Search for Efficient Formulations of non-Abelian Lattice Gauge Theories for Hamiltonian Simulation

Indrakshi Raychowdhury,
Zohreh Davoudi, Andrew Shaw
University of Maryland, College Park

26 July, 2021

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Lattice QCD

- Certain inaccessible regime: SIGN PROBLEM.

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Lattice QCD

- Certain inaccessible regime: SIGN PROBLEM.

GOAL

Quantum simulating or quantum computing for Lattice QCD

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Lattice QCD

- Certain inaccessible regime: SIGN PROBLEM.

GOAL

Quantum simulating or quantum computing for Lattice QCD

Lattice gauge theory Computations

Classical Computation Era — Quantum Computation Era

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are generally different.

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Lattice QCD

- Certain inaccessible regime: SIGN PROBLEM.

GOAL

Quantum simulating or quantum computing for Lattice QCD

Lattice gauge theory Computations

Classical Computation Era

Quantum Computation Era

Monte Carlo simulation

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are entirely different.

Hamiltonian simulation

Framework: Hamiltonian Formalism

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York and Tel Aviv University, Ramat Aviv, Israel and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Quantum simulation of LATTICE-QCD: A BRIEF STATUS REPORT

Quantum Computing/Simulating QCD

Gauge theory,
SU(3) in 3+1 dimension

Too complicated to start with!

Simpler, yet similar theories:

U(1) gauge theory: Quantum Electrodynamics (QED)

Schwinger Model: QED in 1+1d

Simple theory: discrete gauge theories

$\mathbb{Z}_N$ gauge theory; $\mathbb{Z}_2$ gauge theory in 2+1 dimensions

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Schwinger Model: QED in 1+1d

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

Quantum-classical computation of Schwinger model dynamics using quantum computers

N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage
Phys. Rev. A **98**, 032331 – Published 28 September 2018

Towards analog quantum simulations of lattice gauge theories with trapped ions

Zohreh Davoudi,[1,2] Mohammad Hafezi,[3,4] Christopher Monroe,[3,5] Guido Pagano,[3,5,6] Alireza Seif,[3] and Andrew Shaw[1]

Quantum Algorithms for Simulating the Lattice Schwinger Model

Alexander F. Shaw[1,5], Pavel Lougovski[1], Jesse R. Stryker [2], and Nathan Wiebe[3,4]

and many more..

Experimental Demonstration:

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer (2016)

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

A scalable realization of local U(1) gauge invariance in cold atomic mixtures (2020)

Alexander Mil[1,*], Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]

Observation of gauge invariance in a 71-site Bose–Hubbard quantum simulator (2020)

Bing Yang, Hui Sun, Robert Ott, Han-Yi Wang, Torsten V. Zache, Jad C. Halimeh, Zhen-Sheng Yuan, Philipp Hauke & Jian-Wei Pan

Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices (2019)

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch & Monika Aidelsburger

Realization of density-dependent Peierls phases to engineer quantized gauge fields coupled to ultracold matter (2019)

Frederik Görg, Kilian Sandholzer, Joaquín Minguzzi, Rémi Desbuquois, Michael Messer & Tilman Esslinger

Simple theory: discrete gauge theories $\mathbb{Z}_N$ gauge theory; $\mathbb{Z}_2$ gauge theory in 2+1 dimensions

Quantum simulation of LATTICE-QCD: A BRIEF STATUS REPORT

Simplest, non-abelian gauge theory: **SU(2) gauge theory**

Proposals:

PRL **110,** 125304 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Cold-Atom Quantum Simulator for SU(2) Yang-Mills Lattice Gauge Theory

Erez Zohar,[1] J. Ignacio Cirac,[2] and Benni Reznik[1]

PRL **110,** 125303 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Atomic Quantum Simulation of U(N) and SU(N) Non-Abelian Lattice Gauge Theories

D. Banerjee,[1] M. Bögli,[1] M. Dalmonte,[2] E. Rico,[2,3] P. Stebler,[1] U.-J. Wiese,[1] and P. Zoller[2,3]

PRL **112,** 120406 (2014) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2014

Constrained Dynamics via the Zeno Effect in Quantum Simulation: Implementing Non-Abelian Lattice Gauge Theories with Cold Atoms

K. Stannigel,[1] P. Hauke,[1,*] D. Marcos,[1] M. Hafezi,[2] S. Diehl,[1,3] M. Dalmonte,[1,3,†] and P. Zoller[1,3]

PRL **115,** 240502 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

A. Mezzacapo,[1,2] E. Rico,[1,3] C. Sabín,[4] I. L. Egusquiza,[5] L. Lamata,[1] and E. Solano[1,3]

No Experimental Demonstration Yet!

Digital implementation:

PHYSICAL REVIEW D **101,** 074512 (2020)

SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers

Natalie Klco,[*] Martin J. Savage,[†] and Jesse R. Stryker[‡]

IQuS@UW-21-001

A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis

Anthony Ciavarella,[1,*] Natalie Klco,[2,†] and Martin J. Savage[1,‡]

SU(2) hadrons on a quantum computer

Yasar Atas *,[1,2,†] Jinglei Zhang *,[1,2,‡] Randy Lewis,[3] Amin Jahanpour,[1,2] Jan F. Haase,[1,2,§] and Christine A. Muschik[1,2,4]

Way out?

Preliminary implementations!

Too restricted!

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Phys. Rev. D **103**, 094501 IQuS@UW-21-001

A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis

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Plenary

Friday, 30 July 2021

KLCO, Natalie (Caltech)

10:40 [638] SU(3) gauge theory on quantum hardware

Way out?

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Too restricted!

arXiv:2009.11802v1

Search for Efficient Formulations for Hamiltonian Simulation of non-Abelian Lattice Gauge Theories

Zohreh Davoudi,[1,2] Indrakshi Raychowdhury,[1] and Andrew Shaw[1]

[1] *Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA*

[2] *RIKEN Center for Accelerator-based Sciences, Wako 351-0198, Japan*

Readily available toolbox: **Classical computation**

Simplest theory to analyze: **SU(2) LGT in 1+1 dimension**

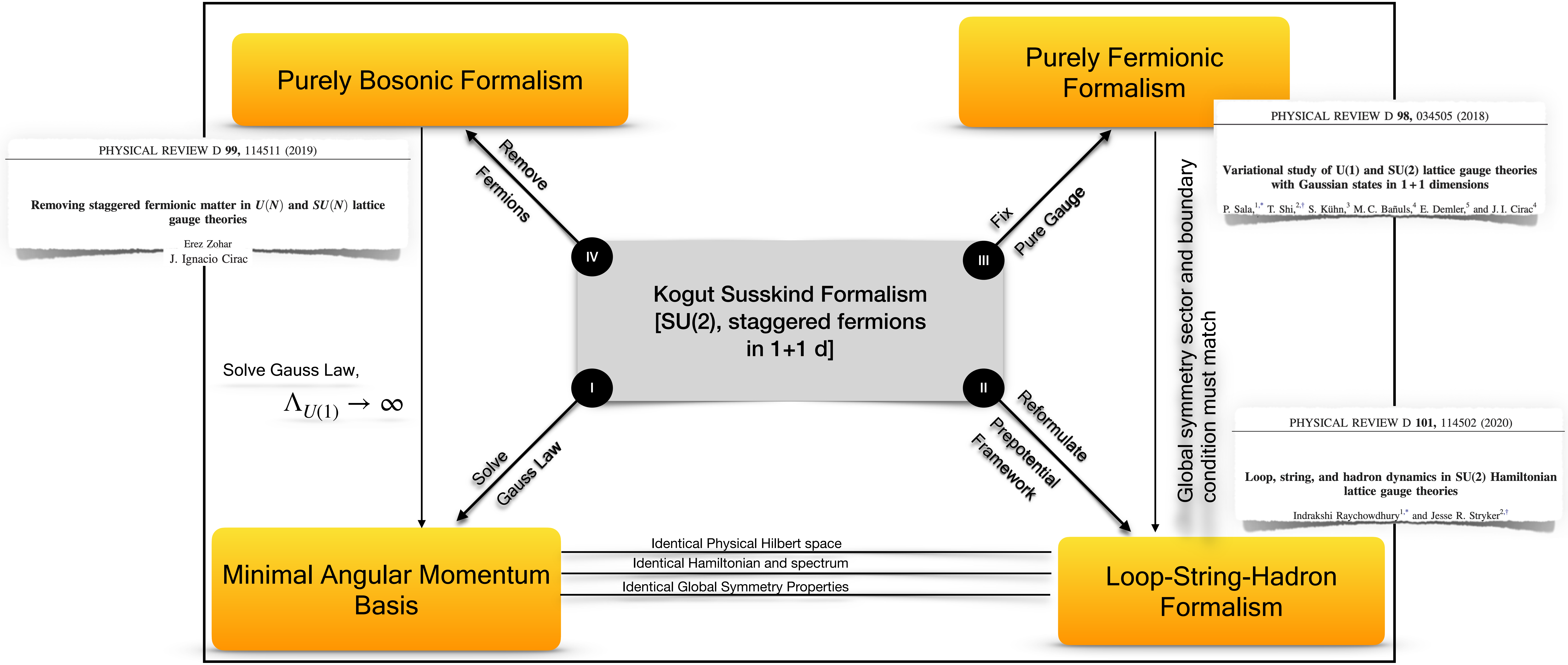
Computational technique: **Exact diagonalization**

Other technique: tensor network calculation: talk by Aniruddha Bapat

Tensor network simulations of a manifestly gauge-invariant SU(2) lattice gauge theory formulation

Thursday, July 29, 2021 9:15 PM (15 minutes)

Renewed interest in Hamiltonian LGT



Alternate model: Quantum link model, topic of the next talk in this session

Renewed interest in Hamiltonian LGT

	Pros.	Cons.
Angular Momentum Basis	• Minimal and physical basis	• Additional cost for imposing Gauss' law. • Calculation involves SU(2) CG coefficients, SU(3) generalization is nontrivial. • Physical basis is linear combination of angular momentum basis, exponential cost.
Purely Bosonic Formalism	• No fermionic degrees of freedom, useful in higher dimensions.	• Additional U(1) gauge field is introduced (i.e additional cut-off effect), at the cost of removing fermions using Gauss law. • All the non trivialities of angular momentum basis still exists.
Purely Fermionic Formalism	•No bosonic degrees of freedom, no cut-off effect.	• Only valid in 1 spatial dimensional lattice with open boundary condition.
Loop-String-Hadron Formalism	• Minimal and physical basis. • Local description of gauge invariant Hilbert space • States are 1-sparse. • Valid for any dimensions and any boundary condition.	• Involves extra lattice-sites and links in higher dimension.

