Nonperturbative infrared finiteness in super-renormalisable scalar quantum field theory

Image: ESA and Planck Collaboration

Andreas Jüttner

JULY 26-30 2021, ZOOM/GATHER@MIT

LatCos Collaboration

Guido Cossu	Edinburgh, now Braid Technologies
Luigi Del Debbio	Edinburgh
Elizabeth Dobson	Edinburgh, now University of Graz
Elizabeth Gould	Southampton, now in Queen's Uni, CA
Ben Kitching-Morley	Southampton
Andreas Jüttner	CERN, Southampton
Joseph K.L. Lee	Edinburgh
Valentin Nourry	Edinburgh and Southampton, now University de Paris
Antonin Portelli	Edinburgh
Henrique Bergallo Rocha	Edinburgh
Kostas Skenderis	Southampton

Papers

- Nonperturbative infrared finiteness in super-renormalisable scalar quantum field theory, Phys.Rev.Lett. 126 (2021) 22, 221601
- Renormalisation of the energy-momentum tensor in three-dimensional scalar SU(N) theories using the Wilson flow, Phys.Rev.D 103 (2021) 11, 114501

Related talks at this conference:

- Henrique Bergallo Rocha, Wed 14:15-30
- Joseph Lee, Wed 14:30-14:45

Motivation: Cosmology

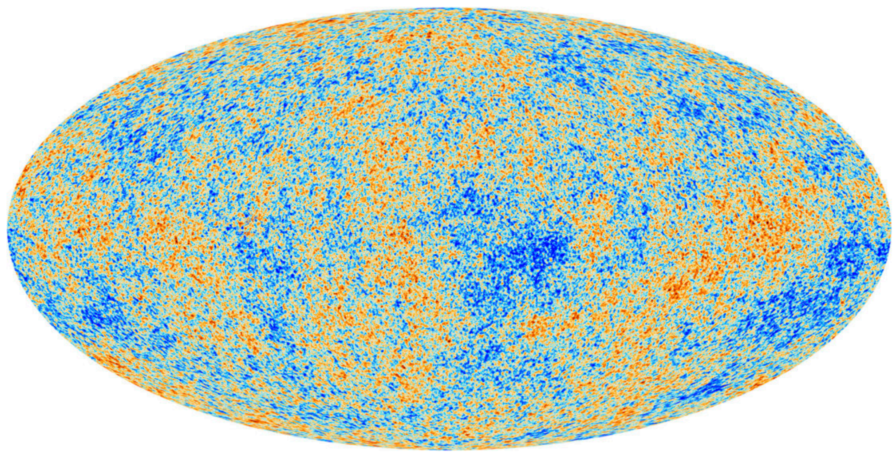
Problems in standard cosmology solved by inflation:

- Horizon problem
- Flatness problem
- Formation of structure

The particle physics interpretation of inflation seems ad hoc:

- Fine tuning of inflation potential and inflaton
- Initial conditions
- EFT — what about UV completion towards the very early universe (initial singularity)?

Famous testable prediction in many models of inflation:
power-law primordial power spectrum



ESA and the Planck Collaboration

Holographic Cosmology

Idea: Quantum gravity description of early universe in terms of Holographic dual QFT

Cosmological observables are mapped to correlation functions of the dual QFT

Witten (2001)] [Strominger (2001) (dS/CFT correspondence)
Maldacena (2002) (wavefunction of the universe)
McFadden and Skenderis (2009) (Holographic Cosmology)

Dual theory is 3d QFT with large-N scaling with massless scalars, fermions and gauge fields

