrm{Tr}\,(A_\nu A_\nu \partial_\mu A_\mu) \qquad \mathrm{Tr}\,(A_\mu A_\nu \partial_\mu A_\nu)$$

4 different quartic terms

$$(\mathrm{Tr}\, A_\mu A_\mu)^2, \qquad \mathrm{Tr}\,(A_\mu A_\nu)\,\mathrm{Tr}\,(A_\mu A_\nu)$$
$$\mathrm{Tr}\,(A_\mu A_\mu A_\nu A_\nu)\,, \qquad \mathrm{Tr}\,(A_\mu A_\nu A_\mu A_\nu)$$

Most general $\mathscr{L}$

8 terms $\rightarrow$ 7 independent couplings $(z, h_1, h_2, g_1, g_2, g_3, g_4)$

$$
\begin{aligned}
\mathscr{L} &= \frac{1}{2}\partial_\mu A^a_\nu \partial_\mu A^a_\nu - \frac{1}{2}\left(1 - \frac{1}{z}\right)(\partial_\mu A^a_\mu)^2 + h_1 \tilde{\mathscr{O}}_1 + h_2 \tilde{\mathscr{O}}_2 + \mathscr{V} \\
\tilde{\mathscr{O}}_1 &= A^a_\mu A^b_\nu \partial_\mu A^c_\nu d_{abc} \\
\tilde{\mathscr{O}}_2 &= A^a_\mu A^b_\nu \partial_\mu A^c_\nu f_{abc} \\
\mathscr{V} &= \sum_{i=1}^{4} g_i \mathscr{O}_i \\
\mathscr{O}_1 &= \frac{1}{8} A^a_\mu A^b_\mu A^c_\nu A^g_\nu d_{abe} d_{cge} = \mathrm{Tr}\,(A_\mu A_\mu A_\nu A_\nu) - \frac{1}{N}(\mathrm{Tr}\,(A_\mu A_\mu))^2 \geq 0 \\
\mathscr{O}_2 &= \frac{1}{8N}(A^a_\mu A^a_\mu)^2 = \frac{1}{2N}(\mathrm{Tr}\,(A_\mu A_\mu))^2 \geq 0 \\
\mathscr{O}_3 &= \frac{1}{8N} A^a_\mu A^b_\mu A^a_\nu A^b_\nu = \frac{1}{2N}\mathrm{Tr}\,(A_\mu A_\nu)\mathrm{Tr}\,(A_\mu A_\nu) \geq 0 \\
\mathscr{O}_4 &= \frac{1}{4} A^a_\mu A^b_\mu A^c_\nu A^g_\nu f_{ace} f_{bge} = -\frac{1}{2}\mathrm{Tr}\,[A_\mu, A_\nu]^2 \geq 0 \,,
\end{aligned}
$$

Most general $\mathscr{L}$

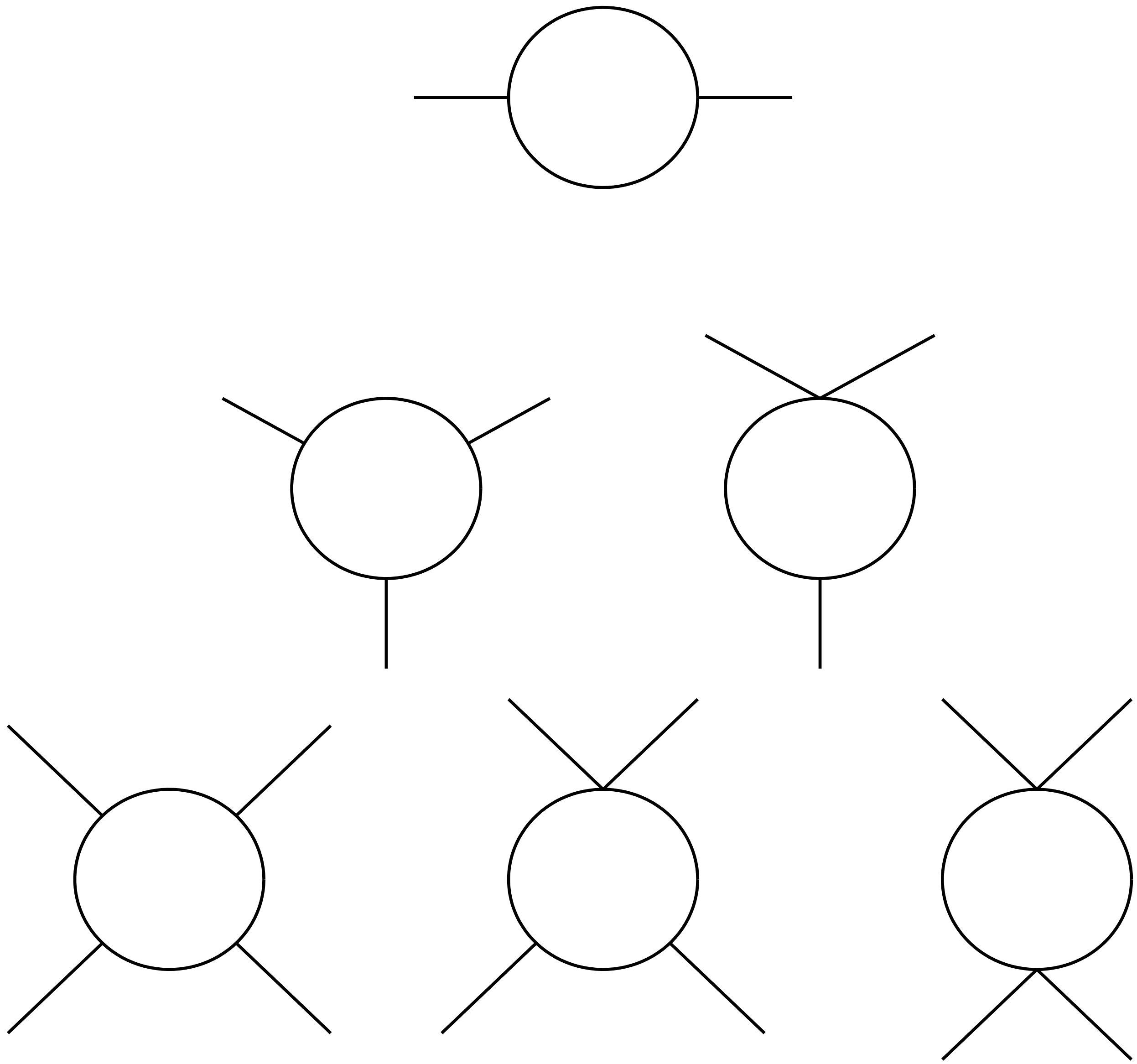
Yang-Mills special case $(z, h_1, h_2, g_1, g_2, g_3, g_4) = (\infty, 0, g, 0, 0, 0, g^2)$