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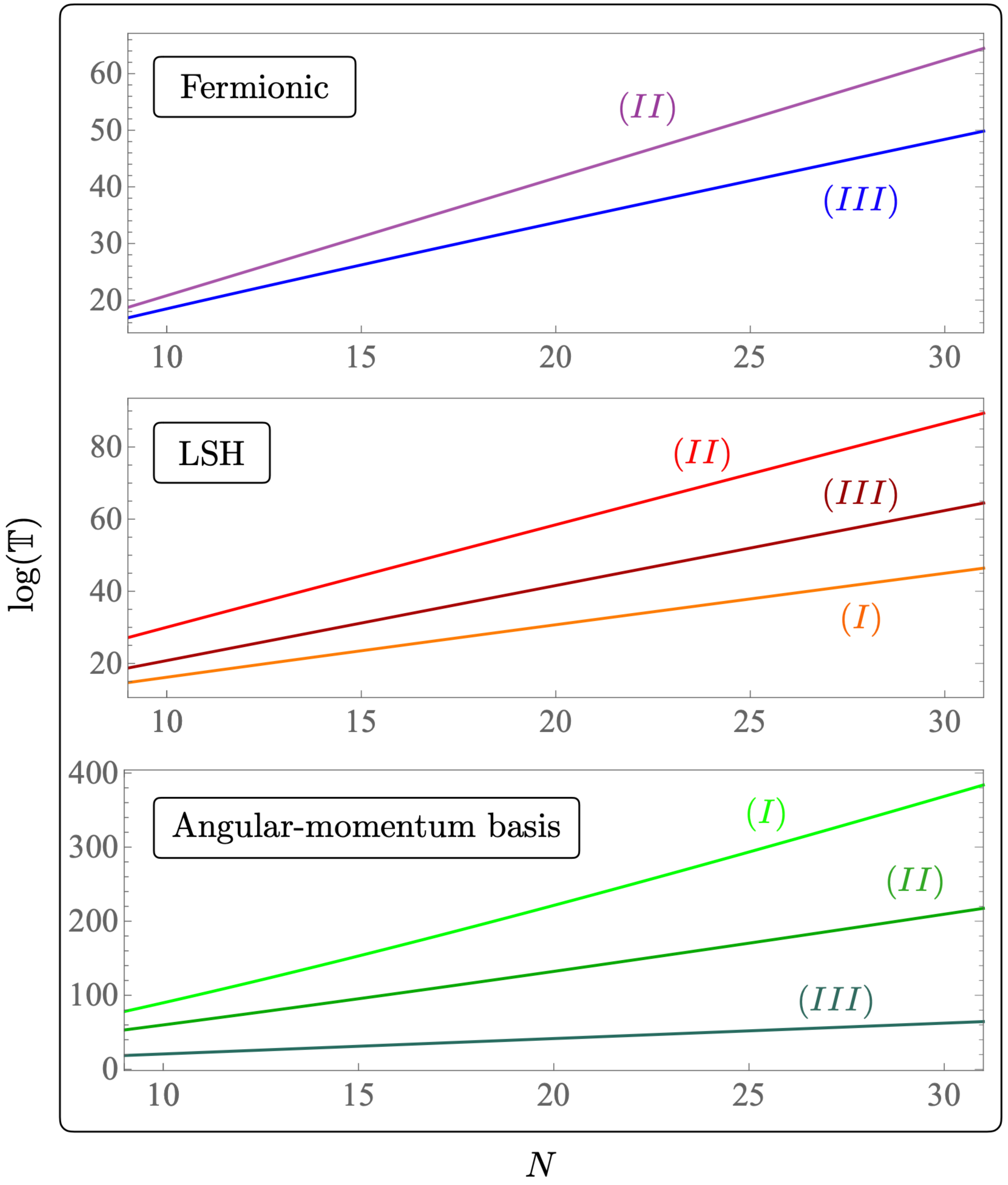
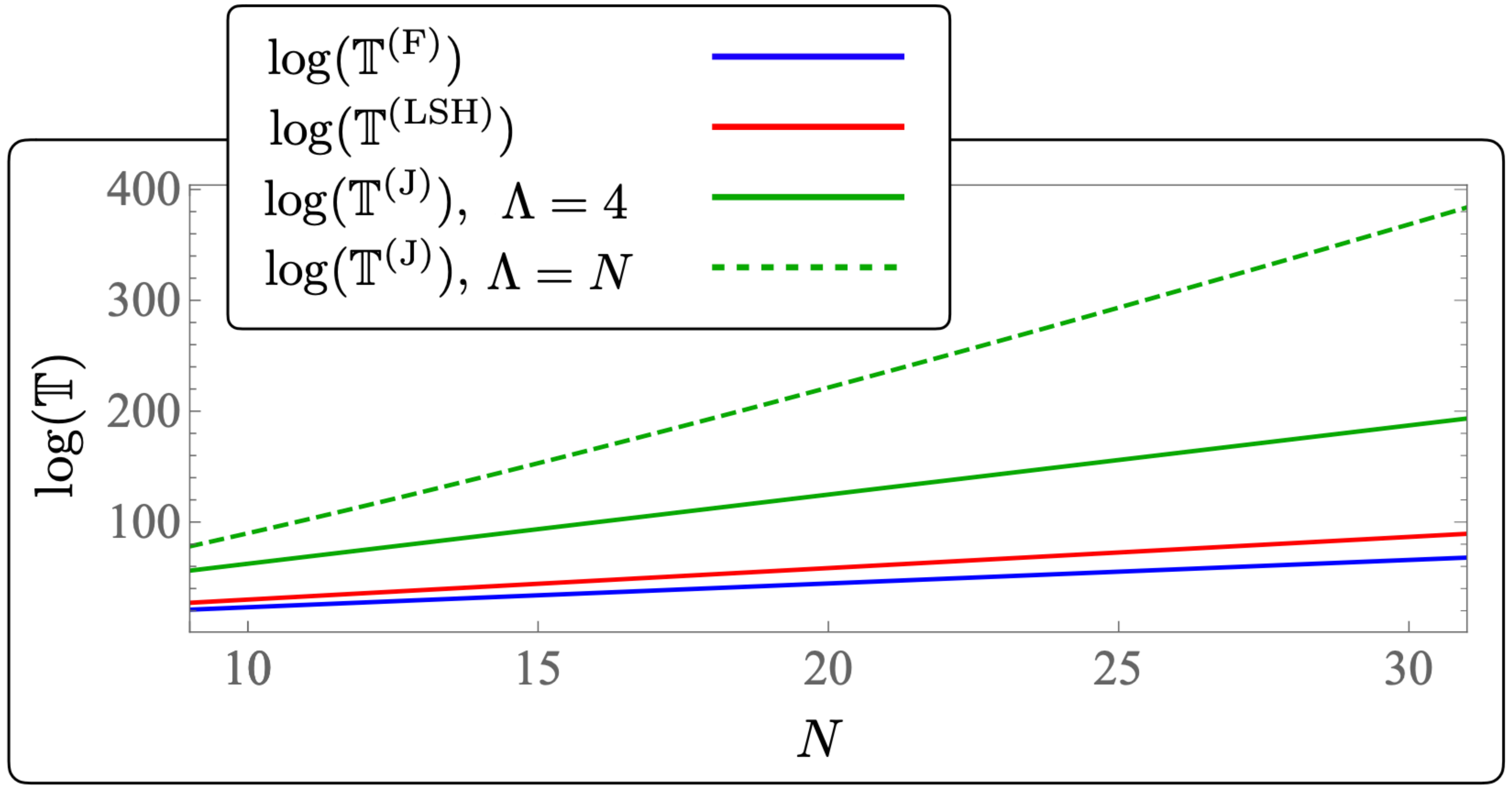
Should be more useful for Hamiltonian simulation.

Time-complexity of Hamiltonian simulation



(I) Hilbert-space construction,
(II) Hamiltonian generation,
(III) Observable computation.



Cumulative cost of Hamiltonian simulation

An explicit comparison: for N = 20,
Angular-momentum formulation (with $\Lambda = N$) requires 160 orders of magnitude larger computing resources than the LSH formulation, while the fermionic formulation requires 20 orders of magnitude lesser resources than the LSH formulation.

Conclusion:

Loop-String-Hadron formalism is more convenient to work with, at least in lower dimension.

An explicit example: 2 staggered site lattice with open boundary condition

Angular Momentum Basis

$$1)\ \ [|0,0\rangle\,|0,0\rangle\,|0,0\rangle]^{(0)} \otimes [|0,0\rangle\,|1,1\rangle\,|0,0\rangle]^{(1)}$$

$$\begin{aligned}2)\ & \frac{1}{2}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|0,0\rangle\right]^{(1)} \\ & - \frac{1}{2}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|1,0\rangle\,|0,0\rangle\right]^{(1)} \\ & - \frac{1}{2}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|0,0\rangle\right]^{(1)} \\ & + \frac{1}{2}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|1,0\rangle\,|0,0\rangle\right]^{(1)},\end{aligned}$$

$$\begin{aligned}3)\ & \frac{1}{\sqrt{6}}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|1,0\rangle\,|1,-1\rangle\right]^{(1)} \\ & - \frac{1}{2\sqrt{3}}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|1,0\rangle\right]^{(1)} \\ & - \frac{1}{2\sqrt{3}}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|1,0\rangle\,|1,0\rangle\right]^{(1)} \\ & + \frac{1}{\sqrt{6}}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|0,1\rangle\,|1,1\rangle\right]^{(1)} \\ & - \frac{1}{\sqrt{6}}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|1,0\rangle\,|1,-1\rangle\right]^{(1)} \\ & + \frac{1}{2\sqrt{3}}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|1,0\rangle\right]^{(1)} \\ & + \frac{1}{2\sqrt{3}}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|1,0\rangle\,|1,0\rangle\right]^{(1)} \\ & - \frac{1}{\sqrt{6}}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},-\tfrac{1}{2}\rangle\,|0,1\rangle\,|1,1\rangle\right]^{(1)},\end{aligned}$$

