Thermalization of gauge theories from their Entanglement Spectrum

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arXiv:2107.11416

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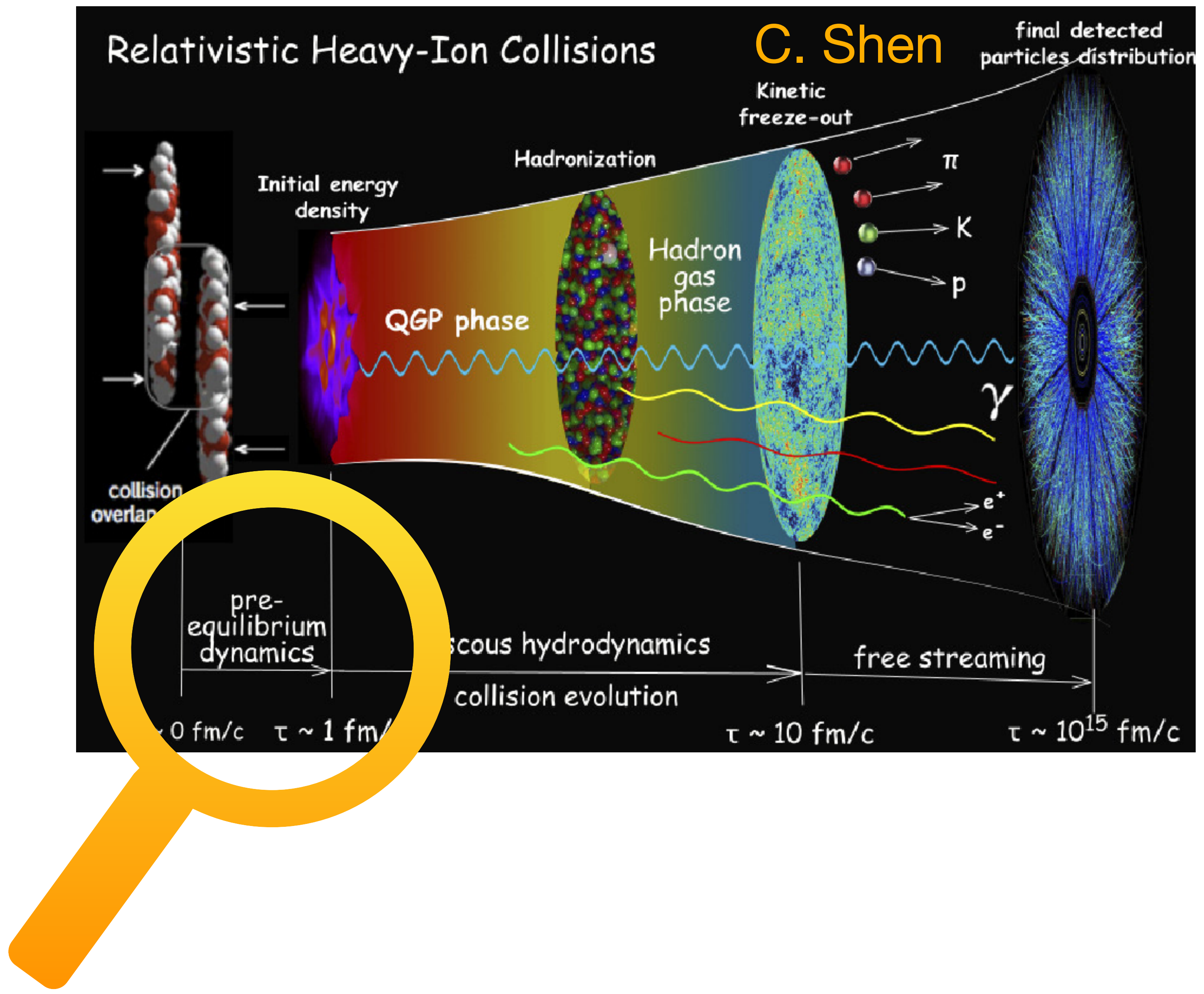
Heidelberg

38th International Symposium on Lattice Field Theory **July 27 2021**

QCD: How does it thermalize?



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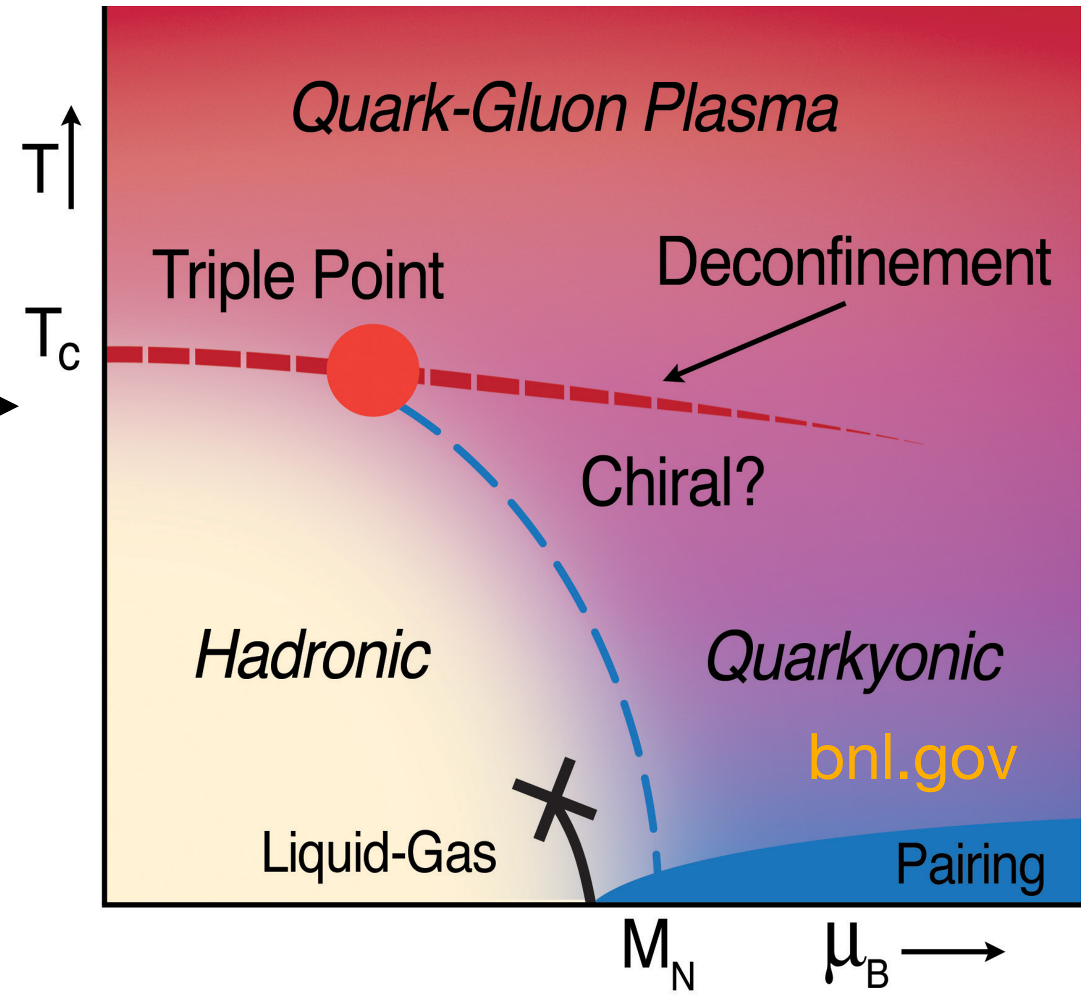
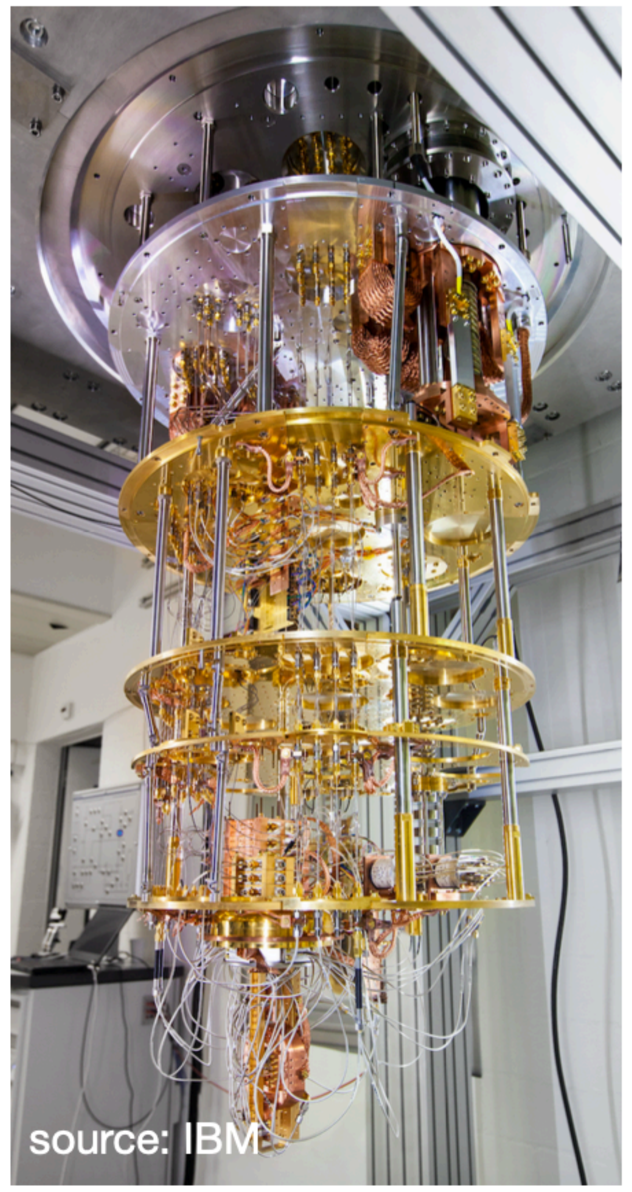


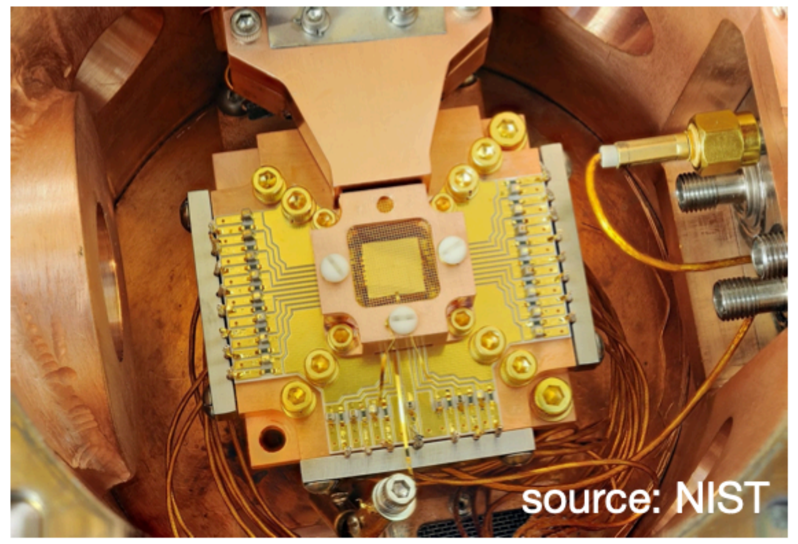
pre-equilibrium dynamics:
some weak (and strong) coupling insights

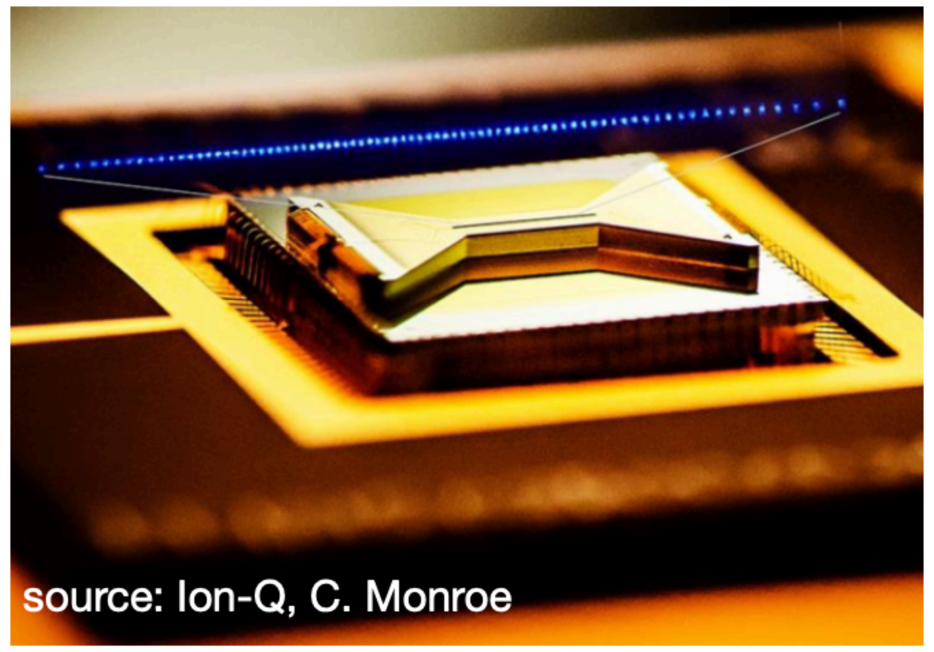
- …
- \`bottom-up': Baier, Mueller, Schiff and Son PLB 502, 51 (2001)
- Berges, Boguslavski, Schlichting Venugopalan, PRD 89, 074011 (2014), PRD 89, 114007 (2014)
- Kurkela, Lu, PRL 113, 182301
- Keegan, Kurkela, Romatschke, van der Schee, JHEP 2016(4), 1
  ….
- Berges, Heller, Mazeliauskas, Venugopalan 2005.12299 *(review)*

