Quirks of QCD

Twist-2 Operators on the Lattice

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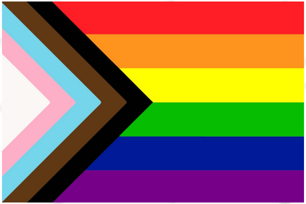
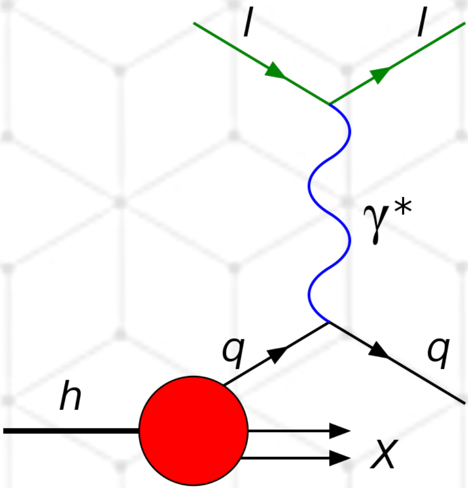


Deep Inelastic Scattering (DIS)

- Study the structure of nucleons using electrons as a probe
- First experimental evidence of quarks
- At very high energies, the electron breaks the nucleon into different hadrons
- Calculation involves the Operator Product Expansion (OPE)
- A helpful tool for solving these types of problems are the Twist-2 Operators, where twist= dimension-spin
- Twist-2 Operators related to PDFs

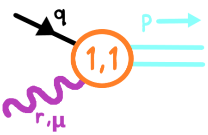


The Gradient Flow

- break rotational symmetry $\Rightarrow$ twist isn't a good quantum number
- smear the fields in physical units, as opposed to lattice units [Lüsher, Weisz 2011]
- $B_\mu|_{\tau=0} = A_\mu$
- $\partial_\tau B_\mu = D_\nu G_{\nu\mu} + \alpha_0 D_\mu \partial_\nu B_\nu$ where $D_\mu = \partial_\mu + [B_\mu, .]$
- $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$
- new propagators, quark and gluon kernels, flowed vertices
- QCD vertices are at $\tau = 0$.

$$= e^{-(t-s)p^2}\theta(t-s)$$

$$= \frac{\delta^{\alpha\beta}\delta_{\mu\nu}\theta(t-s)}{p^2}$$



$$= X^{(1,1)}(p,q,r)^a_\mu$$
$$= gt^a(r_\mu + 2q_\mu)$$

Our Goal

Demonstrate that applying the gradient flow to the lattice controls power-divergent mixing of twist-2 operators.

Symmetry of the Lattice

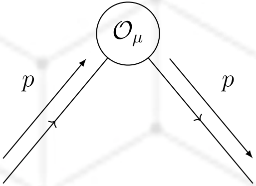
- lattice symmetry group $H(4)$
- twist-2 operators in different irreducible representation of H(4) will not mix under renormalization [Göckeler et al. 1996]
- twist-2 operators in the same irrep will mix?
- take simplest example

$$\widetilde{\mathcal{O}}_\mu = \bar{\psi}\gamma_\mu\psi$$

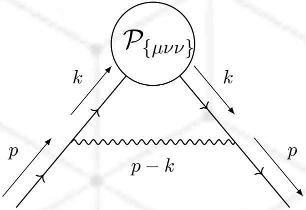
$$\widetilde{\mathcal{P}}_{\{\mu\nu\nu\}} = \frac{1}{\sqrt{2}}\sum_{\nu=1}^{4}\bar{\psi}\gamma_{\{\mu}\overleftrightarrow{D}_\nu\overleftrightarrow{D}_{\nu\}}\psi = \frac{1}{\sqrt{2}}\sum_{\nu=1}^{4}\left(\bar{\psi}\gamma_\mu\overleftrightarrow{D}_\nu\overleftrightarrow{D}_\nu\psi + \bar{\psi}\gamma_\nu\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu\psi + \bar{\psi}\gamma_\nu\overleftrightarrow{D}_\nu\overleftrightarrow{D}_\mu\psi\right)$$

- in irreducible representation $\tau_4^{(1)} = I^-$

Continuum without the Gradient Flow



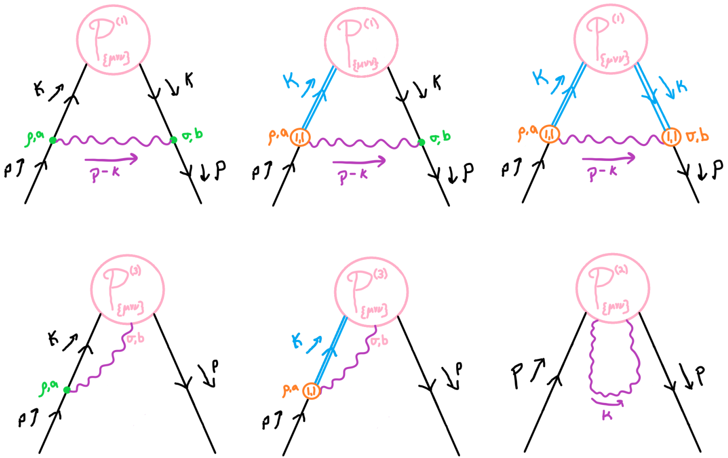
$$= \gamma_\mu$$



$$= \frac{-2i\sqrt{2}g^2 C_F}{(4\pi)^2}(\gamma^\mu p^2 + 2\not{p}p^\mu)$$

Continuum with the Gradient Flow

$$\Gamma^{\text{tree}}_{\mathcal{O}} = \gamma_\mu e^{-2\tau p^2} \text{ and } \Gamma^{(\text{tree})}_{\mathcal{P}} = -2\sqrt{2}(\gamma_\mu p^2 + 2\not{p} p_\mu) e^{-2\tau p^2},$$



Continuum with the Gradient Flow

$$\Gamma^{\text{tree}}_{\mathcal{O}} = \gamma_\mu e^{-2\tau p^2}$$

$$\Gamma^{\mu}_{\mathcal{P}A} = 2\sqrt{2}g^2 C_F[a(p^2\gamma^\mu + 2p^\mu \not{p}) + b\not{p}\gamma^\mu]$$

$$a = m^2 p^2 K_{2,4}(2t) + \left(\frac{(D-2)(D+1)}{D}p^2 + m^2\right) K_{2,2}(2t) - \frac{(D-2)^2}{D} K_{2,0}(2t) \quad (1)$$

$$b = \frac{2imp^2(4-D)}{D} K_{2,2}(2t), \text{ and } K_{n,l}(x) = \int_k \frac{e^{-xk^2}}{(k^2+m^2)^n k^l}. \quad (2)$$

Result

We still see evidence of power-divergent mixing in the continuum even with the gradient flow, but this mixing is now controlled by the flow time, τ.

Conclusion

So Far

- demonstarted that, in the continuum, **without** the gradient flow, our example operators have power divergent mixing
- demonstrated that, in the continuum **with** the gradient flow, our example operators have power divergent mixing, but the mixing is controlled by the flow time

Next Steps

- redo the calculation on the lattice **without** the gradient flow
- redo the calculation on the lattice **with** the gradient flow
- investigate the power divergence on the lattice. Not obvious because two things going on. check if our method controls the power divergent mixing between the two operators

Thanks!