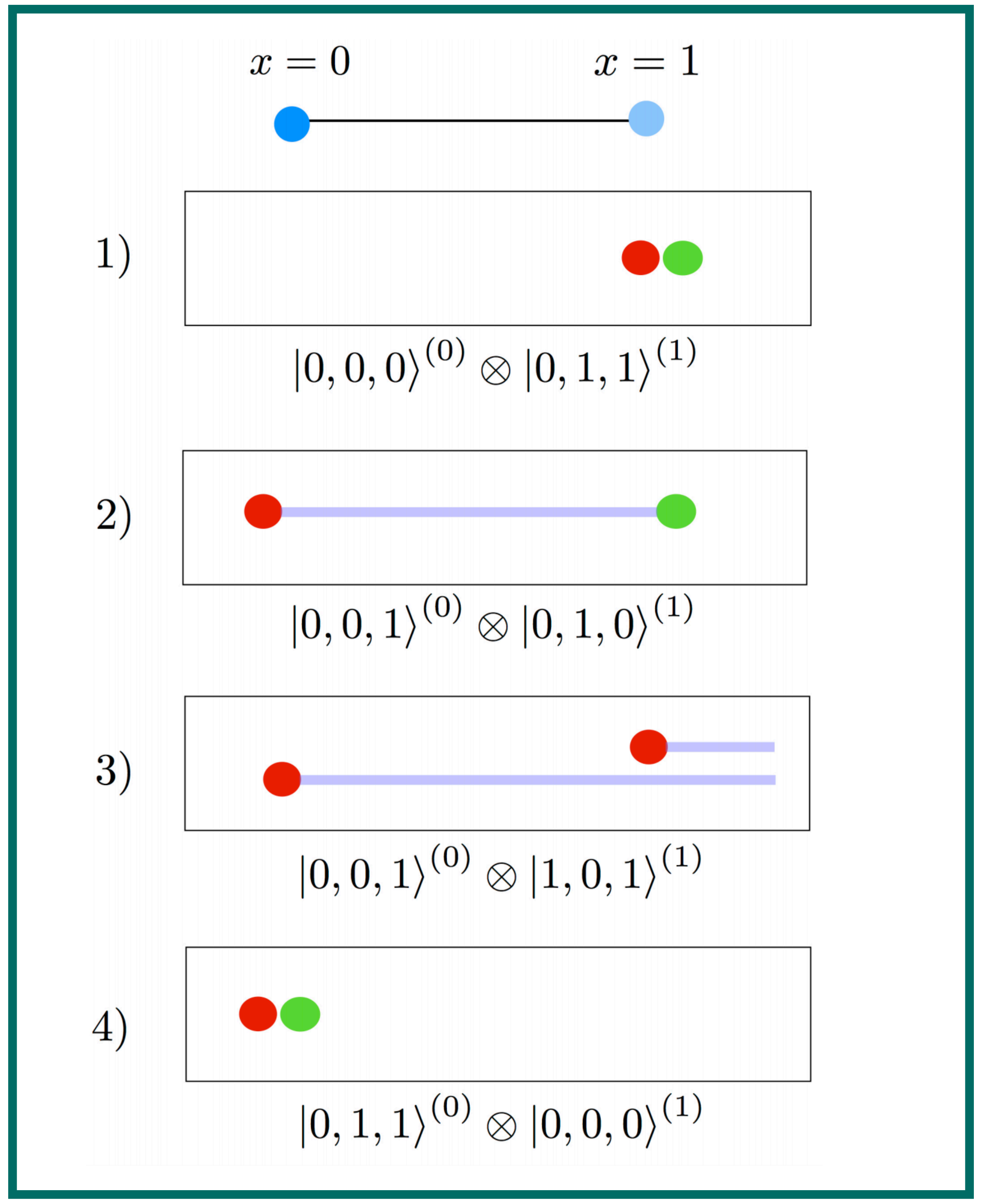
$$4)\ \ [|0,0\rangle\,|1,1\rangle\,|0,0\rangle]^{(0)} \otimes [|0,0\rangle\,|0,0\rangle\,|0,0\rangle]^{(1)}$$

Purely Fermionic Formalism

$$\begin{aligned}&1)\ |0,0\rangle^{(0)} \otimes |1,1\rangle^{(1)}, \\ &2)\ |0,1\rangle^{(0)} \otimes |0,1\rangle^{(1)}, \\ &3)\ |0,1\rangle^{(0)} \otimes |1,0\rangle^{(1)}, \\ &4)\ |1,0\rangle^{(0)} \otimes |0,1\rangle^{(1)}, \\ &5)\ |1,0\rangle^{(0)} \otimes |1,0\rangle^{(1)}, \\ &6)\ |1,1\rangle^{(0)} \otimes |0,0\rangle^{(1)}.\end{aligned}$$

Has redundancies in physical degrees Hilbert space

Loop-String-Hadron Formalism



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An explicit example: 2 staggered site lattice with open boundary condition

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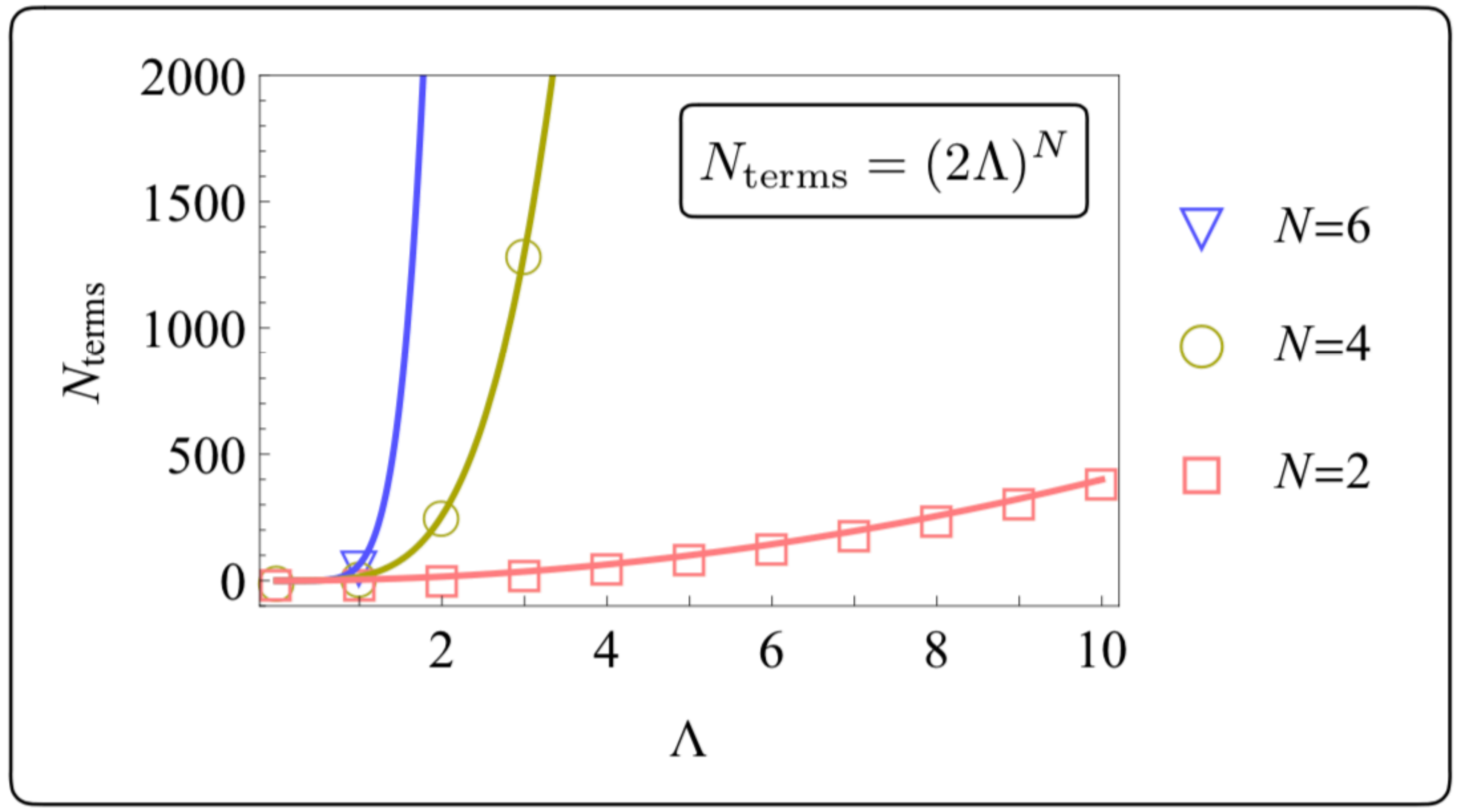
2) $\frac{1}{2}\left[|0,0\rangle\,|1,0\rangle\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|0,0\rangle\right]^{(1)}$



$+\frac{1}{2\sqrt{3}}\left[|0,0\rangle\,|0,1\rangle\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right]^{(0)} \otimes \left[|\tfrac{1}{2},\tfrac{1}{2}\rangle\,|0,1\rangle\,|1,0\rangle\right]^{(1)}$

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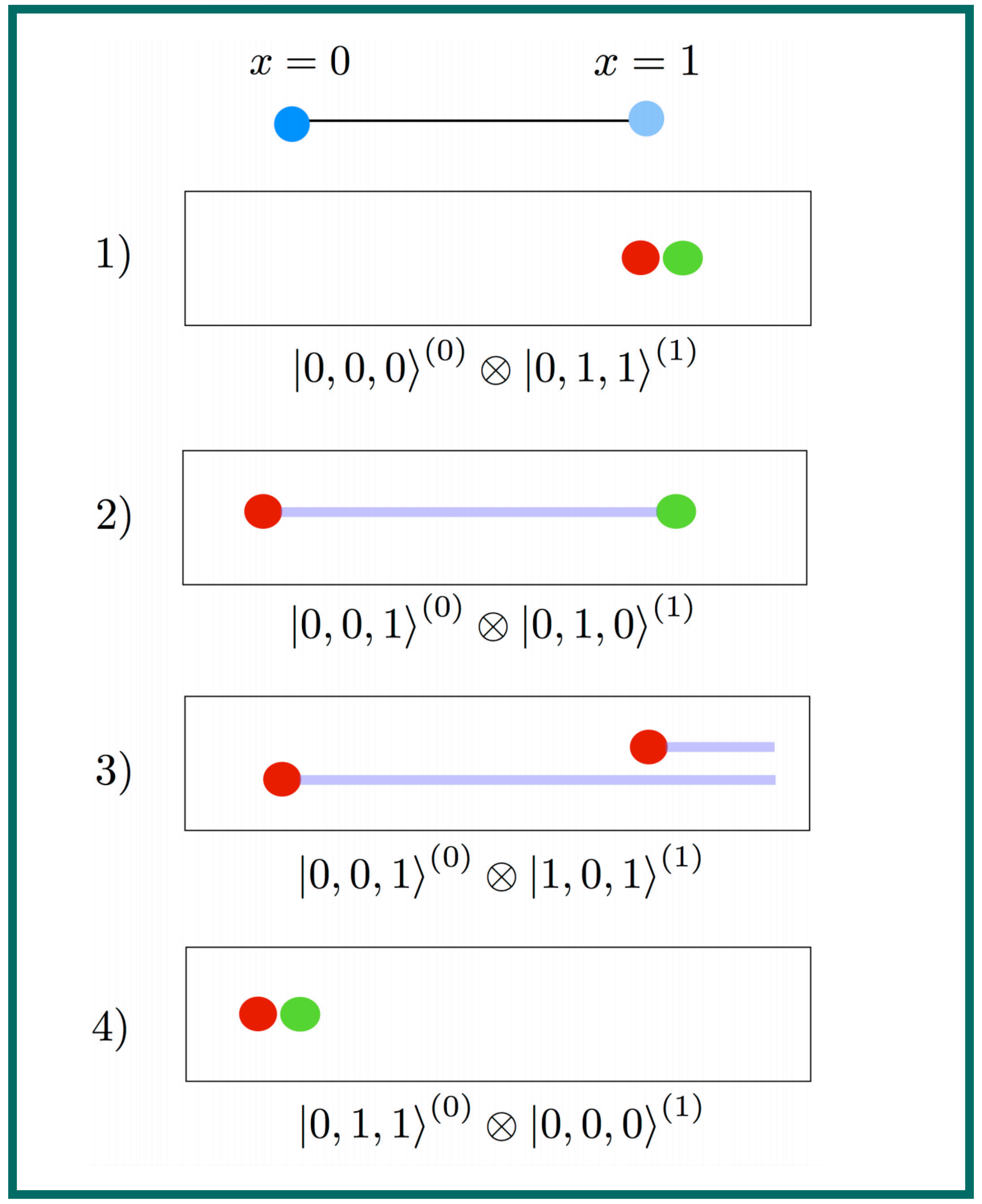
as redundancies in physical degrees Hilbert space