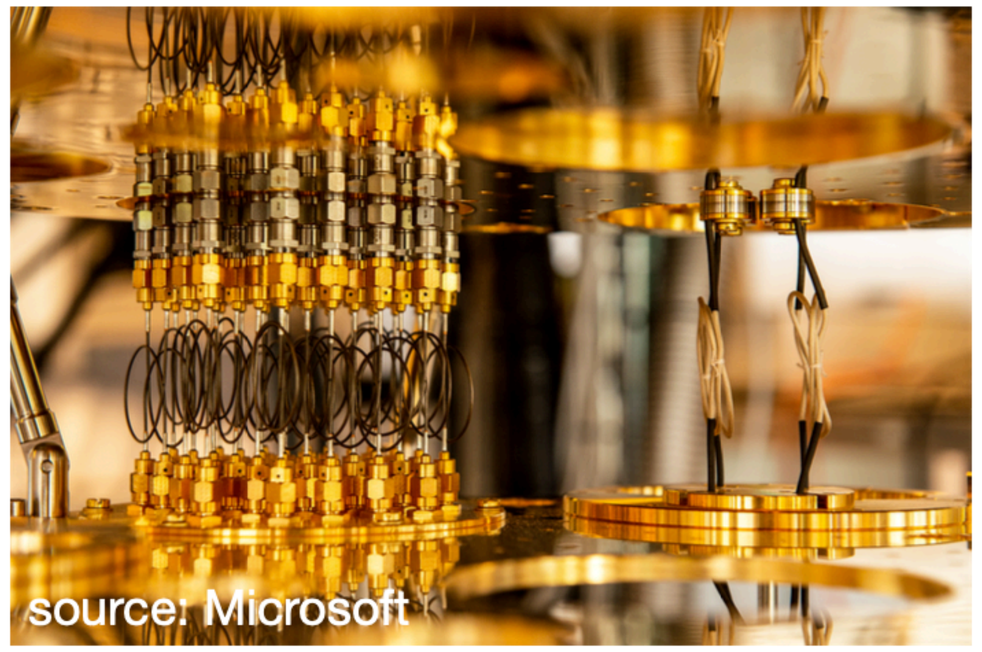
- **real-time dynamics**
  aka. "basically impossible for classical computers"

- **a look at the future:**
  digital quantum computers and analog simulators

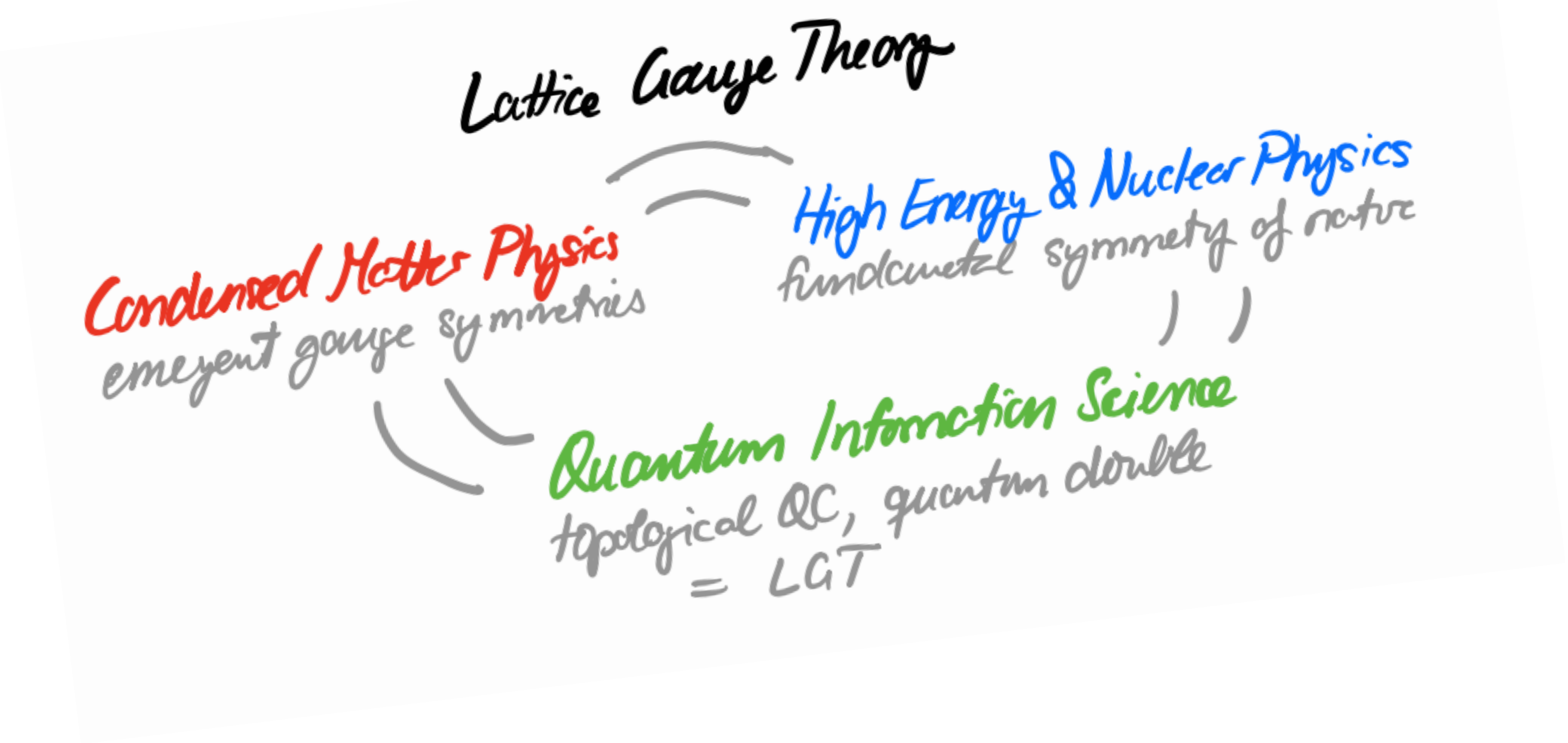








Thermalization of isolated quantum systems

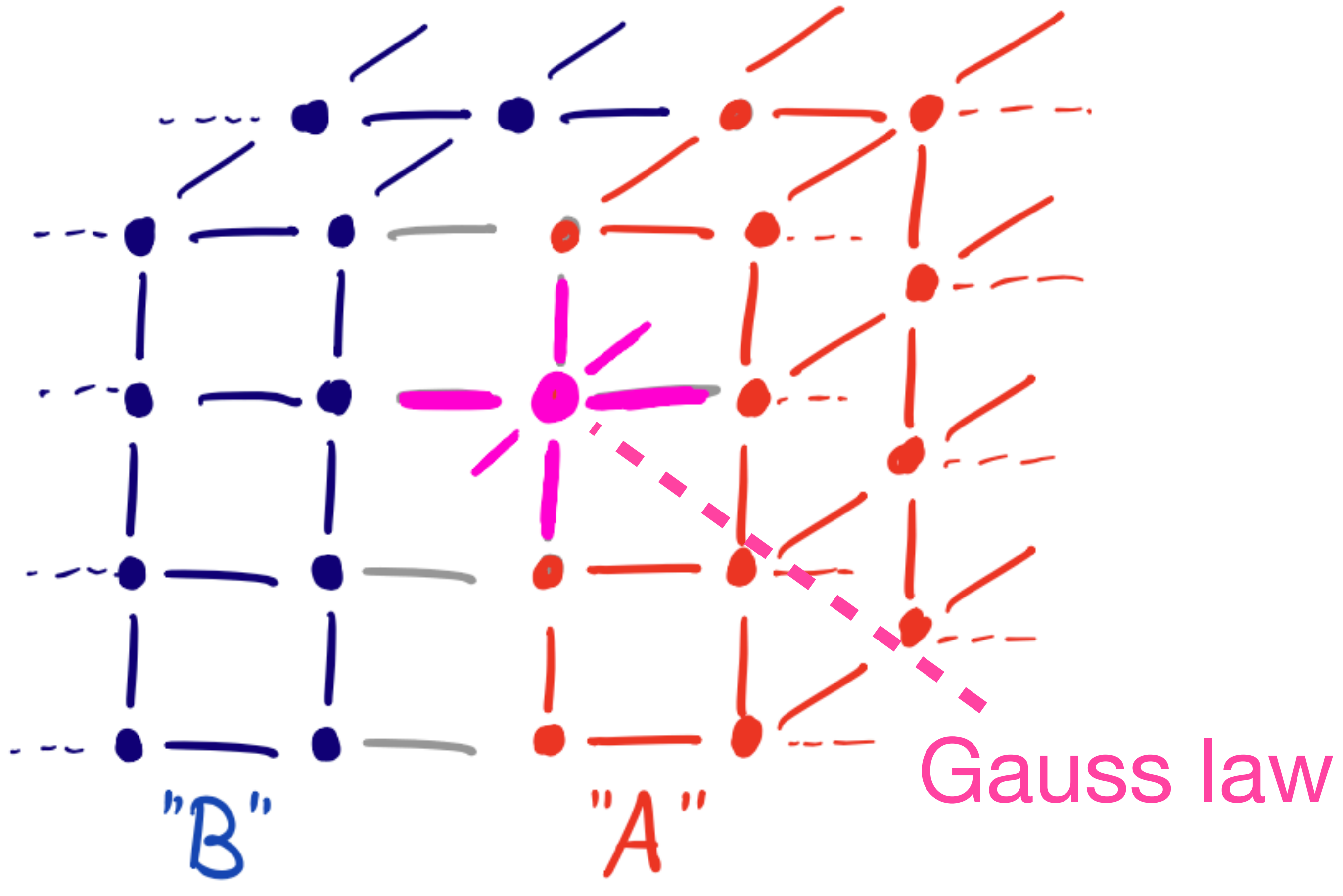


- … by no means a QCD problem alone,
  but very much a Lattice problem!

- **Eigenstate Thermalization Hypothesis**
  J. Deutsch PRA 43, 2046 (1991), M. Srednicki PRE 50, 888 (1994)

- **Entanglement Entropy of lattice <u>gauge</u> theories?**

$$\rho_A = \mathrm{Tr}_B(\rho)\,?$$

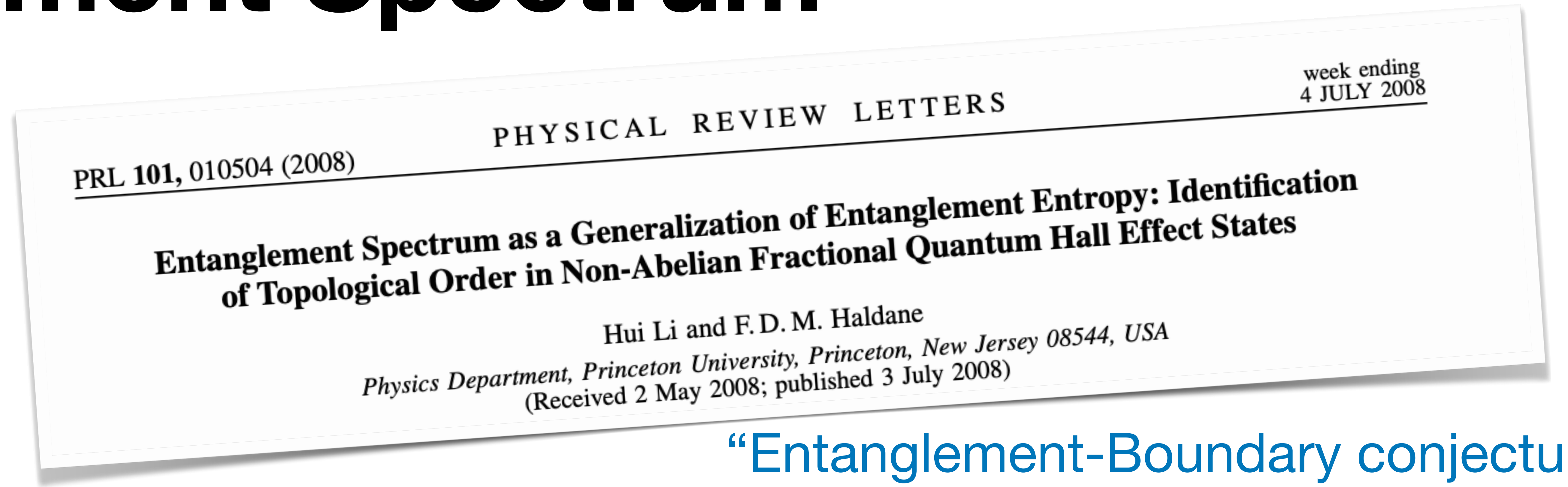


Buividovich, Polikarpov PLB 670, 141 (2008)
Casini, Huerta, Rosababl PRD 89, 085012 (2014)
Aoki, Iritanni, Nozaki, Numasawa, Shiba, Tasaki JHEP 2015
Gosh, Soni, Trivedi JHEP 2015

Entanglement Spectrum

- **Entanglement Structure via Schmidt spectrum**

$$|\psi\rangle = \sum_i e^{-(1/2)\xi_i} |\psi^A\rangle \otimes |\psi^B\rangle$$


PRL 101, 010504 (2008) PHYSICAL REVIEW LETTERS week ending 4 JULY 2008

Entanglement Spectrum as a Generalization of Entanglement Entropy: Identification of Topological Order in Non-Abelian Fractional Quantum Hall Effect States

Hui Li and F. D. M. Haldane

Physics Department, Princeton University, Princeton, New Jersey 08544, USA

(Received 2 May 2008; published 3 July 2008)

"Entanglement-Boundary conjecture"

Li, Haldane, PRL 101, 010504 (2008)

- **Interpret ξ_i as energy levels of `Entanglement Hamiltonian'**

$$H^{\text{ent.}} = -\log(\rho_A) \qquad \rho_A = \mathbf{Tr}_B(\rho)$$

- **Thermalization**

Garrison, Grover PRX 8, 021026 (2018)

$\rho_A^{\text{therm.}} = \mathrm{Tr}_B(e^{-\beta H})$	vs.	$\rho_A = e^{-H^{\text{ent.}}}$
canonical ensemble		Entanglement Structure

