Semiclassical gravitational collapse of a massless, scalar quantum field

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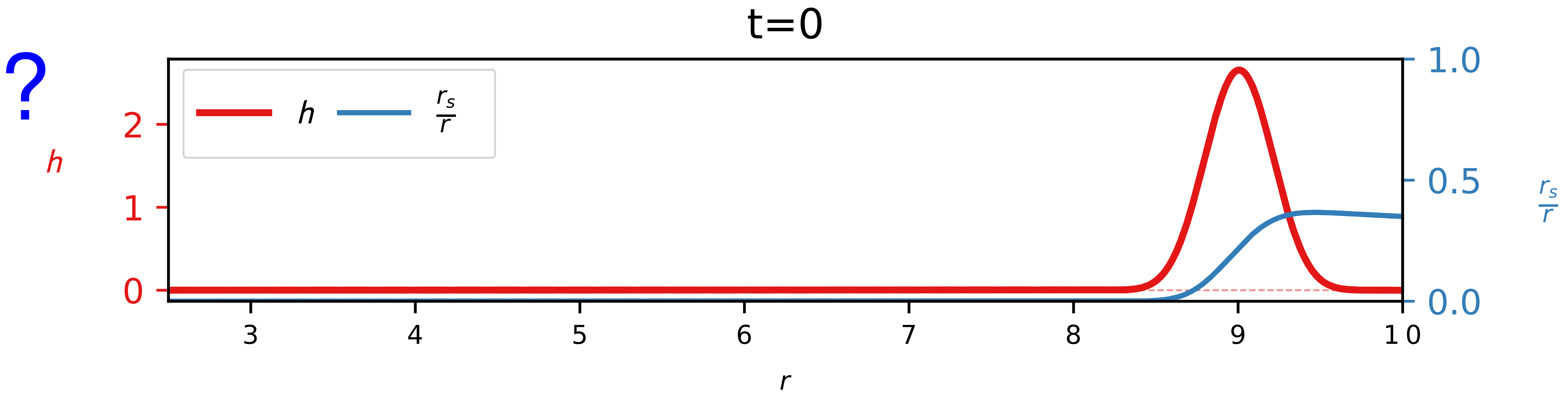
MOTIVATION

- How does a block hole evaporate (semiclassically)?
  - ? Is the time evolution unitary?
  - ? Does a horizon form?



  - ? Where does Hawking radiation originate from exactly?
- Ideal numerical treatment:
  - ☞ Real time quantum field theory
  - ☞ Self consistency: Field is the only source of curvature

SETUP

- Real time QFT:
  - ☞ Explicit time evolution in Fock basis
  - ☞ Usual problem: Interesting systems require Hamiltonian truncation
- Massless scalar field, coupled semiclassically to gravity:
  - ✓ Scalar field part of time evolution trivial: No Hamiltonian truncation
  - ✓ Gravitational part of time evolution traceable for coherent states
- Radial symmetry to simplify problem further

PARAMETERIZATION

- **Metric:** $d\tau^2 = \alpha^2(t,r)\,dt^2 - a^2(t,r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)d\Omega^2$
  Christodou '86, Choptuik '92

- **Scalar field:** $\bar{\phi}(t,r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \bar{\phi}_{lm}(t,r)\; Y_{lm}(\theta,\varphi)$

- **Field equations:**

$$\frac{a'}{a} + \frac{\alpha'}{\alpha} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \delta_{\varepsilon lm}\left(r\,\frac{a^2}{\alpha^2}\,\dot{\bar{\phi}}^{\dagger}_{lm}\,\dot{\bar{\phi}}_{lm} + r\,\bar{\phi}^{\dagger\prime}_{lm}\,\bar{\phi}'_{lm}\right)$$

$$\frac{a'}{a} - \frac{\alpha'}{\alpha} - \frac{1-a^2}{r} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \delta_{\varepsilon lm}\,\frac{a^2}{r}\,l(l+1)\,\bar{\phi}^{\dagger}_{lm}\,\bar{\phi}_{lm}$$

$$\frac{2\dot{a}}{ar} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \delta_{\varepsilon lm}(\dot{\bar{\phi}}^{\dagger}_{lm}\,\bar{\phi}'_{lm} + \bar{\phi}^{\dagger\prime}_{lm}\,\dot{\bar{\phi}}_{lm})$$

$\delta_{\varepsilon lm}\ldots$optional regularization factor (e.g. angular point splitting)

SCALAR FIELD EVOLUTION

- Redefine scalar field: $\phi_{lm} = \bar{\phi}_{lm}\, r \sqrt{\frac{a_0}{2\,\alpha_0}}$
- Scalar field Hamiltonian (density):

$$q_{0f} = \begin{pmatrix} -1 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 4/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 5/6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 6/7 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathscr{H}_{lm} = \Pi_{lm} A\, \Pi^{\dagger}_{lm} + \phi^{\dagger}_{lm} A^{-\frac{1}{2}} K_l A^{-\frac{1}{2}} \phi_{lm} \qquad K_l = q^{\dagger} q + \frac{\alpha^2}{r^2} l(l+1)$$

$$q = r\sqrt{\frac{\alpha}{a}}\,\partial_r \frac{1}{r}\sqrt{\frac{\alpha}{a}} \qquad A = \frac{a_0\,\alpha}{\alpha_0\, a}$$

- Diagonalize Hamiltonian for initial time: $K_l(t_0) = V_l\, \omega_l^2\, V_l$

HEISSENBERG PICTURE

- Parameterize time evolution: $\phi_{lm} = \frac{u_l^\dagger(t)\, b_{lm+}^\dagger + u_l^T(t)\, b_{lm-}}{\sqrt{2}}$ $\Pi_{lm} = \frac{b_{lm+}\, v_l(t) - b_{lm-}^\dagger\, v_l^*(t)}{\sqrt{2}}$
- Time evolution $v_l(t)$ and $u_l(t)$ in coefficients:

$$u_l(t_0) = \frac{1}{\sqrt{\omega_l}} V_l^T \qquad v_l(t_0) = \sqrt{\omega_l}\, V_l^T$$

$$\dot{u}_l = -i\, v_l A \qquad \dot{v}_l = -i\, u_l \frac{1}{\sqrt{A}} K_l \frac{1}{\sqrt{A}}$$

- Bogolyubov identities: $\mathrm{Re}(v_l^\dagger u_l) = \mathrm{Re}(u_l^\dagger v_l) = \mathbb{1}$ $\mathrm{Im}(u_l^\dagger u_l) = \mathrm{Im}(v_l^\dagger v_l) = 0$