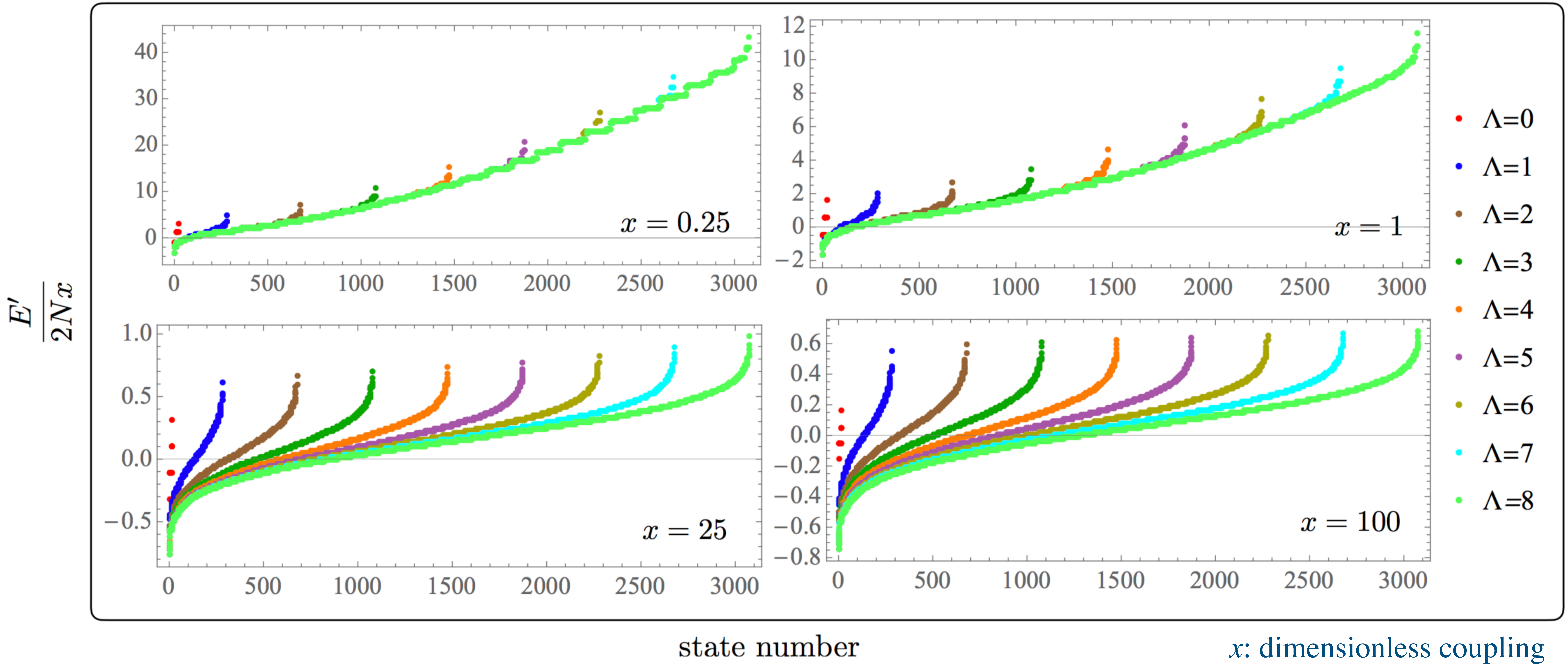
Loop-String-Hadron Formalism



Explicit calculations using the most convenient framework:

Spectrum

N=6, PBC, the symmetry sector connected to strong coupling vacuum



x: dimensionless coupling

Explicit calculations using exact diagonalization

N=6, PBC, the symmetry sector connected to strong coupling vacuum

Effect of finite cut-off

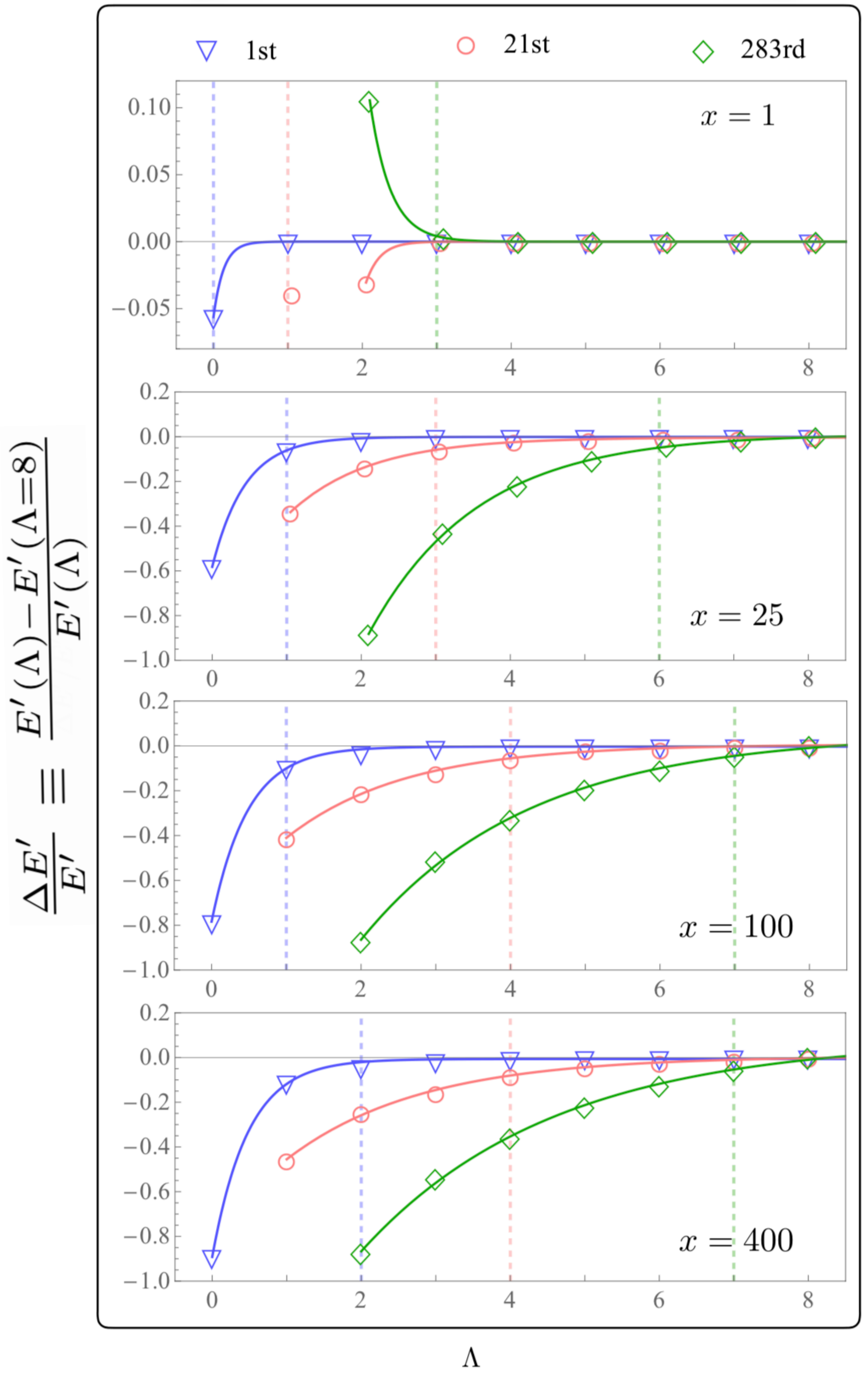
Important to analyze for any bosonic Hilbert space such as, LSH or angular momentum basis

Quantifying truncation error, Asymptotic scaling behavior matches previous studies

Explicit calculations using exact diagonalization

N=6, PBC, the symmetry sector connected to strong coupling vacuum

In the spectrum



The dashed lines denote the first Λ values at which the corresponding scaled energies become equal or less than 10% of their values at Λ = 8 (which are approximated as the infinite cut-off)