A particular line in the space of couplings

QFT from $\mathscr{L}$

- Dimensional regularization and $\overline{\text{MS}}$
- Renormalize to 1-loop
- Compute β-functions
- Study RG flows

1-loop diagrams



β-functions

$$
\begin{aligned}
\beta_z &= z \times linear(h_1^2, h_2^2) \\
\beta_{h_i} &= h_i \times linear(h_1^2, h_2^2, g_1, g_2, g_3, g_4) \quad i = 1, 2 \\
\beta_{g_i} &= quadratic(h_1^2, h_2^2, g_1, g_2, g_3, g_4) \quad i = 1, 2, 3, 4
\end{aligned}
$$

Non-trivial polynomial dependence on z

z, h_1, h_2 renormalize multiplicatively

RG flows

We have the β-functions to 1-loop explicitly

Line of Gaussian fixed points: $(z, h_1, h_2, g_1, g_2, g_3, g_4) = (z, 0, 0, 0, 0, 0, 0)$

Can search for asymptotically free RG flows: any flow which in the UV ends up on the above line

For simplicity set $h_1 = 0$ and $h_2^2 = g_6$ (we have 6 couplings now)

Look for RG flows such that for $\mu \to \infty$

$$
\begin{aligned}
z(\mu) &\to const \\
g_i(\mu) &\sim 16\pi^2 \frac{C_i}{\log \frac{\mu}{\Lambda}}, \qquad i = 1, 2, 3, 4, 6
\end{aligned}
$$

RG flows

$$g_i(\mu) \sim 16\pi^2 \frac{C_i}{\log\frac{\mu}{\Lambda}}, \qquad i = 1, 2, 3, 4, 6$$

g_6 must be non-zero, otherwise $\lambda\phi^4$ type model

$r_{1,2,3,4} = \frac{g_{1,2,3,4}}{g_6}$ is constant towards UV

Look for asymptotically free g_6 and constant z, $r_{1,2,3,4} = \frac{g_{1,2,3,4}}{g_6}$

Fixed points for the ratios $\rightarrow$ solve system of polynomial equations

Solutions from β-functions depend on N

Large-N limit, $N \rightarrow \infty$, $Ng_{1,2,3,4,6}$ fixed

Once solutions are found, check 2 things

- $\mathscr{V}$ should be stable, $\mathscr{V} \geq 0$ for well-defined Euclidean QFT, necessary and sufficient condition on $g_{1,2,3,4}$ not obvious

