Finite Volume Study of Flavor Singlet Scalar Meson in SU(3) Nf=8 Gauge Theory

Lattice Strong Dynamics (LSD) Collaboration

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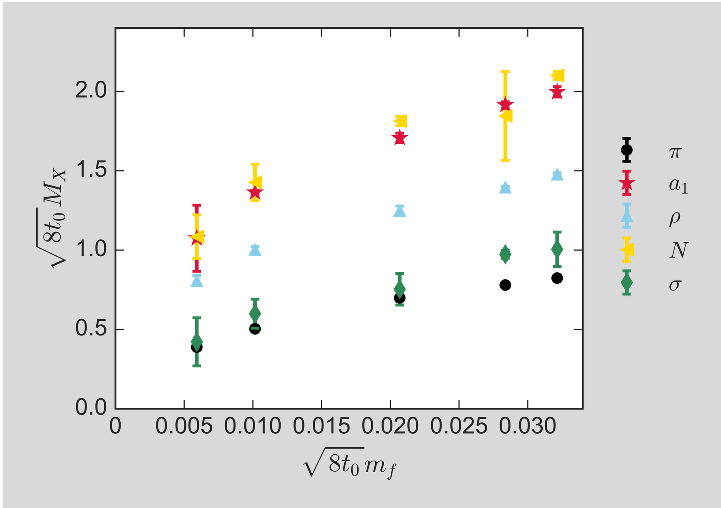
Previous LSD Results

Earlier study [1] of flavor singlet scalar meson at fixed Mπ×L≈5.3



Finite difference correlator used to avoid fitting constant:

Δσ(t+½) = Cσ(t+1) - Cσ(t)

Problems:

Δσ(t+½) reduced signal

Large systematics due to fitting range variation

Recent I=2 $\pi\pi$ scattering [2] did finite volume study for Mπ , Fπ

$$M_m(L) = M_m(\infty)\left(1+\alpha_m\,\frac{M_\pi^2}{4\pi F_\pi^2}\,\frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}\right)$$

$$F_m(L) = F_m(\infty)\left(1+\beta_m\,\frac{M_\pi^2}{4\pi F_\pi^2}\,\frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}\right)$$

am	aM_π	aF_π	$M_\pi^2/(4\pi F_\pi^2)$
0.00889	0.22525(27)	0.052491(59)	1.4654(48)
0.00750	0.205680(20)	0.048233(20)	1.4471(12)
0.00500	0.16575(10)	0.03982(28)	1.379(19)
0.00222	0.1085(12)	0.02742(32)	1.246(40)
0.00125	0.08115(82)	0.02165(23)	1.118(33)

Extrapolation is very accurate.
Coefficients are somewhat large:
$\alpha\pi$=19.23(98), $\beta\pi$=-20.4(1.5)

Features Added to Current Analysis

Model averaging [3] over choice of fitting ranges

$$\log p(M|D) \approx -\frac{1}{2}\chi^2 - N_p - \frac{N_t}{2} + (t_{\max} - t_{\min})$$

Shrinkage estimator [4,5] for covariance matrix

$$\Sigma_s(i,j) = (1-\lambda)\Sigma(i,j) + \lambda\Sigma(i,i)\delta(i,j)$$

New subtraction scheme for p=0 correlator

$$C_s(t) = C(t) - \frac{1}{N_t}\sum_{t'=0}^{N_t-1} C_s(t')$$

New correlators at non-zero spatial momentum, i.e. p=(1,0,0) do not require subtraction. Ground state energies E(p) translated to rest masses M using lattice dispersion relation

$$\widehat{E}^2 = \widehat{p}^2 + \widehat{M}^2$$

$$\widehat{E} = 2\sinh\frac{E}{2}\ , \quad \widehat{p}_i = 2\sin\frac{2\pi n_i}{2N_s}\ , \quad \widehat{M} = 2\sinh\frac{M}{2}$$

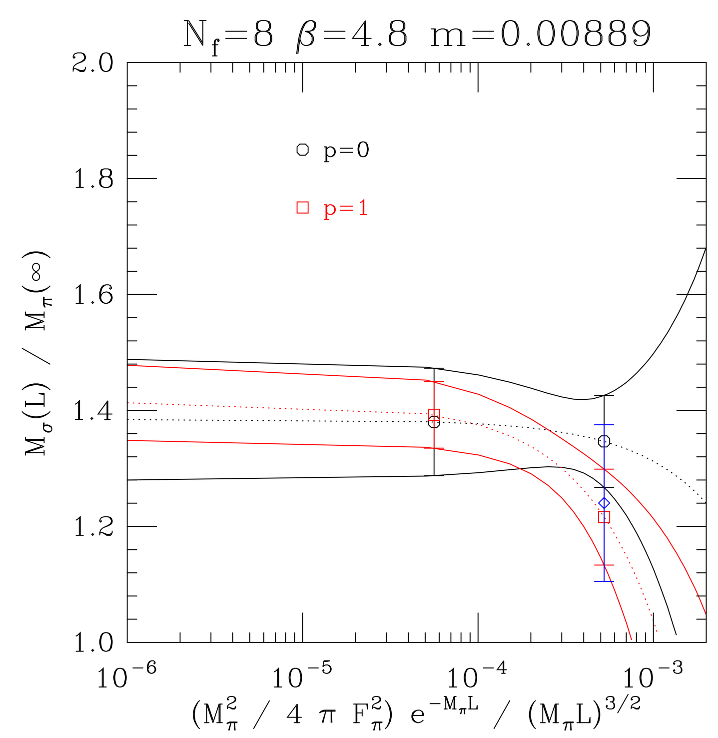
New ``relativistic'' normalization [6] for amplitudes related to residues of poles in frequency domain by discrete Fourier transform