Start with study of simplest model

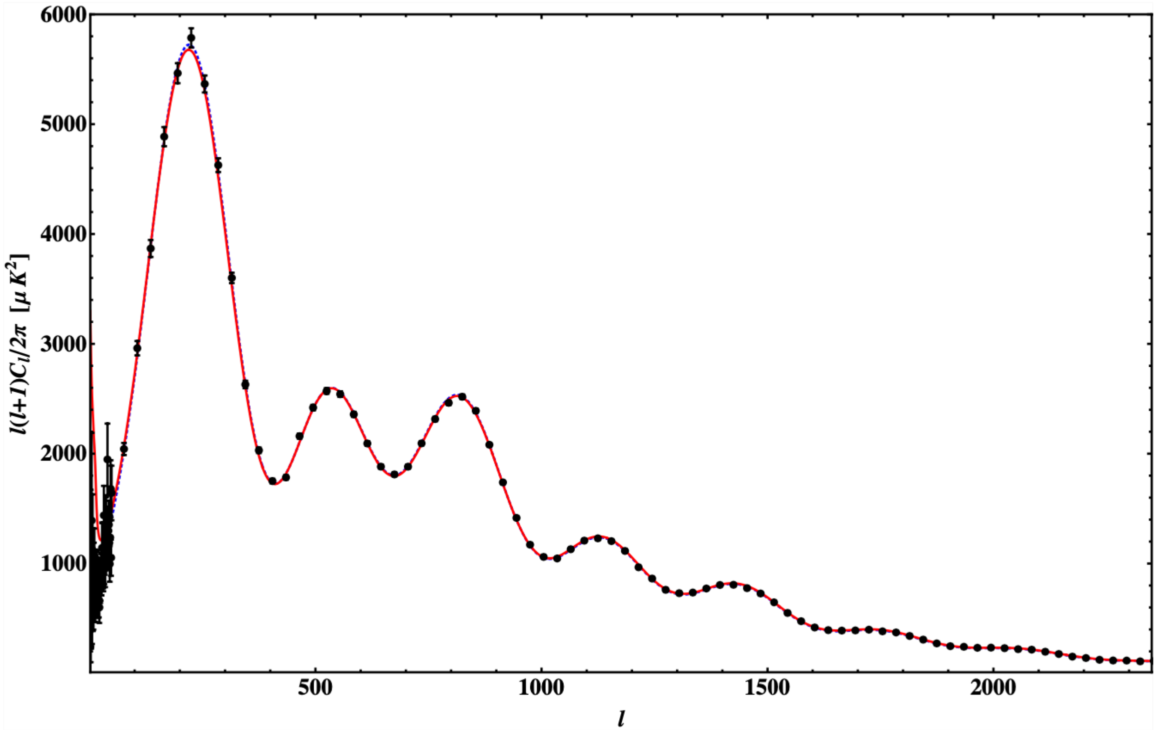
massless 3d scalar SU(N) matrix ϕ^4 theory

$$\phi(x) = \phi^a(x) T^a$$

$$S = \frac{N}{g} \int d^3x \mathrm{Tr} \left((\partial_\mu \phi(x))^2 + \phi^4(x) \right)$$

- $[\phi] = 1, [g] = 1$
- Mass UV divergent
- With each order in PT UV(IR) deg. of. div. decreases(increases)
- IR divergent in PT
- $N = 2$ model equivalent to $O(3)$ vector model

Start with study of simplest model



Model studied in PT at 2-loop

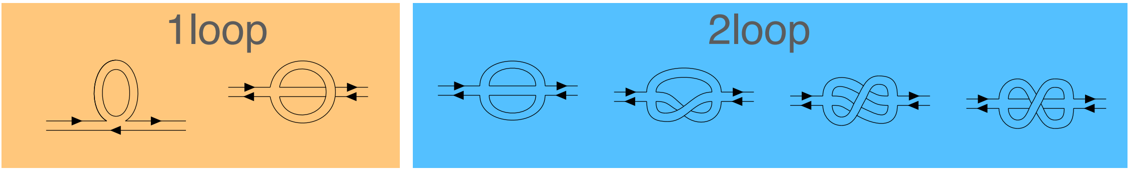
Coriano, Delle Rose, Skenderis EPJC 81 (2021) 2, 174

- $g^2_{\rm eff} > 1$ for $l < 260$
- For $l > 260$ model fits better than ΛCDM

Understanding the low-l region better requires a non-perturbative study → lattice study

Massless point — perturbatively

$$S = \frac{a^3 N}{g} \sum_{x \in \Lambda^3} \mathrm{Tr}\left((\delta_\mu \phi(x))^2 + (m^2 - m_c^2)\phi^2(x) + \phi^4(x) \right)$$



$Z_0 = 0.252731\ldots$

$$\frac{Z_0}{a} = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \frac{1}{\hat{p}^2}$$

$$D(p) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{\hat{k}^2\, \hat{q}^2\, \hat{r}^2}$$

