Density of States for Gravitational Waves

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Composite Dark Matter Model

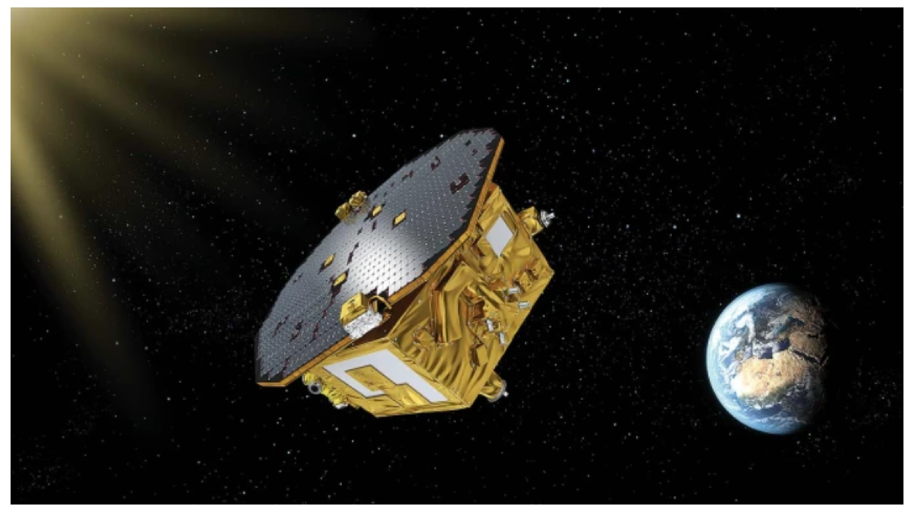
Pure gauge $SU(N)$ or $SU(N)$ coupled to N new fundamental fermions

Dark Matter candidate either "dark glueball" or "dark baryon"

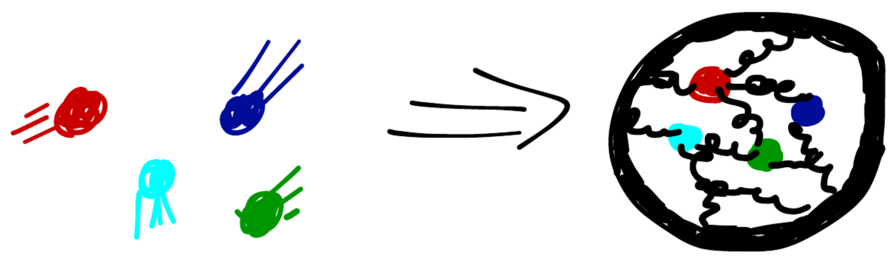
Compelling features:

- naturally explains mass
- naturally explains stability
- multiple viable explanations for the abundance of the Dark Matter candidate
- evades all current constrains from experiments

New way to probe Dark Matter:
Gravitational wave detectors!



Dark Matter undergoes **confinement** transition.



1st order phase transition → gravitational waves

Numerical Tool: LLR

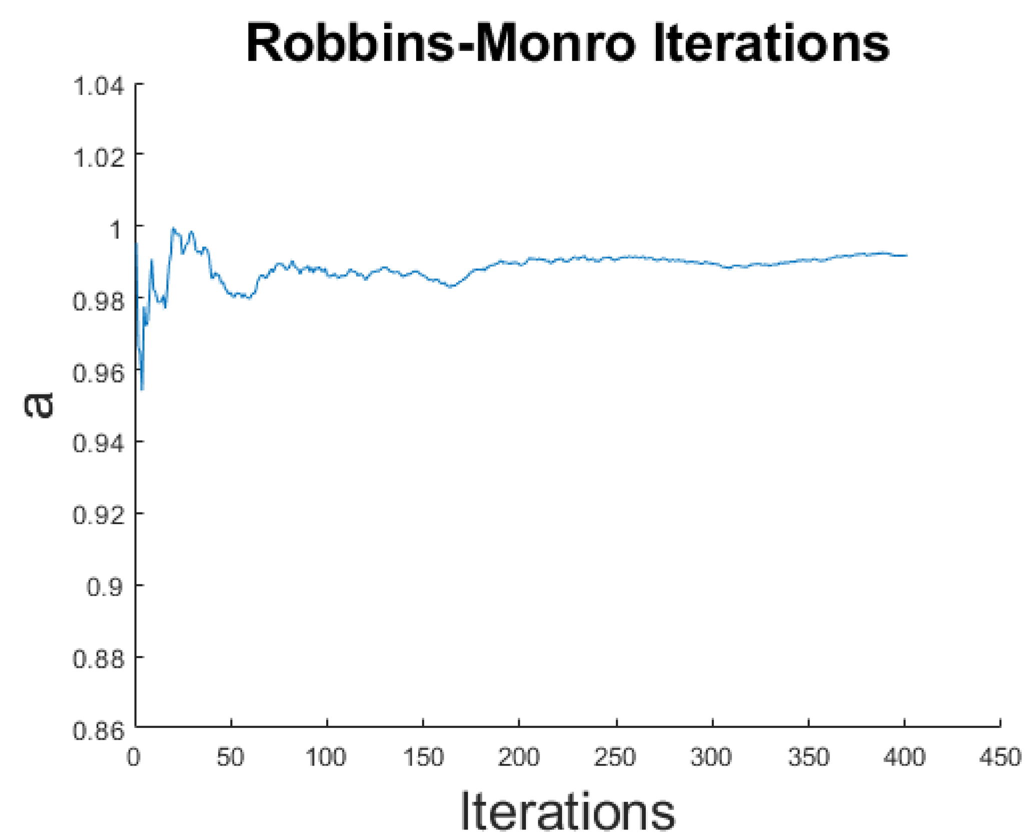
Markov Chain Monte Carlo performs poorly at 1st order P.T.

