Chiral and Continuum Limit of Large Nc QCD

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Overview

- Large N_c expansion captures nonperturbative features of QCD: confinement, chiral symmetry breaking, mesonic spectrum...
- The objective is to study 2-flavor SU(N_c) gauge theory in the chiral and continuum limit. We consider $N_c = 3, 4, 5$, and study the scaling of χPT LECs with N_c.
- Compare with other approaches: 4-flavor (Hernandez et al); Quenched (Bali); small volume, very large N_c (Gonzalez-Arroyo).
- This talk: present preliminary results.

WχPT

- Wilson Chiral Perturbation Theory is χPT with Wilson-type fermions on a lattice.
- Lattice spacing a is additional parameter in EFT expansion.
- Generic Small Mass (GSM) power counting scheme: $p^2 \sim m_q \sim a$.
- Predicts at NLO:

$$M_\pi^2 = 2Bm_q\left(1 + \frac{1}{2}\frac{2Bm_q}{8\pi^2F^2}\log(2Bm_q/\mu^2)\right) + C_m m_q^2 + W_{qq} a m_q + W_q a^2 \tag{1}$$

$$f_\pi = F\left(1 - \frac{2Bm_q}{8\pi^2F^2}\log(2Bm_q/\mu^2)\right) + C_F m_q + W_F a \tag{2}$$

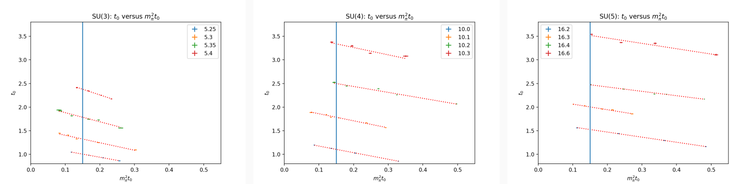
- Large N_c predicts $B \sim O(N_c^0)$, $f_\pi^2 \sim O(N_c^1)$.

Simulation Details

- We have collected data for $N_c = 3, 4, 5$ over a range of β and κ values. Mostly $16^3 \times 32$, some $24^3 \times 32$.
- HMC algorithm with Clover fermions and NHYP links.
- For each data point, we have between 400 and several thousand trajectories, keeping every 10th configuration. Autocorrelations appear controlled according to jackknife analysis.
- Most datasets have $m_\pi L > 4$, a few are below this threshold.

Scale Setting

- Wilson flow is used for scale setting: $a(\beta) = t_0(\beta)^{-0.5}$.
- To fix lattice spacing in terms of lattice coupling, we interpolate $m_\pi^2 t_0$ versus t_0 to $m_\pi^2 t_0 = 0.15$.
- Lattice spacings in the range $\sim 0.5 - 1.0$.



Analysis Methodology — Model Averaging

- Issue: many data sets.
- Model Averaging (Jay-Neil) is used to select fit ranges in spectroscopic analysis: weight $\propto e^{-\frac{1}{2}(\chi^2 - 2N_{\text{points}})}$.
- Jackknife for errors and correlations between observables.
- lsqfit and gvar are used for fitting and error propagation.

Chiral Formulae Revisited

With the choice $\mu = \frac{3}{N_c} 8\pi^2 F^2$ and the replacement $2Bm_q/8\pi^2F^2 \to m_\pi^2/8\pi^2 f_\pi^2 \equiv \xi$, the NLO chiral formulae (1) and (2) become:

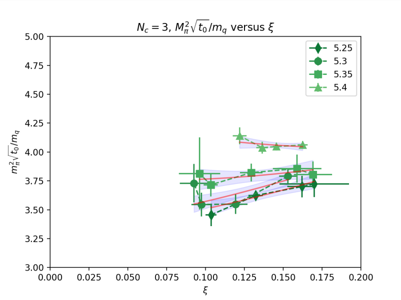
$$m_\pi^2/m_q = 2B(1 + \tfrac{1}{2}\xi \log \tfrac{N_c}{3}\xi) + L_M\xi + W_{qq}a + \tilde{W}_q a^2/\xi \qquad (3)$$

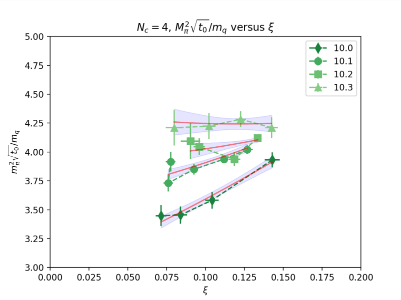
$$f_\pi = F(1 - \xi \log \tfrac{N_c}{3}\xi) + L_F\xi + W_F a \qquad (4)$$

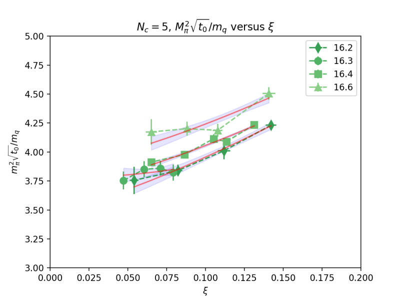
In this form:

- LEC dependence decouples, can fit equations independently.
- At fixed lattice spacing, $B \to B(a) = B + \frac{1}{2} W_{qq} a$.
  $F \to F(a) = F + W_F a$.

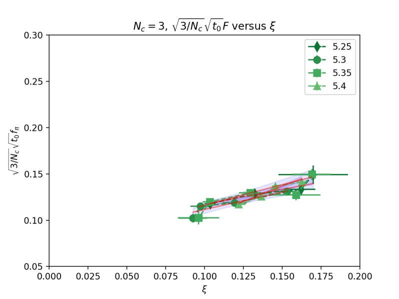
Pion Mass Plots — 2 Parameter Fits, beta by beta

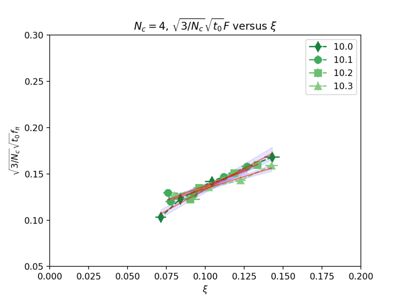


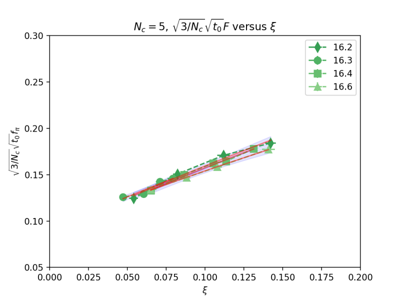




Decay Constant Plots — 2 Parameter Fits, beta by beta







Status

- B is a-dependent; F less so.
- m_π^2/m_q and f are smooth, nearly linear in ξ.
- $f \sim \sqrt{N_c}$, $B \sim N_c^0$ as expected.
- $N_c = 3$ is problematic: ξ is too big. Squeezed between breakdown of χPT and finite volume effects.

Conclusion

- We are still collecting data.
- Need to do a better job on $N_c = 3$ — smaller masses and larger volumes.

But:

- This is a case where large N_c is easier!
- Chiral expansion is in $\xi = m_\pi^2/8\pi^2 f_\pi^2 \sim m_q/N_c$.
- Range of applicability of χPT grows with N_c, as ξ falls at fixed m_q.

Thank you!