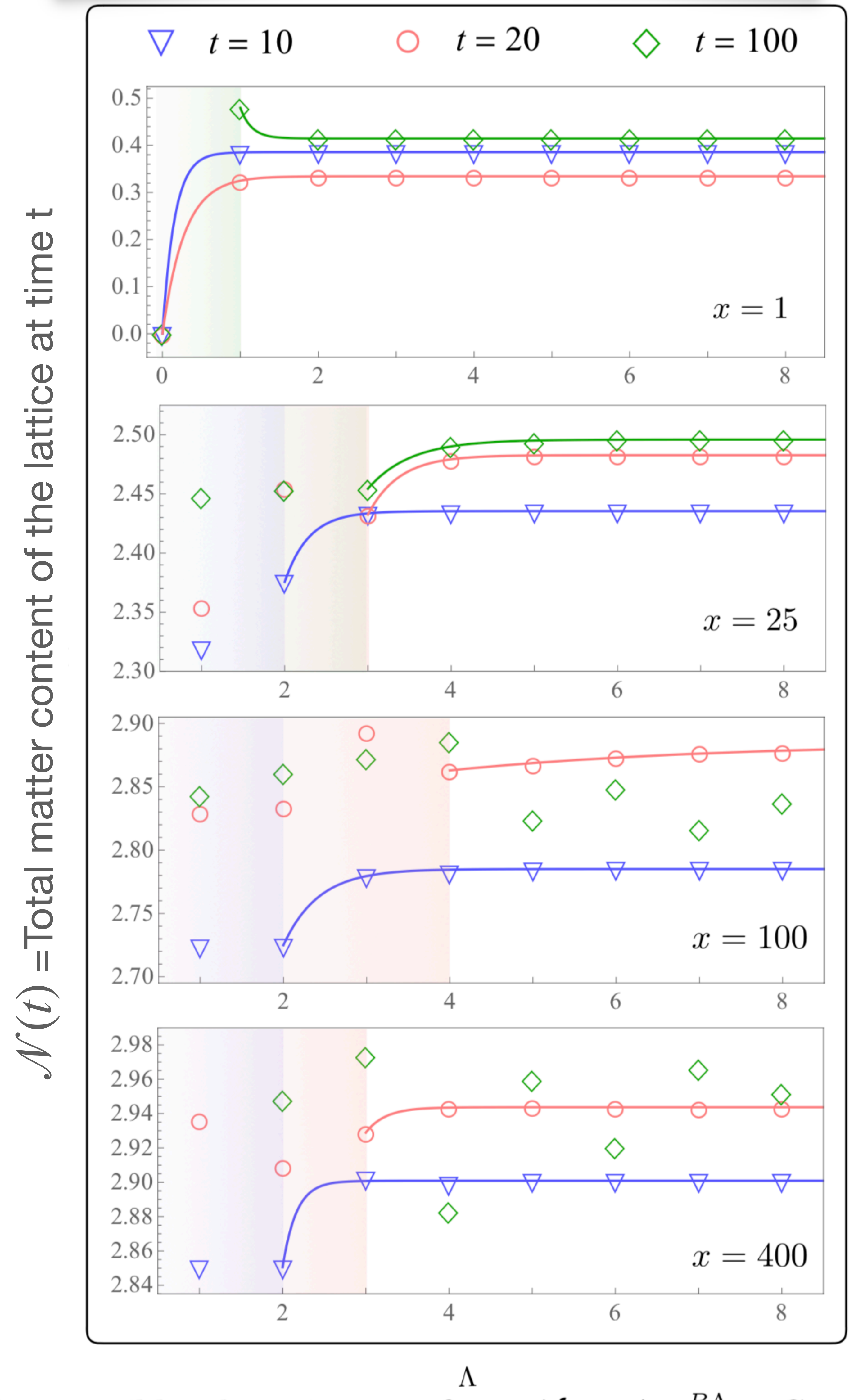
Effect of finite cut-off

Important to analyze for any bosonic Hilbert space such as, LSH or angular momentum basis

Quantifying truncation error, Asymptotic scaling behavior matches previous studies

x: dimensionless coupling

In the real time dynamics

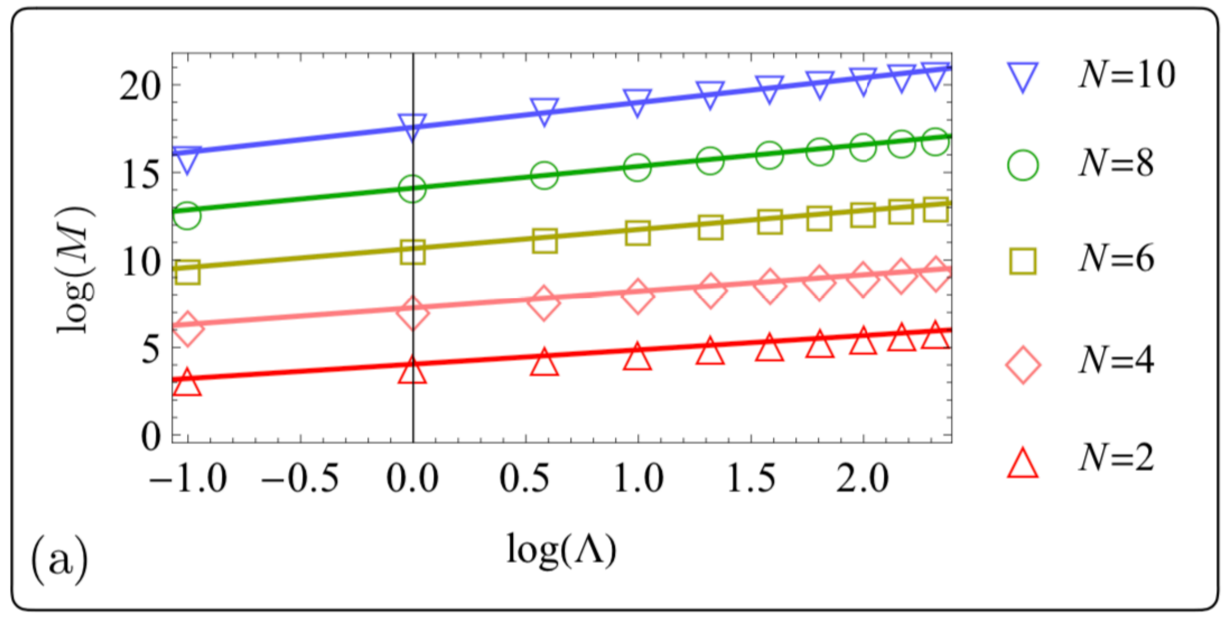


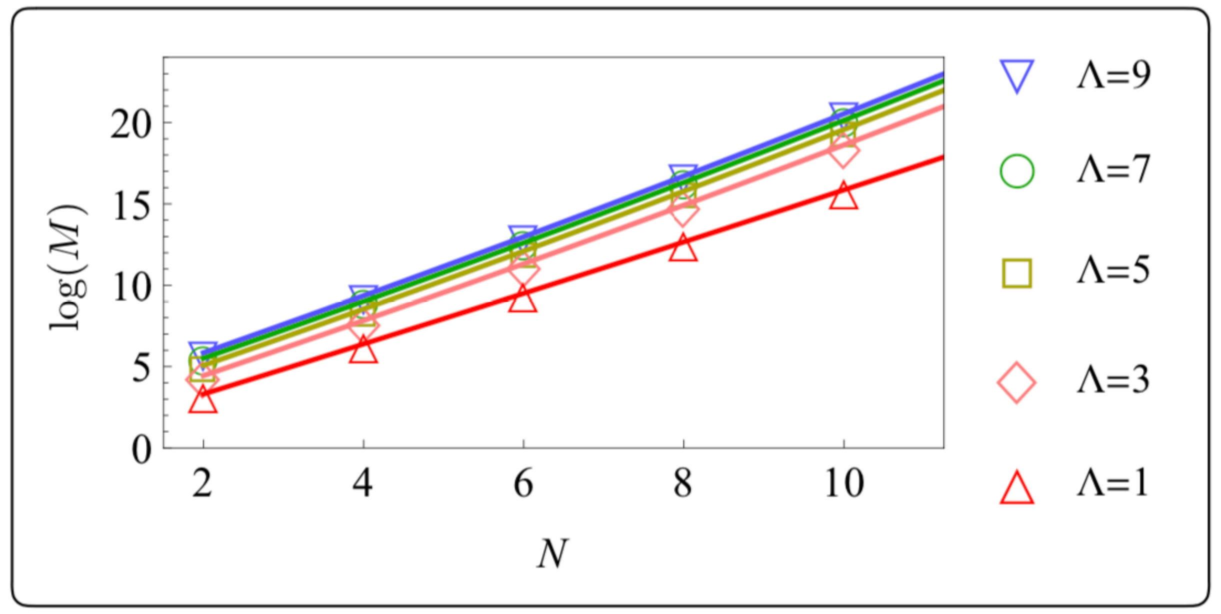
When possible, the points are fit to $\mathcal{N} = Ae^{-B\Lambda} + C$ and the colored regions associated with each t are excluded from such fits.

Continuum limit:

Explicit calculations using exact diagonalization

- bulk limit ⟶ outside the scope of exact diagonalization





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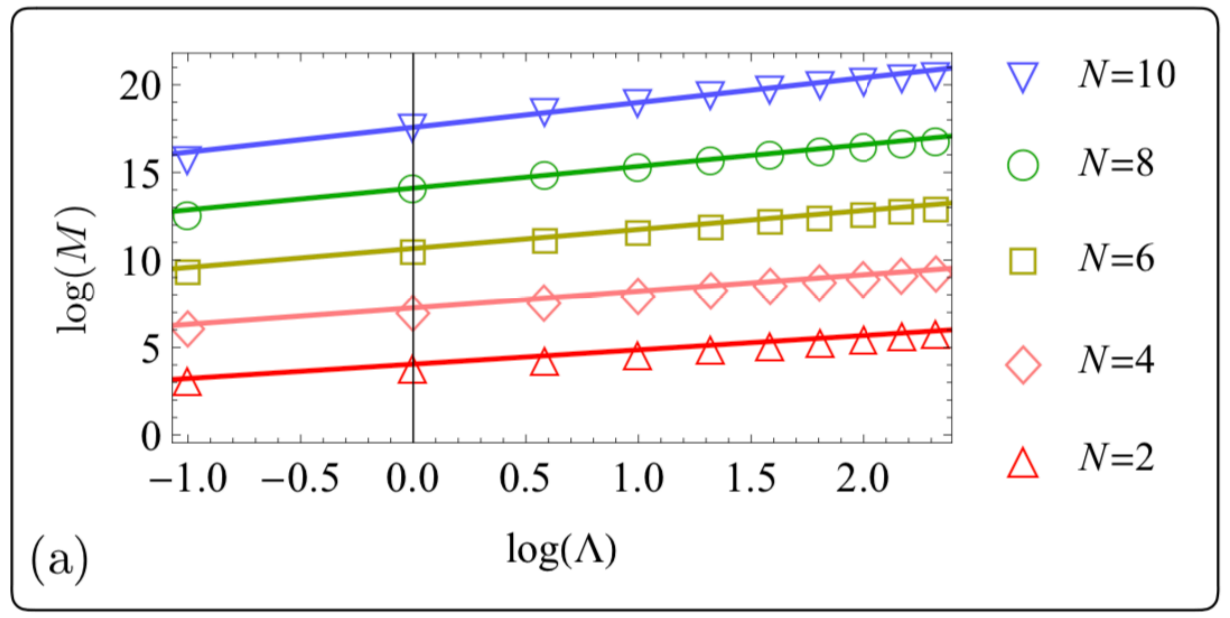