$(r = -k - q - p)$

$$m_c^2 = -g \frac{Z_0}{a}\left(2 - \frac{3}{N^2}\right) + g^2 D\,(p = \Lambda_{\mathrm{IR}})\left(1 - \frac{6}{N^2} + \frac{18}{N^4}\right)$$

M. Laine and A. Rajantie, Nucl. Phys. B 513, 471 (1998),
LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

IR behaviour

Log divergent $D(ap) \overset{p\to 0}{=} -\frac{\log(ap)^2}{(4\pi)^2}$ power divergences at higher orders

Lüscher, Weisz, *NPB* 445 (1995) 429-450
LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

IR divergence would render theory and therefore whole idea of Holographic Cosmology useless

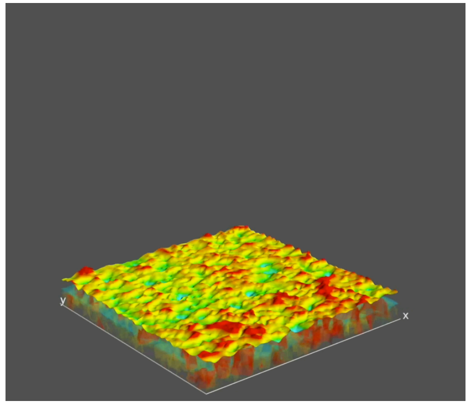
It is however expected that theory non-perturbatively IR-finite and the dimensional coupling g plays the roles of the IR regulator

Jackiw, Templeton PRD 23 1981,
Applequist, Pisarski PRD 23 1981

HERE: address question of IR finiteness and study critical properties

Lattice study of scalar SU(N) matrix QFT

- Theory and observables implemented in Grid
- Ensemble generation on SKL cluster (STFC DiRAC Distributed Research utilizing Advanced Computing and U. of Southampton Iridis5)
- O(100k) trajectories per ensemble



Simulation parameters:

N	2,4
g	0.1,0.2,0.3,0.5,0.6
L	8,16,32,48,64,96,128
m^2	many masses in vicinity of 2-loop

Data and analysis code on Zenodo

Data	DOI 10.5281/zenodo.4266114
Analysis code	DOI 10.5281/zenodo.4290508

Guidance for lattice study from finite-volume EFT

Interpretation of lattice results guided by effective theory of magnetisation

$$M = \frac{1}{L^3}\int d^3x\,\phi(x)$$

$$S_{\text{eff}} = \frac{L^3 N}{g}\left[(m^2 - m_c^2)\operatorname{Tr}[M^2] + \operatorname{Tr}[M^4]\right]$$

Quantisation
$$\langle O[M]\rangle = \frac{1}{\mathcal{Z}_{\text{eff}}}\int_{\mathfrak{su}(N)} dM\, O[M]\exp(-S_{\text{eff}}[M])$$

Analytical result for exp.val.:
$$\langle \operatorname{Tr}(M^k)^l\rangle = \left(\frac{g}{L^3 N}\right)^{\frac{kl}{4}} \frac{\Psi_{kl}\left(m^2\sqrt{\frac{NL^3}{g}}\right)}{\Psi_{00}\left(m^2\sqrt{\frac{NL^3}{g}}\right)}$$

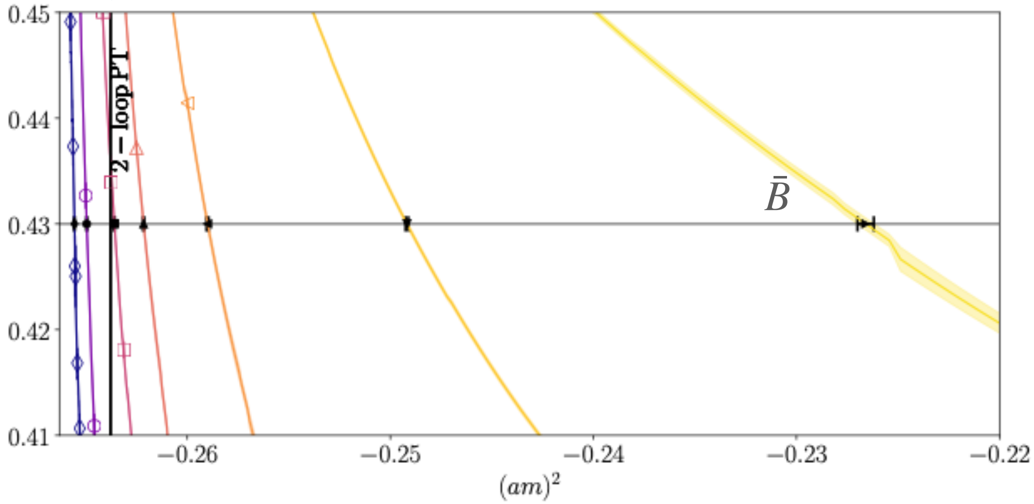
LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Finite-size scaling study

