Three-dimensional Gross-Neveu model with two flavors of staggered fermions

Sandip Maiti

Saha Institute of Nuclear Physics

In collaboration with:
Debasish Banerjee, Shailesh Chandrasekharan, Marina Krstic Marinkovic

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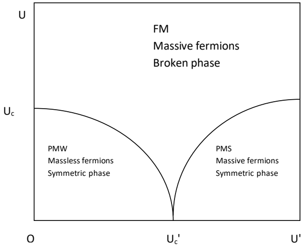
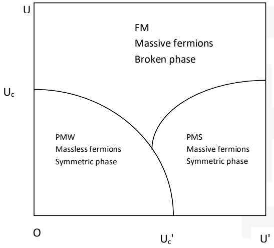
Motivation

- In the traditional paradigm, massless fermions acquire a mass through the spontaneous symmetry breaking(SSB) of chiral symmetries. This is forced due to t'Hooft anomalies.
- In lattice field theory alternate mechanisms of mass generation were known for fermions become massive without SSB. (Ayyar V, Chandrasekharan Shailesh: PhysRevD.91.065035)
- These exotic mechanisms have become interesting recently. (Razamat and Tong: Phys.Rev.X.11.011063)
- In this work we explore a three dimensional Gross-Neveu model with two couplings such that both mechanisms of fermion mass generation are possible and can compete.
- The goal is to understand how the three phases,(1) the massless fermion phase (2) the traditional massive phase and (3) the exotic massive phase, connect with each other.

Possible Phase diagrams





Action

$$S = S_0 + S_1^I + S_2^I,$$

Where,

$$S_0 = \sum_{x,\alpha} \frac{1}{2} \eta_{x,\alpha} \{ (\bar{u}_x u_{x+\alpha} - \bar{u}_{x+\alpha} u_x) + (\bar{d}_x d_{x+\alpha} - \bar{d}_{x+\alpha} d_x) \},$$

$\Uparrow$

staggered phase factor

S_0 is invariant under $SU(4) \times U(1)$ symmetry

$$S_1^I = -U \sum_{x,\alpha} \left(\bar{u}_x u_x \bar{u}_{x+\alpha} u_{x+\alpha} + \bar{d}_x d_x \bar{d}_{x+\alpha} d_{x+\alpha} \right) ,$$

$\longrightarrow$ invariant under $SU(2) \times U(1) \times SU(2) \times U(1)$

$$S_2^I = -U' \sum_{x} \left(\bar{u}_x u_x \bar{d}_x d_x \right) \longrightarrow SU(4)$$

- But S_2^I breaks $U(1)$ chiral symmetry explicitly.

Fermion Bag Idea

- In order to study our GN model using Monte Carlo methods we use the fermion bag approach.
- Idea: Group fermion worldlines inside regions called fermion bags.
  (Shailesh Chandrasekharan: Eur.Phys.J.A 49 (2013) 90)
- The partition function is then written as a sum over configurations of fermion bags.
- One possible way to identify fermion bags is to introduce new variables, the "dimers" $d_{x,\alpha}$ for nearest neighbour interactions and "instantons" i_x for single site interactions.
  (Ayyar, Chandrasekharan, and Rantaharju: Phys.Rev.D 97 (2018) 5, 054501)

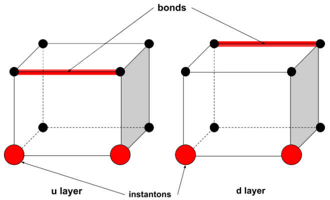
$$Z = \int [D\bar{u}DuD\bar{d}Dd]\; e^{-\sum_{x,y}(\bar{u}_x M_{x,y} u_y + \bar{d}_x M_{x,y} d_y) + U\sum_{x,y}(\bar{u}_x u_x \bar{u}_y u_y + \bar{d}_x d_x \bar{d}_y d_y) + U' \sum_x \bar{u}_x u_x \bar{d}_x d_x}$$

$$e^{U\bar{u}_x u_y \bar{u}_y u_x} = \prod_{x,y}(1 + U\bar{u}_x u_x \bar{u}_y u_y) = \prod_{x,y} \sum_{d_{x,y}=0,1} (U\bar{u}_x u_x \bar{u}_y u_y)^{d_{x,y}}$$

$$e^{U' \bar{u}_x u_x \bar{d}_x d_x} = \prod_x (1 + U' \bar{u}_x u_x \bar{d}_x d_x) = \prod_x \sum_{i_x=0,1} (U' \bar{u}_x u_x \bar{d}_x d_x)^{i_x}$$

The partition function then becomes the sum over all configurations of [i] and [d]

$$Z = \sum_{[i]} U'^{N_i} \Big(\sum_{[u][d]} U^{N_u} U^{N_d} det(W_u[f]) det(W_d[f]) \Big)$$



