The 38th International Symposium on Lattice Field Theory

Near-Physical Point Isospin-Breaking Corrections to $K_{\ell 2}/\pi_{\ell 2}$

Andrew Zhen Ning Yong
University of Edinburgh
26th July 2021

RBC/UKQCD COLLABORATION

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner (Southampton)

Columbia University
Norman Christ
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda

University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.

OVERVIEW

In experiments, inclusive rates are measured

$\mathcal{O}(\alpha)$ and $\mathcal{O}(m_u \neq m_d)$
correction

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) \equiv \Gamma^{\rm tree} + \delta\Gamma = \Gamma^{\rm tree}(1 + \boxed{\delta R_P})$$

where $\quad P = K, \pi$

where…

$$\Gamma^{\rm tree} = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_{\ell^+}^2 \left(1 - \frac{m_{\ell^+}^2}{M_{P^+}^2}\right)^2$$

f_P defined in isospin-symmetric QCD theory ($\alpha = 0, m_u = m_d$)…

Interested in the following:

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \underbrace{\boxed{\frac{\Gamma(K^+ \to \mu^+ \nu_{\mu^+}[\gamma])}{\Gamma(\pi^+ \to \mu^+ \nu_{\mu^+}[\gamma])} \frac{M_{K^+}^3}{M_{\pi^+}^3} \frac{(M_{\pi^+}^2 - m_{\mu^+}^2)^2}{(M_{K^+}^2 - m_{\mu^+}^2)^2}}}_{\text{Experiment}} \underbrace{\boxed{\frac{(f_\pi / f_K)^2}{1 + (\delta R_K - \delta R_\pi)}}}_{\text{Theory}}$$