- **so one might suspect…**

$$H \sim H^{\text{ent.}}$$
$$\xi_i \sim E_i$$

… if thermal

Entanglement Structure of LGTs

arXiv:2107.11416

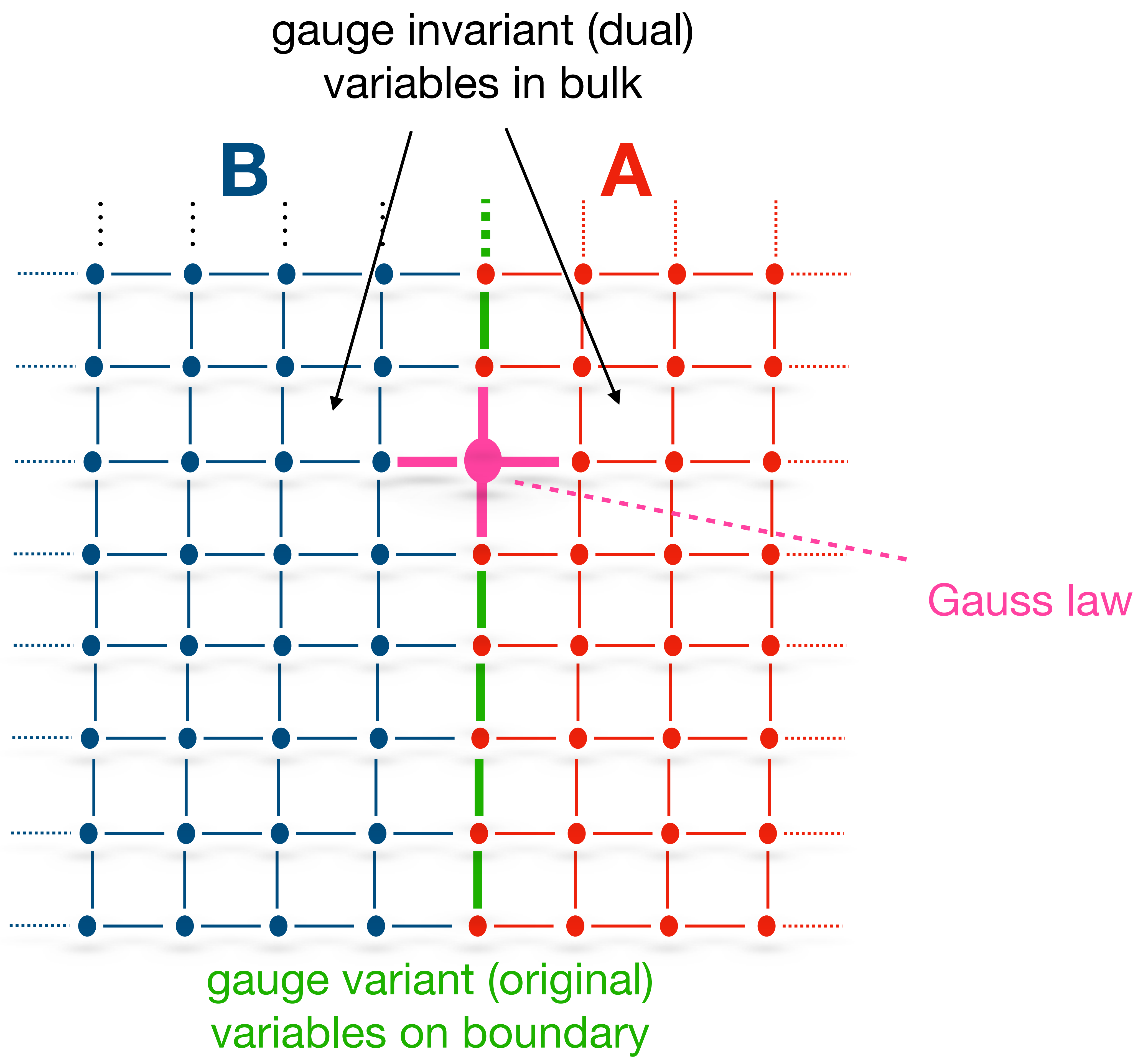
- Z_2 **LGT in (2+1)d** Wegner J. Math Phys. 12, 2259 (1971)

$$H = -\sum_{\mathbf{n}} \sigma^z_{\mathbf{n},x}\sigma^z_{\mathbf{n}+\hat{x},y}\sigma_{\mathbf{n}+\hat{y},x}\sigma^z_{\mathbf{n},y} - \epsilon \sum_{\mathbf{n},i} \sigma^x_{\mathbf{n},i}$$

"plaquette term" "electric field term"

$$G_{\mathbf{n}} = \sigma^x_{\mathbf{n},x}\sigma^x_{\mathbf{n}-\hat{x},x}\sigma^x_{\mathbf{n},y}\sigma^x_{\mathbf{n}-\hat{y},y}$$

"Gauss law"

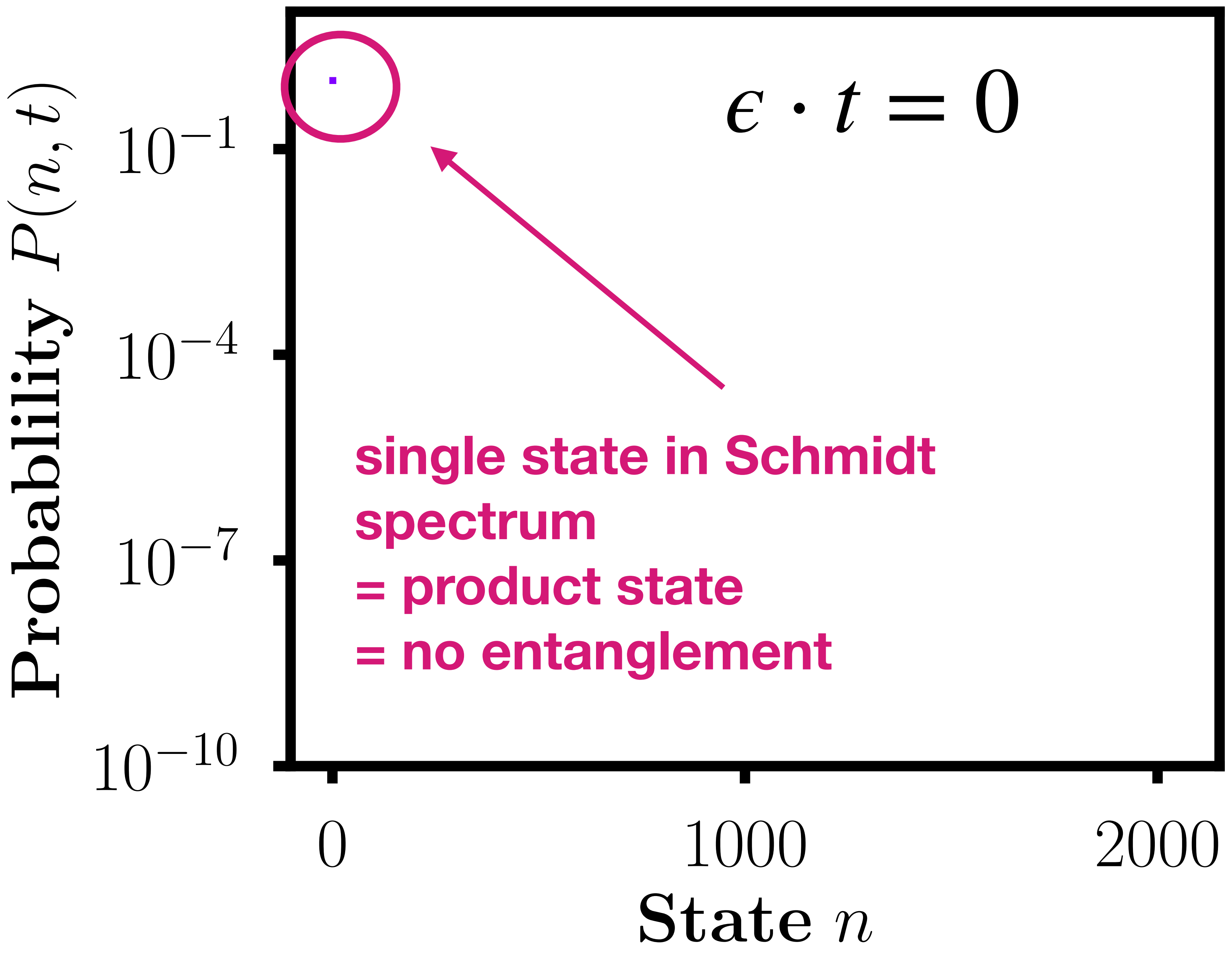


- **our approach:**
  dual theory 'embedded' into unphysical Hilbertspace along entanglement cuts

Thermalization of Gauge Theories

arXiv:2107.11416

- **Quench dynamics** $\epsilon = "\infty" \to 1$
- **Schmidt/Entanglement spectrum** $P(n,t) \equiv \exp\{-\xi_n(t)\}$



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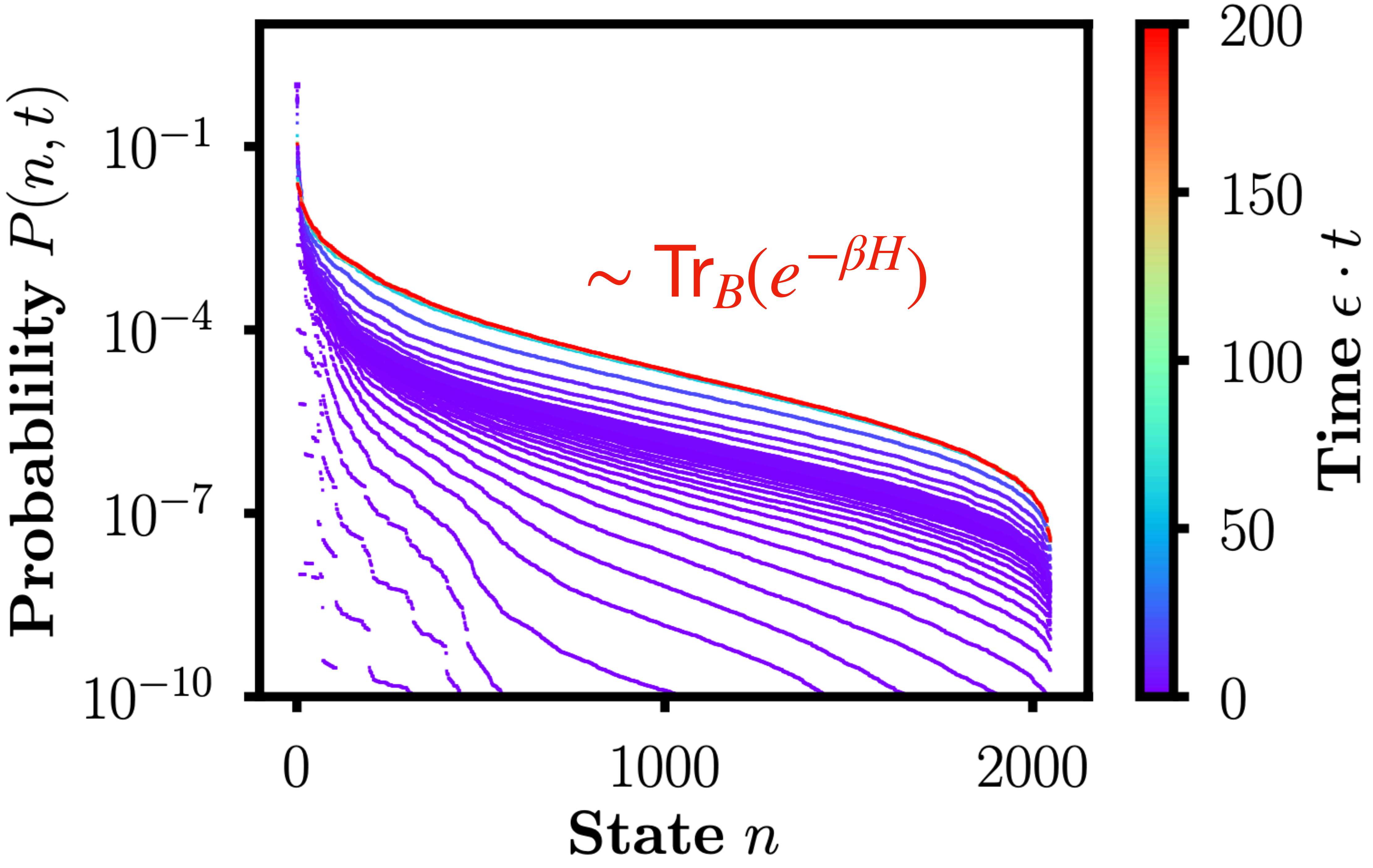


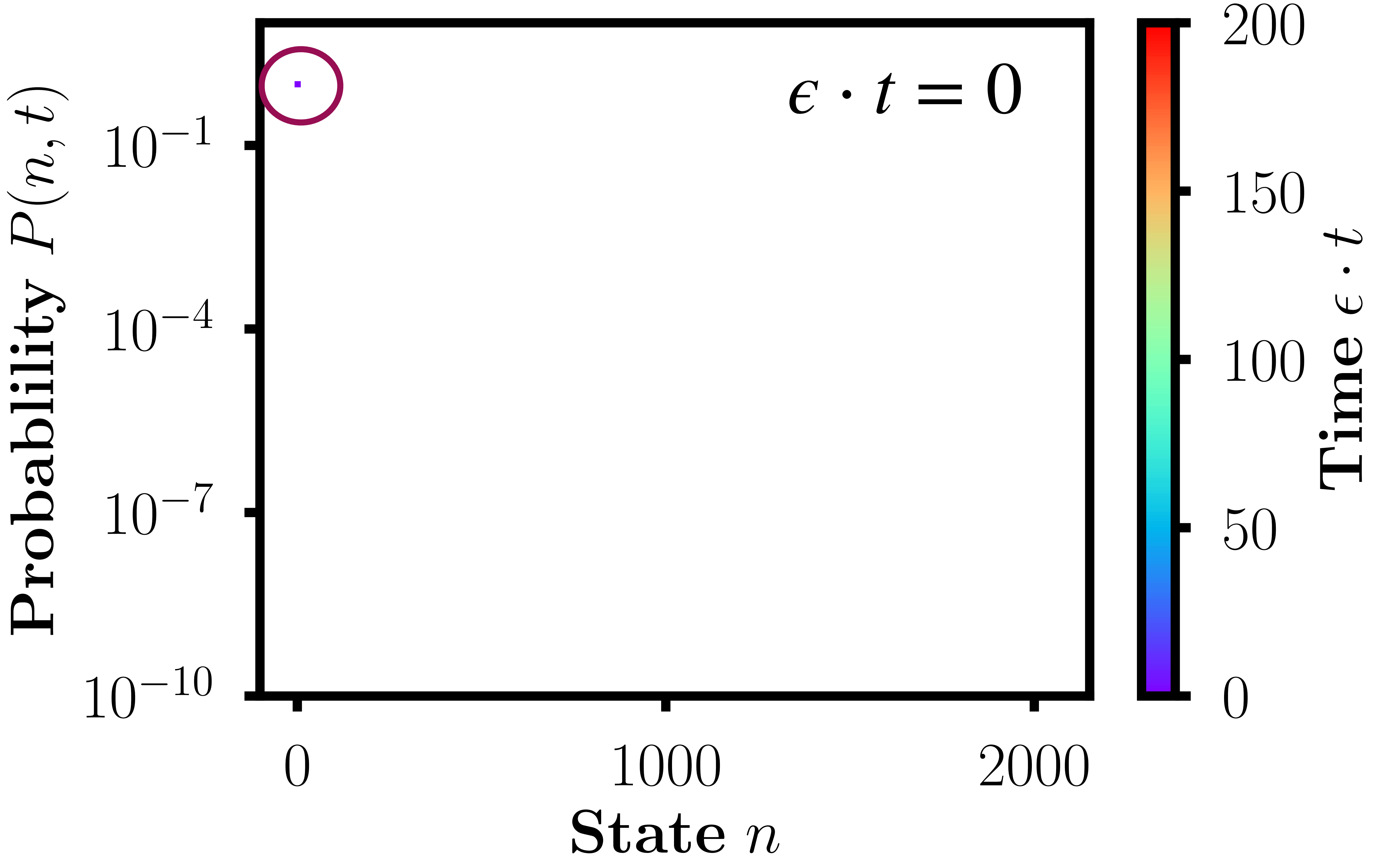
- initial product state,
- no entanglement,
- pure RDM