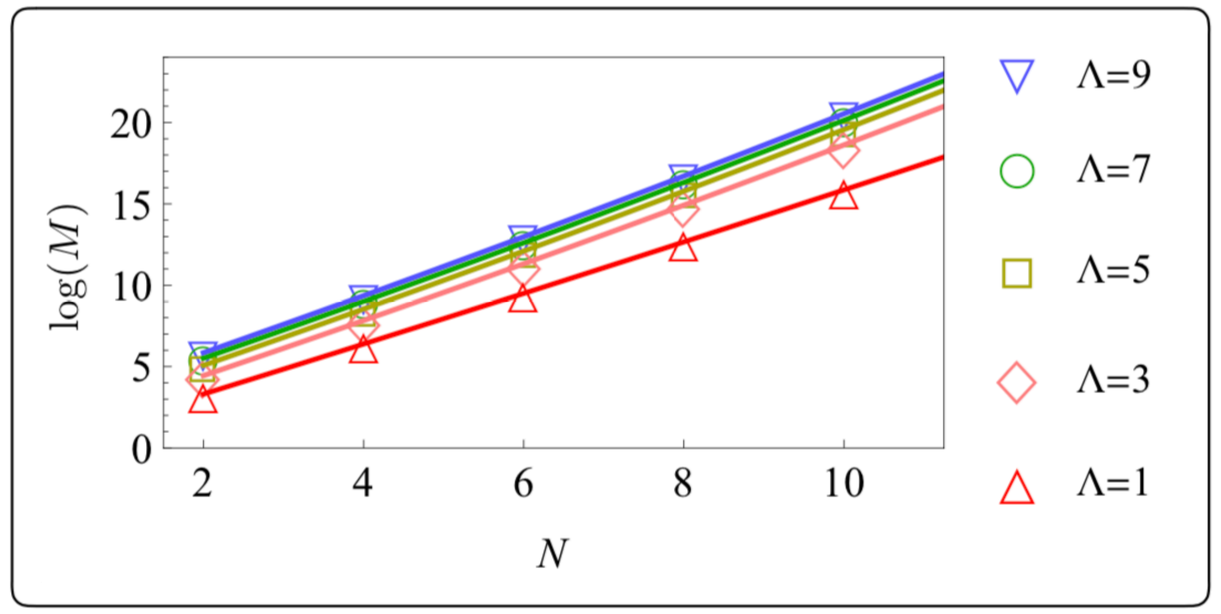

Other technique: tensor network calculation: talk by Bapat, A

Tensor network simulations of a manifestly gauge-invariant SU(2) lattice gauge theory formulation

Thursday, July 29, 2021 9:15 PM (15 minutes)

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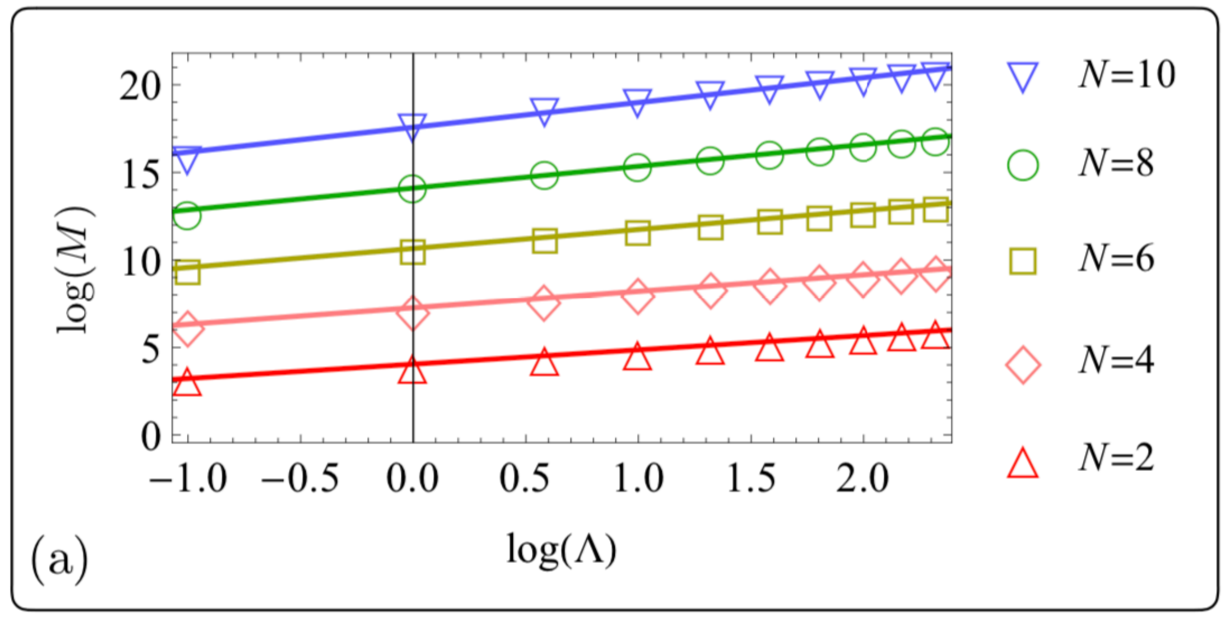
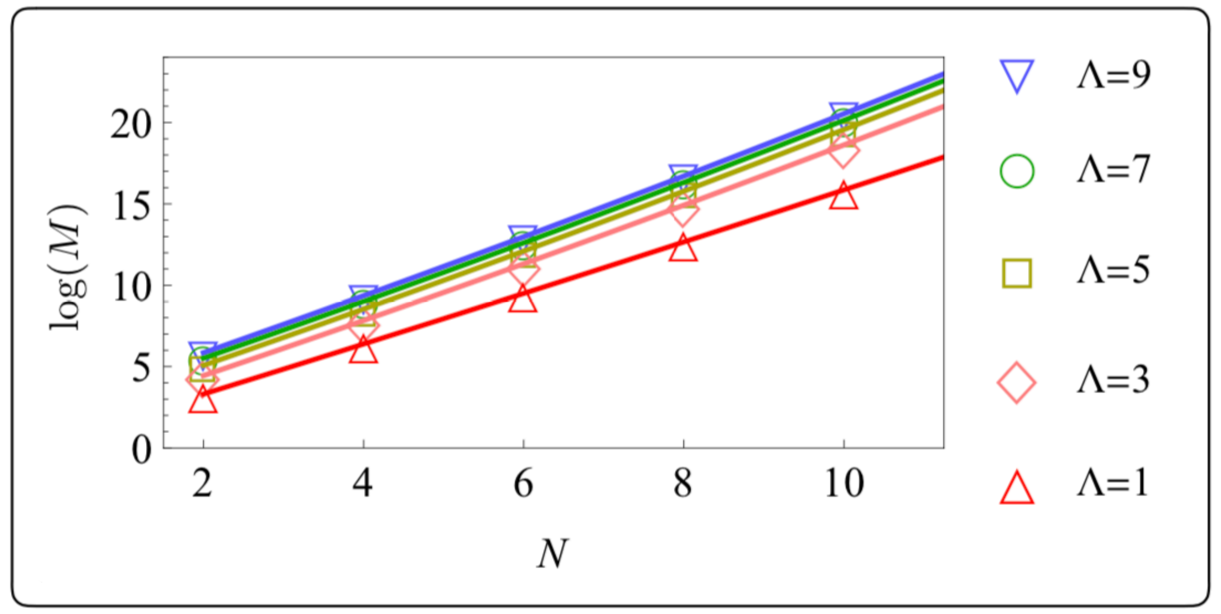
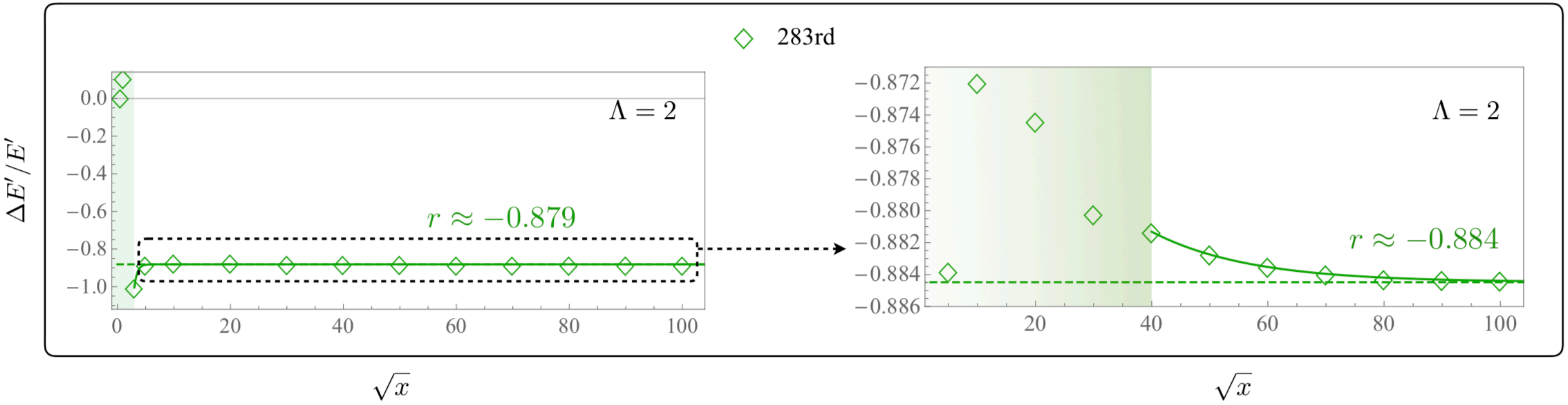




- weak coupling limit



The asymptotic values of the quantity r, are obtained from an exponential fit.

Other technique: tensor network calculation: talk by Bapat, A

Tensor network simulations of a manifestly gauge-invariant SU(2) lattice gauge theory formulation

Thursday, July 29, 2021 9:15 PM (15 minutes)

Remarks:

Hamiltonian simulation of non-Abelian LGT demands for convenient framework and basis.

With the original Kogut-Susskind formalism: beyond Schwinger model is extremely difficult.

Among many available formalisms of the theory, the Loop-String-Hadron formalism is demonstrated to be particularly useful.

Immediate and straightforward applications both in analog and digital simulation has demonstrated profound advantages over any other framework

PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

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Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

Jesse R. Stryker†
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

arXiv:2009.13969

UMD-PP-020-8

Cold Atom Quantum Simulator for String and Hadron Dynamics in Non-Abelian Lattice Gauge Theory

Raka Dasgupta[1] and Indrakshi Raychowdhury[2]

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Looking forward:
LSH formalism of QCD

Thank You

Backup slide# 1

$d = 1$

Hamiltonian, describing dynamics of loops, strings and hadrons.

$$H^{(\mathrm{LSH})} = H_I^{(\mathrm{LSH})} + H_E^{(\mathrm{LSH})} + H_M^{(\mathrm{LSH})}$$

$$H_I^{(\mathrm{LSH})} = \frac{1}{2a}\sum_n \Bigg\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) + 1}} \times \left[\hat{S}_o^{++}(x)\hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x)\hat{S}_i^{--}(x+1)\right] \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1)) + 1}} + \mathrm{h.c.}\Bigg\},$$

$$H_E^{(\mathrm{LSH})} = \frac{g^2 a}{2}\sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} + 1\right)\right],$$

$$H_M^{(\mathrm{LSH})} = m\sum_n (-1)^x(\hat{n}_i(x) + \hat{n}_o(x)),$$

$$\begin{aligned}
\hat{S}_o^{++} &= \hat{\chi}_o^{+}(\lambda^+)^{\hat{n}_i}\sqrt{\hat{n}_l + 2 - \hat{n}_i},\\
\hat{S}_o^{--} &= \hat{\chi}_o^{-}(\lambda^-)^{\hat{n}_i}\sqrt{\hat{n}_l + 2(1 - \hat{n}_i)},\\
\hat{S}_o^{+-} &= \hat{\chi}_i^{+}(\lambda^-)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 2\hat{n}_o},\\
\hat{S}_o^{-+} &= \hat{\chi}_i^{-}(\lambda^+)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 1 + \hat{n}_o},
\end{aligned}$$

$$\begin{aligned}
\hat{S}_i^{+-} &= \hat{\chi}_o^{-}(\lambda^+)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 1 + \hat{n}_i},\\
\hat{S}_i^{-+} &= \hat{\chi}_o^{+}(\lambda^-)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 2\hat{n}_i},\\
\hat{S}_i^{--} &= \hat{\chi}_i^{-}(\lambda^-)^{\hat{n}_o}\sqrt{\hat{n}_l + 2(1 - \hat{n}_o)},\\
\hat{S}_i^{++} &= \hat{\chi}_i^{+}(\lambda^+)^{\hat{n}_o}\sqrt{\hat{n}_l + 2 - \hat{n}_o}.
\end{aligned}$$