Solution: Wang-Landau algorithm
Reconstruct density of states $\rho(E)$

1. Define re-weighted expectation value $\langle\langle E - E_0 \rangle\rangle_\delta(a) = \frac{1}{N}\int \mathcal{D}\phi\theta_{E_0,\delta}(E - E_0)e^{-aS}$
   - **Energy restriction** $\theta = 1$ if $E = E_0 \pm \frac{\delta}{2}$, 0 else
   - **New Boltzmann weight** e^{-aE}
   - New parameter a
2. Set $\langle\langle E - E_0 \rangle\rangle_\delta(a) = 0$ and $\delta \to 0$
3. Solve for a: $a = \frac{\mathrm{d}ln(\rho(E))}{\mathrm{d}E}\Big|_{E=E_0}$
4. Integrate to obtain $\rho(E)$
5. $\langle \mathcal{O} \rangle = \frac{1}{Z(\beta)}\int \mathrm{d}E\mathcal{O}\rho(E)e^{\beta E}$
   Simple 1 dimensional integral!

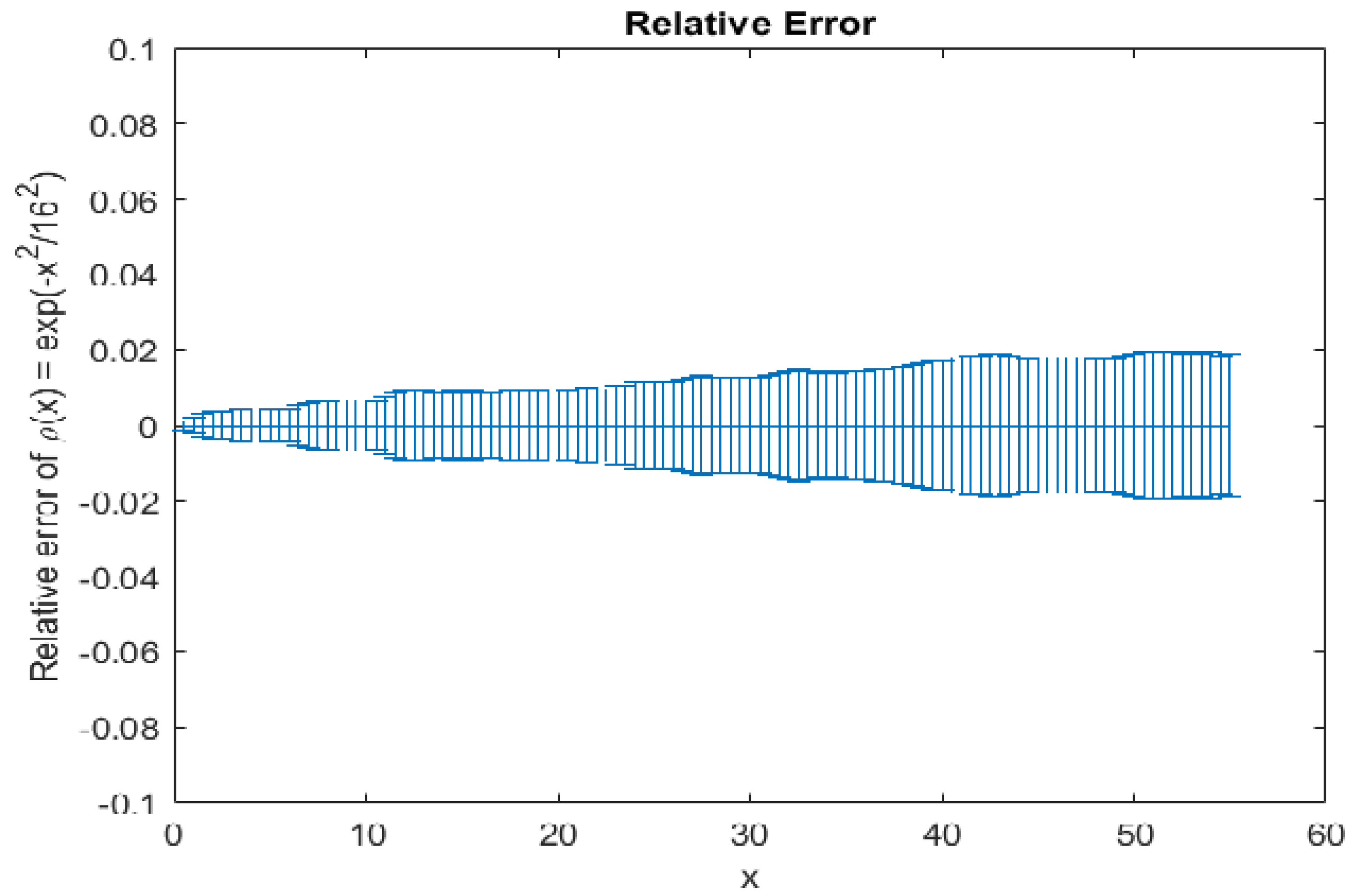
Solve $\langle\langle E - E_0 \rangle\rangle_\delta(a) = 0$ iteratively

