Critical behaviour and phase structure of 3d Scalar+Gauge Field Theories in the adjoint representation

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LatCos Collaboration

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- Ben Kitching-Morley (Southampton)
- Andreas Jüttner (Southampton, CERN)
- Joseph K.L. Lee (Edinburgh)
- Valentin Nourry (Edinburgh and Southampton, now Université de Paris)
- Antonin Portelli (Edinburgh)
- Henrique Bergallo Rocha (Edinburgh)
- Kostas Skenderis (Southampton)

Holographic Cosmology

Introduction to Holography

We are interested in a class of conjectured gauge/gravity dualities of the form

Gravity in $d+1$ dimensions $\Leftrightarrow$ QFT in d dimensions

An essential aspect of these holographic models is that

Weakly coupled gravity $\Leftrightarrow$ Strongly coupled QFT
Strongly coupled gravity $\Leftrightarrow$ Weakly coupled QFT

Time evolution in the gravitational theory corresponds to inverse RG flow in the QFT.

In McFadden and Skenderis' paper on Holographic cosmology [1], it is shown that the power spectrum of a gravitational theory may be mapped to the observable

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle \tag{1}$$

in the QFT. The best we can do is start with an ansatz for the QFT, constrain its features and then try to match the observables!

[1] Paul McFadden and Kostas Skenderis. "The Holographic Universe". In: i (2010). DOI: 10.1088/1742-6596/222/1/012007. arXiv: 1001.2007. URL: http://arxiv.org/abs/1001.2007http://dx.doi.org/10.1088/1742-6596/222/1/012007

Ansatz as general as possible:

$$S = \frac{1}{g_{YM}^2} \int d^3x \, \mathrm{Tr} \left[\frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \bar{\psi}^L \gamma_\mu D^\mu \psi^L \right. \\ \left. + \lambda_{M_1 M_2 M_3 M_4} \phi^{M_1} \phi^{M_2} \phi^{M_3} \phi^{M_4} + \mu^{\alpha\beta}_{M \, L_1 L_2} \phi^M \psi^{L_1}_\alpha \psi^{L_2}_\beta \right] \quad (2)$$

- N_A gauge fields
- N_ϕ non-minimal scalars
- N_ψ fermions

The Holographic Universe

Dictionary:

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl} \tag{3}$$

$$\Delta_S^2(q) = \frac{-q^3}{16\pi^2 \text{Im} B(-iq)} \tag{4}$$

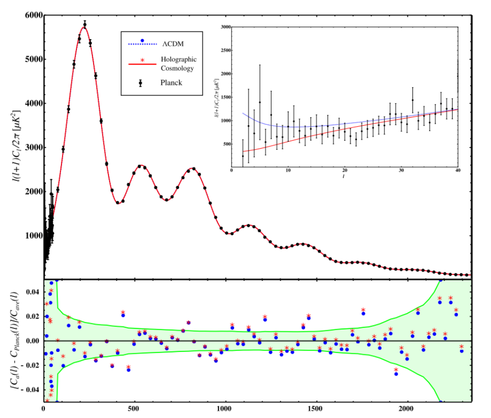
$$\Delta_T^2(q) = \frac{-q^3}{\pi^2 \text{Im} A(-iq)} \tag{5}$$

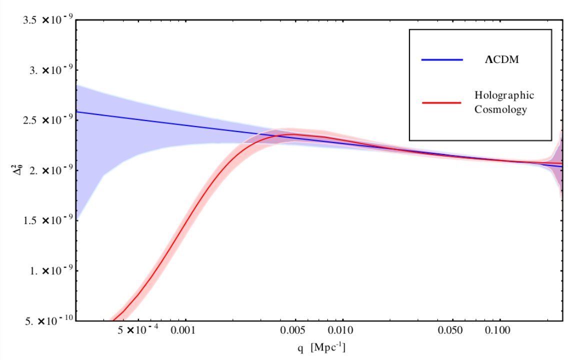
Universal form at 2 loops:

$$\Delta_R^2(q) = \frac{\Delta_0^2}{1 + \frac{gq*}{q}\log|\frac{q}{\beta g q*}|} \tag{6}$$

Perturbative results apply for $g\frac{q*}{q} \ll 1$

Afshordi et al[2] have been able to compare these models with CMB data:





[2] Niayesh Afshordi et al. "From Planck Data to Planck Era: Observational Tests of Holographic Cosmology". In: *Physical Review Letters* 118.4 (2017), pp. 1–8. ISSN: 10797114. DOI: 10.1103/PhysRevLett.118.041301. arXiv: 1607.04878

The Holographic Universe

- Constraining theory against Planck data $\rightarrow$ no fermions!
- For high multipoles ($l \geq 30$), the two models fit the data equally well and are within 1σ of each other!
- However, for multipoles $2500 \geq l \geq 2$, we have $2 \times 10^{-3} \leq |g \frac{q*}{q}| \leq 2.5$
- We need non-perturbative methods.