- thermal equlibrium at late times,
- von Neumann entropy = thermal entropy,
- mixed state

Thermalization of Gauge Theories

arXiv:2107.11416

$$P(n,t) \equiv \exp\{-\xi_n(t)\}$$



Thermalization of Gauge Theories

arXiv:2107.11416

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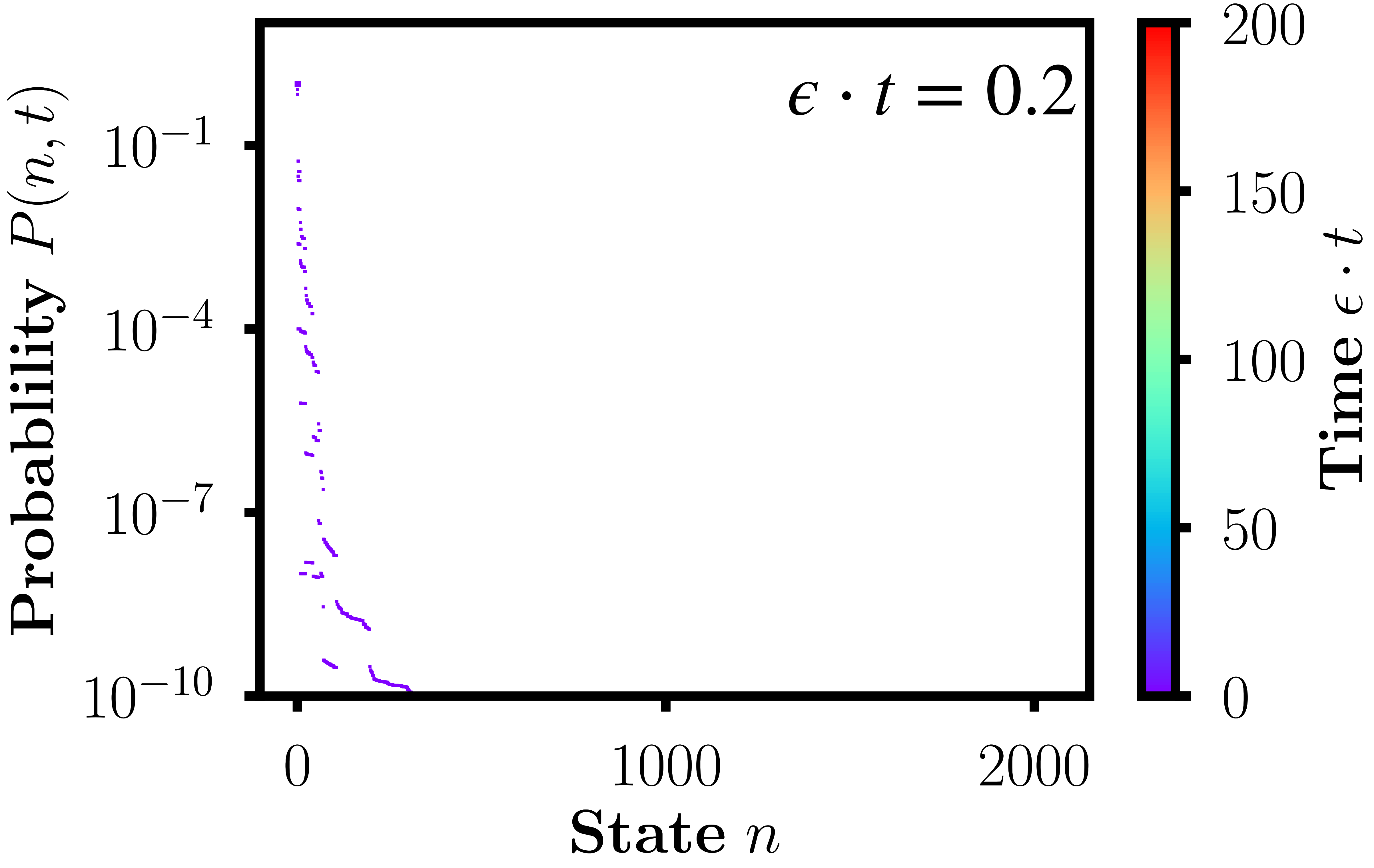

Thermalization of Gauge Theories

arXiv:2107.11416

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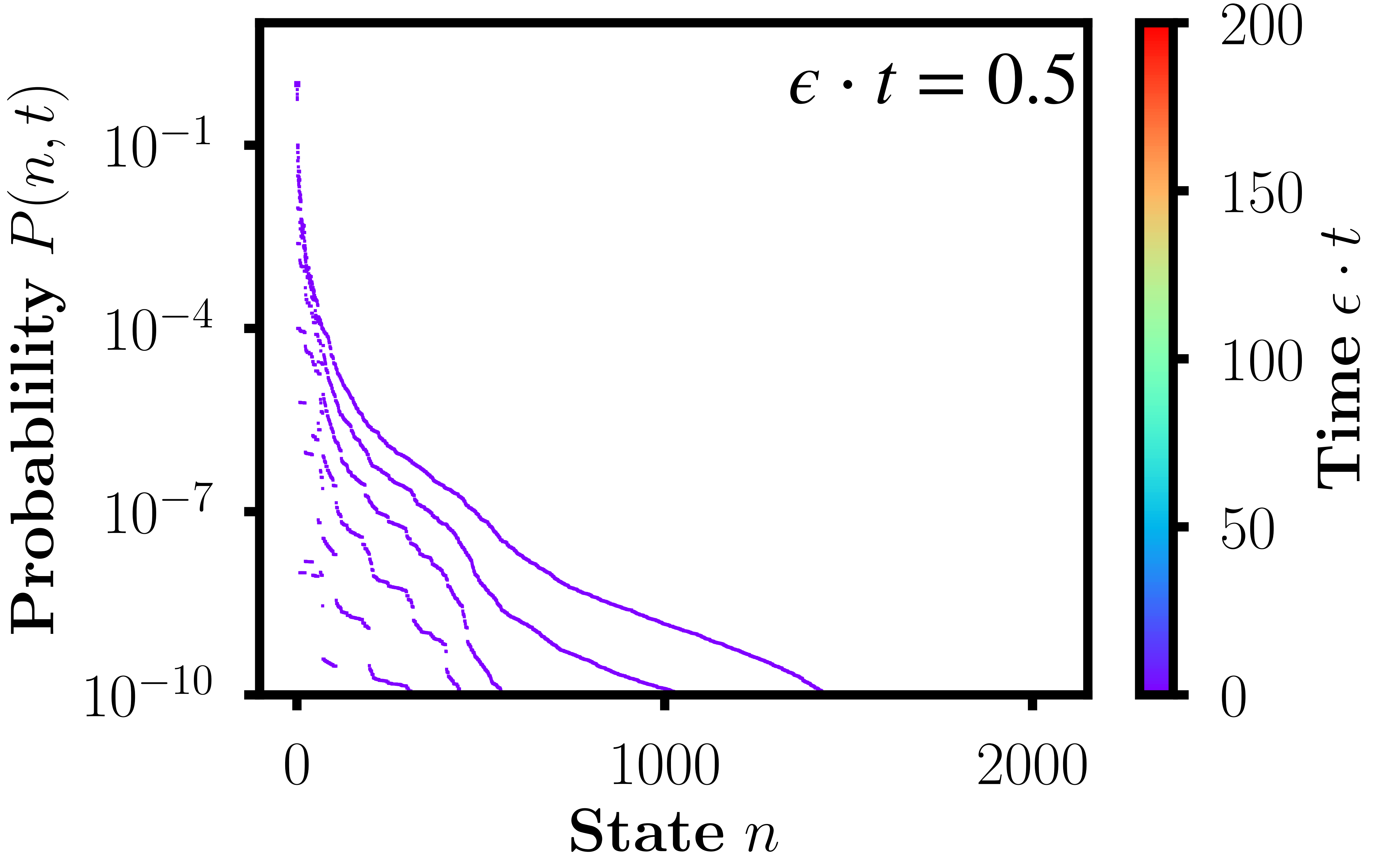
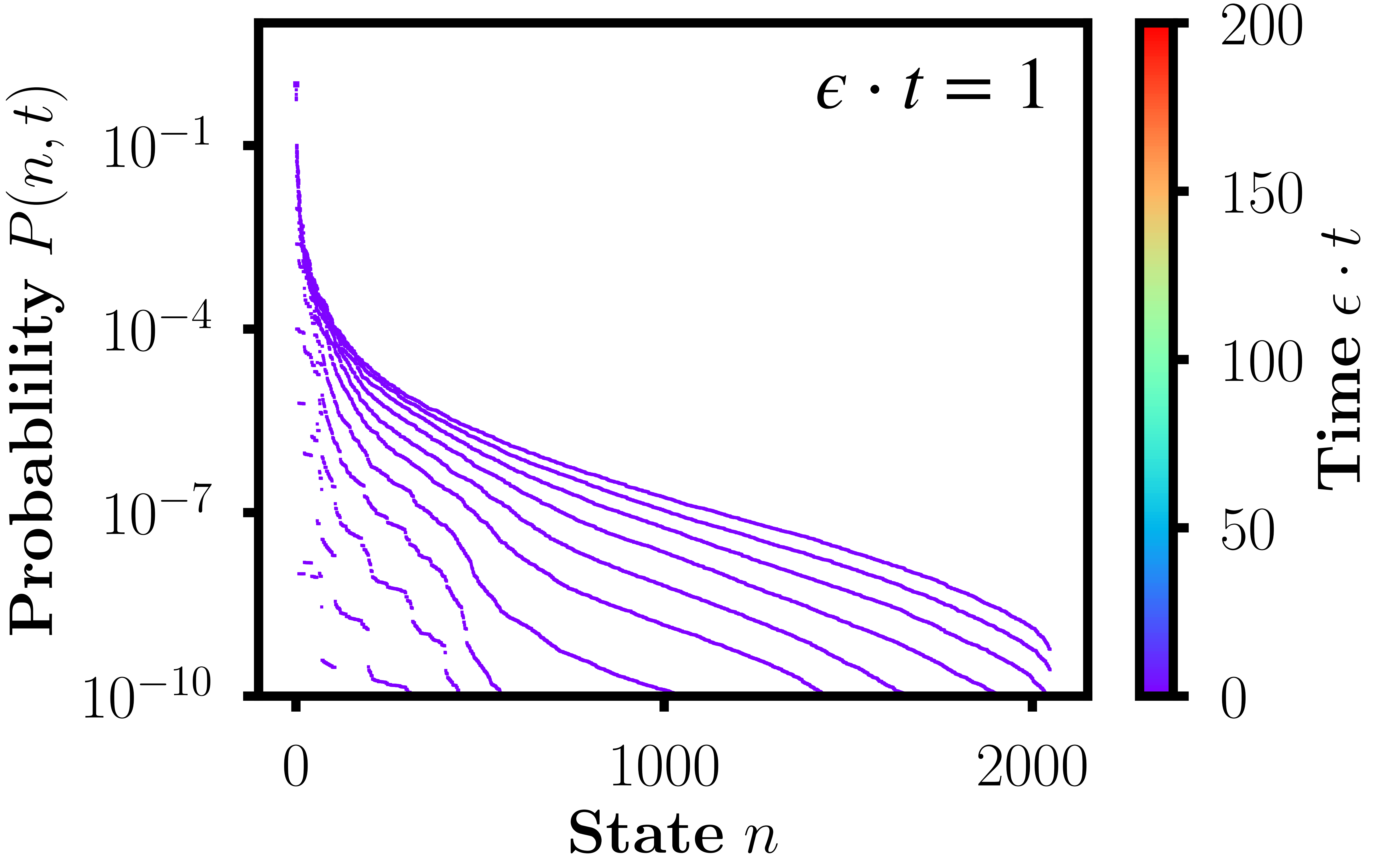