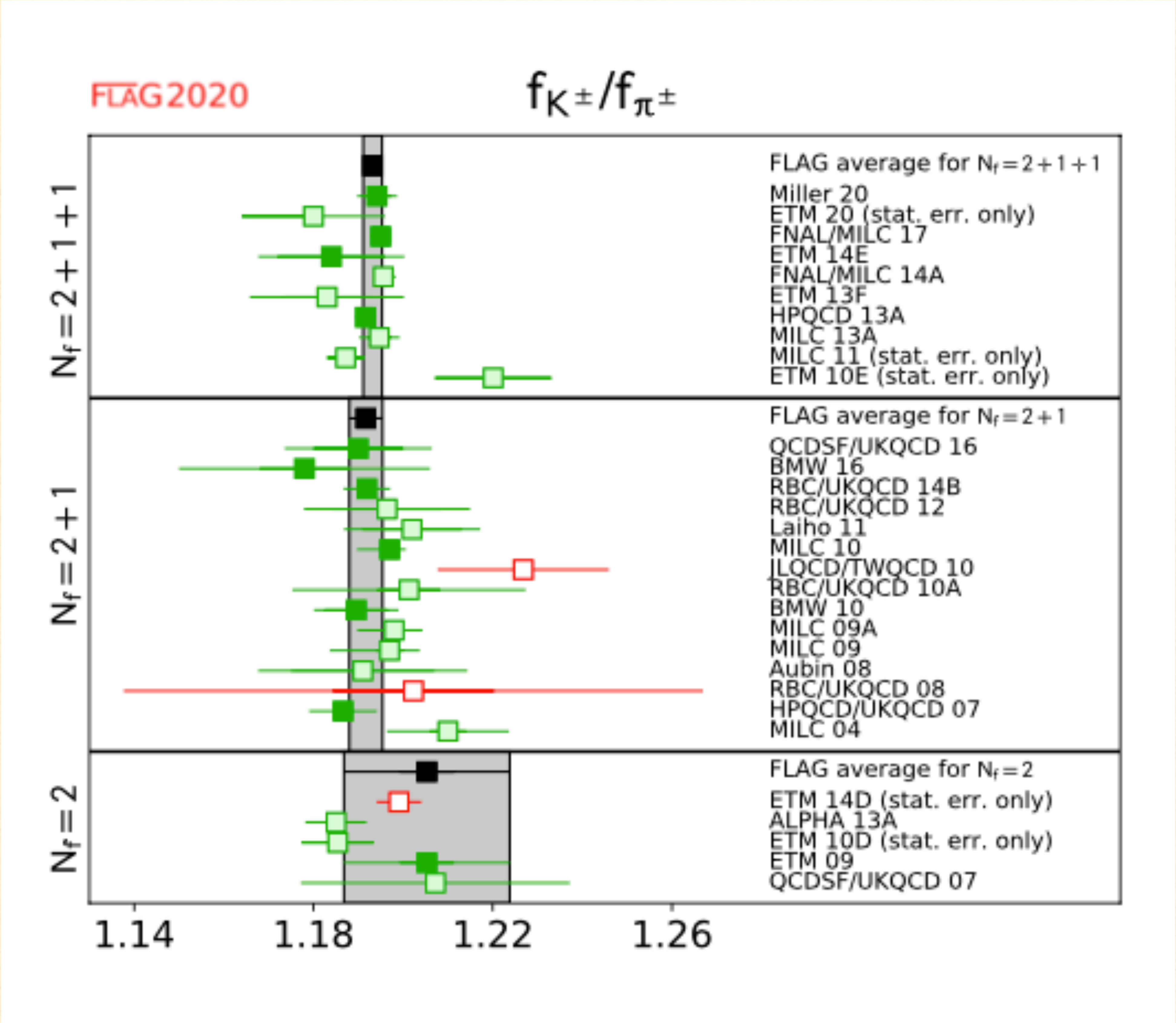
OVERVIEW

Goal: determine $\frac{V_{us}}{V_{ud}}$ using lattice QCD+QED inputs

- Testing unitarity of CKM → search for BSM physics
- From PDG2020…

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

- Theory input (eg. $f_{K^\pm}/f_{\pi^\pm}$) reached percent-level precision or better!
- Need to include isospin-breaking (IB) contributions ($\mathcal{O}(\alpha)$ and $\mathcal{O}(m_u \neq m_d)$) to improve precision on V_{ud} and V_{us}



ISOSPIN-BREAKING CORRECTIONS

Four-fermion operator for this weak decay: $H_W = (\bar{\nu}\gamma_L^\mu \ell)(\bar{q}_1 \gamma_L^\mu q_2)$

$$\mathcal{A}_P^{r,s} \equiv \langle \ell^+, r; \nu_\ell, s | H_W | P^+ \rangle \,, \; \mathcal{A}_P^{0\,r,s} = (\bar{u}^s \gamma_L^\tau v^r)\, p_P^\tau f_P$$

$$\Gamma^{\mathrm{tree}} = K \sum_{r,s} |\mathcal{A}_P^{0\,r,s}|^2$$

$$\delta\Gamma = 2K \sum_{r,s} \mathrm{Re}\left[\mathcal{A}_P^{0\,r,s*} \boxed{\delta \mathcal{A}_P^{r,s}} \right]$$