The strong-coupling vacuum of the LSH Hamiltonian is given by

$$\begin{aligned}
&n_l(x) = 0, \ \text{for all } x,\\
&n_i(x) = 0, \ n_o(x) = 0, \ \text{for } x \text{ even},\\
&n_i(x) = 1, \ n_o(x) = 1, \ \text{for } x \text{ odd}.
\end{aligned}$$

Backup slide# 2

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

$|n_l, n_i, n_o\rangle$

at matter gauge vertex in any dimension

$|l_{12}, l_{23}, l_{31}\rangle$

at pure gauge vertex in $d > 1$

$n_i, n_o \in \{0,\ 1\}$

$n_l, \{l_{ij}\} \in \{0,\ \infty\}$

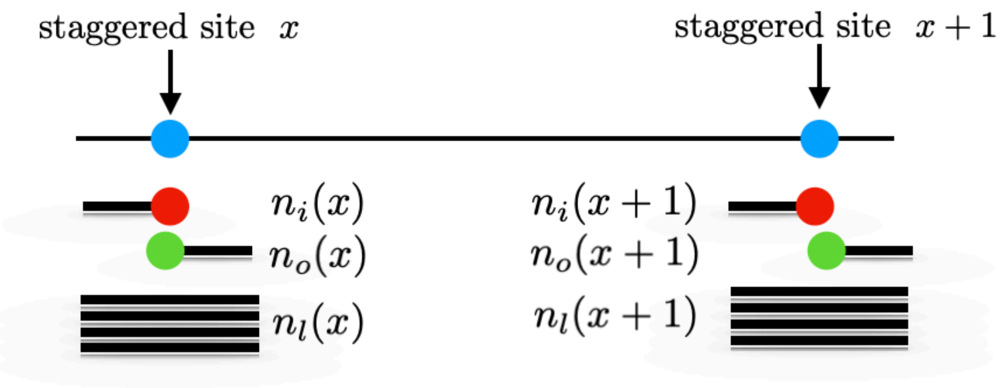
Impose a cut-off $\bar{j}$

$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

qq $\quad d = 1$

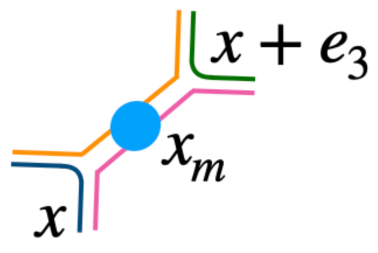
$$n_l + n_o(1 - n_i)\Big|_x = n_l + n_i(1 - n_o)\Big|_{x+1}$$



qg $\quad d = 2$

$$l_{23} + l_{31}\Big|_x = n_l + n_i(1 - n_o)\Big|_{x_m}$$

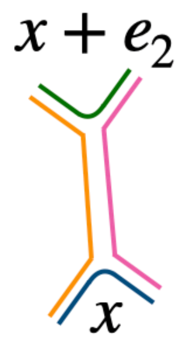
$$l_{23} + l_{31}\Big|_{x+e_3} = n_l + n_o(1 - n_i)\Big|_{x_m}$$



gg

$$l_{12} + l_{31}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{3}\bar{1}}\Big|_{x+e_1}$$

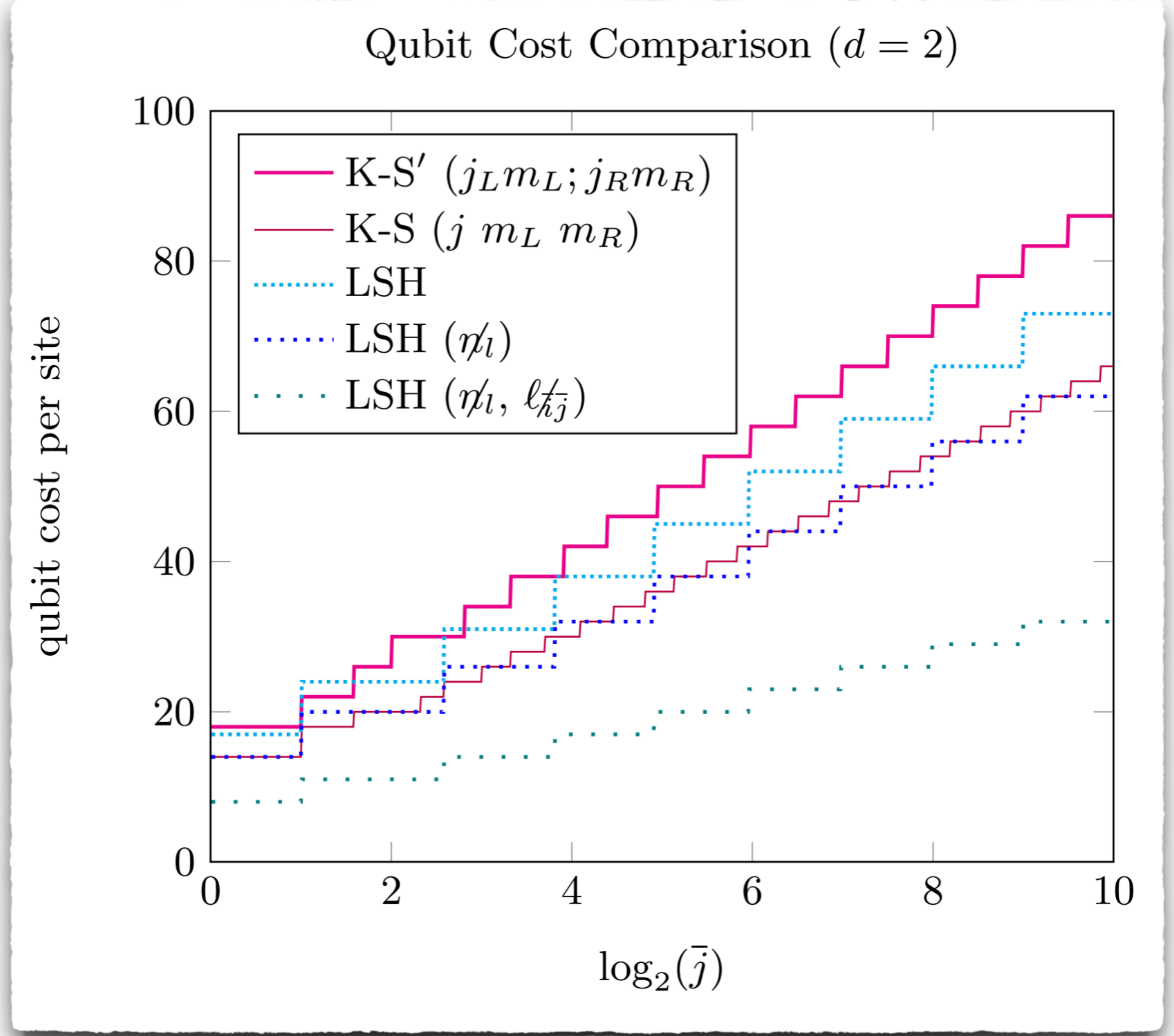
$$l_{12} + l_{23}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{2}\bar{3}}\Big|_{x+e_2}$$