Thermalization of Gauge Theories

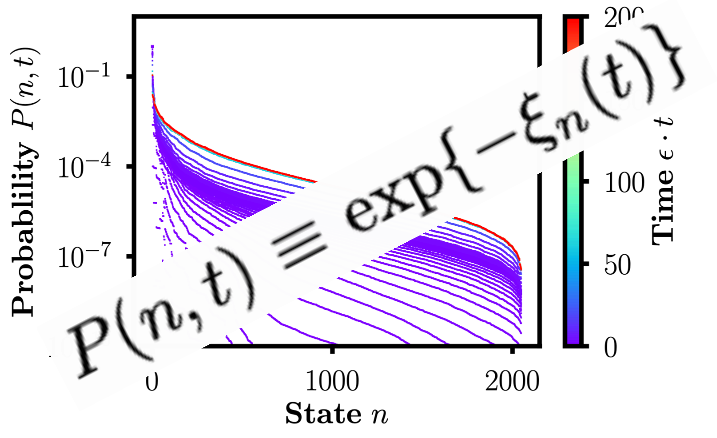
arXiv:2107.11416

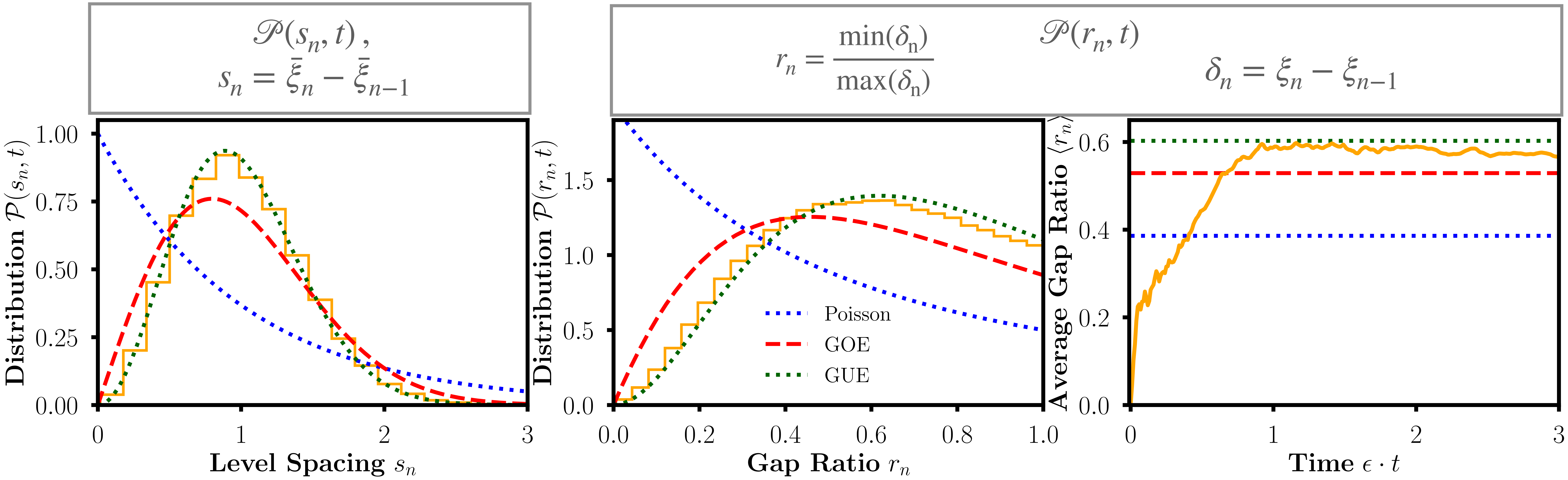
$P(n,t) \equiv \exp\{-\xi_n(t)\}$



Thermalization of Gauge Theories

arXiv:2107.11416

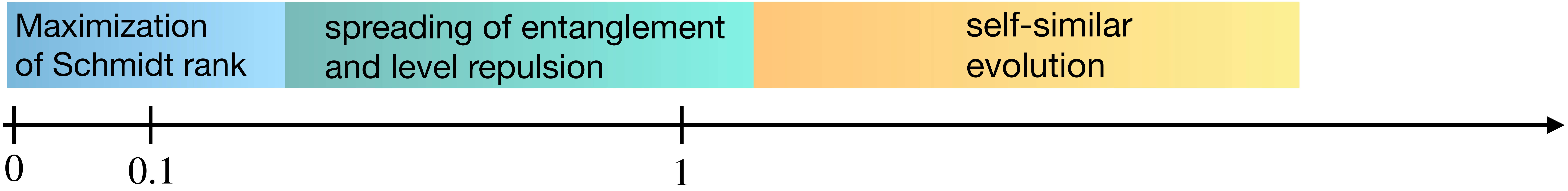
- **Level repulsion from Entanglement spectrum** ()

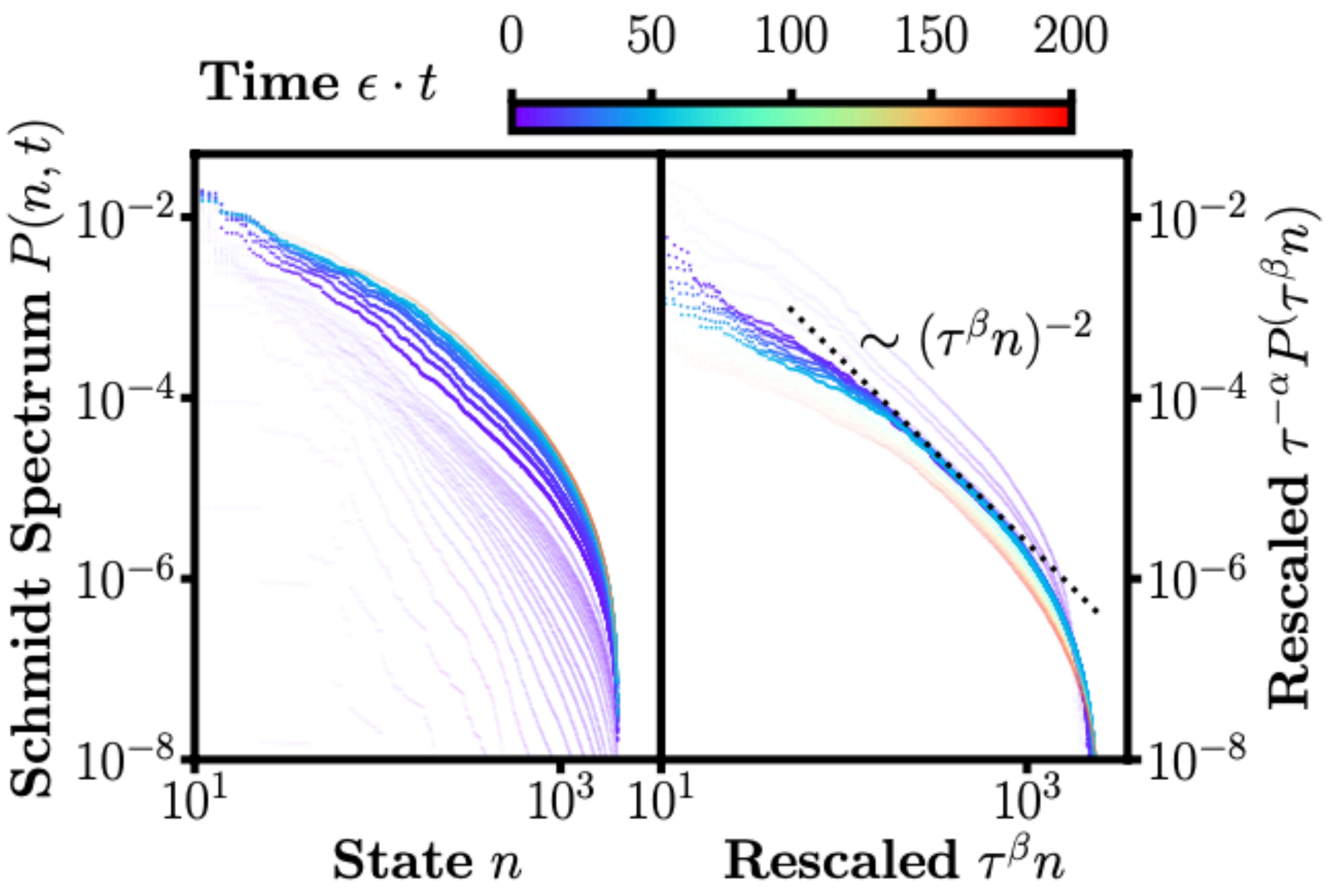


- **Gaussian Orthogonal Ensemble & Gaussian Unitary Ensemble = "chaotic"**
- **Poisson = Many Body Localization**

Thermalization of Gauge Theories

arXiv:2107.11416





- **Self similarity of Schmidt spectrum, quantum chaos**

$$\boxed{P(n,t) = \tau^{-\alpha} P(\tau^{\beta} n)}$$

$\tau = \epsilon(t - t_0)$

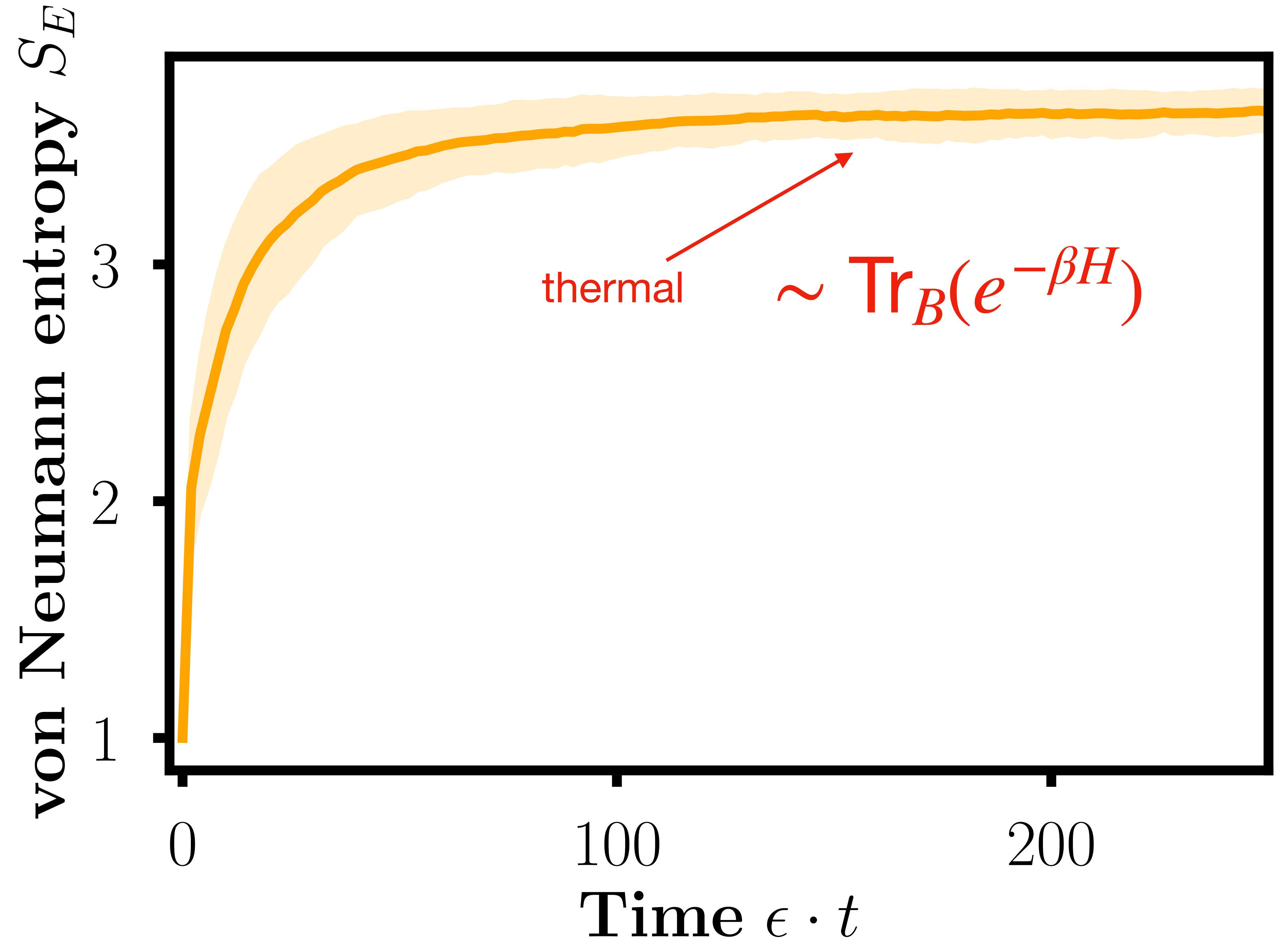
- **(Universal) scaling exponents**

$$\boxed{\alpha = 0.8 \pm 0.1 \quad \beta = 0.05 \pm 0.03}$$

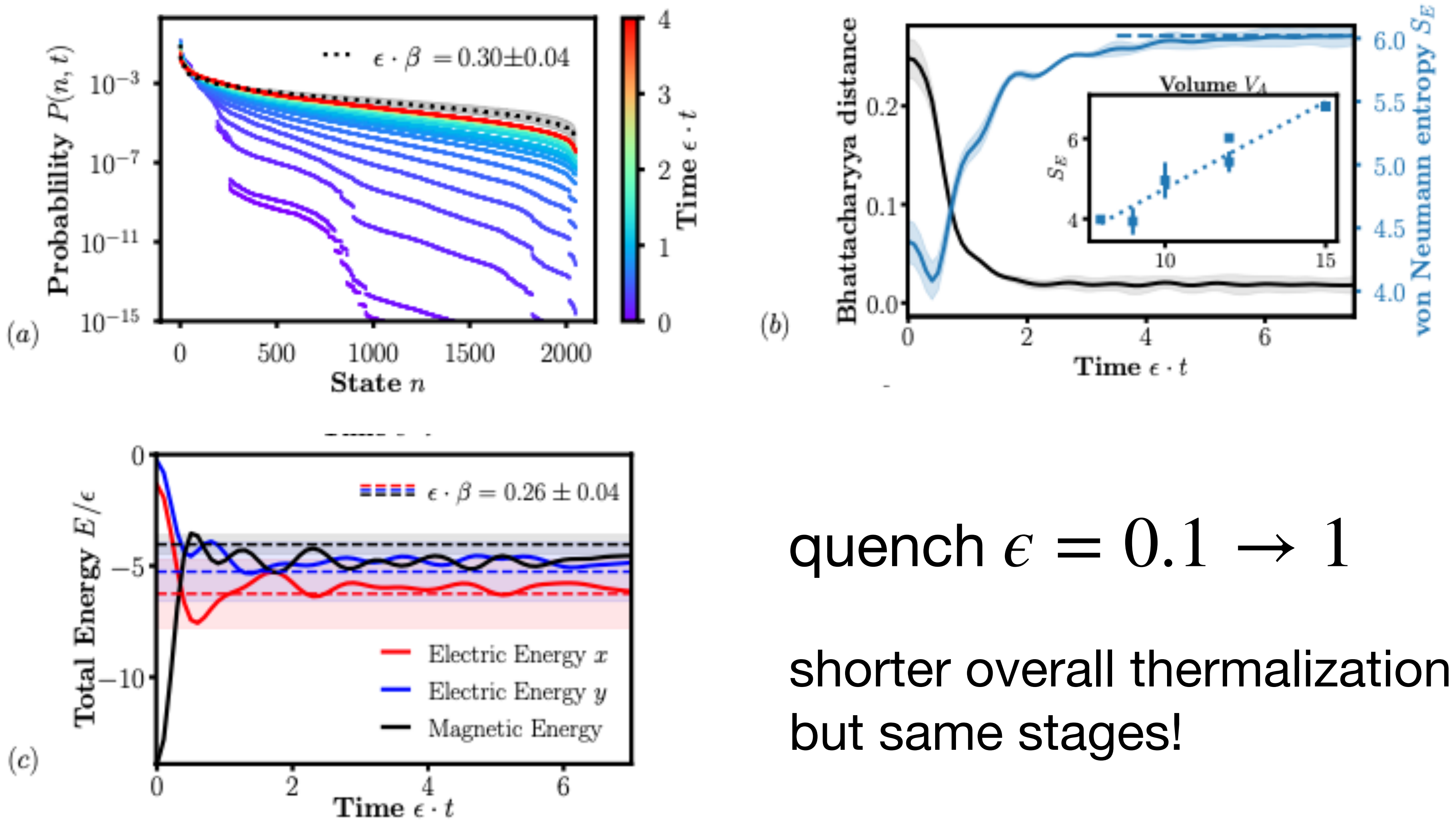
$\epsilon \cdot t_0 = 1.8 \pm 0.2$

Thermalization of Gauge Theories

arXiv:2107.11416



Generic for **any** initial state!



quench $\epsilon = 0.1 \rightarrow 1$

shorter overall thermalization but same stages!