PT with g as IR cutoff

Binder cumulant

$$B = 1 - \frac{N}{3}\frac{\langle \mathrm{Tr}(M^4)\rangle}{\langle \mathrm{Tr}(M^2)\rangle^2}$$



- Reweighted lattice data
- monitor crossing with choice of $\bar{B}$
- scaling according to EFT:

$$f\left[\frac{\sqrt{N}}{g^2}\left[\overline{m}^2(L) - m_c^2\right](gL)^{1/\nu}\right] = \bar{B}\,.$$

LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Finite size scaling study

The EFT motivates the global (g, L) FSS fit-ansatz:

$$\overline{m}^2(\bar{B},g,L) = m_c^2(g)|_{1-\mathrm{loop}} + g^2\alpha + g^2\left((gL)^{-1/\nu}\frac{\bar{B}-f_0}{f_1} + \beta D_{\mathrm{IR}}(\Lambda_{\mathrm{IR}})\mathcal{N}(N)\right)$$

NLO PT (lin. divergence) · finite-size scaling · IR term · color factor

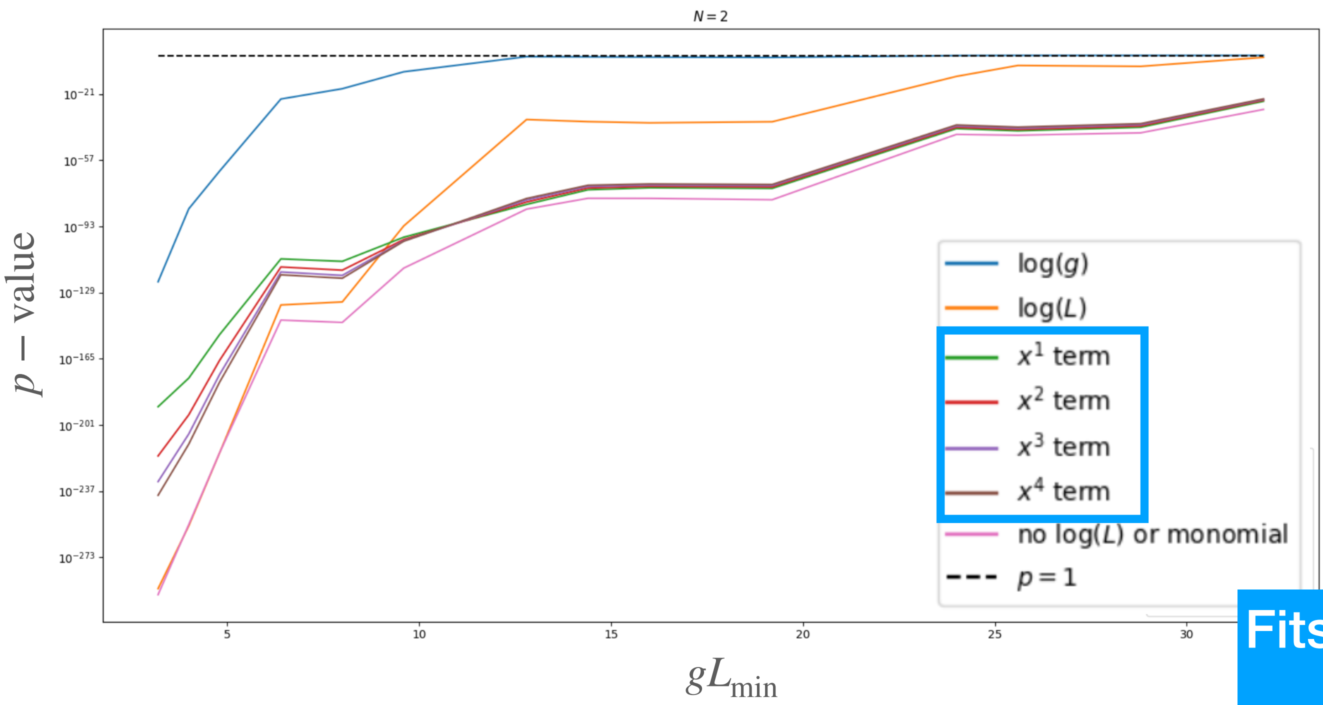
IR power divergent: $D_{\mathrm{IR}} = (gL)^{n-2}$ with $n = 3,4,5,6$