What we need to do

1. Discretise the continuum model
2. Find the massless point
3. Compute the Energy-Momentum tensor[3,4]
4. Compute its 2-point function
5. Compare with Planck data

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[3] Check out Joseph Lee's talk!

[4] Luigi Del Debbio et al. "Renormalization of the energy-momentum tensor in three-dimensional scalar SU(N) theories using the Wilson flow". In: *Physical Review D* 103.11 (2021), p. 114501. ISSN: 2470-0010. DOI: 10.1103/PhysRevD.103.114501. arXiv: 2009.14767. URL: https://link.aps.org/doi/10.1103/PhysRevD.103.114501

Scalar+Gauge Theory

Scalar+Gauge SU(N) Theory

Scalar+Gauge lattice action (a.k.a. 3D Georgi-Glashow model):

$$S[\phi, U] = \frac{N}{g} \sum_{x \in \Lambda^d} \mathrm{Tr} \left[a^3 \left(\sum_{\mu=1}^{d} (\Delta_\mu \phi(x))^2 + m^2 \phi^2(x) + \lambda \phi^4(x) \right) + \frac{1}{a} \sum_{\mu<\nu} \mathrm{Re} \left[\mathbb{1} - P_{\mu\nu} \right] \right] \quad (7)$$

- 4d theory of interest due to Higgs sector. Also interesting due to t'Hooft-Polyakov monopoles!
- 4d $\rightarrow$ 3d via dimensional reduction at finite temperature.
- 3d theory with variable-length adjoint scalars has been the subject of some study in $N = 2, 3$[5] . Theoretical and numerical analyses predict crossover behaviour and first-order transitions[6]. Also a conjectured critical point[7].

[5] K. Kajantie et al. "3d SU(N) + adjoint Higgs theory and finite-temperature QCD". In: *Nuclear Physics B* 503.1-2 (1997), pp. 357–384. ISSN: 05503213. DOI: 10.1016/S0550-3213(97)00425-2. arXiv: 9704416 [hep-ph]

[6] Sudhir Nadkarni. "The SU(2) adjoint Higgs model in three dimensions". In: *Nuclear Physics, Section B* 334.2 (1990), pp. 559–579. ISSN: 05503213. DOI: 10.1016/0550-3213(90)90491-U

[7] A. Hart et al. "On the phase diagram of the SU(2) adjoint Higgs model in 2+1 dimensions". In: *Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics* 396.1-4 (1997), pp. 217–224. ISSN: 03702693. DOI: 10.1016/S0370-2693(97)00104-4. arXiv: 9612021 [hep-lat]

Algorithm

- We use the Grid[a] library for our lattice simulations.
- We apply the Heatbath-Overrelaxation (HBOR) algorithm, with Kennedy-Pendleton[b] updates for gauge fields and using the Bunk[c] method for scalar updates.
- Adjoint scalar-gauge interactions are enforced via a Metropolis step.

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[a] https://github.com/paboyle/Grid

[b] A. D. Kennedy and B. J. Pendleton. "Improved heatbath method for Monte Carlo calculations in lattice gauge theories". In: *Physics Letters B* 156.5-6 (1985), pp. 393–399. ISSN: 03702693. DOI: 10.1016/0370-2693(85)91632-6

[c] B. Bunk. "Monte-Carlo methods and results for the electro-weak phase transition". In: *Nuclear Physics B (Proceedings Supplements)* 42.1-3 (1995), pp. 566–568. ISSN: 09205632. DOI: 10.1016/0920-5632(95)00313-X

Algorithm

- Undergoing optimisation for A100 GPUs → Tursa (DiRAC) commissioning in Edinburgh.
- Grid classes needed some restructuring like rewriting gauge methods to allow for arbitrary dimensionality and number of colours.