- **von-Neumann entropy = thermal entropy**

Summary

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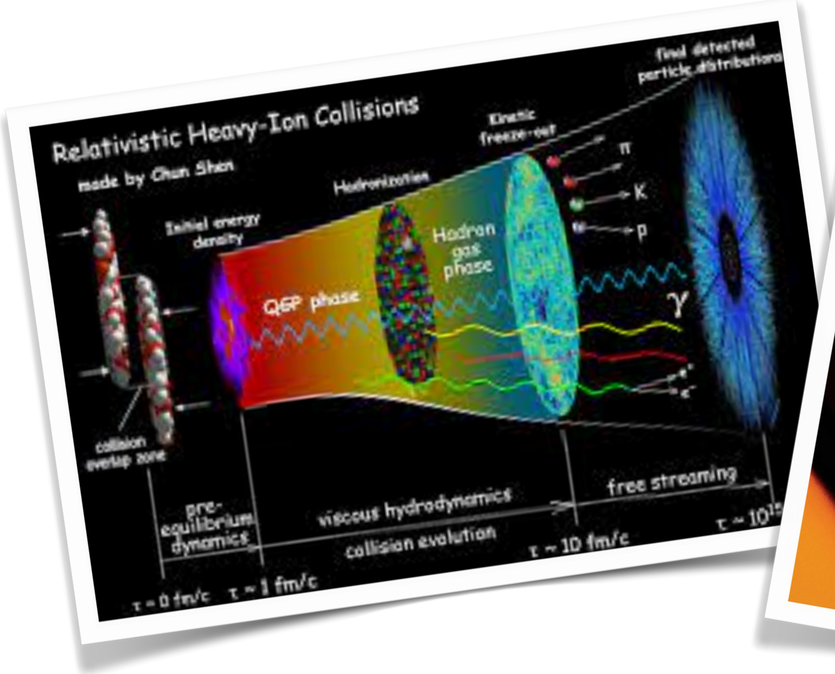
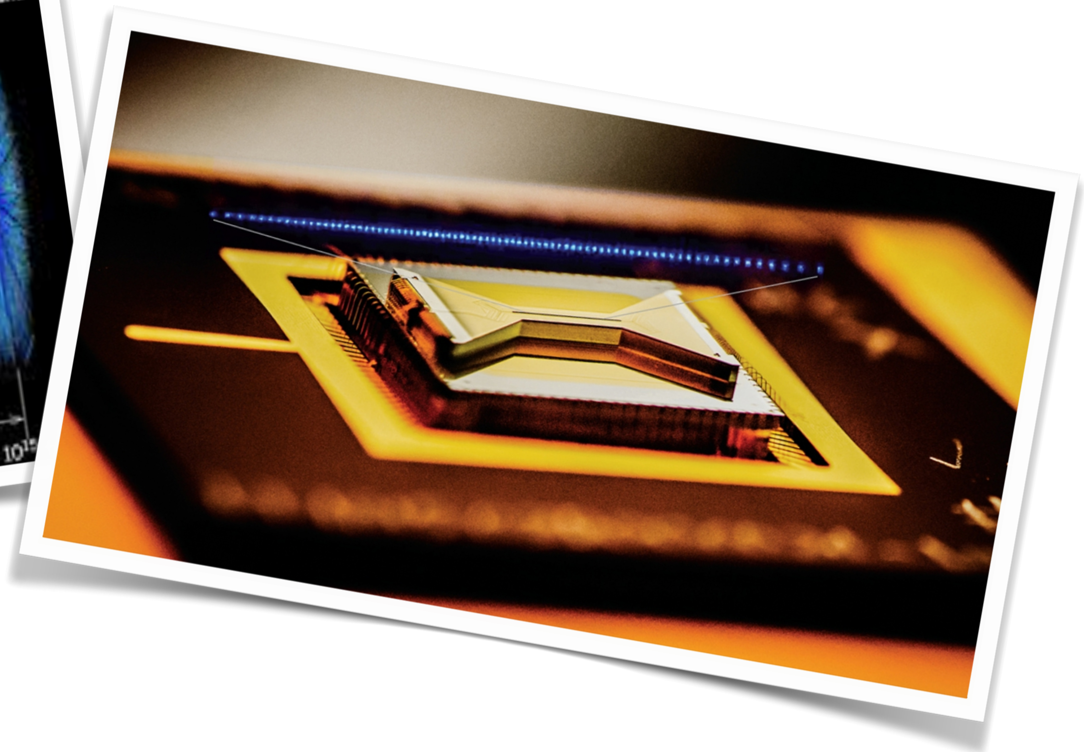
U.S. DEPARTMENT OF ENERGY Office of Science
ÖAW AUSTRIAN ACADEMY OF SCIENCES
SIMONS FOUNDATION

- **Entanglement Spectrum key to understand thermalization of (lattice) gauge theories**





- **reconciliation of quantum versus classical thermalization paradigms**

classical
- non-linear time evolution
- ergodicity, chaos, universality

Entanglement Structure

quantum mechanics
- time evolution linear
- Eigenstate Thermalization via matrix elements of observables
- "complexity comes from energy eigenfunctions"

J. Deutsch PRA 43, 2046 (1991), M. Srednicki PRE 50, 888 (1994)

- **Important goal for digital quantum computers and analog simulators!**
Extension to Z_n and U(1) easy, non-Abelian LGTs more work

Just a theorist's fantasy?

Entanglement Hamiltonian Tomography in Quantum Simulation

Christian Kokail,[1,2] Rick van Bijnen,[1,2] Andreas Elben,[1,2] Benoît Vermersch,[1,2,3] and Peter Zoller[1,2]
[1] Center for Quantum Physics, University of Innsbruck, Innsbruck, Austria
[2] Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Innsbruck, Austria
[3] Univ. Grenoble Alpes, CNRS, LPMMC, 38000 Grenoble, France

Entanglement is the crucial ingredient of quantum many-body physics, and characterizing and quantifying entanglement in closed system dynamics of quantum simulators is an outstanding challenge in today's era of intermediate scale quantum devices. Here we discuss an efficient tomographic protocol for reconstructing reduced density matrices and entanglement spectra for spin systems. The key step is a parametrization of the reduced density matrix in terms of an entanglement Hamiltonian involving only quasi local few-body terms. This ansatz is fitted to, and can be independently verified from, a small number of randomised measurements. The ansatz is suggested by Conformal Field Theory in quench dynamics, and via the Bisognano-Wichmann theorem for ground states. Not only does the protocol provide a testbed for these theories in quantum simulators, it is also applicable outside these regimes. We show the validity and efficiency of the protocol for a long-range Ising model in 1D using numerical simulation
quantum simulators [Brydges *et al.*, Science,
of the entanglement spectrum in quench dyna

PHYSICAL REVIEW X **6**, 041033 (2016)

Measurement Protocol for the Entanglement Spectrum of Cold Atoms

Hannes Pichler,[1,2,3,*] Guanyu Zhu,[4,†] Alireza Seif,[4] Peter Zoller,[3,5] and Mohammad Hafezi[4,6,7]
[1] ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA
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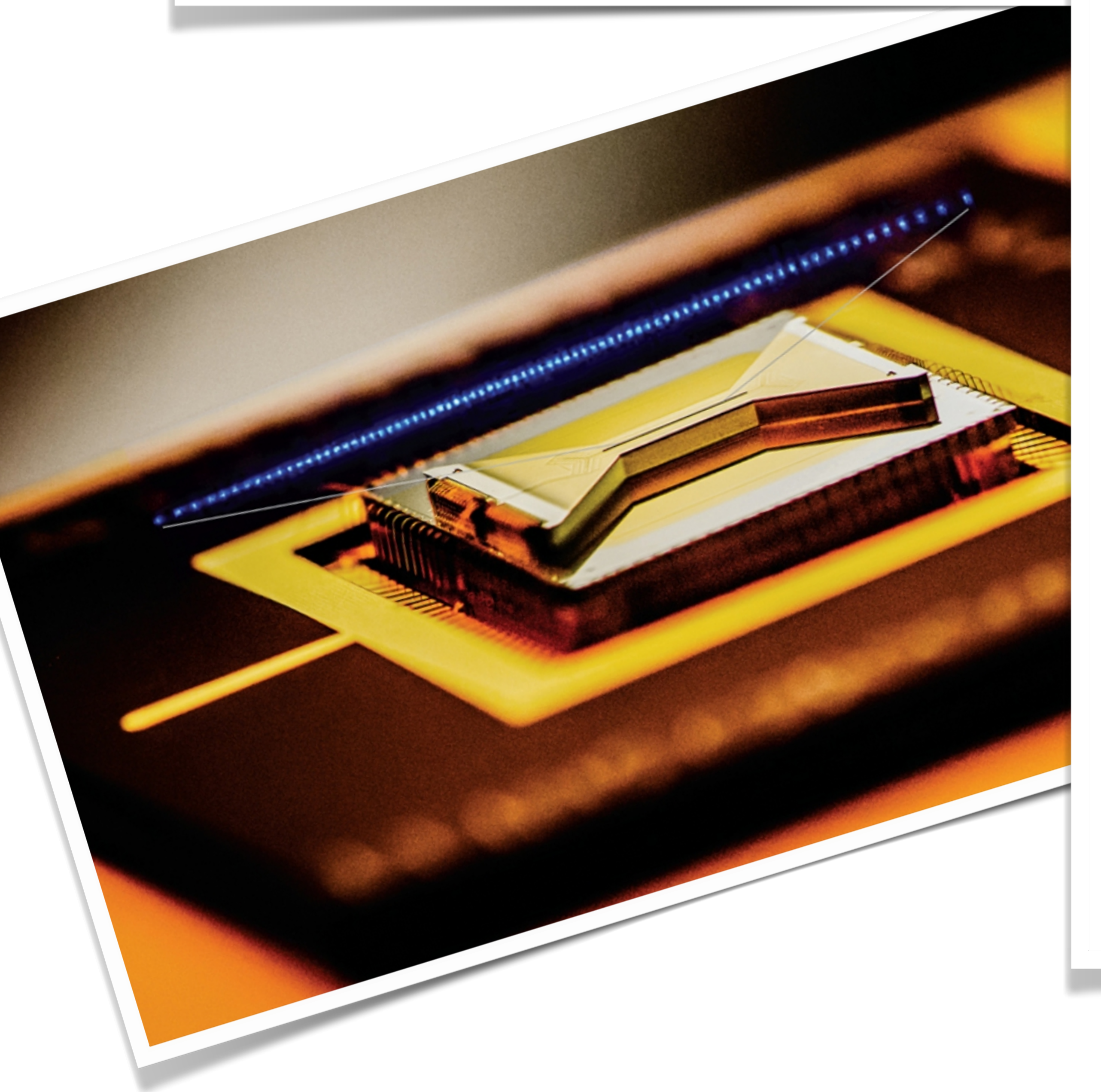
[7] Department of Electrical and Computer Engineering and Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA
(Received 9 June 2016; revised manuscript received 23 September 2016; published 17 November 2016)

Entanglement, and, in particular, the entanglement spectrum, plays a major role in characterizing many-body quantum systems. While there has been a surge of theoretical works on the subject, no experimental
to date because of the lack of an implementable measurement scheme.
protocol to access the entanglement spectrum of many-body states in
tical lattices. Our scheme effectively performs a Ramsey spectroscopy of
d is based on the ability to produce several copies of the state under
ssibility to perform a global swap gate between two copies conditioned
We show how the required conditional swap gate can be implemented



Mixed-State Entanglement from Local Randomized Measurements

Andreas Elben,[1,2,*] Richard Kueng,[3,*] Hsin-Yuan (Robert) Huang,[4,5] Rick van Bijnen,[1,2] Christian Kokail,[1,2] Marcello Dalmonte,[6,7] Pasquale Calabrese,[6,7,8] Barbara Kraus,[9] John Preskill,[4,5,10,11] Peter Zoller,[1,2] and Benoît Vermersch[1,2,12]
[1] Center for Quantum Physics, University of Innsbruck, Innsbruck A-6020, Austria
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[5] Department of Computing and Mathematical Sciences, Caltech, Pasadena, California 91125,
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[9] Institute for Theoretical Physics, University of Innsbruck, A6020 Innsbruck, Austria
[10] Walter Burke Institute for Theoretical Physics, Caltech, Pasadena, California 91125, USA
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[12] Université Grenoble Alpes, CNRS, LPMMC, 38000 Grenoble, France

(Received 22 July 2020; accepted 20 October 2020; published 11 November 2020)

We propose a method for detecting bipartite entanglement in a many-body mixed state bas
estimating moments of the partially transposed density matrix. The estimates are obtained by perfo
local random measurements on the state, followed by postprocessing using the classical sh
framework. Our method can be applied to any quantum system with single-qubit control. We p
a detailed analysis of the required number of experimental runs, and demonstrate the protocol
existing experimental data [Brydges *et al.*, Science **364**, 260 (2019)].