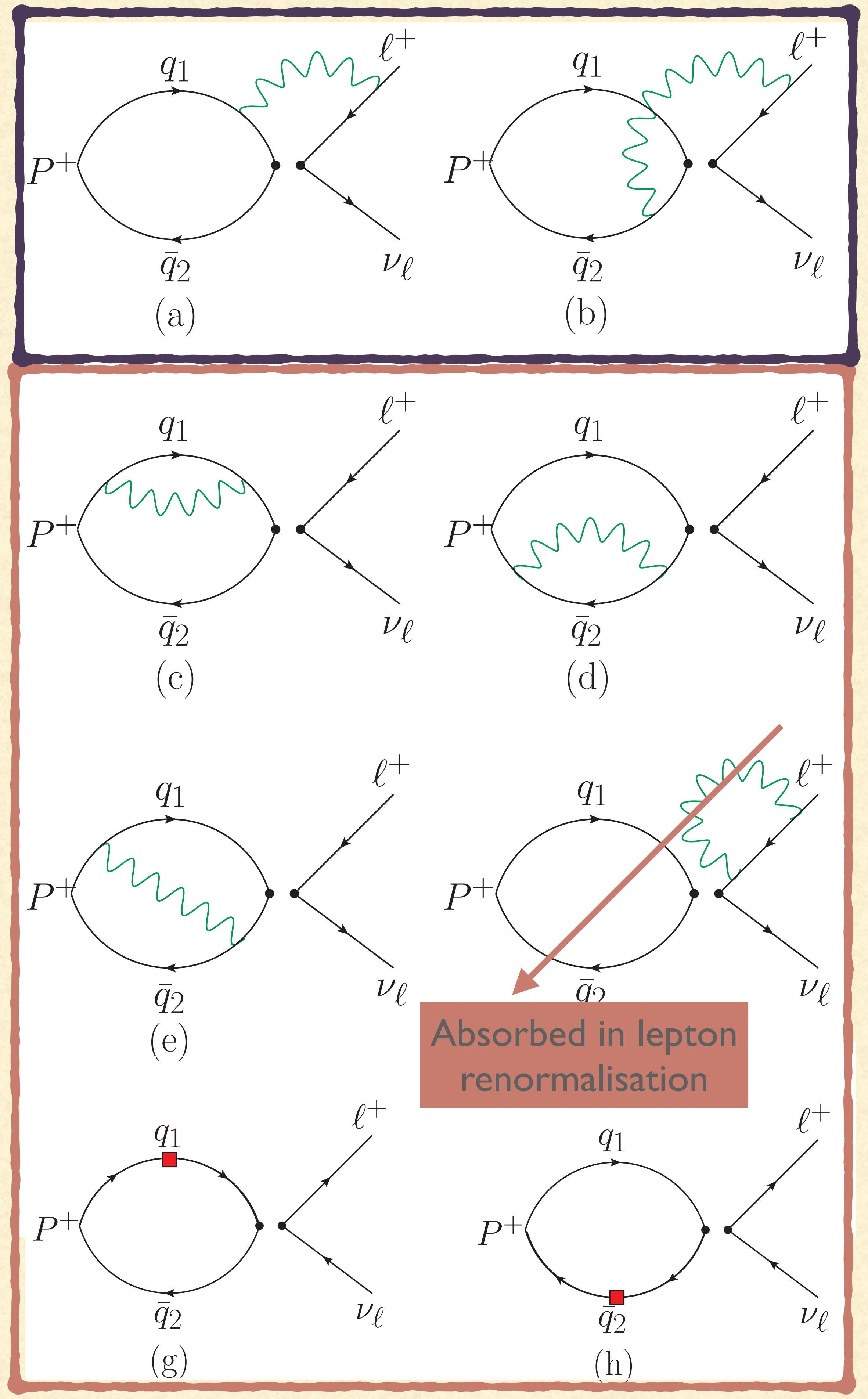
IB correction to matrix element

Perturbative expansion in α and bare quark mass shift $\Delta m = m - m^0$ [1]:

$$\langle O \rangle = \langle O \rangle_0 + \alpha \frac{\partial}{\partial \alpha} \langle O \rangle \Big|_{\alpha=0} + \sum_f \Delta m_f \frac{\partial}{\partial m_f} \langle O \rangle \Big|_{m_f = m_f^0} + \ldots$$

Non-factorisable correction

Factorisable correction



[1] G. M. De Divitis *et* al. Phys. Rev. D 87, 114505

STRATEGY

New challenge: Compute $\delta R_{K\pi}$ in (near) physical point simulation

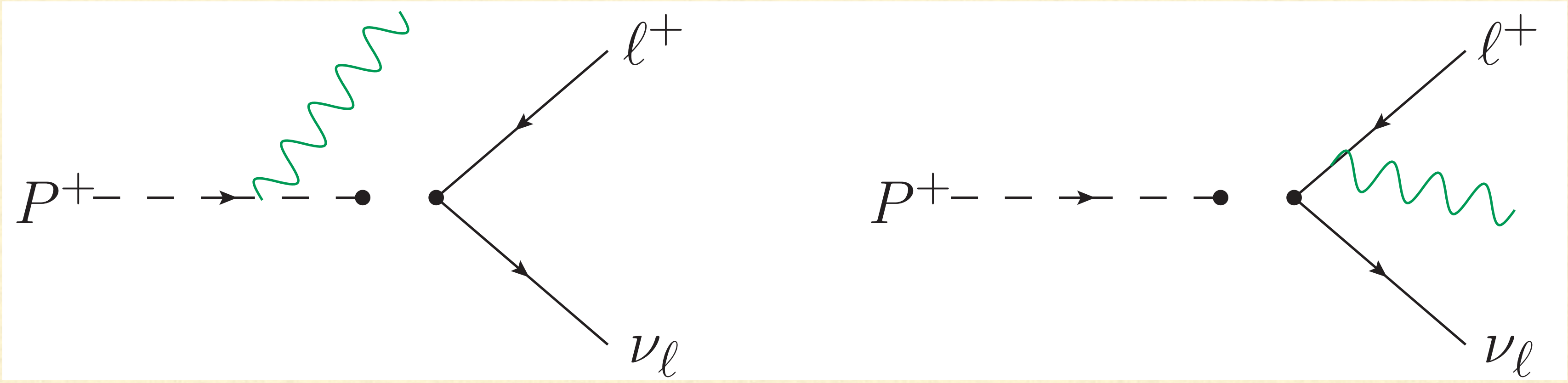
- Virtual photon correction to amplitude → Infrared divergence
- Pioneering work done by the RM123-Southampton collaboration, see…
  - PoS, CD15:023, June-July 2016
  - PhysRevLett., 120.072001, Feb 2018
  - Phys. Rev. D, 100:034514, Aug 2019

