Symmetric mass generation in lattice gauge theory

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Lattice 2021

Outline

Can fermions acquire mass without breaking flavor symmetries ?

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$SO(4)$ invariant reduced staggered fermion model in $D = 4$ has massless and massive phases without any intervening symmetry broken phase

Action and Symmetries

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With non-zero κ we can achieve a direct transition between the massless and massive phase without breaking the global $SO(4)$ symmetry and the discrete Z_2 symmetry.

Phase structure from numerical simulation

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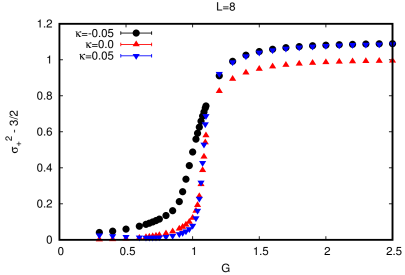

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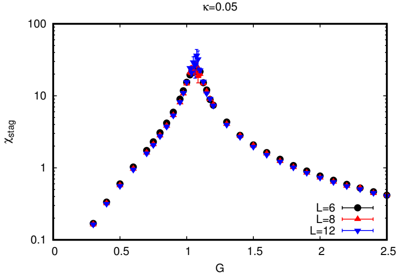

$\chi stag$

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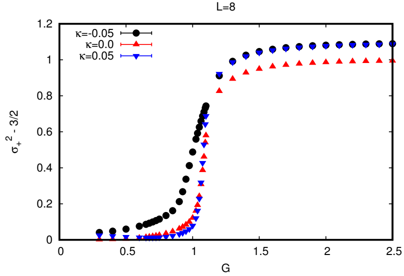

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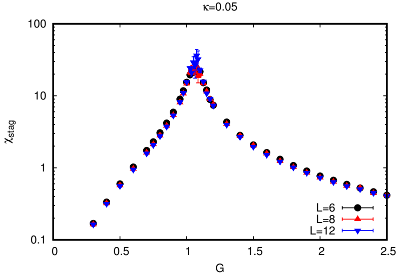

$\chi stag$

Connected susceptibility
$\chi_{stag} = \frac{1}{V}\sum_{x,y} < \epsilon(x)\psi^a(x)\psi^b(x)\epsilon(y)\psi^a(y)\psi^b(y) >$ shows no volume scaling while we have evidence for four-fermion condensate for non-zero κ

Extending this to a gauge model

Is it possible to retain this phase structure if we make one of the $SU(2)$ subgroups a gauge symmetry.

Action

$S_F = \sum_{x,\mu} \frac{1}{2}\eta_\mu(x) Tr[\psi^\dagger(x)U_\mu(x)\psi(x+\mu)V_\mu^\dagger(x) - \psi^\dagger(x)U_\mu^\dagger(x-\mu)\psi(x-\mu)V_\mu(x-\mu)]$

Symmetries

$$\psi \rightarrow G(x)\psi(x)H^\dagger$$
$$U_\mu(x) \rightarrow G(x)U_\mu(x)G^\dagger(x+\mu)$$
$$V_\mu(x) \rightarrow H(x+\mu)V_\mu H^\dagger(x)$$

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We gauge the symmetry under which the scalar is a singlet.

Yukawa term and auxiliary field action

$S_Y = \sum_x Tr(\phi(x)\psi(x)\psi^\dagger(x)) + \frac{1}{2\lambda^2}\sum_x Tr(\phi(x))^2$

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Integrating out the scalar field generates four-fermion term $\sum_x Tr(\psi(x)\psi^\dagger(x)\psi(x)\psi^\dagger(x))$

Continued

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Reduction to $SO(4)$ model

Using the pseudo-reality condition for $SU(2)$ elements $\psi^\dagger = \sigma_2 \psi^T \sigma_2$ and $\psi = \sigma_\mu \chi_\mu$ it is easy to show that this model reduces to the aforementioned $SO(4)$ model in $\beta_G, \beta_H \to \infty$ limit.

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As β_H increases the lattice spacing *a* decreases and the physical temperature $T = \frac{1}{La}$ increases with de-confinement once temperature exceeds the strong scale.

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As β_H increases the lattice spacing a decreases and the physical temperature $T = \frac{1}{La}$ increases with de-confinement once temperature exceeds the strong scale. We keep Yukawa coupling small and scan for four-fermion condensate as β_H is varied.

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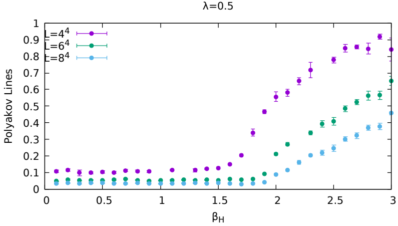

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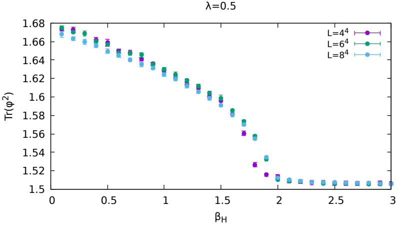

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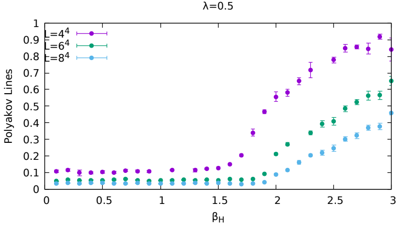

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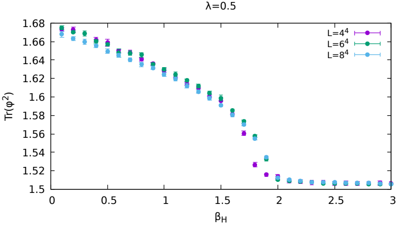

$\langle Tr(\phi^2) \rangle$

$Tr(\phi^2)$ is used as a proxy for four-fermion condensate. Here the four-fermion condensate is a result of gauge interactions since the Yukawa coupling is below the critical λ_c to drive the transition.

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- Spectrum of composite states in the confining regime requires further work.
- Can this mechanism be used to decouple (have infinitely massive) right-handed states and obtain a chiral theory in the continuum limit ?

Thank you!

Collaborators

Simon Catterall , Goksu Can Toga

Funding and computing resources

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