IR regularised or log-divergent: $D_{\mathrm{IR}} = \log \Lambda_{\mathrm{IR}}$ with $\Lambda_{\mathrm{IR}} \sim g, 1/L$

Test which IR-scaling is manifest in the theory by testing on simulation data

LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Global fits assuming IR power divergence



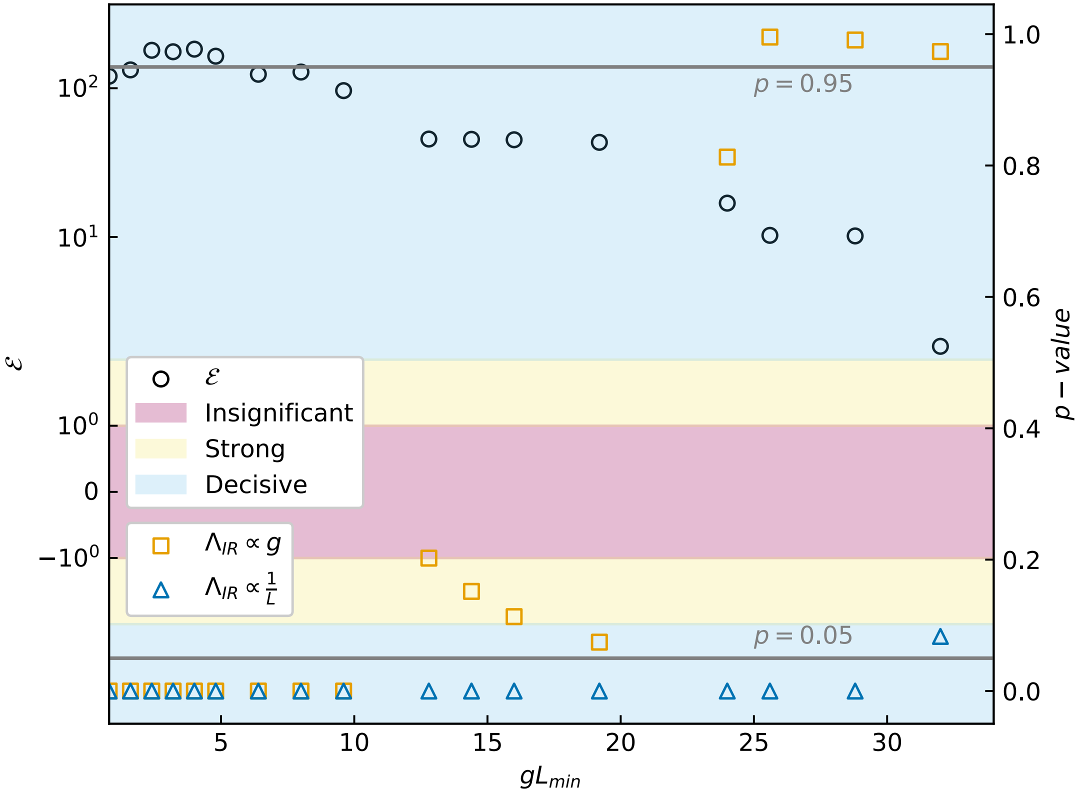
Left axis: p-value

Horizontal axis: values with $gL > gL_{\text{min}}$ Included in the fit

Fits suggest NO IR power divergences — vanishing $p-$value

LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Global fits assuming IR behaviour $\sim \log(1/L)$ or $\sim \log(g)$