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) = \Gamma_0 + \Gamma_1,$$
$$= \lim_{L\to\infty} \boxed{\left(\Gamma_0(L) - \Gamma_0^{\rm pt}(L)\right)} + \boxed{\lim_{m_\gamma \to 0} \left(\Gamma_1(m_\gamma, \Delta E) + \Gamma_0^{\rm pt}(m_\gamma)\right)}$$

$\mathcal{O}(1/L)$ universal FVE removed †

Analytic calculation when $E_{res} < E_\gamma < \Delta E$

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) = \Gamma^{\rm tree}(1 + \delta R_P)$$



† For structure-dependent FVE, see N. Hermansson-Truedsson talk.

SIMULATION DETAILS

- Physical point Möbius Domain-Wall Fermions (DWF) [2]
- $N_f = 2+1, \quad 48^3 \times 96, \quad L_S = 24, \quad a^{-1} \simeq 1730\mathrm{MeV} \qquad \Rightarrow M_\pi = 139.15(36)\,\mathrm{MeV}$
- Valence light: physical mass ZMöbius, $L_s = 10$
- 60 configurations (20 trajectory-spacing)

[2] RBC/UKQCD Collaboration PhysRevD.93.074505 (2014)

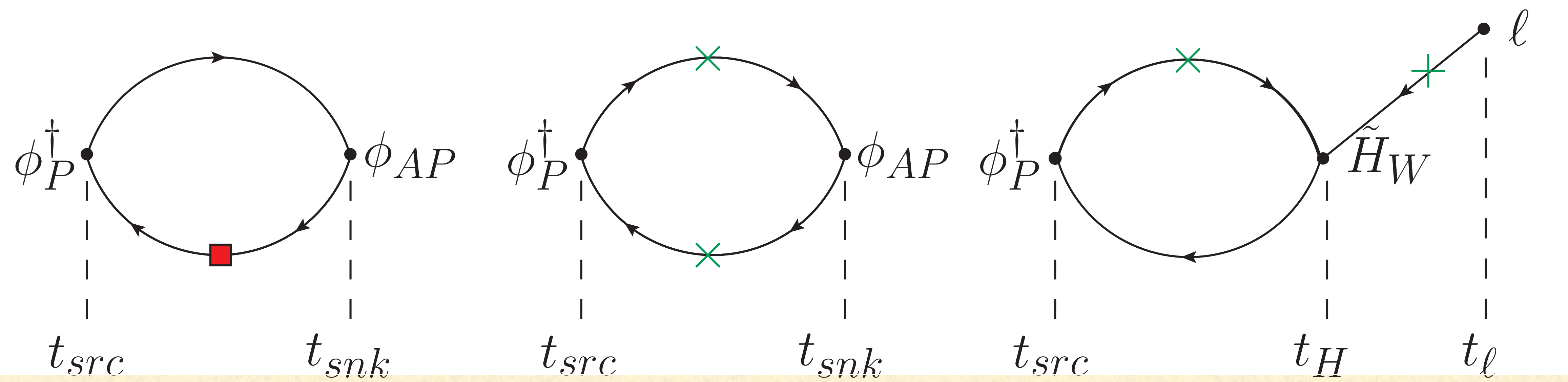
IMPLEMENTATION

Interpolators from standard 2pt analysis:

$$\phi_P = \bar{q}_2\gamma^5 q_1, \quad \phi_{AP} = \bar{q}_2\gamma^0\gamma^5 q_1$$