$$a^{(n+1)} = a^{(n)} + \frac{12}{\delta^2(n+1)}\langle\langle E - E_0 \rangle\rangle_\delta(a^{(n)})$$

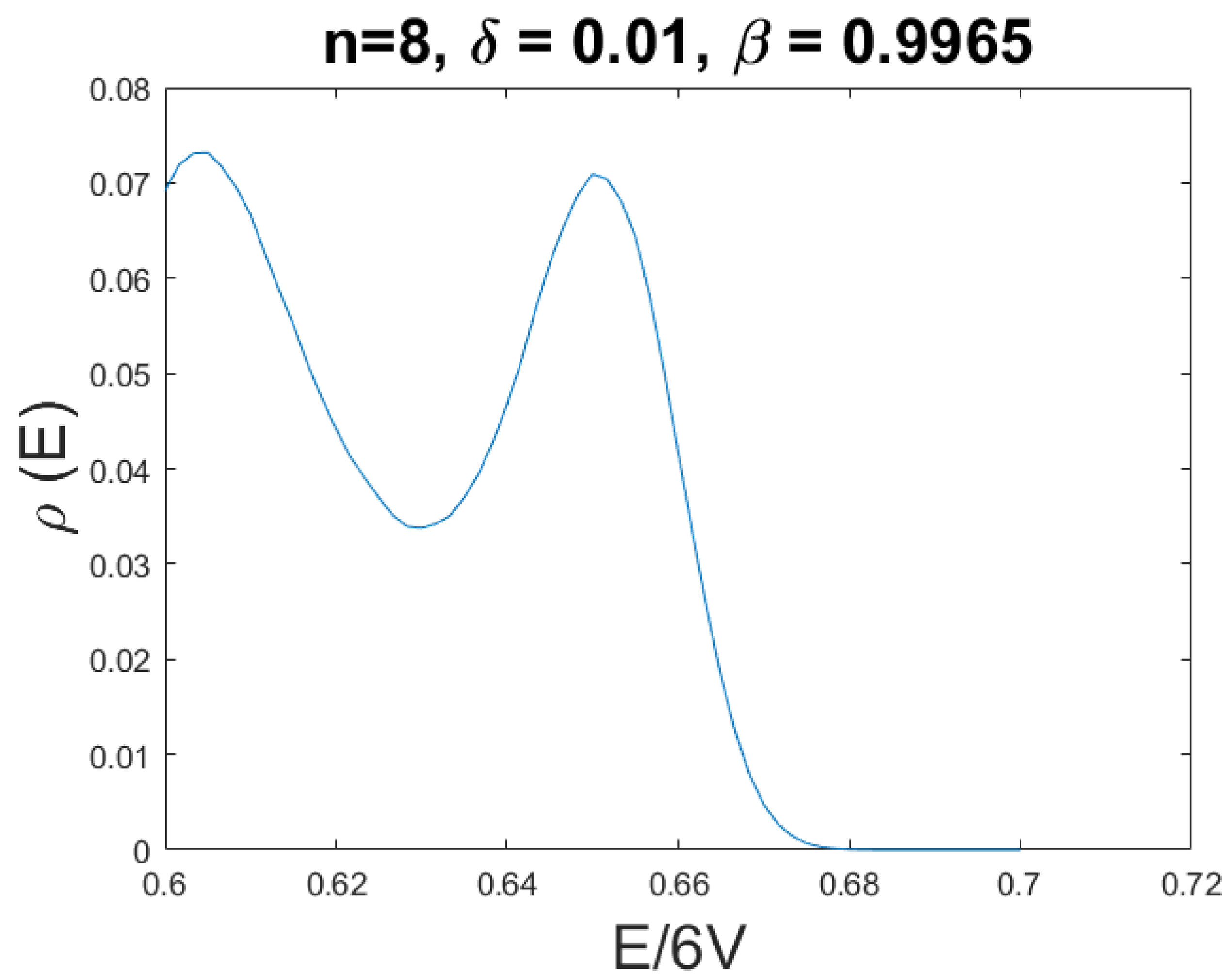


Near Constant Error

Reproduce Gaussian Distribution $e^{-\frac{x^2}{16^2}}$ with LLR



Density of States



Density of states ρ for $U(1)$; critical temperature $\beta = 0.9965$; lattice volume $V = 8^4$; energy interval size of $\delta = 0.01/V$.