Left axis: Bayesian evidence $\mathcal{E}$
positive: $\Lambda_{\mathrm{IR}} \sim g$
negative: $\Lambda_{\mathrm{IR}} \sim 1/L$

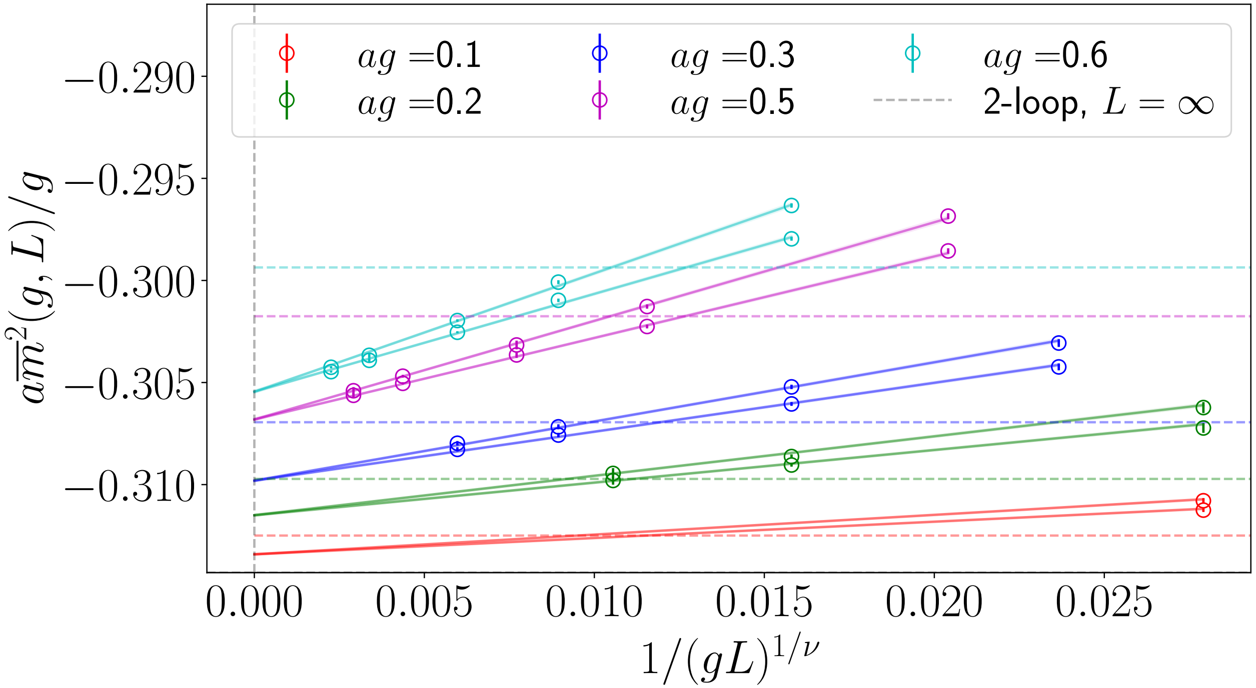
Right axis: p-value

Horizontal axis: values with $gL > gL_{\mathrm{min}}$
Included in the fit

Evidence for IR-finiteness of the theory based on both Bayesian and Frequentist analysis

LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Central fit - nonperturbative results



- Only ansatz capable of the data is with $\log(g)$
- $p-$value 0.2 | ($\chi^2/dof = 1.2, dof = 31$)
- Good agreement with 2-loop result with $\Lambda_{\rm IR} \sim g$

LatCos Phys.Rev.Lett. 126 (2021) 22, 221601

Conclusions

- Exciting project that covers the entire range of lattice QFT, Holography, theoretical and observational Cosmology
- Dual QFT has non-perturbative regime, requires lattice QFT
- **We have shown IR finiteness of the 3d QFT with implications for holographic cosmology**

Really looking forward to see Lattice contributing to cosmology in this novel and falsifiable way!

Related talks at this conference:

- Henrique Bergallo Rocha, Wed 14:15-30
- Joseph Lee, Wed 14:30-14:45