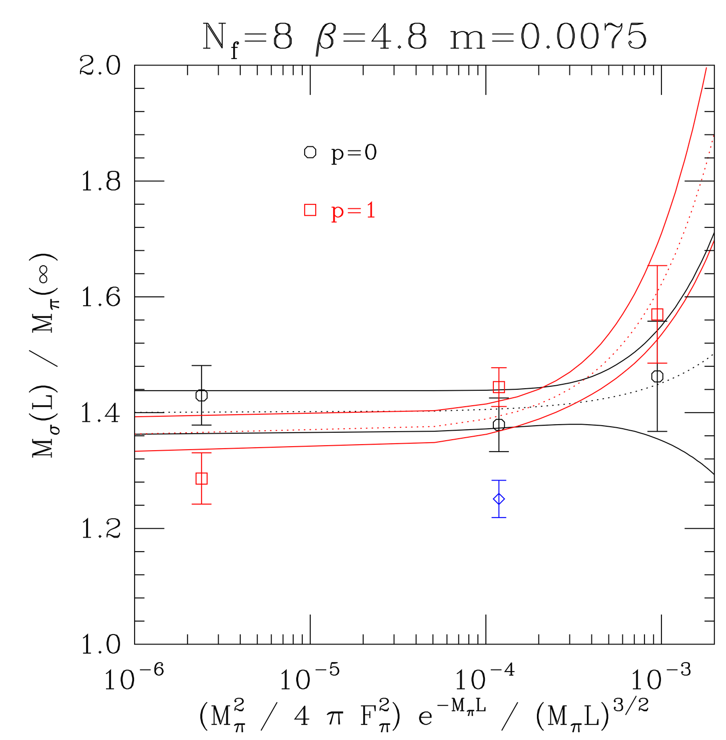
$$\widehat{\omega}_k = 2\sin\frac{2\pi k}{2N_t}\ , \quad \widehat{\omega}'_k = 2\sin\left(\frac{\pi}{2} - \frac{2\pi k}{2N_t}\right)$$