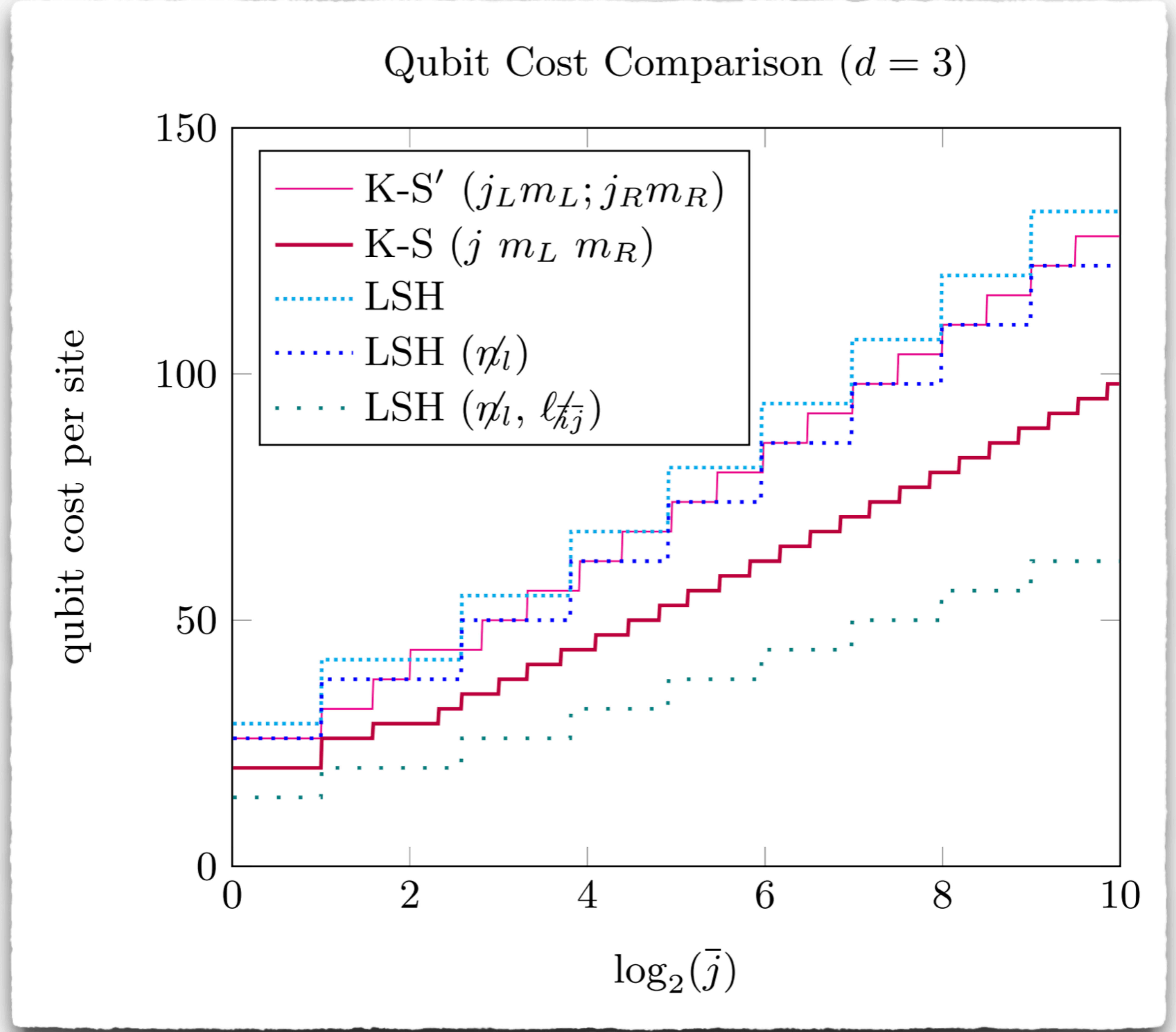


Backup slide# 3

$d > 1$

Qubit Cost Analysis



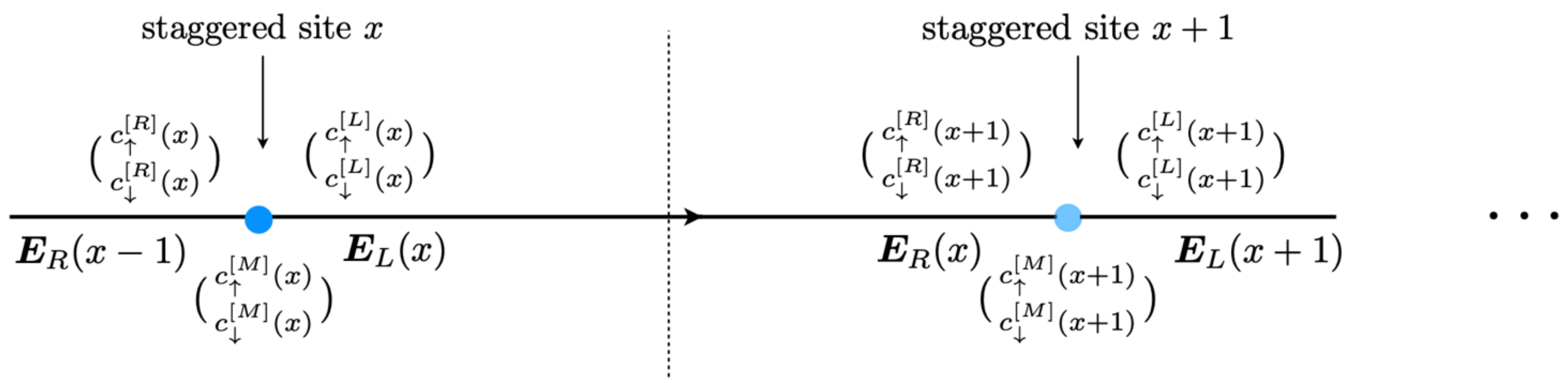


Backup slide# 4

Comparing to Quantum Link Model: SU(2) in 1+1d

$$H^{(\mathrm{QLM})} = H_I^{(\mathrm{QLM})} + H_M^{(\mathrm{QLM})} + H_E^{(\mathrm{QLM})} + H_{\mathrm{break}}^{(\mathrm{QLM})}$$

$$\hat{G}^a(x) = -\hat{E}_L^a(x) + \hat{E}_R^a(x-1) + c_s^{[M]\dagger}(x) T^a_{s,s'} c_{s'}^{[M]}(x)$$



$$U(x) = \begin{pmatrix} c_\uparrow^{[L]}(x) c_\uparrow^{[R]\dagger}(x+1) & c_\uparrow^{[L]}(x) c_\downarrow^{[R]\dagger}(x+1) \\ c_\downarrow^{[L]}(x) c_\uparrow^{[R]\dagger}(x+1) & c_\downarrow^{[L]}(x) c_\downarrow^{[R]\dagger}(x+1) \end{pmatrix}$$

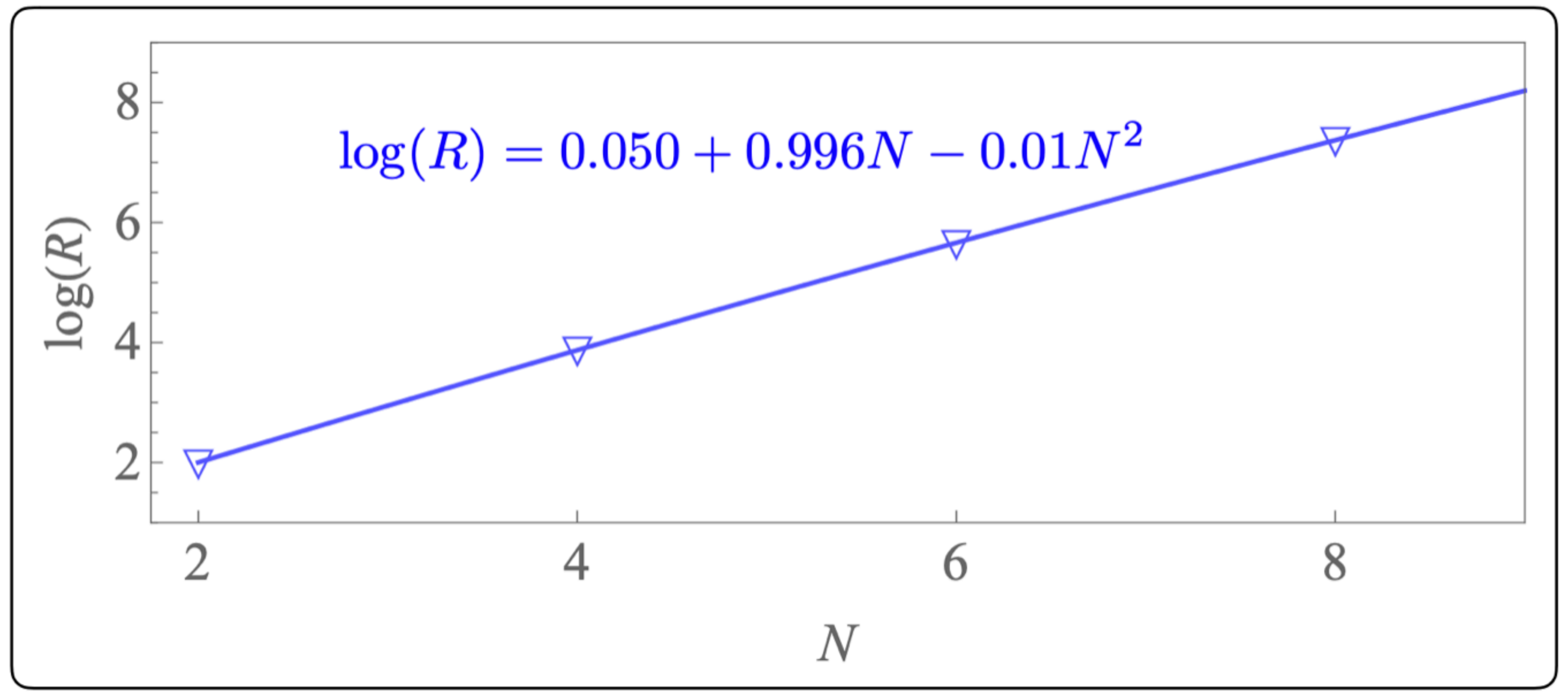
$$H_I^{(\mathrm{QLM})} = t \sum_{x,s,s'} \left[\hat{c}_s^{[M]\dagger}(x) \hat{U}_{s,s'}(x) \hat{c}_{s'}^{[M]}(x+1) + \mathrm{h.c.} \right]$$
$$= t \sum_{x,s,s'} \left[\hat{c}_s^{[M]\dagger}(x) \hat{c}_s^{[L]}(x) \hat{c}_{s'}^{[R]\dagger}(x+1) \hat{c}_{s'}^{[M]}(x+1) + \mathrm{h.c.} \right]$$

$$H_M^{(\mathrm{QLM})} = m \sum_x (-1)^x \left[\hat{n}_\uparrow^{[M]}(x) + \hat{n}_\downarrow^{[M]}(x) \right]$$

$$H_E^{(\mathrm{QLM})} = \frac{g_0^2}{2} \sum_x \left[\hat{\boldsymbol{E}}_R^2(x-1) + \hat{\boldsymbol{E}}_L^2(x) \right]$$
$$\equiv \frac{3g_0^2}{8} \sum_x \Big[\Big(\hat{n}_\uparrow^{[R]}(x-1) + \hat{n}_\downarrow^{[R]}(x-1) - 2\hat{n}_\uparrow^{[R]}(x-1) \hat{n}_\downarrow^{[R]}(x-1) \Big) + \Big(\hat{n}_\uparrow^{[L]}(x) + \hat{n}_\downarrow^{[L]}(x) - 2\hat{n}_\uparrow^{[L]}(x) \hat{n}_\downarrow^{[L]}(x) \Big) \Big]$$

Spectrum of $H^{(\mathrm{QLM})} \neq$ Spectrum of $H^{(\mathrm{LSH})}$

Exponentially expensive than LSH in 1+1d



$R =$ ratio of the dimension of the physical Hilbert space in the QLM to that in the KS formulation (or equivalently the LSH formulation) when the cutoff is set to its saturating value with OBC.

Finite dimensional Hilbert space, popular as the framework for simulating gauge theories.

In 1+1d, with open boundary conditions, LSH/KS Hilbert space has much smaller dimensions.

QLM in higher dimension may become useful to simulate the physics of interest.

LSH is efficient and accurate for near term quantum simulations.

$$\sum_{s=\uparrow,\downarrow} \left[n_s^{[M]}(x) + n_s^{[L]}(x) + n_s^{[R]}(x) \right] = \mathrm{const.}$$

$$H_{\mathrm{break}}^{(\mathrm{QLM})} = \frac{\epsilon}{2} \sum_x \left[\det \hat{U}(x, x+1) + \mathrm{h.c.} \right]$$
$$= \epsilon \sum_x \left[\hat{c}_\uparrow^{[L]\dagger}(x) \hat{c}_\downarrow^{[L]\dagger}(x) \hat{c}_\downarrow^{[R]}(x+1) \hat{c}_\uparrow^{[R]}(x+1) + \mathrm{h.c.} \right]$$
$$n_\uparrow^{[L]}(x) + n_\downarrow^{[L]}(x) + n_\uparrow^{[R]}(x+1) + n_\downarrow^{[R]}(x+1) = 2.$$

S. Chandrasekharan and U.J. Wiese, NPB, 1997;
R. Brower, S. Chandrasekharan, and U.J. Wiese PRD, 1999.