Still many improvements that can be done for GPU runs, like checkerboarding steps on the GPU, gauge staples using stencil routines, RNG on the GPU, etc...

```
~~~~~~~~~~~~~~~
Scalar HB step times:
Total Time (s)                = 0.277
Nearest neighbours time (s) = 0.187 (67.6% of step)
Complex RNG time (s)          = 0.00117 (0.422% of step)
Lie Algebra RNG time (s)      = 0.0148 (5.33% of step)
Accept/reject time (s)        = 0.00299 (1.08% of step)
PickCheckerboard time (s)     = 0.0118 (4.26% of step)
SetCheckerboard time (s)      = 0.00591 (2.13% of step)
I/O time (s)                  = 0.0001 (0.0361% of step)
Other time (s)                = 0.053 (19.1% of step)
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```

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~~~~~~~~~~~~~~~
Gauge HB step times:
Total Time (s)                       = 1.28
Staple calculation time (s)          = 0.726 (56.9% of step)
Scalar nearest neighbours time (s) = 0.108 (8.43% of step)
Complex RNG time (s)                 = 0.0477 (3.74% of step)
Accept/reject time (s)               = 0.0333 (2.61% of step)
PickCheckerboard time (s)            = 0.155 (12.2% of step)
SetCheckerboard time (s)             = 0.00665 (0.521% of step)
Lorentz index peek time (s)          = 0.000878 (0.0688% of step)
Lorentz index poke time (s)          = 0.015 (1.17% of step)
SU(2) subgroup extract time (s)      = 0.001 (0.0785% of step)
SU(2) subgroup insert time (s)       = 0.0183 (1.44% of step)
I/O time (s)                         = 0.0217 (1.7% of step)
Other time (s)                       = 0.143 (11.2% of step)
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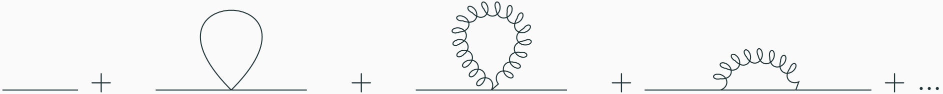
The literature[8] and our own lattice PT calculations can give us the mass counterterms from the self-energy of the scalar propagator:

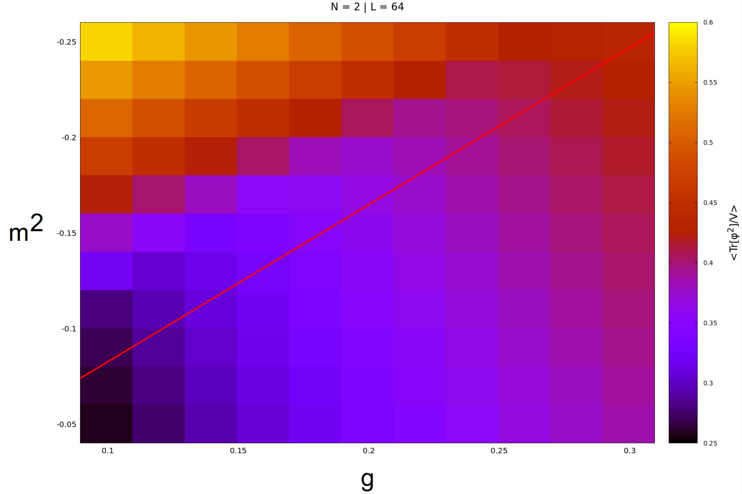


(8)

$$\delta m^2 = -\frac{gZ_0}{a}\left[2(1+\lambda) - \frac{3}{N^2}\lambda\right]$$
$$-\frac{g^2}{16\pi^2}\left[2\kappa_1 - \kappa_4 + 4\pi^2 Z_0\left(\frac{1}{2} - \frac{4}{3N^2}\right) + 10\pi^2 Z_0^2 + 2\lambda\left(2 - \frac{3}{N^2}\right)\left(4\pi^2 Z_0^2 - \delta\right) - 4(\delta+\rho)\right.$$
$$\left. - \lambda\left(\lambda - 4 - \frac{6}{N^2}(\lambda - 1) + \frac{18}{N^4}\lambda\right)\left(\zeta + \log\frac{6}{a\mu}\right)\right] \quad (9)$$

[8] M. Laine and A. Rajantie. "Lattice-continuum relations for 3d SU(N)+Higgs theories". In: *Nuclear Physics B* 513.1-2 (1998), pp. 471–489. ISSN: 05503213. DOI: 10.1016/S0550-3213(97)00709-8. arXiv: 9705003 [hep-lat]