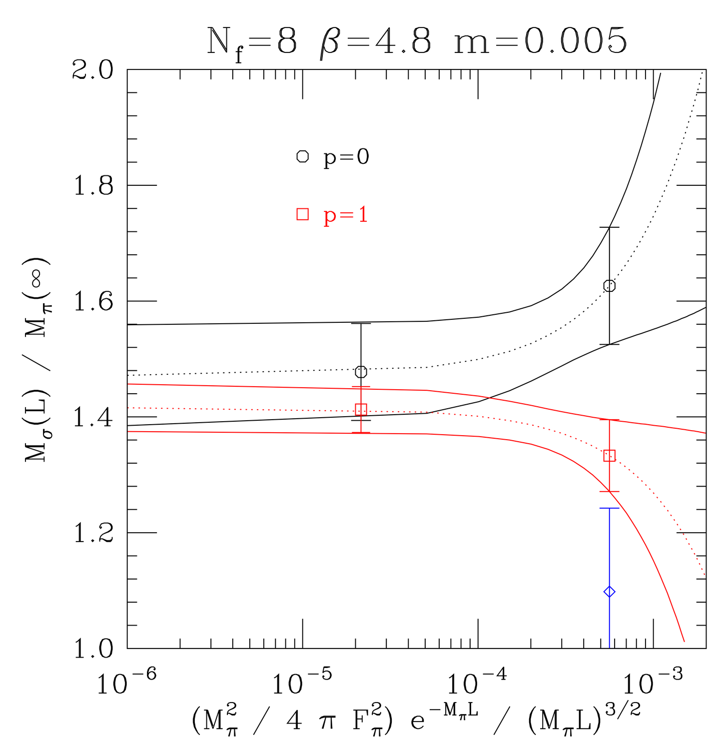
New larger volume ensembles

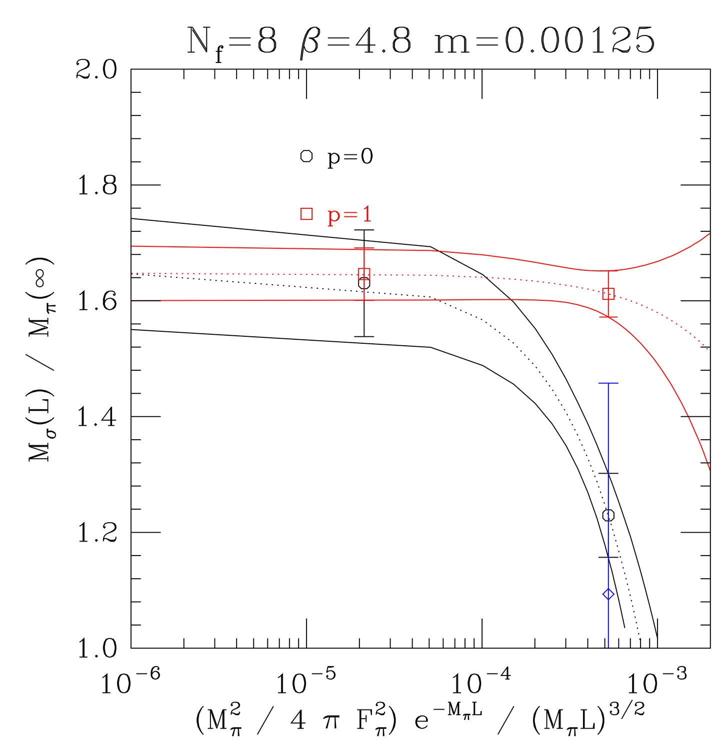
Very Preliminary Results

Model averaged results for finite volume study of 0++ mass using one non-oscillating and one oscillating exponential.









Model Functions for Staggered Mesons

$$C(t) = c_0 + \sum_n \frac{c_n}{2\left(1-e^{-M_n N_t}\right)\sinh(M_n)}\left[e^{-M_n t} + e^{-M_n(N_t-t)}\right] + (-1)^t \sum_j \frac{c'_j}{2\left(1-e^{-M'_j N_t}\right)\sinh(M'_j)}\left[e^{-M'_j t} + e^{-M'_j(N_t-t)}\right]$$

$$C_s(t) = c'_0 + \sum_{n=1}^{n_{\max}} \frac{c_n}{2\left(1-e^{-M_n N_t}\right)\sinh(M_n)}\left[e^{-M_n t} + e^{-M_n(N_t-t)}\right] - \frac{1}{N_t}\frac{c_n}{\widehat{M}_n^2} + (-1)^t \sum_{j=1}^{j_{\max}} \frac{c'_j}{2\left(1-e^{-M'_j N_t}\right)\sinh(M'_j)}\left[e^{-M'_j t} + e^{-M'_j(N_t-t)}\right] - \frac{1}{N_t}\frac{c'_j}{4+\widehat{M}'^2_j}$$

$$c'_0 = -\frac{1}{N_t}\sum_{n=n_{\max}+1}^{\infty}\frac{c_n}{\widehat{M}_n^2} - \frac{1}{N_t}\sum_{j=j_{\max}+1}^{\infty}\frac{c'_j}{4+\widehat{M}'^2_n}$$

$$\widetilde{C}(k) = c_0\delta_{k,0} + \frac{1}{N_t}\sum_n \frac{c_n}{\widehat{M}_n^2 + \widehat{\omega}_k^2} + \frac{1}{N_t}\sum_j \frac{c'_j}{\widehat{M}'^2_j + \widehat{\omega}'^2_k}$$

Volume	Mass	Try	MDTU	Period (MDTU)	Block (MDTU)	N_{blk}
$24^3\times48$	0.00889	1	[250,25000]	10	**100**	**247**
$32^3\times64$		1	[1040,7000]	40	80	75
		2	[1040,7000]	40	80	75
		3	[1040,7000]	40	80	75
		4	[1040,7000]	40	80	75
		C			**80**	**300**
$24^3\times48$	0.0075	1	[350,10000]	10	**90**	**107**
$32^3\times64$		1	[255,1395]	10	100	
			[1400,25160]	5	100	**249**
$48^3\times96$		1	[250,9990]	10	70	139
		2	[250,9990]	10	70	139
		C			**70**	**278**
$32^3\times64$	0.005	1	[251,29641]	5	100	293
		2	[20011,22815]	2	100	28
		3	[29001,31653]	2	100	26
		4	[10001,13293]	2	100	32
		C			**100**	**379**
$48^3\times96$		1	[250,4200]	10	50	79
		2	[250,3390]	10	50	63
		C			**50**	**142**
$64^3\times128$	0.00125	r0	[200,2060]	10	120	15
		r1	[200,1990]	10	120	15
		r2	[200,2010]	10	120	15
		r3	[200,2070]	10	120	15
		s0	[8436,17088]	12	120	72
		s1	[7644,17472]	12	120	82
		s2	[7212,17412]	12	120	86
		C			**120**	**300**
$96^3\times128$		2	[500,3144]	2	80	34
		3	[500,3282]	2	80	35
		C			**80**	**69**

Discussion

Current fits tend to estimate higher masses than published results. Could be due to excited state contamination. Fits in progress.

Two different methods of dealing with constant agree in larger volumes (Mπ×L ~ 6.6-9.9) but don't yet agree in smaller volumes (Mπ×L ~ 5.3)

Still unclear even on the sign of the finite volume effect. Even higher statistics on smaller volumes needed.

References

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