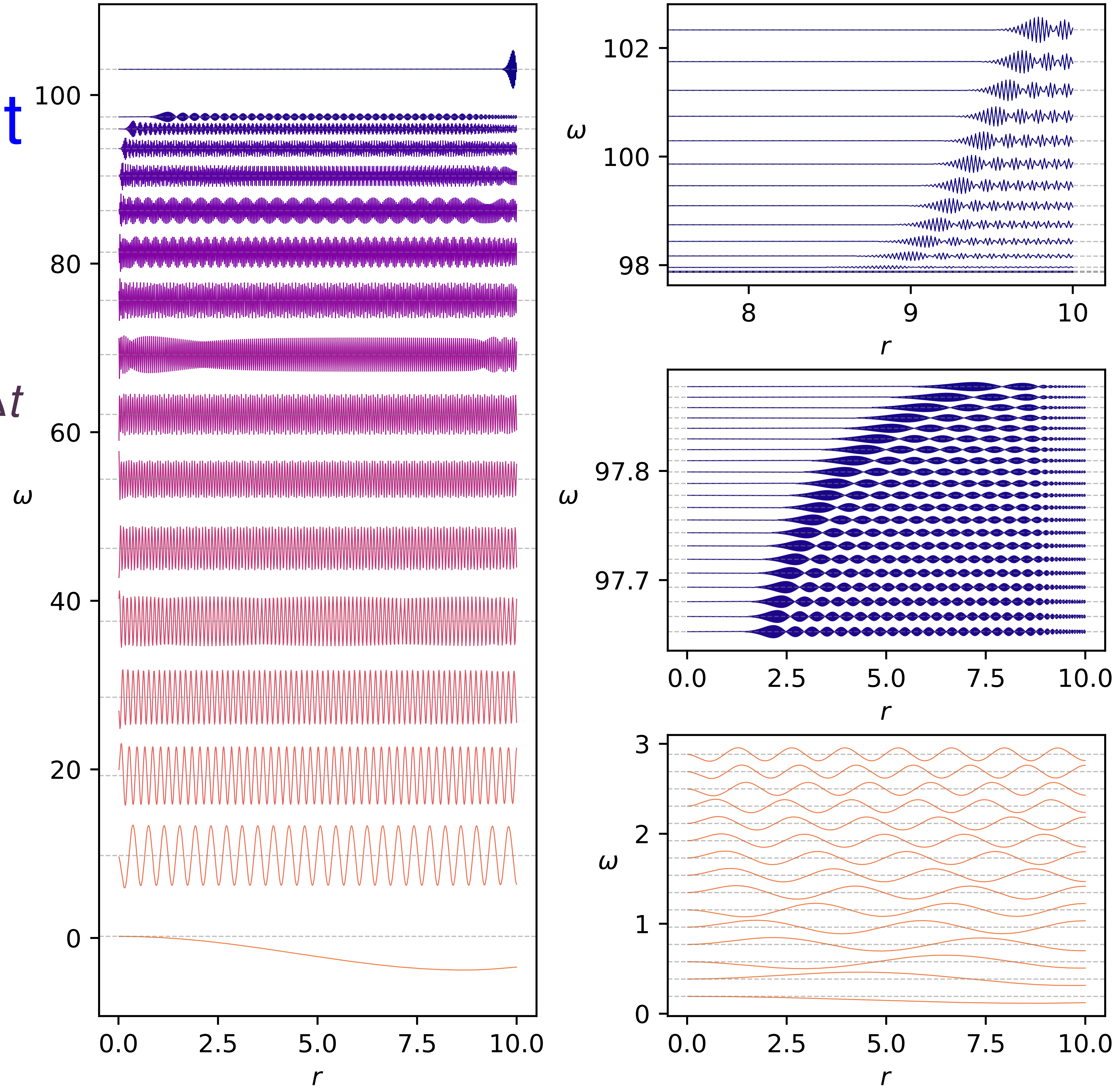
NUMERICAL IMPLEMENTATION

- Finite time evolution as exact Bogolyubov transformation:

$$\begin{pmatrix} u_{t+\Delta t} & v_{t+\Delta t} \end{pmatrix} = \begin{pmatrix} u_t & v_t \end{pmatrix} e^{-i\begin{pmatrix} 0 & \frac{1}{\sqrt{A}} K_l \frac{1}{\sqrt{A}} \\ A & 0 \end{pmatrix}\Delta t}$$

- Need explicit diagonalization:

$$K_l = \overline{V}_l\, \overline{\omega}_l^2\, \overline{V}_l$$



EXPECTATION VALUES

- Vacuum expectation values (e.g. Hamiltonian density):

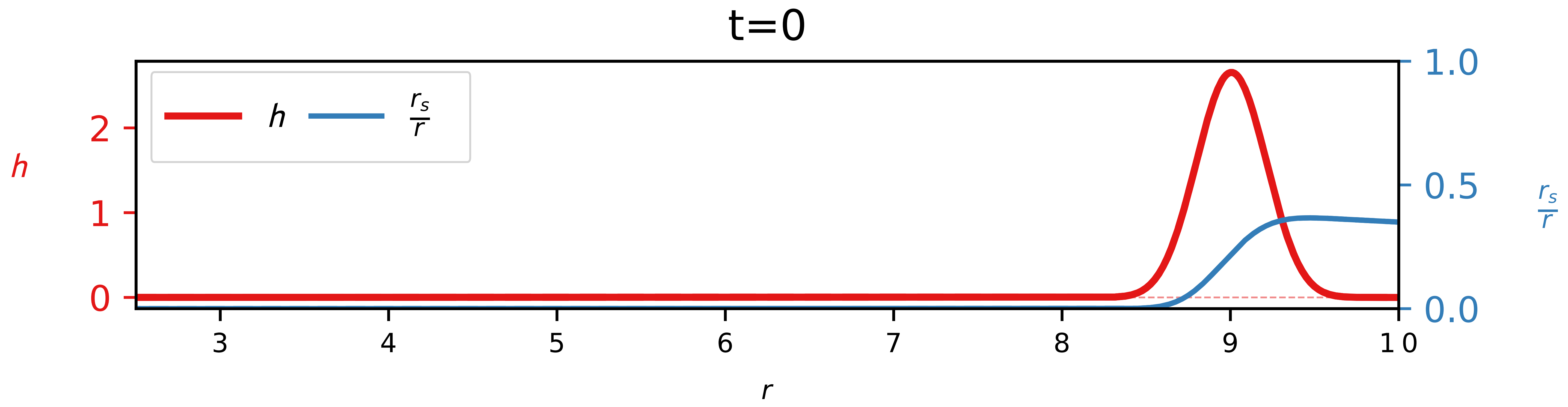
$$\langle 0|\mathscr{H}_{lm}|0\rangle_r = \frac{N_f}{2}\sqrt{A}\left(q_0 u_l^\dagger\, u_l\, q_0^T + u_l^\dagger\, u_l \frac{l(l+1)}{r^2}\frac{\alpha^2}{A^2} + v_l^\dagger v_l\right)_{rr}\sqrt{A}$$

- Initially coherent state (only $l = 0$):

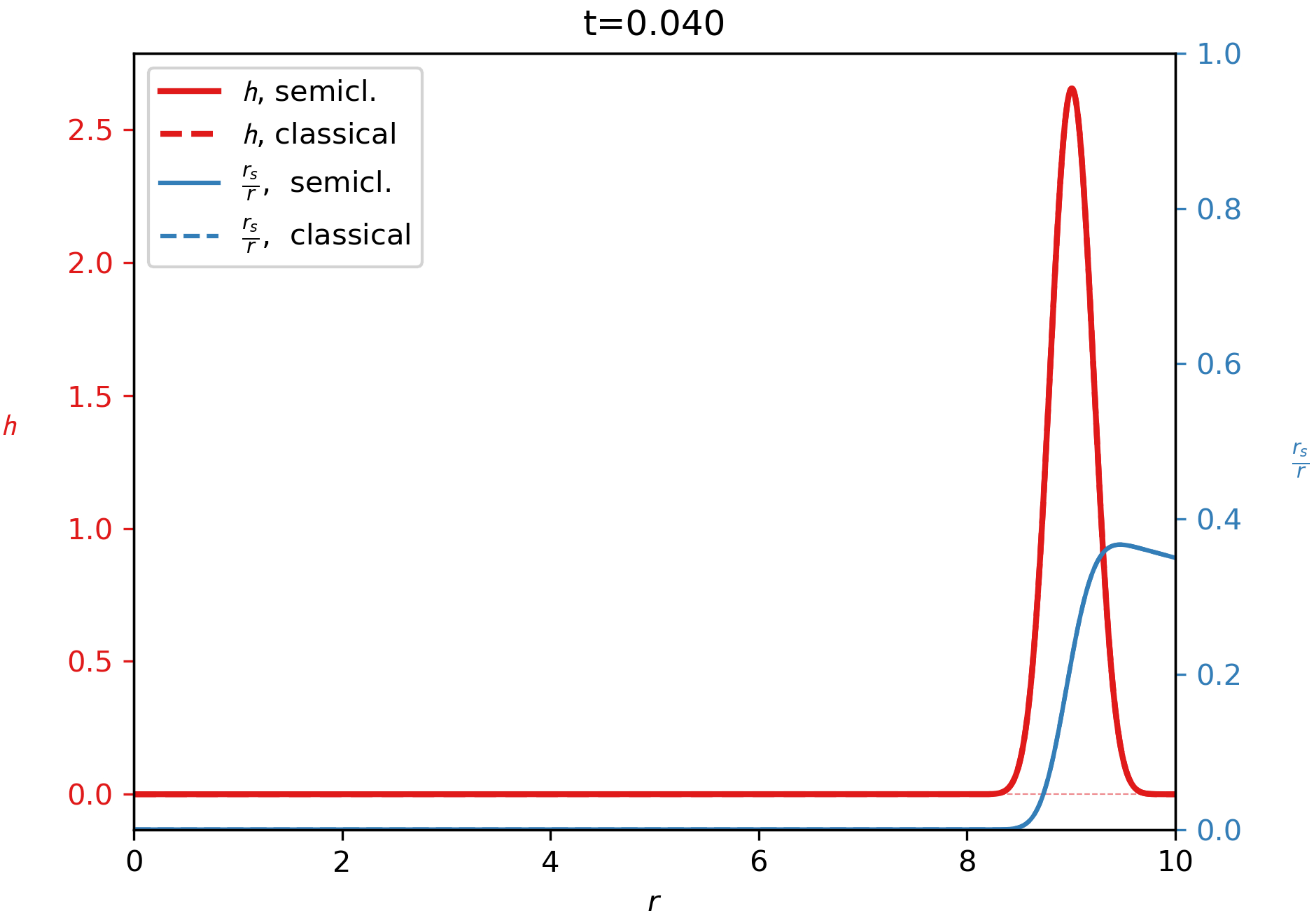
$$h_r = \langle\lambda|\mathscr{H}|\lambda\rangle_r = \langle 0|\mathscr{H}|0\rangle_r + h_r^{\text{class}}$$