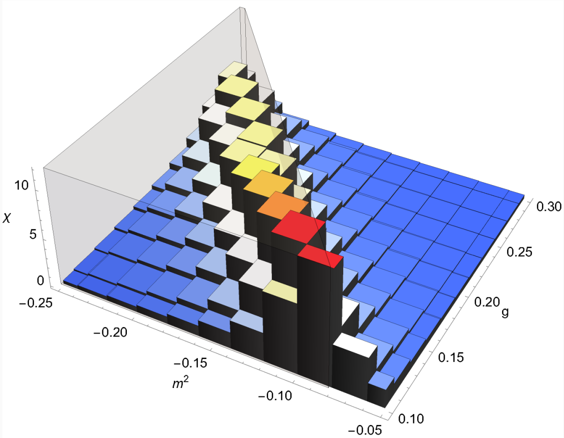
Phase Diagram Simulation



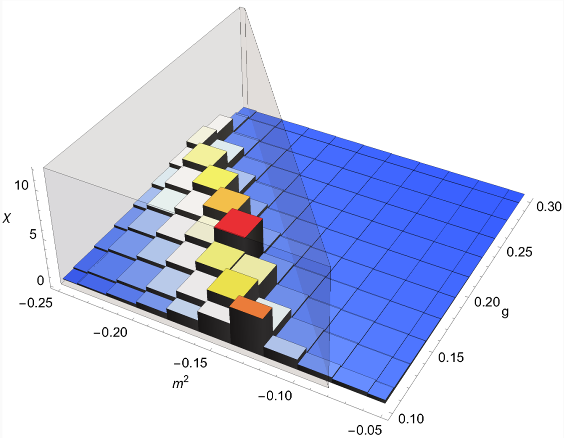
PRELIMINARY

Lattice PT predictions seem to match, also possible crossover behaviour.

Phase Diagram Simulation



$L = 8$

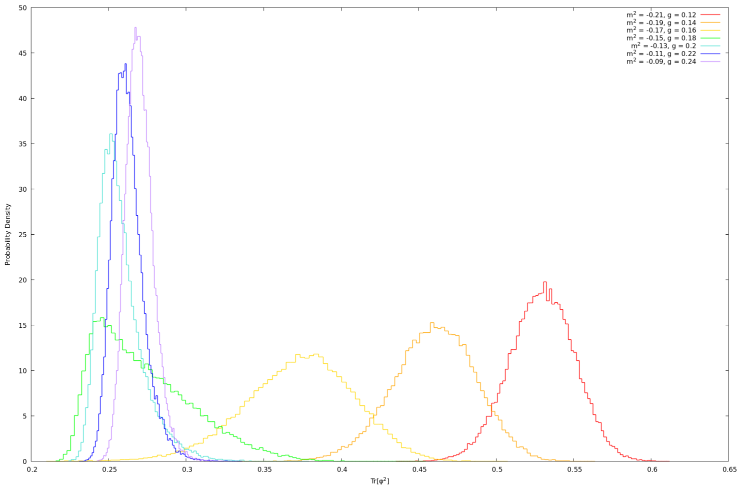


$L = 64$

PRELIMINARY

Phase transition does not appear to be second-order, in contrast with pure scalar case.

Phase Diagram Simulation



PRELIMINARY

Histogram overlap in the symmetric phase means m^2 reweighting is possible.

Conclusions

Conclusions and Outlook

- The SU(N) adjoint scalar superrenormalisable theory in 3D has a rich phase structure which has not yet been extensively explored, especially for $N > 3$.
- Finding regions of critical behaviour is crucial for Holographic Cosmology, so a more thorough exploration of the phase diagram is in order.
- Non-perturbative effects are much more prominent in the adjoint Higgs theory than in the fundamental representation, so Lattice is a particularly important tool here.

- We have published compelling evidence for the IR-finiteness of the pure scalar theory[9][10]. Next logical step is doing the same for the Scalar+Gauge case.
- This involves doing a finite-size scaling analysis at the critical line, measuring critical exponents, understanding better the phase structure at higher N.
- Ultimately, the goal is to be able to measure the 2-point function of $T_{\mu\nu}$ and from there match the model against the CMB spectrum.

Thank You!

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[9] Check out Andreas Jüttner's talk!

[10] Guido Cossu et al. "Nonperturbative Infrared Finiteness in a Superrenormalizable Scalar Quantum Field Theory". In: *Physical Review Letters* 126.22 (2021), p. 221601. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.126.221601. arXiv: 2009.14768. URL: https://link.aps.org/doi/10.1103/PhysRevLett.126.221601