Continuum limit of two-dimensional multiflavor scalar gauge theories

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References:
PRD 101, 054503 (2020)
PRD 102, 034512(2020)
JHEP 05 (2021) 18

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Introduction

- The interplay between global and local symmetries is crucial to determine the main structure of a theory: the nature of the spectrum, the degeneracy of the energy levels, the phase diagram, the nature of the universality class in the presence of a critical phenomenon..

- Two-dimensional theories of Multiflavor Scalar Chromodynamics (MSC) with non-Abelian local and global invariance are a paradigmatic class of toy-models where we can test the role of symmetries at criticality

- These models are asymptotically free. At criticality, we will show that these models share the same critical properties as Non-Linear Sigma Models (NLSMs) defined on symmetric spaces with the same global symmetry

NLSMs and asymptotic freedom

- 2D NLSMs, e.g. $\mathrm{O}(N_f)$ and $\mathrm{U}(N_f)$ theories, are the simplest scalar models with non-Abelian global symmetry

$$\boxed{H = -N_f \sum_{x,\mu} \mathrm{Tr}\, \phi_x^\dagger \phi_{x+\mu}} \qquad \mathrm{Tr}\, \phi_x^\dagger \phi_x = 1\,.$$

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- However, these models exhibit asymptotic freedom in the limit $\beta \to +\infty$, developing an exponentially divergent correlation length

$$\left\langle \mathrm{Tr}\, \phi_x^\dagger \phi_y \right\rangle_c \sim e^{-|x-y|/\xi}, \qquad \xi \sim \beta^p e^{c\beta}\,.$$

2D Multiflavor scalar chromodynamics (MSC)

- From now on, we take as main candidate of our models real MSC theories in the fundamental of the gauge group, but generalizations to $\mathbb{C}$ or other group representations are straightforward

$$\boxed{H_{\rm MSC} = -N_f \sum_{x,\mu} {\rm Tr}\, \phi_x^t V_{x,\mu} \phi_{x+\mu} - \frac{\gamma}{N_c} \sum_x {\rm Tr}\, \Box_x} \qquad {\rm Tr}\, \phi_x^t \phi_x = 1$$

- ϕ_x^{if} being $N_c \times N_f$ matrices transforming in the fundamental of $SO(N_c)$
- $V_{x,\mu}^{ij}$ are link variables $\in SO(N_c)$

- The theory is characterized by a $SO(N_c)$ non-abelian local invariance ($N_c \geq 3$)

$$V_{x,\mu} \to W_x^t V_{x,\mu} W_{x+\mu}, \qquad \phi_x \to W_x \phi_x, \qquad \text{with } W_x \in SO(N_c)\,,$$

and $O(N_f)$ non-abelian global invariance ($N_f \geq 3$)

$$V_{x,\mu} \to V_{x,\mu}, \qquad \phi_x \to \phi_x O, \qquad \text{with } O \in O(N_f)\,.$$

The conjecture

- What is the role of the local symmetry in determining the continuum limit of the lattice model?

Conjecture

MSC theories have the same continuum limit as NLSMs defined on a symmetric space with the same global symmetry. Simplest models with such properties are given by theories defined on projective space

$$\mathbb{R}P^{N_f-1}$$

Note: $\mathbb{R}P^{N_f-1}$ theories are $O(N_f)$ models characterized by $\mathbb{Z}_2$ gauge invariance.

- The critical behavior of the MSC crucially depends on the minimum energy configurations over which low-energy excitations take place in the asymptotic low-temperature regime.

Minimum energy configurations in the $\beta \to +\infty$ limit

- A feasible observable that underlays the structure of the scalar field vacuum in the $\beta \to +\infty$ limit is $\mathrm{Tr}\, B_x^2$

$$\boxed{B_x^{fg} = \phi_x^{if}\phi_x^{ig}}$$

(N_c, N_f)	$\mathrm{Tr}\, B_x^2$
(3, 3)	1.0000(1)
(3, 4)	1.00000(1)
(4, 3)	1.00000(1)

Note: In $\mathbb{R}P^{N_f-1}$ models, by definition the bilinear $P_x^{fg} = \phi_x^f \phi_x^g$ satisfies $\mathrm{Tr}\, P_x^2 = 1$

- At low-temperatures, the MSC is effectively described as a theory of projectors in terms of the SO(N_f) order parameter B_x^{fg} (apart from constants)

$$\boxed{\mathrm{H}_{\mathrm{eff}} = -\kappa \sum_{x,\mu} \mathrm{Tr}\, B_x B_{x+\mu}} \qquad \mathrm{Tr}\, B_x = \mathrm{Tr}\, B_x^2 = 1$$

which is an alternative definition of $\mathbb{R}P^{N_f-1}$ models on a lattice

Observables and Finite-size scaling

- To study numerically the nature of the critical behavior, we consider correlations of the spin-2 order parameter Q_x for the breaking of SO(N_f)

$$Q_x^{fg} = B_x^{fg} - \frac{\delta^{fg}}{N_f} \,,$$

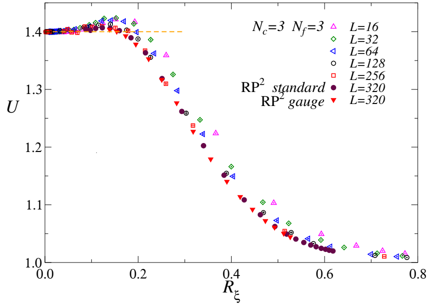
  in particular we considered the RG invariant quantities U and $R_\xi = \xi/L$.