Four-fermion operator with neutrino leg amputated:

$$\tilde{H}_{W,\beta} = (\gamma_L^\mu \ell)_\beta(\bar{q}_2\gamma_L^\mu q_1), \quad \gamma_L^\mu = \gamma^\mu(\mathbf{1}-\gamma^5)$$



Examples of sequential insertions for (non-)factorisable correlation functions

- $\mathrm{QED_L}$ theory
- Electro-quenched calculation
- Sequential insertion of scalar (S) current and local QED current (renormalised by Z_V) in Feynman gauge on quark propagators
- 96 gauge-fixed wall sources per config.
- Muon propagator generated using free DWF, with twisted B.C. for 4-mom conservation
- 8 $t_\ell - t_H$ separations
- Neutrino propagator included during data processing

FROM SIMULATION TO NATURE

- Linear expansion to physical/isosymmetric QCD point using derivatives w.r.t simulated bare parameters
- For an observable, X (eg hadronic mass),

$$X = X^0 + \alpha \partial_\alpha X|_{\alpha=0} + \sum_f \Delta m_f \partial_{m_f} X|_{m_f = m_f^0}, \quad \text{where} \quad \partial_y X = \frac{\partial X}{\partial y}$$

Tuning to physical point $(\alpha, m_u \neq m_d)$

- Using $\alpha = \alpha_{\text{EM}}$, tune $\{\Delta m_f\}$ such that $M^2_{\pi^+}, M^2_{K^+}, M^2_{K^0}$ match their experimental masses
- Scale setting with Ω^- baryon

Tuning to QCD point $(\alpha = 0)$

- Introduce unphysical mesons as proxy [3,4]:

$$M^2_{ud} = \frac{1}{2}(M^2_{\bar{u}u} + M^2_{\bar{d}d}) \approx 2Bm_{ud} + \ldots,$$

$$\Delta M^2 = M^2_{\bar{u}u} - M^2_{\bar{d}d} \approx 2B(m_u - m_d) + \ldots,$$

$$M^2_{K\chi} = \frac{1}{2}\left(M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}\right) \approx 2Bm_s + \ldots$$

- Set $\alpha = 0$, tune $\{\Delta m'_f\}$ to the following values:

$$M^2_{ud} = (M^{exp}_{\pi^0})^2, \quad M^2_{K\chi} = \frac{1}{2}\left((M^{exp}_{K^+})^2 + (M^{exp}_{K^0})^2 - (M^{exp}_{\pi^+})^2\right),$$

$$\Delta M^2 = \begin{cases} 0 & \text{if} \quad m_u = m_d \\ (\Delta M^2)^{phys} & \text{if} \quad m_u \neq m_d \end{cases}$$

[3] BMW Collaboration PhysRevLett.111.252001 (2013), [4] BMW Collaboration Nature 593 (2021)

FROM SIMULATION TO NATURE

- $(\Delta M^2)^{phys} \approx 2B(m_u - m_d) \rightarrow$ check $\dfrac{m_u}{m_d}$