DOI: 10.1103/PhysRevLett.125.200501

nature physics
ARTICLES
https://doi.org/10.1038/s41567-018-0151-7

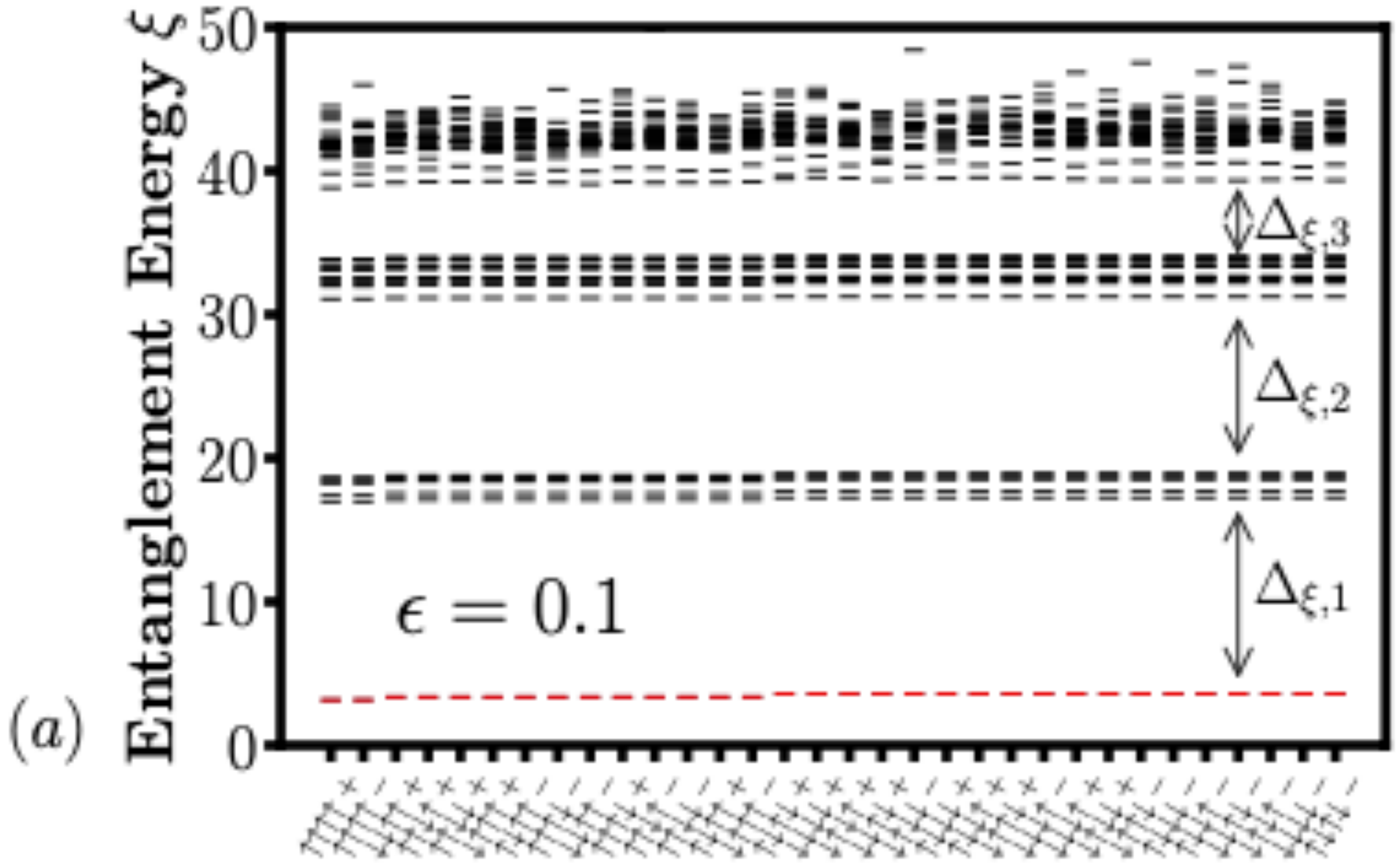
Quantum simulation and spectroscopy of entanglement Hamiltonians

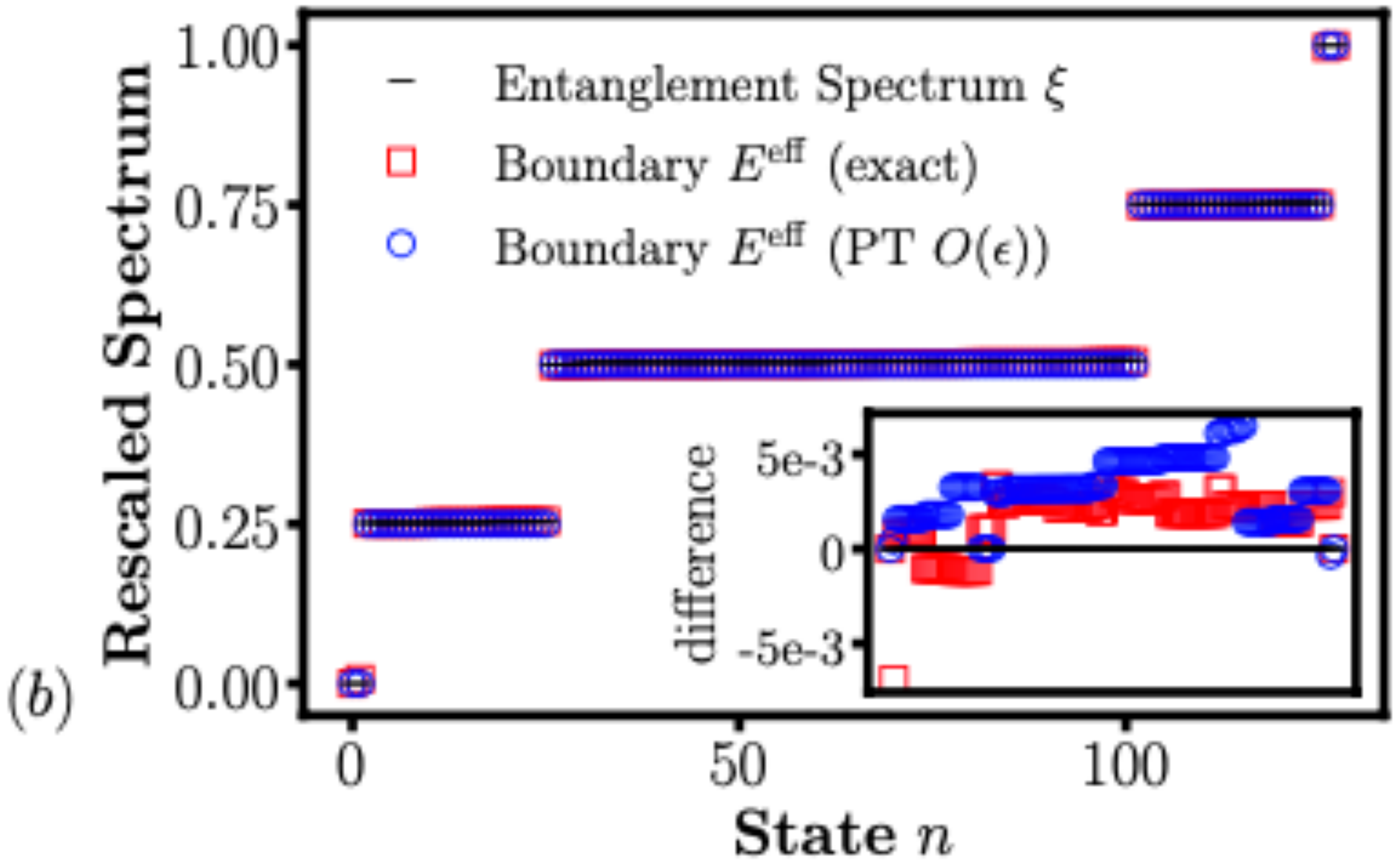
M. Dalmonte[1,2*], B. Vermersch[3,4] and P. Zoller[3,4]

The properties of a strongly correlated many-body quantum system, from the presence of topological order to the onset of quantum criticality, leave a footprint in its entanglement spectrum. The entanglement spectrum is composed by the eigenvalues of the density matrix representing a subsystem of the whole original system, but its direct measurement has remained elusive due to the lack of direct experimental probes. Here we show that the entanglement spectrum of the ground state of a broad class of Hamiltonians becomes directly accessible via the quantum simulation and spectroscopy of a suitably constructed entanglement Hamiltonian, building on the Bisognano–Wichmann theorem of axiomatic quantum field theory. This theorem gives an explicit physical construction of the entanglement Hamiltonian, identified as the Hamiltonian of the many-body system of interest with spatially varying couplings. On this basis, we propose a scalable recipe for the measurement of a system's entanglement spectrum via spectroscopy of the corresponding Bisognano–Wichmann Hamiltonian realized in synthetic quantum systems, including atoms in optical lattices and trapped ions. We illustrate and benchmark this scenario on a variety of models, spanning phenomena as diverse as conformal field theories, topological order and quantum phase transitions.

Bonus I: Demonstration of Li and Haldane's conjecture for Gauge Theories

- ground state $\epsilon = 0.1$, $V_x = V_y = 1$ (topological order)





arXiv:2107.11416

- can even show analytically in perturbation theory

$$H_A^{\text{eff}} = -\epsilon \sum_{n_y=0}^{N_y-1} \sigma^x_{(0,n_y),y}\sigma^x_{(0,n_y-1),y}\, \mu^x_{\mathbf{n}}\, \mu^x_{\mathbf{n}-\hat{y}} + O(\epsilon^2)\,.$$

(effective boundary Hamiltonian)

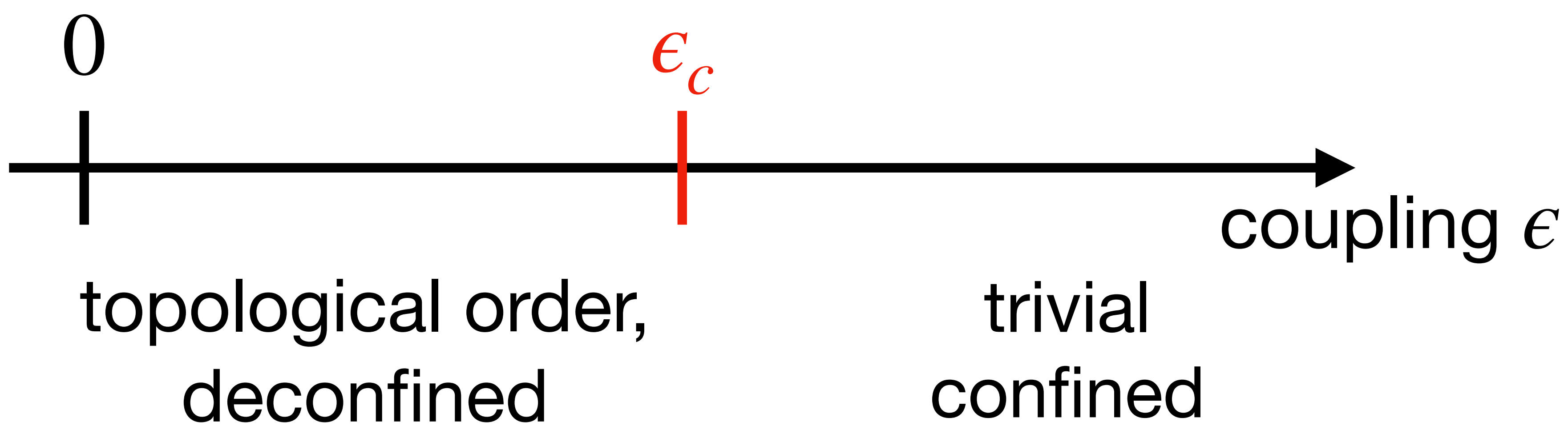
=

$$H_A^{\text{ent}} = N_y \log(2) + \frac{\epsilon}{2} \sum_{n_y=0}^{N_y-1} \sigma^x_{(0,n_y),y}\sigma^x_{(0,n_y-1),y}\, \mu^x_{\mathbf{n}}\, \mu^x_{\mathbf{n}-\hat{y}}\,,$$

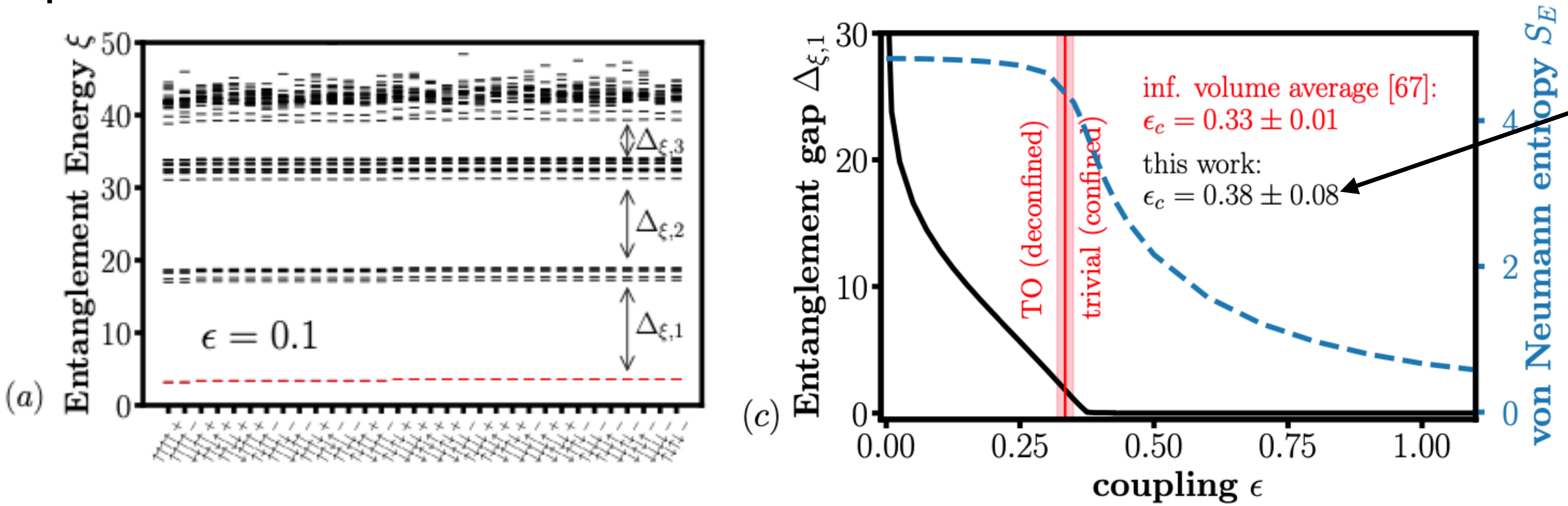
(EntanglementHamiltonian)

Bonus II: Extracting Quantum Phase Transitions from Entanglement Spectrum

- phase diagram (T=0)



- phase transition without local order parameter



6×4 lattice!