- Amplify vacuum effects: only excite one component ϕ_i out of N_f

FIRST ATTEMPT

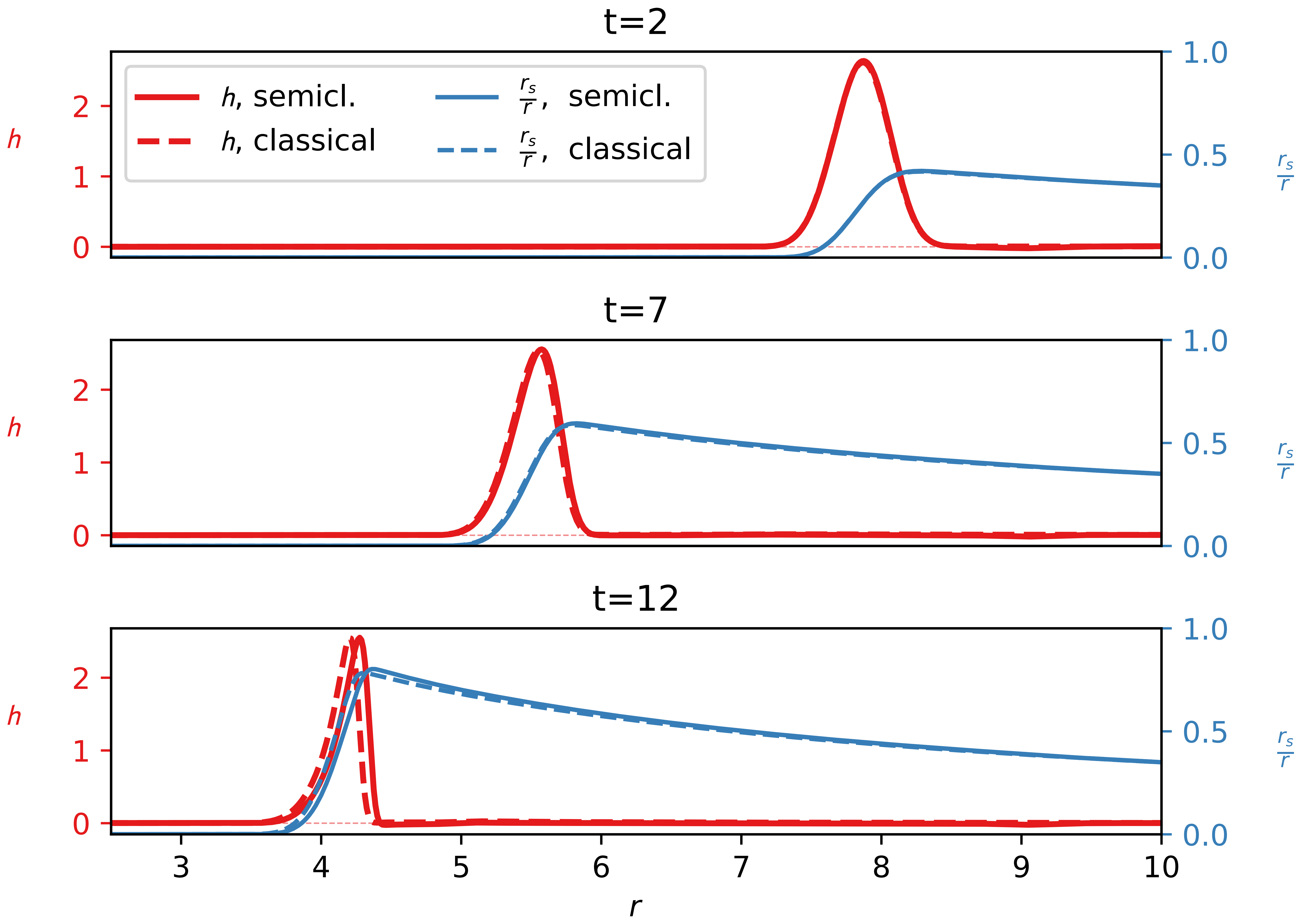


- Truncate to $l = 0$:
  - ☞ Initial coherent state ϕ^{class} and momentum Π^{class} (Nuttall window)
  - ☞ Normal order at initial time
  - ☞ Integrate metric radially from h_r alone
- Leapfrog radial metric integration with scalar field evolution (symplectic)



t=0.040
h, semicl.
h, classical
$\frac{r_s}{r}$, semicl.
$\frac{r_s}{r}$, classical
h
$\frac{r_s}{r}$
r
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

FIRST ATTEMPT



$N_r = 800$

$\Delta t = 0.004$

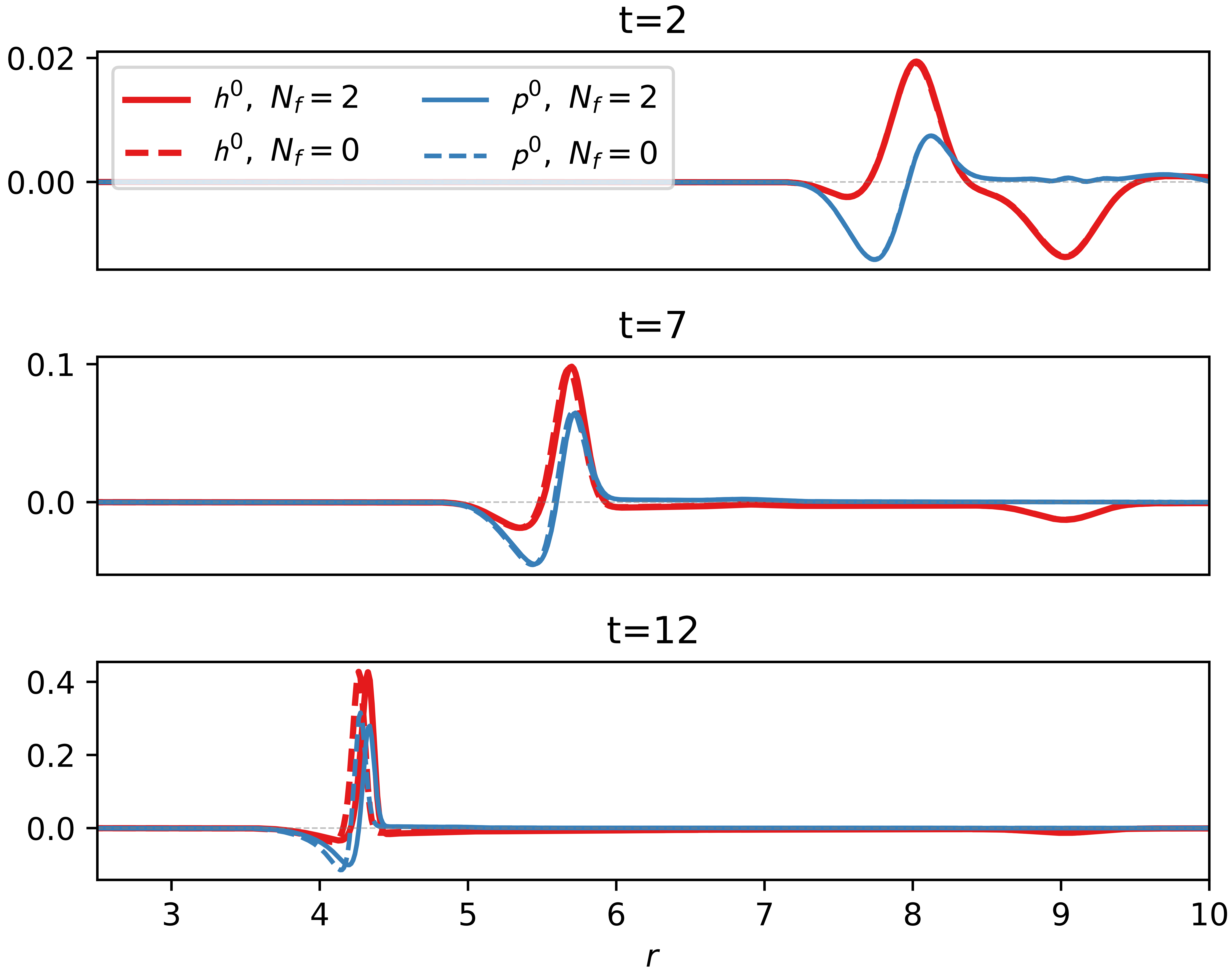
$N_f = 2$

$r_s = 3.5$ (total energy content)