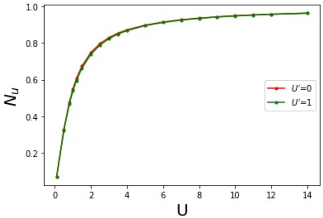
Observables

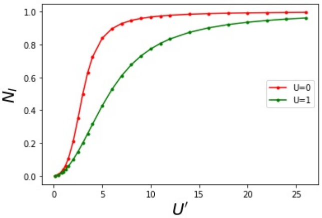
- We focus on four interesting observables.
- Four-fermion condensates:
  $N_u = -\frac{1}{V}\sum_{x,y}\langle \bar{u}_x u_y \bar{u}_y u_x \rangle$, $N_d = -\frac{1}{V}\sum_{x,y}\langle \bar{d}_x d_y \bar{d}_y d_x \rangle$,
  $N_i = \frac{1}{V}\sum_{x}\langle \bar{u}_x u_x \bar{d}_x d_x \rangle$.
- Fermion bilinear susceptibilities:
  $\chi_1 = \frac{1}{V}\sum_{x}\langle \bar{u}u_0 \bar{u}u_x \rangle$, $\chi_2 = \frac{1}{V}\sum_{x}\langle \bar{u}u_0 \bar{d}d_x \rangle$,
- Susceptibilities will diverge as L^d in the broken phase with a fermion bilinear condensate and as $L^{2-\eta}$ at a second order critical point but saturate in the massless phase or the exotic phase.

Results

We present some exact results on a 2^3 lattice.

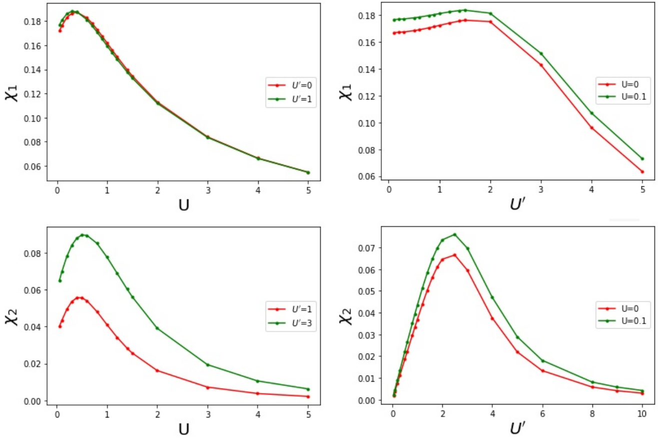
$$N_i = \frac{1}{V}\sum_x \langle\, \bar{u}_x u_x \bar{d}_x d_x \,\rangle, \qquad N_u = -\frac{1}{V}\sum_{x,y} \langle\, \bar{u}_x u_y \bar{u}_y u_x \,\rangle$$





$$\chi_1 = \frac{1}{V}\sum_x \langle \bar{u}u_0\bar{u}u_x\rangle, \quad \chi_2 = \frac{1}{V}\sum_x \langle \bar{u}u_0\bar{d}d_x\rangle,$$

[for 2^3 lattice]



Outlook

- We study the three dimensional Gross-Neveu model for two flavors of staggered fermions and discuss about the alternative mechanism of mass generation. Then we have shown some results.
- We are implementing the algorithm to explore the phase structure of our model.

Thank You !

Back-ups!

Symmetries

$$S_0 = \sum_{x,y} \left(\bar{u}_x M_{xy} u_y + \bar{d}_x M_{xy} d_y\right)$$

$SU(4) \times U(1)$ symmetry:

$$\begin{bmatrix} u_x \\ \bar{u}_x \\ d_x \\ \bar{d}_x \end{bmatrix} \rightarrow Ve^{i\theta} \begin{bmatrix} u_x \\ \bar{u}_x \\ d_x \\ \bar{d}_x \end{bmatrix} \text{, even sites}$$

$$\begin{bmatrix} \bar{u}_x & u_x & \bar{d}_x & d_x \end{bmatrix} \rightarrow \begin{bmatrix} \bar{u}_x & u_x & \bar{d}_x & d_x \end{bmatrix} V^\dagger e^{-i\theta} \text{, odd sites}$$

Symmetries

$$S_1^I = -U\sum_{x,y}\left(\bar{u}_x u_x \bar{u}_y u_y + \bar{d}_x d_x \bar{d}_y d_y\right)$$

$SU(2) \times U(1)$ symmetry:

$$\begin{bmatrix} u_x \\ \bar{u}_x \end{bmatrix} \to V e^{i\theta} \begin{bmatrix} u_x \\ \bar{u}_x \end{bmatrix}$$, even sites

$$\begin{bmatrix} \bar{u}_x & u_x \end{bmatrix} \to \begin{bmatrix} \bar{u}_x & u_x \end{bmatrix} V^\dagger e^{-i\theta}$$, odd sites

For two flavors S_1^I is invariant under $SU(2) \times U(1) \times SU(2) \times U(1)$

Symmetries

$$S_2^{\mathrm{I}} = -U' \sum_x \left(\bar{u}_x u_x \bar{d}_x d_x \right)$$

$SU(4)$ symmetry:

$\chi_1 = \bar{u}$, $\chi_2 = u$, $\chi_3 = \bar{d}$ and $\chi_4 = d$

$\chi_1 = \sum_\alpha V_{1\alpha}\chi_\alpha$, $\chi_2 = \sum_\beta V_{2\beta}\chi_\beta$, $\chi_3 = \sum_\gamma V_{2\gamma}\chi_\gamma$ and $\chi_4 = \sum_\delta V_{4\delta}\chi_\delta$

If $\alpha \neq \beta \neq \gamma \neq \delta$

$$\chi_1\chi_2\chi_3\chi_4 \longrightarrow SU(4)$$