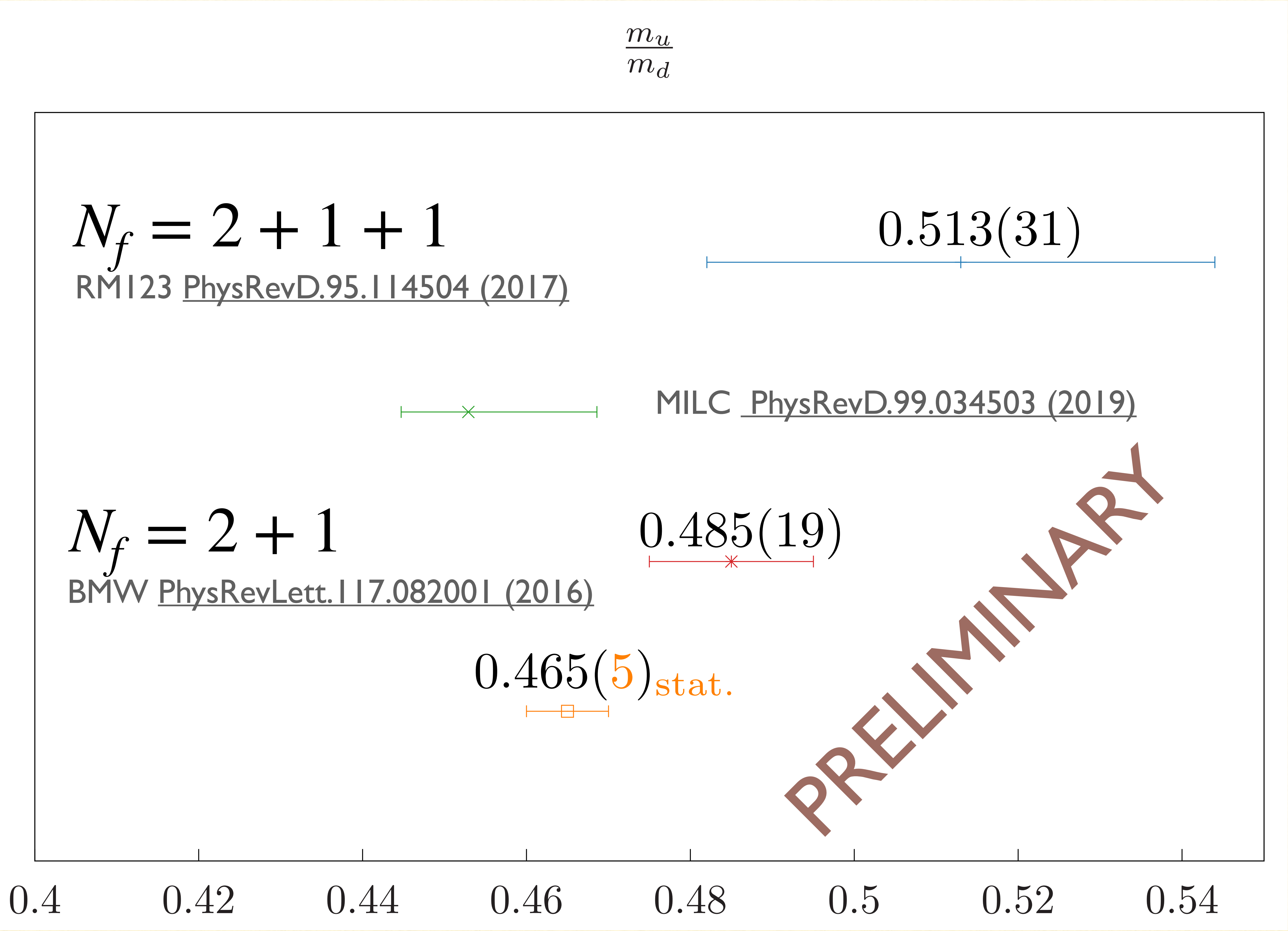
$$(\Delta M^2)^{phys} = -13570(96)_{\text{stat.}}\ \text{MeV}^2$$

- Using inputs in $\overline{\text{MS}} = 2\text{GeV}\ldots$

$$B = 2804(34)\text{MeV}, \quad [5]$$
$$m_{ud} = 3.315(40)\text{MeV} \quad [2]$$

(!) Single lattice spacing; no cont. extrapolation result:

$$\frac{m_u}{m_d} = 0.465(5)_{\text{stat.}}$$ PRELIMINARY



- Comparison with FLAG results (see fig.)…

[2] RBC/UKQCD Collaboration PhysRevD.93.074505 (2014) [5] RBC/UKQCD Collaboration Phys.Rev.D93.054502 (2016)

FROM SIMULATION TO NATURE

- Procedure to separate QCD and QED
  → Correction to Dashen's Theorem

$$\epsilon = \frac{\left(M_{K^+}^2 - M_{K^0}^2\right)^\gamma}{M_{\pi^+}^2 - M_{\pi^0}^2} - 1$$