$r_{\max} = 10$

initial bump around $r = 9$

VACUUM CONTRIBUTION



$l = 0$ vacuum contribution:

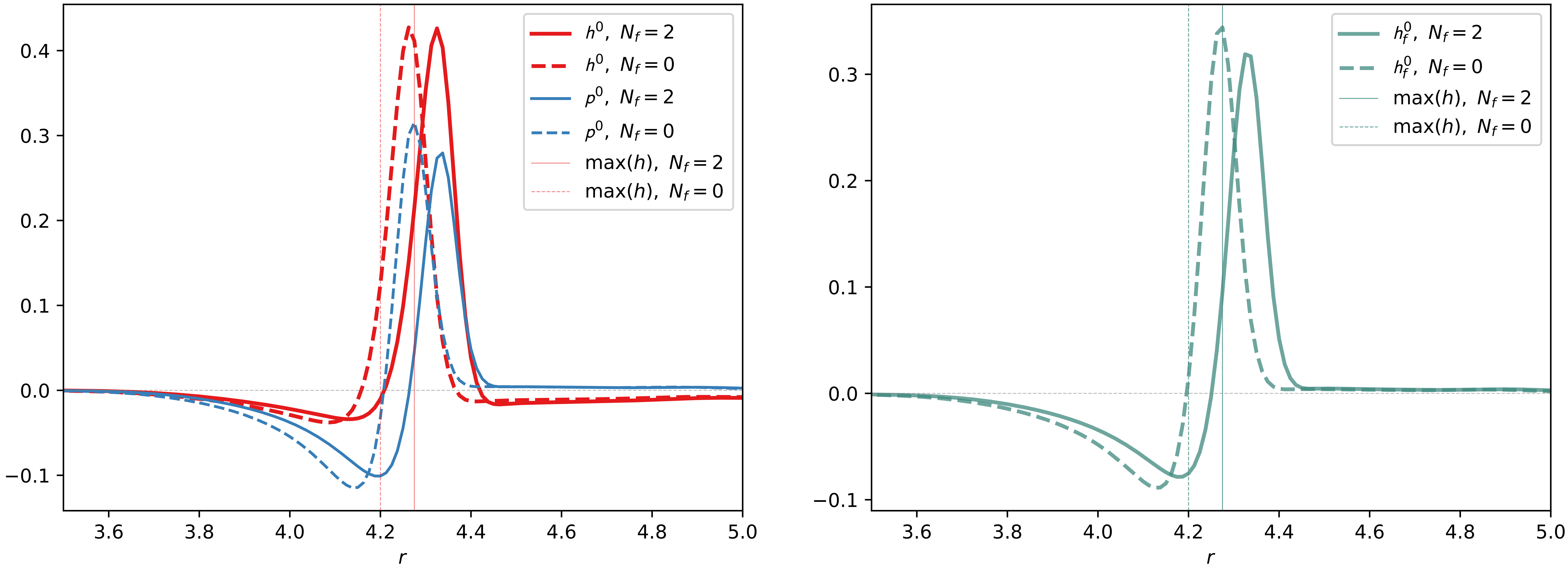
Energy flow (p^0) towards peak

From inside and outside

Horizon formation accelerated

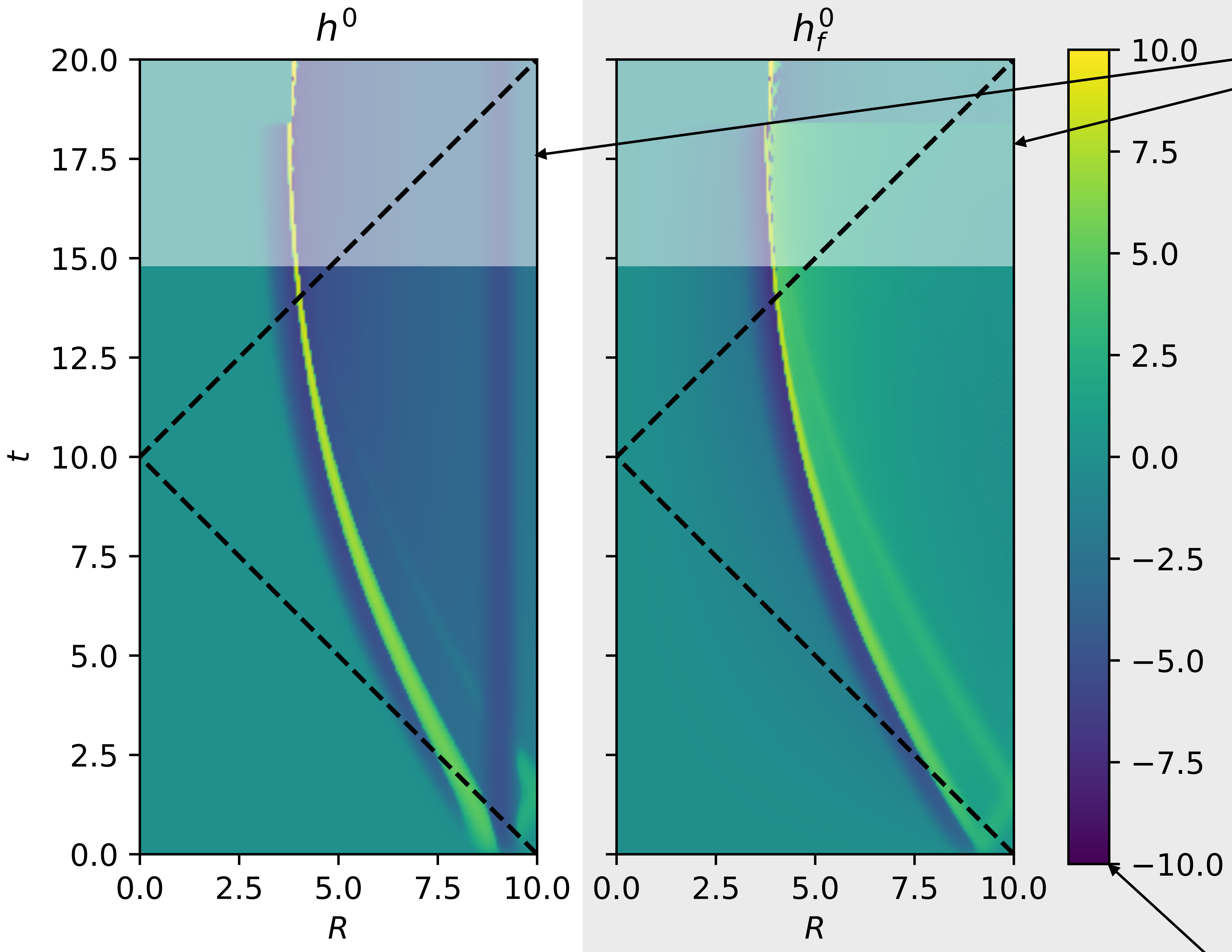
Small backreaction effect

VACUUM CONTRIBUTION ($t = 12$)