$\epsilon_c = 0.38 \pm 0.08$

- phase transition from closure of “entanglement gap” in Entanglement Spectrum (did not compute Wilson line expectation value!)

arXiv:2107.11416

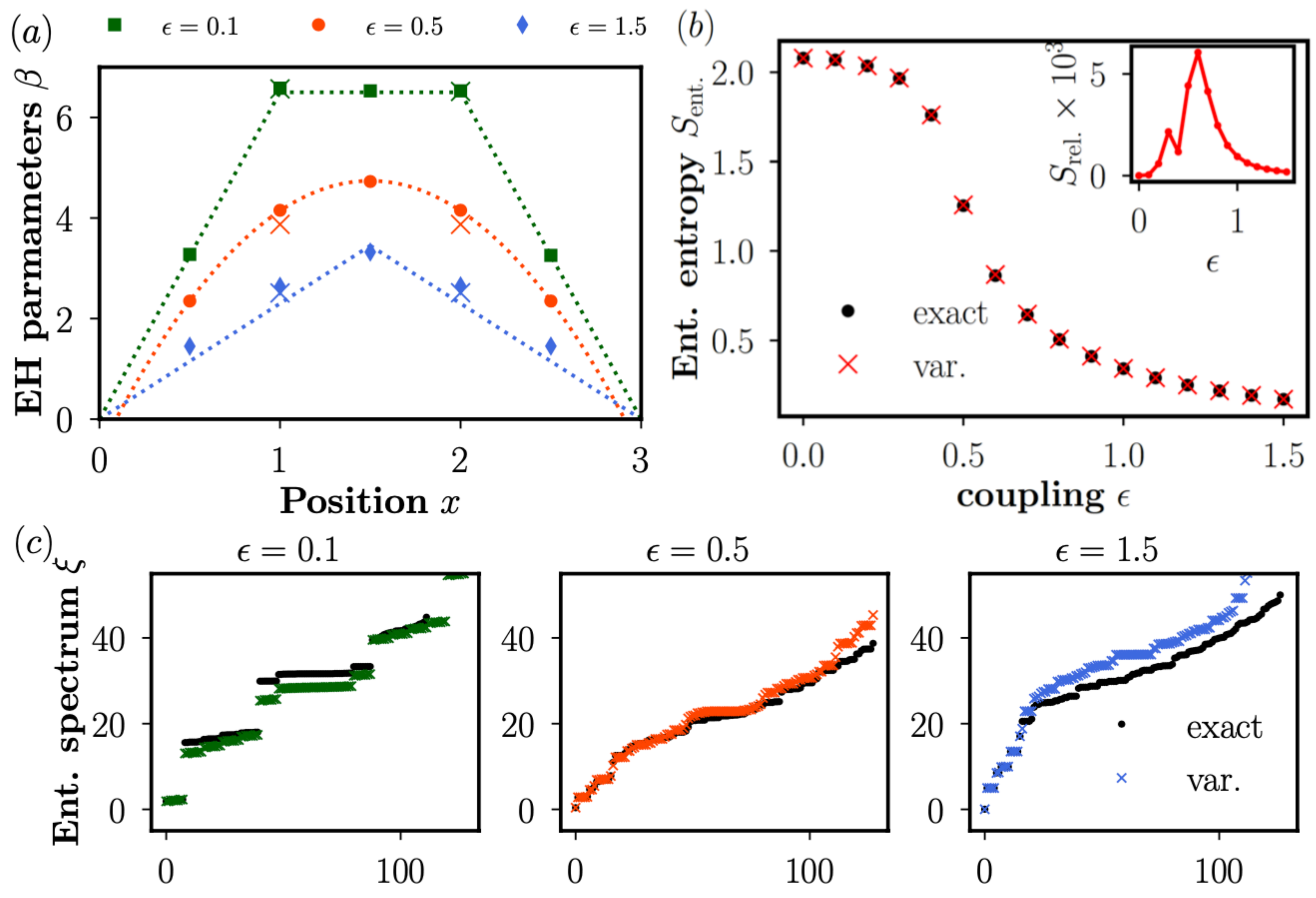
Bonus III: Bisognano-Wichmann theorem for Gauge Theories

- ansatz for reduced DM & Entanglement Hamiltonian $\rho_A \approx \sigma_A$

$$\sigma_A \equiv \frac{1}{Z} e^{-\sum_{\mathbf{n}\in A} \beta_{\mathbf{n}} h_{\mathbf{n}}} \,, \quad Z = \mathrm{Tr}\left[e^{-\sum_{\mathbf{n}\in A} \beta_{\mathbf{n}} h_{\mathbf{n}}}\right]$$

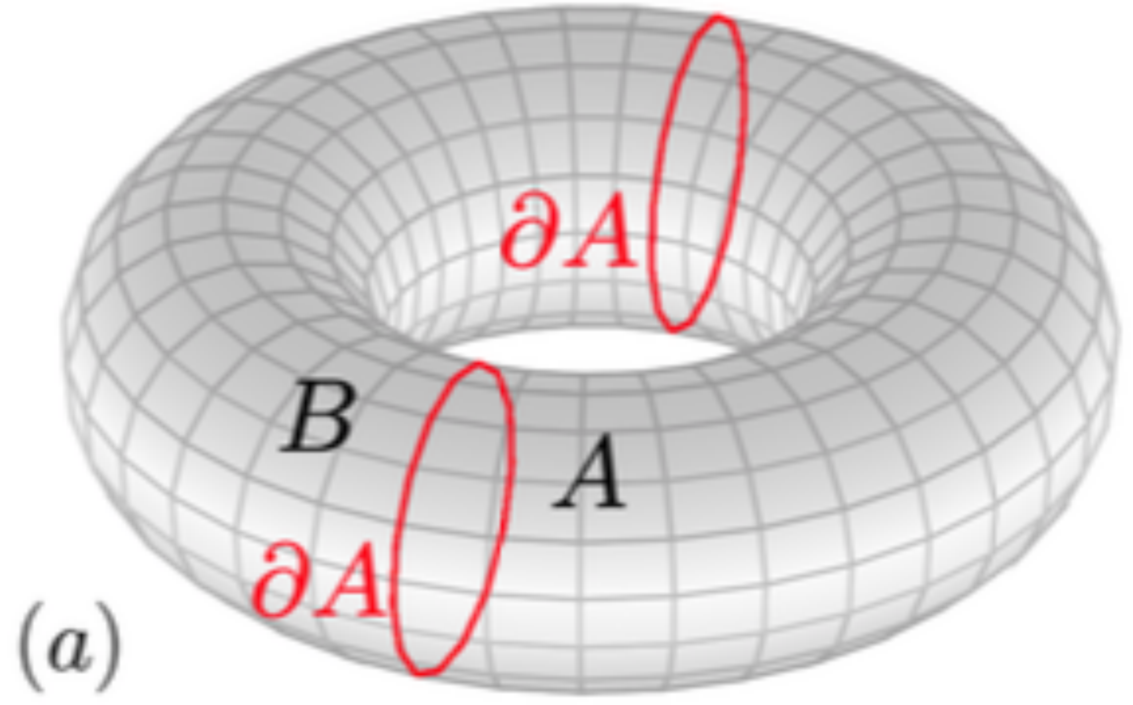
- minimization of relative entropy

$$\begin{aligned} S(\rho_A || \sigma_A) &\equiv \mathrm{Tr}\left[\rho_A \left(\log \rho_A - \log \sigma_A\right)\right] \\ &= -S(\rho_A) + \log Z + \sum_{\mathbf{n}} \beta_{\mathbf{n}} \langle h_{\mathbf{n}} \rangle \,, \end{aligned}$$



arXiv:2107.11416

Backup I: dual formulation



- Z_2 LGT in 2+1d

$$H=-\sum_{\mathbf{n}}\sigma^z_{\mathbf{n},x}\sigma^z_{\mathbf{n}+\hat{x},y}\sigma^z_{\mathbf{n}+\hat{y},x}\sigma^z_{\mathbf{n},y}-\epsilon\sum_{\mathbf{n},i=x,y}\sigma^x_{\mathbf{n},i}\,,$$

- Dual formulation, $N_x \times N_y = (N_x^A + N_x^B) \times N_y$, entanglement cuts at $n_x = 0, N_x^A - 1$

$$H = H_A + H_B + H_{AB}$$

$$\begin{aligned}
H_A \equiv &-\mu^z_{(0,0)}-\sum_{n_x=1}^{N_x^A-3}\sum_{n_y=0}^{N_y-1}\mu^z_{\mathbf{n}}-\sum_{n_y=1}^{N_y-1}\mu^z_{(0,n_y)}\sigma^z_{(0,n_y),y}\\
&-\sum_{n_y=0}^{N_y-1}\mu^z_{(N_x^A-2,n_y)}\sigma^z_{(N_x^A-1,n_y),y}-\epsilon\mu^x_{(0,0)}\\
&-\epsilon\sum_{n_y=1}^{N_y-1}\sigma^x_{(0,n_y),y}-\epsilon\sum_{n_y=0}^{N_y-1}\sigma^x_{(N_x^A-1,n_y),y}\\
&-\epsilon\sum_{n_x=1}^{N_x^A-2}\sum_{n_y=0}^{N_y-1}\mu^x_{\mathbf{n}}\mu^x_{\mathbf{n}-\hat{x}}-\epsilon\sum_{n_x=0}^{N_x^A-2}\sum_{n_y=1}^{N_y-1}\mu^x_{\mathbf{n}}\mu^x_{\mathbf{n}-\hat{y}}\\
&-\epsilon\sum_{n_x=0}^{N_x^A-2}\mu^x_{(n_x,0)}\mu^x_{(n_x,N_x-1)}V_y\,, \qquad (19)
\end{aligned}$$

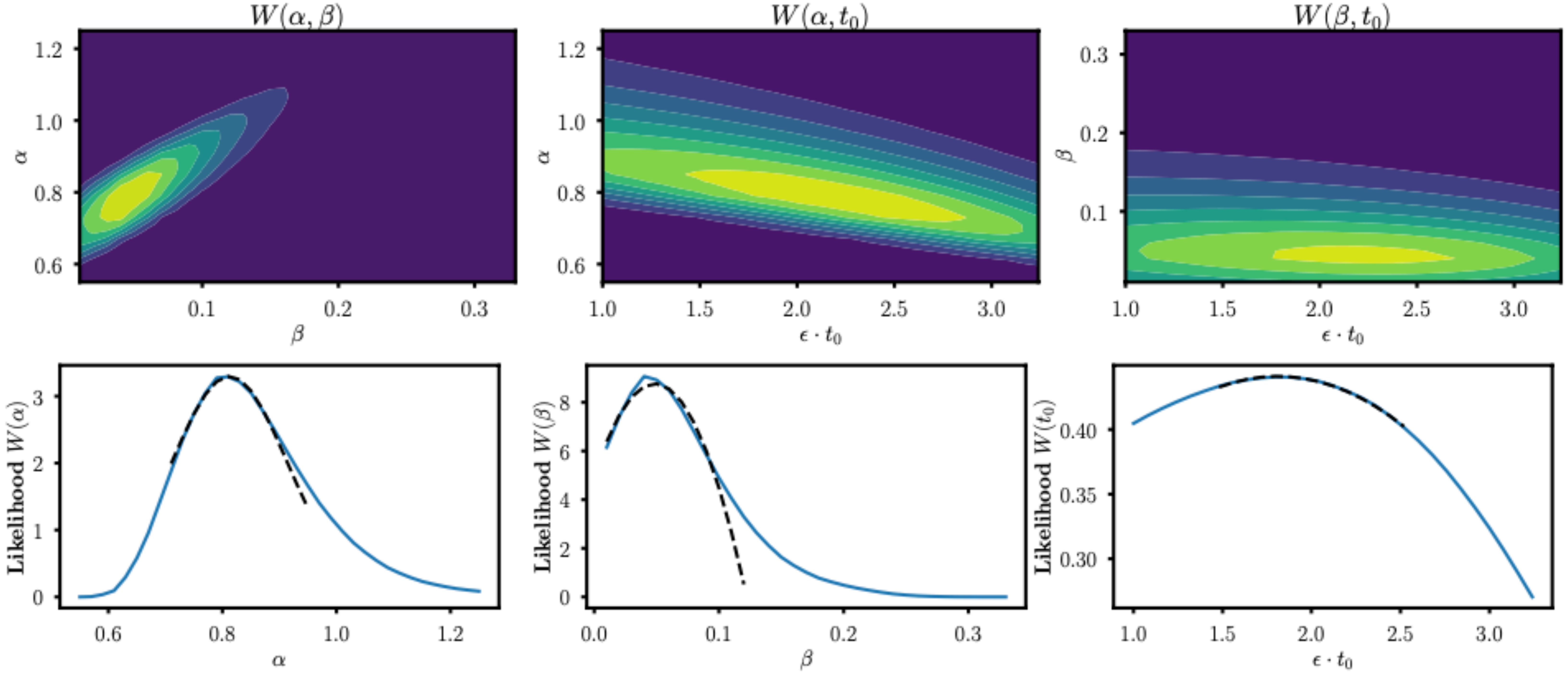
$$\begin{aligned}
H_B \equiv &-\sum_{n_x=N_x^A}^{N_x-2}\sum_{n_y=0}^{N_y-1}\mu^z_{\mathbf{n}}-\epsilon V_y\mu^x_{(N_x-1,N_y-1)}\\
&-\epsilon\sum_{n_y=2}^{N_y-1}\mu^z_{(N_x-1,n_y)}\mu^z_{(N_x-1,n_y-1)}-\epsilon\mu^x_{(N_x-1,1)}\\
&-\epsilon\sum_{n_y=1}^{N_y-1}\mu^x_{(N_x^A-1,n_y)}\mu^x_{(N_x^A-1,n_y-1)}\\
&-\epsilon\mu^x_{(N_x^A-1,0)}\mu^x_{(N_x^A-1,N_y-1)}V_y-\epsilon\sum_{N_x^A}^{N_x-2}\sum_{n_y=1}^{N_y-1}\mu^x_{\mathbf{n}}\mu^x_{\mathbf{n}-\hat{y}}\\
&-\epsilon\sum_{n_x=N_x^A}^{N_x-2}\mu^x_{(n_x,0)}\mu^x_{(n_x,N_y-1)}V_y-\epsilon\mu^x_{(N_x-2,0)}\\
&-\epsilon\sum_{n_x=N_x^A}^{N_x-1}\sum_{n_y=0}^{N_y}\Big|_{\mathbf{n}\neq(N_x-1,0)}\mu^x_{\mathbf{n}}\mu^x_{\mathbf{n}-\hat{x}}\,, \qquad (20)
\end{aligned}$$

$$\begin{aligned}
H_{AB} \equiv &-\sum_{n_y=0}^{N_y-1}\mu^z_{(N_x^a-1,n_y)}\sigma^z_{(N_x^A-1,n_y)}\\
&-\sum_{n_y=1}^{N_y-1}\mu^z_{(N_x-1,n_y)}\sigma^z_{(0,n_y),y}\\
&-\text{(prod. all plaquettes A \& B)} \qquad (21)
\end{aligned}$$

- **No four body terms!!!**

Backup II: scaling analysis

- Scaling analysis as in Mace, Mueller, Schlichting, Sharma PRL 124, 191604 (2020)



- Scaling ansatz

$$P(n,t) = \tau^{-\alpha} P(\tau^{\beta} n)\,, \qquad f_{\rm ref}(t, \hat{n} = \tau^{\beta} n) = \log\{\tau_{\rm ref}^{-\alpha} P(\tau_{\rm ref}^{\beta} n)\}\,. \qquad f_{\rm test}(t,\hat{n}) = \log\{\tau_{\rm test}^{-\alpha} P(\tau_{\rm test}^{\beta} n)\}\,.$$

- Likelihood

$$\chi^2(\alpha,\beta,t_0) = \frac{1}{N_t} \sum_{t \in t_{\rm test}} \frac{\int \frac{d\hat{n}}{\hat{n}} (f_{\rm ref}(\hat{n}) - f_{\rm test}(\hat{n}))^2}{\int \frac{d\hat{n}}{\hat{n}} (f_{\rm ref}(\hat{n}))^2}\,. \qquad W(\alpha,\beta,t_0) = \frac{1}{\mathcal{N}} \exp\left(-\chi^2(\alpha,\beta,t_0)/\chi^2_{\rm min}\right)$$