- Fixed ratios r_i can be stable or unstable in RG-sense

Low $N = 3, 4, 5$

N	z	r_1	r_2	r_3	r_4	NC_6	$\mathscr{V}$
3	0	0.054652	0.122003	0.485317	0.970537	0.138656	stable
3	0	0.064145	0.133021	0.665179	0.964086	0.137153	stable
3	0	-0.647582	-0.580231	1.889786	1.204615	0.138656	unstable
3	0	-0.562664	-0.493787	1.918797	1.173022	0.137153	unstable
3	25/3	0.000334	0.079592	-0.251950	1.020083	0.148484	unstable
3	25/3	0.010673	0.074642	-0.144563	1.004360	0.145542	unstable
3	25/3	-0.108161	-0.028903	-0.034960	1.056248	0.148484	unstable
3	25/3	-0.080316	-0.016348	0.037417	1.034690	0.145542	unstable
4	0	0.044841	0.106784	0.351786	0.979028	0.140948	stable
4	0	0.074162	0.083060	1.368389	0.960858	0.136196	stable
4	25/3	0.004413	0.111209	-0.323177	1.013219	0.146900	unstable
4	25/3	0.016297	0.243636	-0.344606	0.995511	0.145494	unstable
4	25/3	0.017435	0.119096	-0.223217	0.997309	0.144605	unstable
4	25/3	0.017931	0.235838	-0.327356	0.993784	0.145177	unstable
5	0	0.042754	0.103223	0.327436	0.981138	0.141567	stable
5	0	0.054311	1.073479	0.536511	0.957994	0.142046	stable
5	0	0.067257	-0.066910	1.896637	0.967324	0.136857	stable
5	0	0.069027	0.516675	1.600829	0.956705	0.138188	stable
5	25/3	0.012566	0.149475	-0.375377	1.003344	0.145326	unstable
5	25/3	0.021321	0.180212	-0.347564	0.993910	0.144298	unstable

None of them RG-stable

Larger $N > 5$ and $N \to \infty$ (bold face RG-stable)

N	z	r_1	r_2	r_3	r_4	NC_6	$\mathscr{V}$
6	**0**	**0.041817**	**0.101590**	**0.316866**	**0.982127**	**0.141864**	**stable**
6	0	0.048648	1.137578	0.428569	0.966346	0.142530	stable
6	0	0.059916	-0.214070	2.277709	0.972682	0.137748	stable
6	0	0.062649	0.434621	2.043391	0.963808	0.138624	stable
7	**0**	**0.041301**	**0.100682**	**0.311136**	**0.982683**	**0.142032**	**stable**
7	0	0.045944	1.161333	0.383774	0.971232	0.142626	stable
7	0	0.054742	-0.321825	2.541816	0.976034	0.138570	stable
7	0	0.057376	0.412019	2.341096	0.968497	0.139238	stable
10	**0**	**0.040625**	**0.099483**	**0.303720**	**0.983425**	**0.142259**	**stable**
10	0	0.042691	1.184351	0.334451	0.977917	0.142606	stable
10	0	0.047136	-0.495636	2.966144	0.980468	0.140207	stable
10	0	0.048800	0.401667	2.839408	0.975942	0.140564	stable
50	**0**	**0.040047**	**0.098451**	**0.297474**	**0.984071**	**0.142458**	**stable**
50	0	0.040124	1.198242	0.298567	0.983855	0.142474	stable
50	0	0.040300	-0.680516	3.425269	0.983967	0.142360	stable
50	0	0.040376	0.410710	3.418741	0.983752	0.142375	stable
100	**0**	**0.040030**	**0.098420**	**0.297287**	**0.984091**	**0.142464**	**stable**
100	0	0.040049	1.198589	0.297559	0.984037	0.142468	stable
100	0	0.040093	-0.686738	3.440904	0.984065	0.142439	stable
100	0	0.040112	0.411281	3.439259	0.984011	0.142443	stable
$\boldsymbol{\infty}$	**0**	**0.040024**	**0.098409**	**0.297224**	**0.984097**	**0.142466**	**stable**
∞	0	0.040024	1.198704	0.297224	0.984097	0.142466	stable
∞	0	0.040024	-0.688818	3.446135	0.984097	0.142466	stable
∞	0	0.040024	0.411476	3.446135	0.984097	0.142466	stable

Conclusion with vector fields only

For any N, finitely many asymptotically free RG flows

All of them well-defined (non gauge invariant) Euclidean QFT

All of them non-unitary

For $N > 5$ unique RG-stable and stable $\mathscr{V}$ RG flow exists

$z = 0$ selected "dynamically" by requirement of asymptotic freedom $\to \partial_\mu A_\mu^a = 0$ constraint, reduces number of degrees of freedom from 4 to 3

For asymptotic freedom 1-loop is enough

Coupling to ghosts

Can couple to ghosts in a generic $SU(N)$-invariant way

2 more couplings

Can incorporate perturbative gauge theory with particular choice of couplings

Can repeat the β-function calculation (include ghost diagrams), 9 β-functions

Can find finitely many asymptotically free RG flows for any N

Gauge invariance is emergent on a particular RG flow, others are not gauge theories

Conclusions and outlook

The results are perhaps surprising and unexpected

Would be nice to have 2-loop results

For asymptotic freedom 1-loop is enough

Gauge invariance is emergent on a particular RG flow, on generic RG flow we only have finite dimensional symmetry

Diffeomorphism breaking (lattice) formulations of quantum gravity? Diffeomorphisms emerge after tuning a finite number of couplings? If ghosts are included, it seems possible

Dimensionful coupling $\mathscr{L}_{mass} = \frac{1}{2} m^2 A^a_\mu A^a_\mu$ can be added

It was a fun project, not clear what it is directly useful for

Lattice: working directly with A_μ^a, naive differences, etc, asymptotically free Euclidean QFT should be "visible" in simulations, probably non-perturbatively also well-defined, has Λ, fully well-behaved but non-unitary

Thank you for your attention!