Relative to current state vacuum: Only outward transfer

VACUUM CONTRIBUTION



numerically unsafe region

Both energy densities are
- ✔ nontrivial
- ✔ distinct
- ✔ numerically stable

despite several order of magnitude of vacuum subtraction

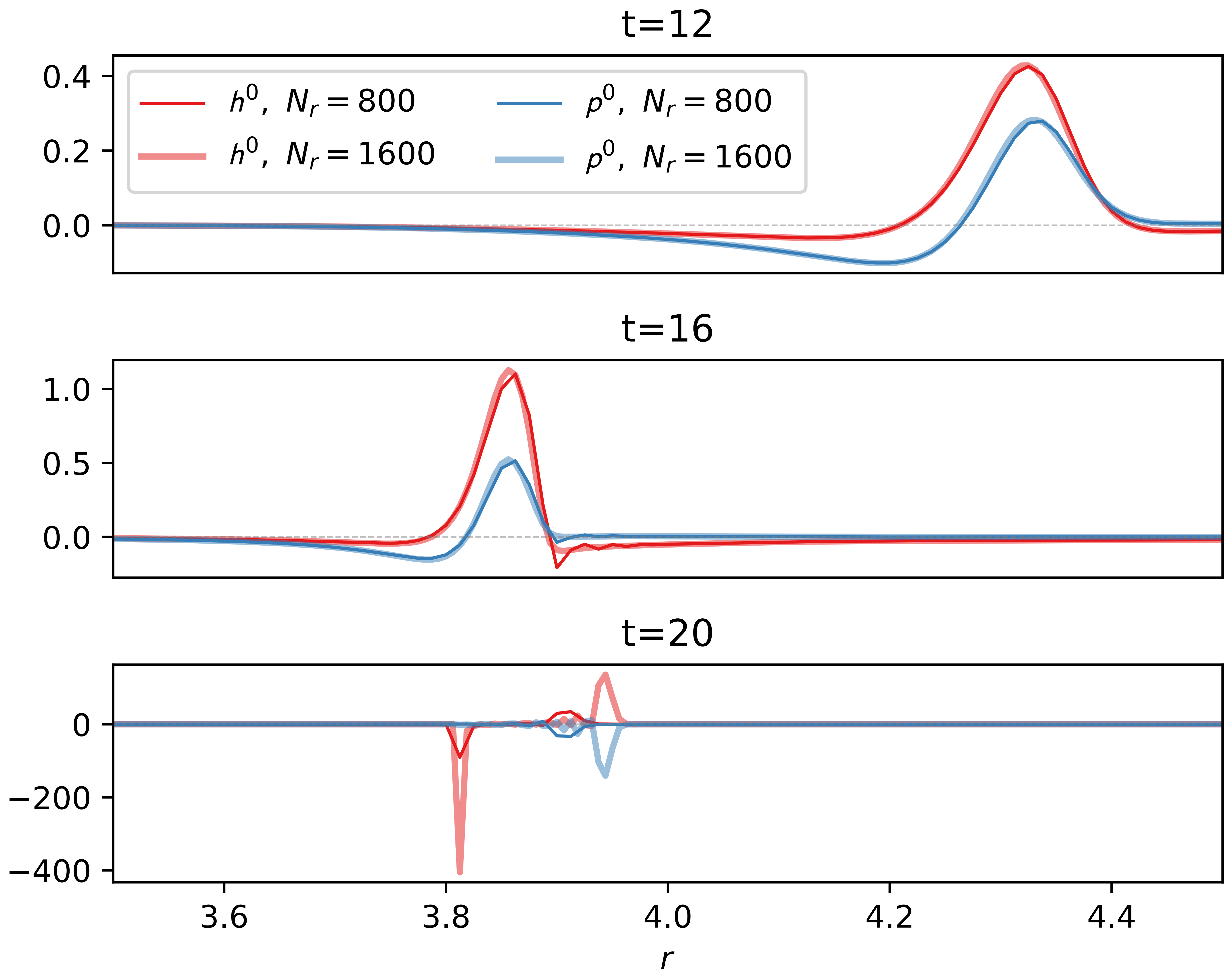
relative to initial state vacuum

relative to current state vacuum

log scale: positive and negative

$\mathrm{sign}(x)\ln(1+10^4|x|)$

CHECKS



Cutoff effects:

$N_r = 800, 1600$

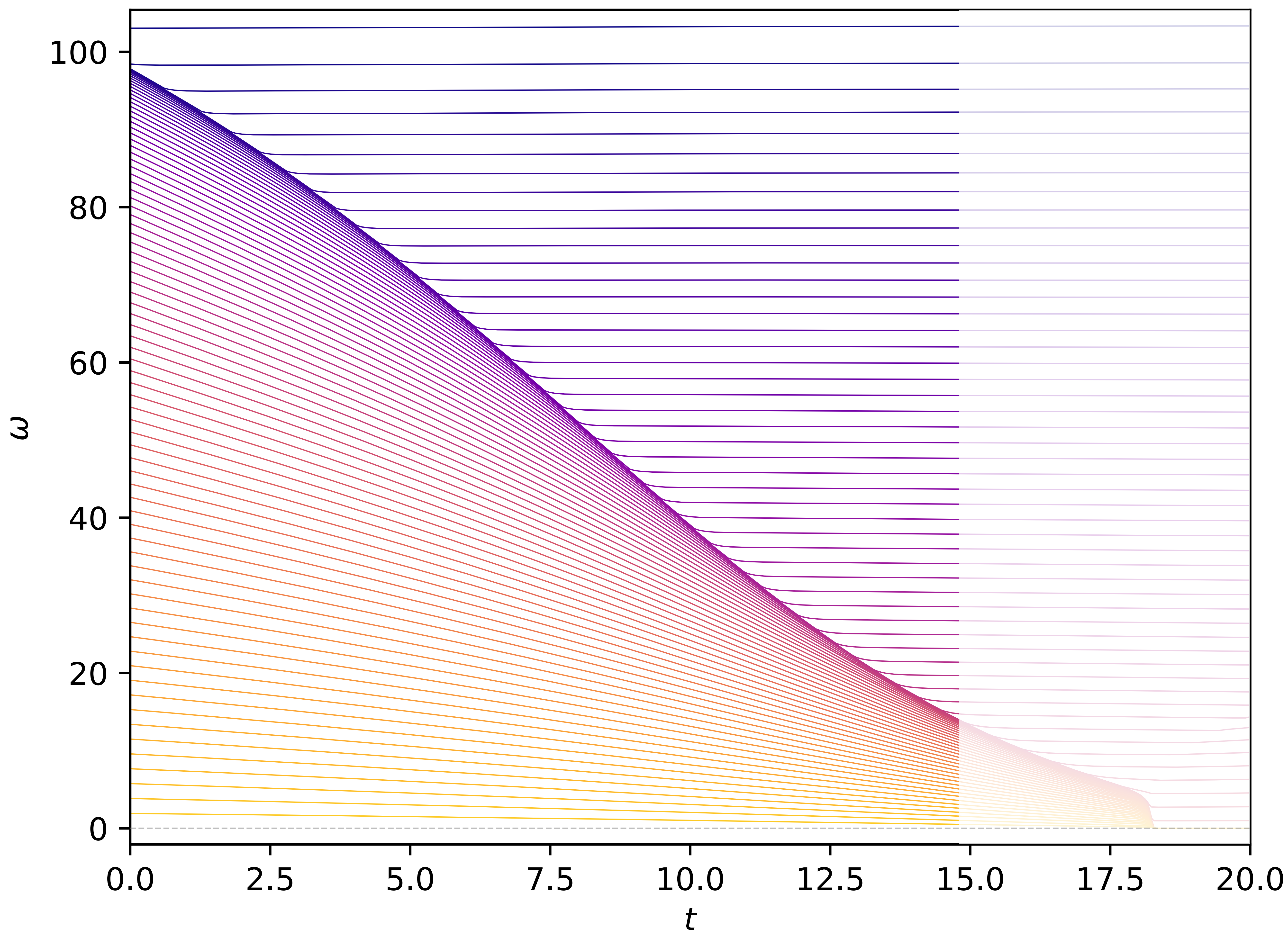
Also checked (not shown):

- finite V
- absorbing/reflecting BC
- bump shapes/positions
- varying Δt
- initial vacuum subtractions
- various radial integrations
- radial discretizations

EIGENMODES

Eigenvalues ω of $q^T q$:

- $\omega \to 0$: horizon formation
- Actual formation: outside validity range



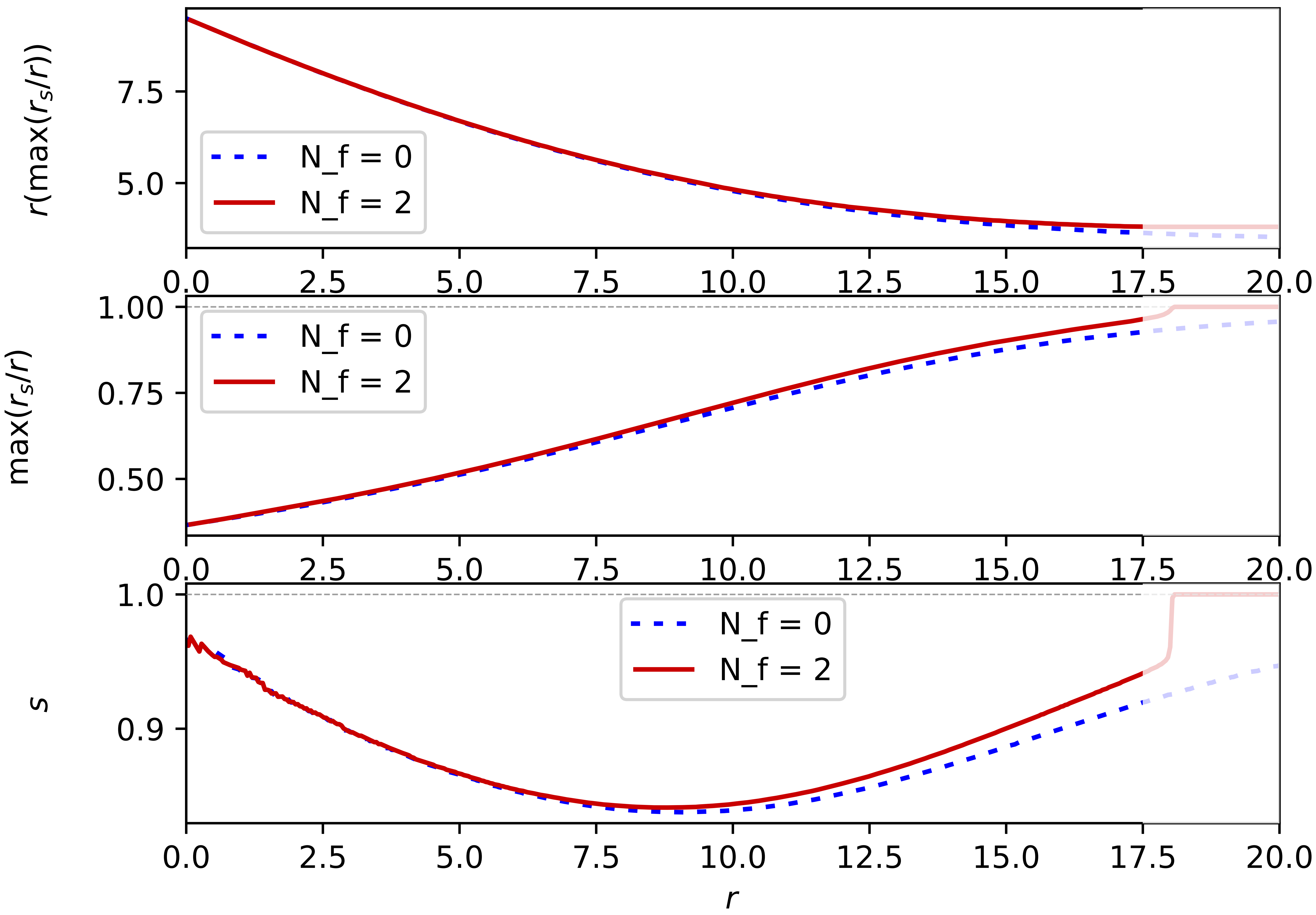
$l = 0$ SUMMARY

$l = 0$ vacuum effects:

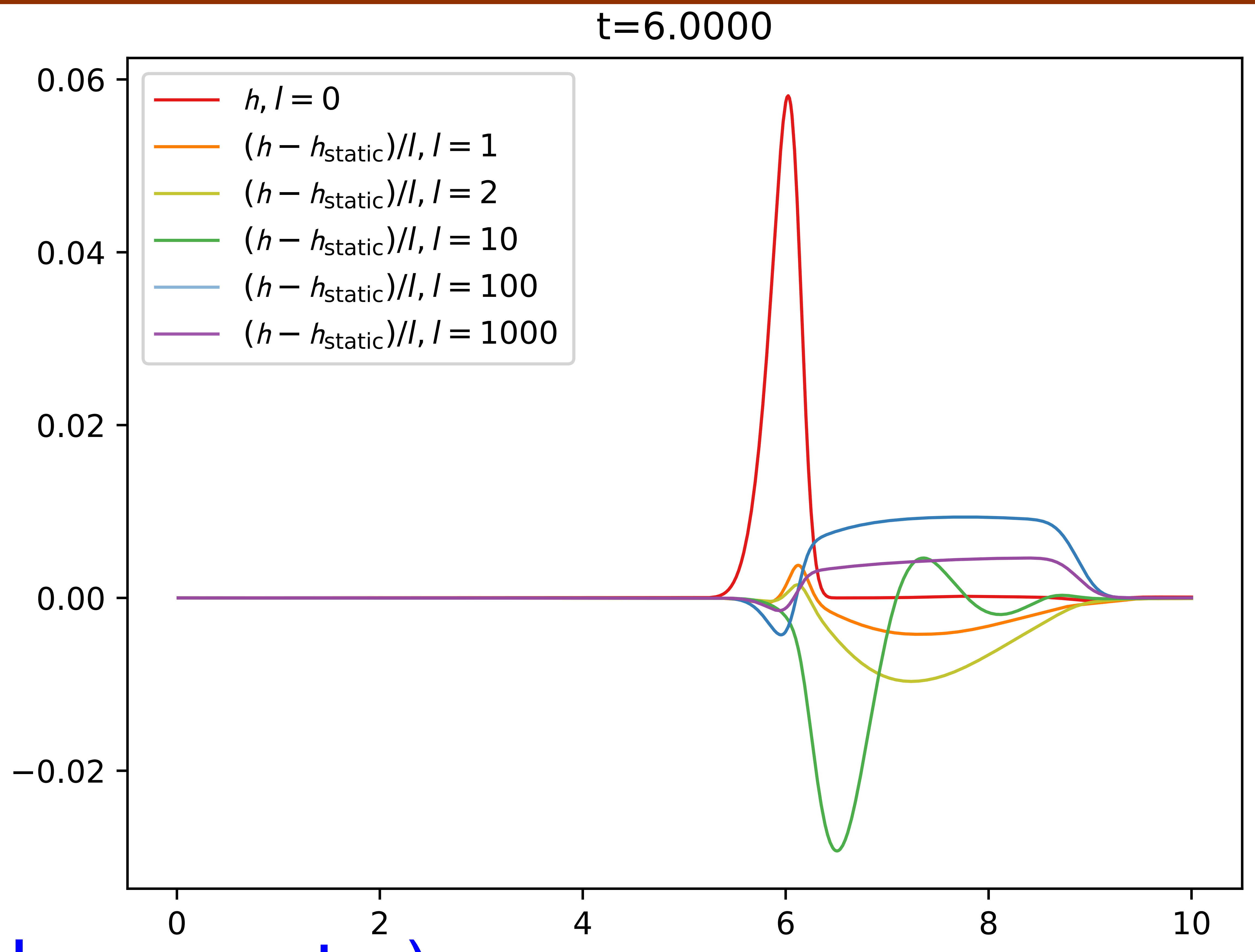
Horizon forms at larger r

Horizon forms more quickly

Modes separate faster



FUTURE WORK



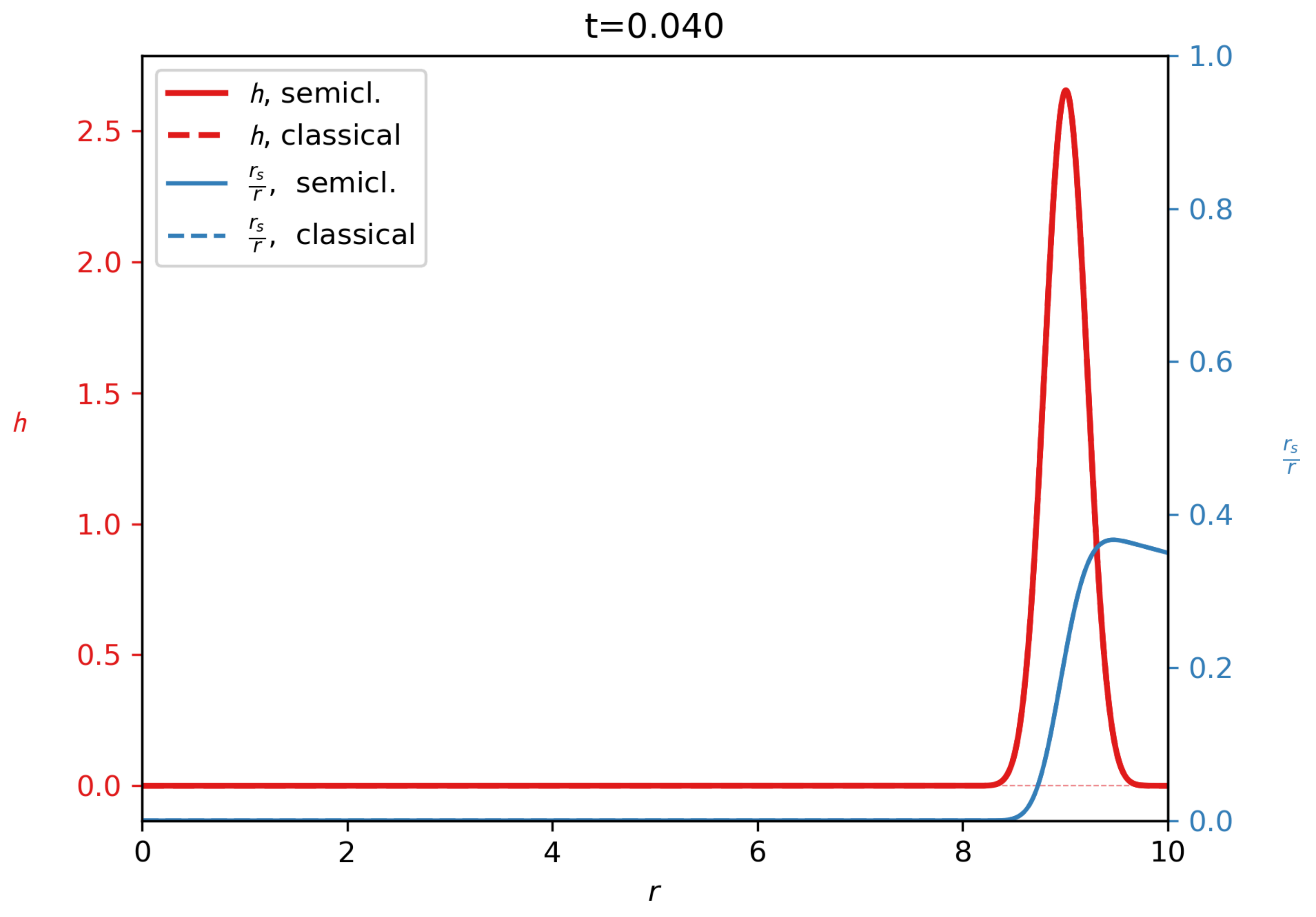
Investigate $l > 0$ modes:

- More complex metric integration
- Vacuum subtraction not sufficient
  - Static term subtraction?
  - Point-split regularization?

More complex scenarios (e.g. no radial symmetry)

Is it unitary? (formation+decay)

THANK YOU



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h, classical
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$\frac{r_s}{r}$, classical
h
$\frac{r_s}{r}$
r

LITERATURE

Classical collapse (selection):

D. Christodoulou, *Commun.Math.Phys.* 105 (1986) 337, Commun. Math. Phys. 105 (1986) 337

M. W. Choptuik *Phys.Rev.Lett.* 70 (1993) 9

Semiclassical,analytic results (selection):

Akira Tomimatsu, *Phys.Rev.D* 52 (1995) 4540

H. Kawai, Y. Yokokura, *Universe* 6 (2020) 6, 77

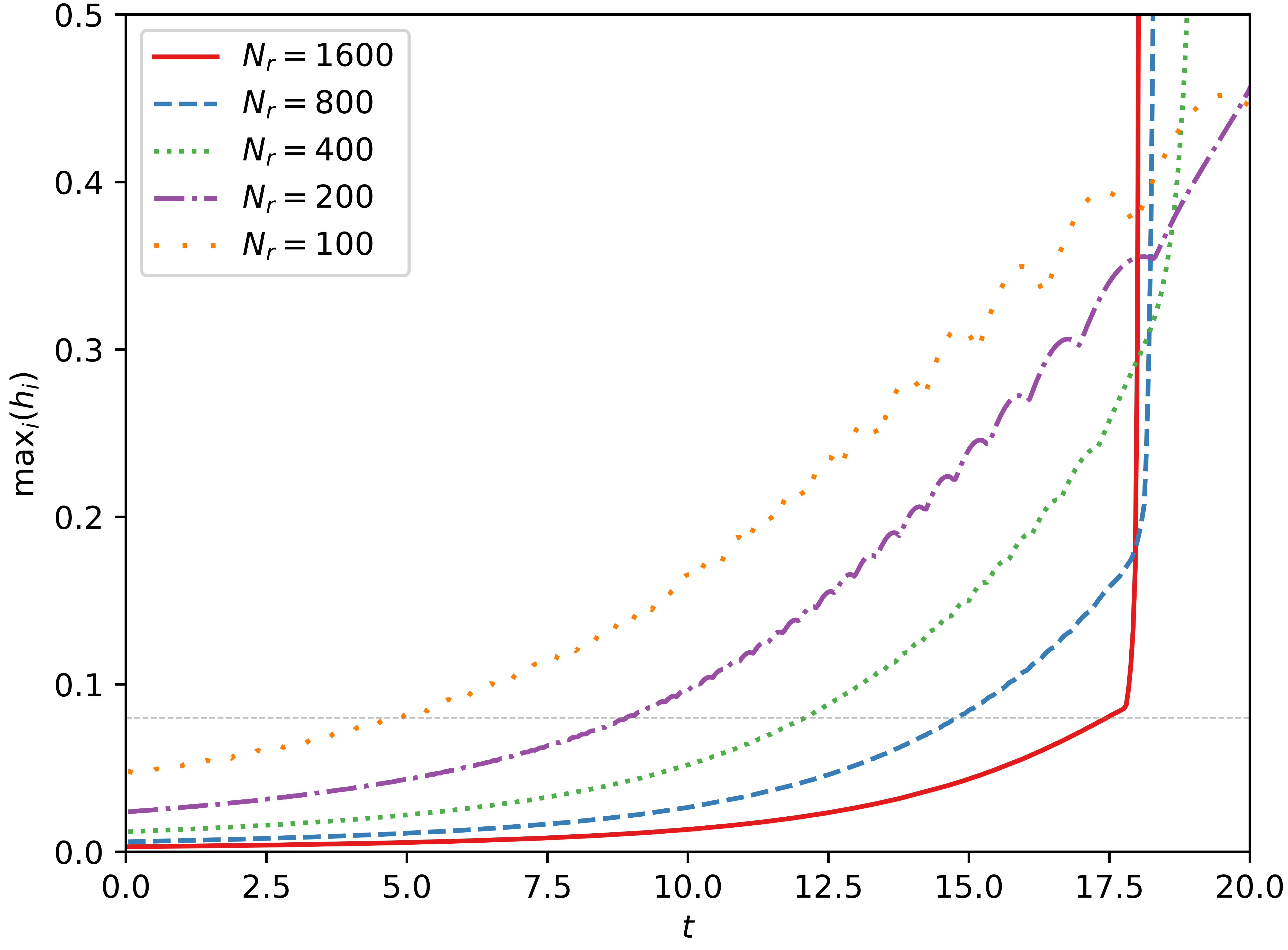
Semiclassical, numerical:

B. Berczi, P. M. Saffin,S. Zhou 2010.10142

NUMERICAL VALIDITY RANGE

Estimate integration accuracy via maximum h_r

Allows determination of validity range from runs themselves



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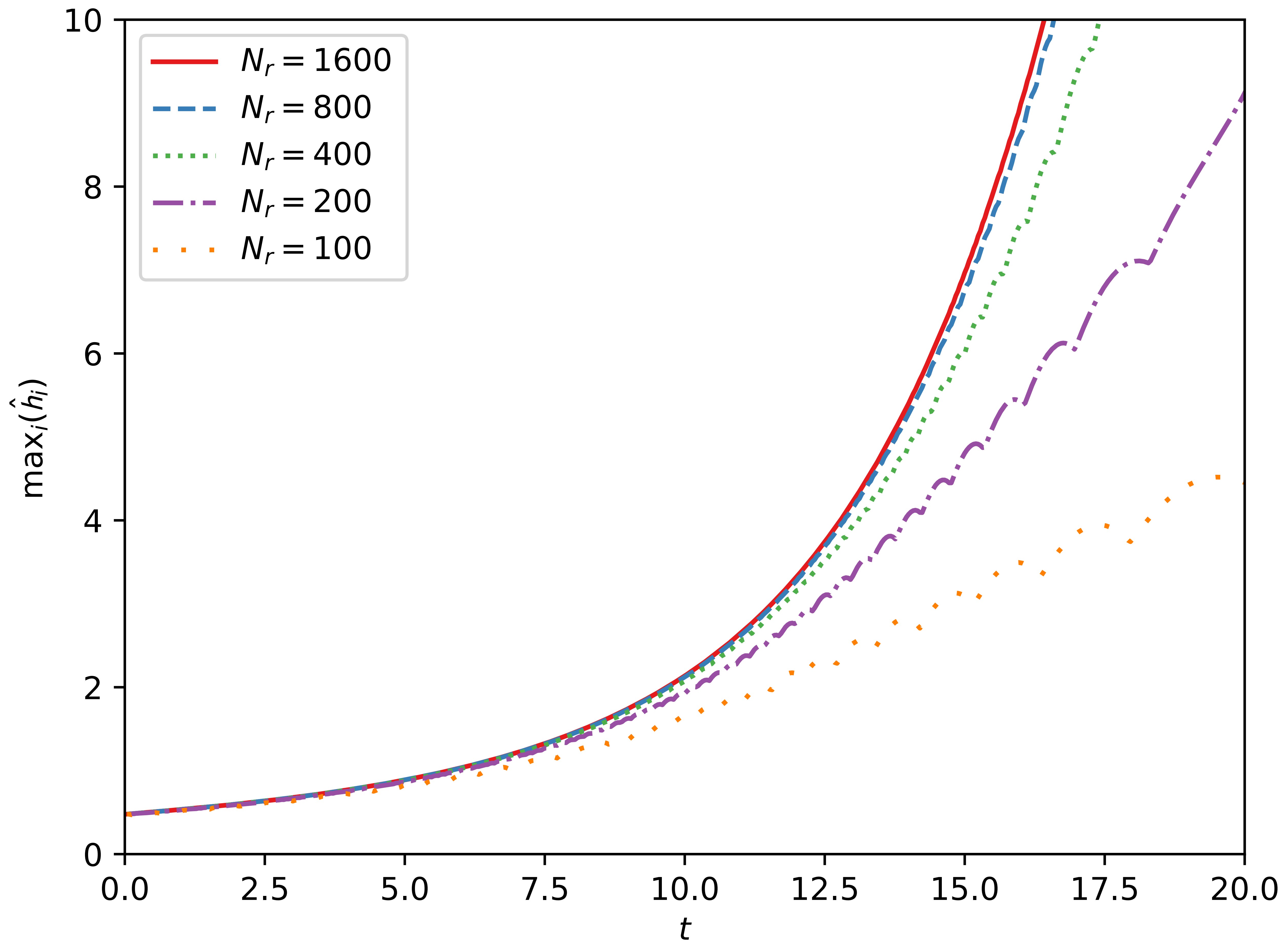
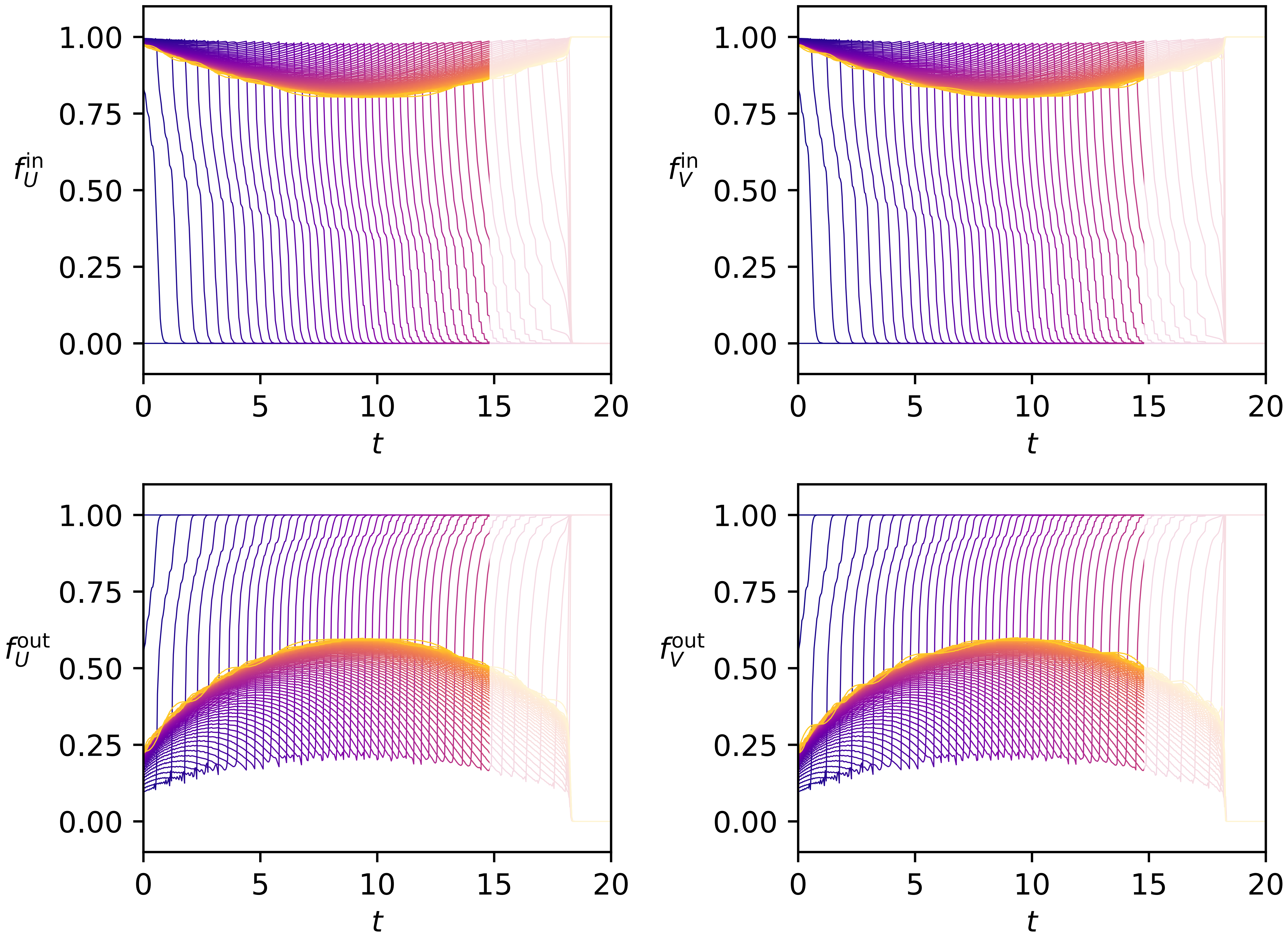


EIGENMODES

Singular vectors of q:

- "inside" vs. "outside" (relative to peak r_s/r)
- diminishing overlap as horizon forms



TIME EVOLUTION

$$u_l(t+\Delta t) = \left(u_l(t)\, A^{-\frac{1}{2}} \overline{V}_l \cos(\overline{\omega}_l \Delta t) - i\, v_l(t) \sqrt{A}\, \overline{V}_l\, \overline{\omega}_l^{-1} \sin(\overline{\omega}_l \Delta t)\right) \overline{V}_l^T \sqrt{A}$$

$$v_l(t+\Delta t) = \left(-i\, u_l(t) A^{-\frac{1}{2}} \overline{V}_l\, \overline{\omega}_l \sin(\overline{\omega}_l \Delta t) + v_l(t) \sqrt{A}\, \overline{V}_l \cos(\overline{\omega}_l \Delta t)\right) \overline{V}_l^T A^{-\frac{1}{2}}$$

CHECK: BIRKHOFF THEOREM