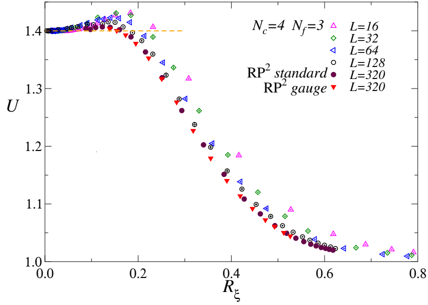
- We compare different models at criticality by plotting U in terms of R_ξ

$$\boxed{U(\beta, L) \approx F(R_\xi)}$$

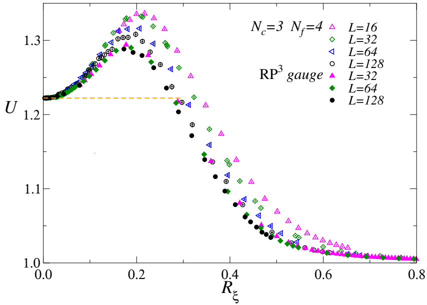
Indeed, $F(R_\xi)$ is a universal scaling function that completely characterizes the universality class. This strategy allows us to compare the critical behavior of different models without tuning any non-universal parameter (in the case at hand MSC and $\mathbb{R}P^{N_f-1}$ scaling curves)

Numerical results: **PRD 102, 034512 (2020)..**





The local symmetry is crucial to identify the gauge-invariant scalar-matter degrees of freedom that develop the critical behavior



However, the continuum limit of MSC theories does not depend on the number of colors of the gauge symmetry!!! But..

In 2D, we obtain results consistent with our conjecture in different ways:

- PRD 101, 054503 (2020): $\mathbb{C}$ MSC in the fundamental of SU(N_c) $\Rightarrow \mathbb{C}P^{N_f-1}$

..but more in general

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$$\boxed{\mathrm{H} = \mathrm{H}_{\mathrm{MSC}} + v \sum_x \mathrm{Tr}\, B_x^2} \qquad B_x^{fg} = \phi_x^{if} \phi_x^{ig}$$

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- C. Bonati & A. Franchi in progress..: $\mathbb{R}$ MSC in the fundamental of $\mathrm{SO}(N_c)$

$$\mathrm{H} = \begin{cases} v \leq 0 : \mathbb{R}P^{N_f-1} \\ v > 0 : \begin{cases} N_c \geq N_f : - \\ N_c < N_f : \frac{\mathrm{O}(N_f)}{\mathrm{O}(N_c)\times\mathrm{O}(N_f-N_c)} \end{cases} \end{cases}$$

Conclusions

To understand the interplay of local and global symmetries at criticality, we studied lattice models of 2D MSC for several combinations of colors-flavors and group representations, both in real and complex space.

We have observed that our numerical results are always consistent with the hypothesis that these models share the same continuum limit (when it exists) as a NLSM defined on a symmetric space with the same global properties.

Last comment: It is worth mentioning that the analyses of the minimum energy configurations in the $\beta \to +\infty$ limit are still valid in 3D. They were used as a starting point for the comprehension of MSC phase diagrams in appropriate limits

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Thank you!

Thank you for your attention!

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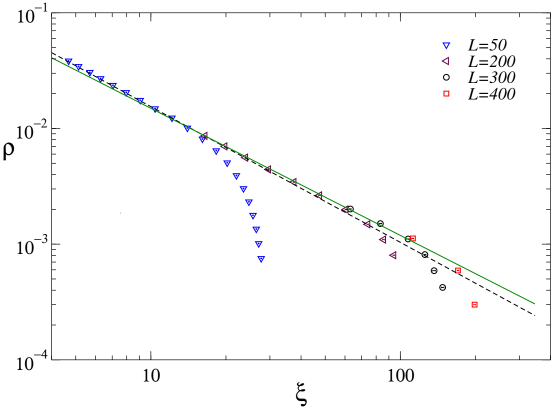
Back up..

$\mathbb{R}P^{n-1}$ universality class

- There is a still open diatribe, questioning on the existence of the $\mathbb{R}P^{n-1}$ universality class in 2D **PRD 102, 034513 (2020)**
- The existence of a $\mathbb{R}P^{n-1}$ class relies on the presence of topological defects

$$\rho := \frac{1}{2}(1 - \langle \Pi_x \rangle) \qquad \rho \sim \xi^{-1}$$



- Let G be a group with an involution $\overline{g}$ defined. We decompose the involutive automorphism $\overline{g}$ into an inner automorphism (g_0) and an involution (*)

$$\overline{g} := g_0^{-1} g^* g_0 \qquad \overline{\overline{g}} = g \qquad (\overline{g_1 g_2}) = \overline{g_1}\,\overline{g_2}$$

- The constraint $\overline{\overline{g}} = g$ implies $g_0^* g_0 \in$ centre of the group $\lambda \mathbb{1}$
- Let H be the subgroup of G such that $\overline{H} = H$
- A realization of the symmetric space G/H in the group space is given by..

$$g = (f^{-1})^* g_0 f \quad \forall f \in G$$

.. multiplying f on the left by $h \in H$, g is not modified

- The group elements satisfy

$$g^* g = \lambda \mathbb{1}$$

- .. We take $G = O(N)$, the $*$=identity, and the centre of $O(N) = \{\pm 1\}$. Two possibilities ($g_0^* g_0 \in$ centre) :
- $\boxed{\lambda = 1}$ We can take g_0 to be diagonal, it has only eigenvalues ± 1 being in $O(N)$. If g_0 has p eigenvalues $+1$ and $N - p$ eigenvalues -1.. it realizes the symmetric space

$$\boxed{O(N)/\big(O(p) \times O(N-p)\big) \underset{p=1}{\equiv} O(N)/\big(O(N-1) \times \mathbb{Z}_2\big) := \mathbb{R}P^{n-1}}$$