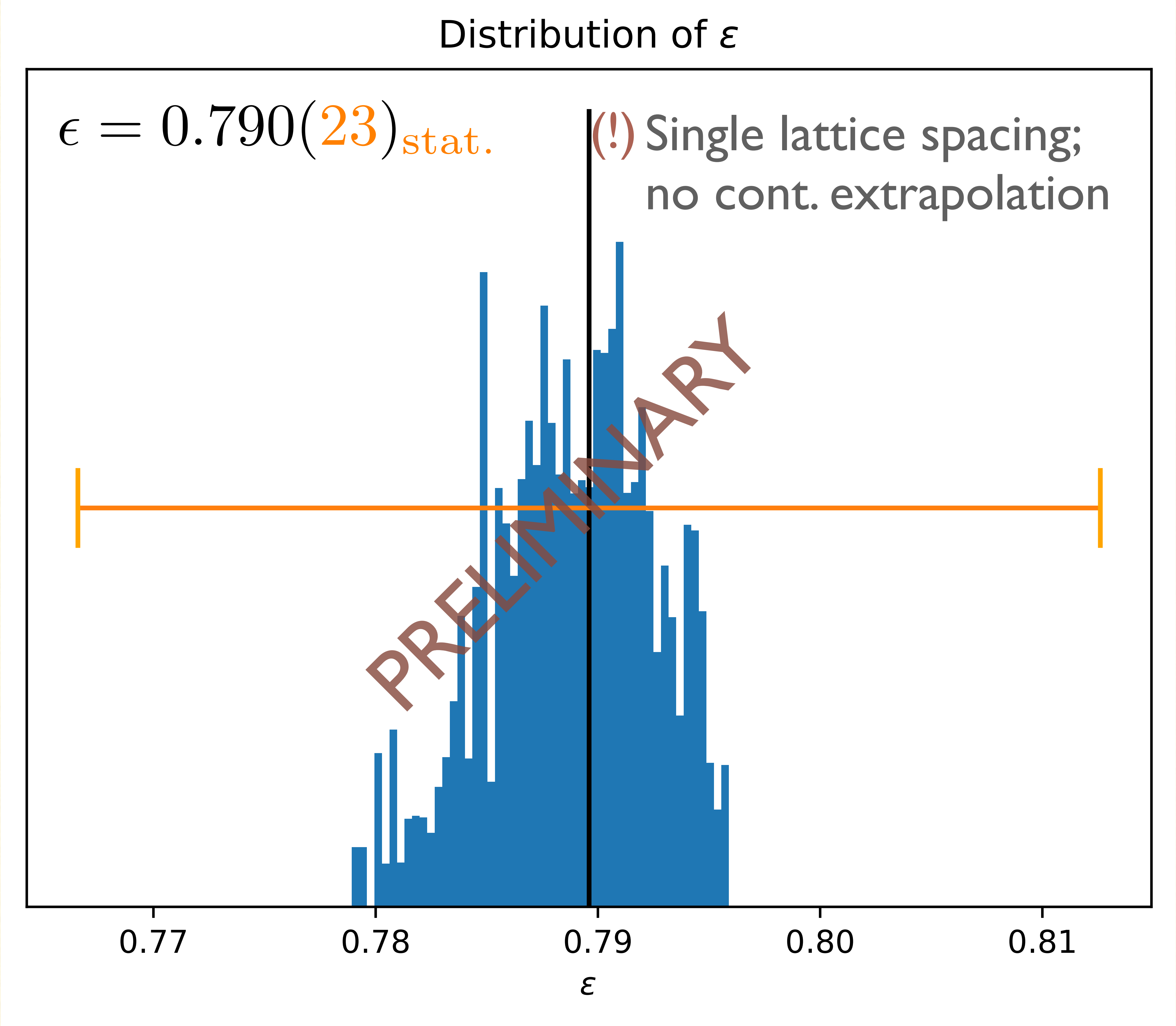
- Comparison with…

$(N_f = 2+1+1):\ \epsilon = 0.79(7)$ RM123 (2017) BMW (2014)

$(N_f = 2+1):\ \epsilon = 0.73(17)$ BMW (2016)

$(N_f = 2+1):\ \epsilon = 0.79(1)_{\text{stat.}}(^{+8}_{-11})_{\text{sys.}}$ MILC (2019)

- Fig: a distribution of ϵ generated by varying fit range of correlation functions



FACTORISABLE AMPLITUDE CORRECTION

- Obtain decay const. (and its correction) from PP (γ^5, γ^5) and PA $(\gamma^5, \gamma^0\gamma^5)$ correlation functions

- Eg. pion tree-level correlator (excl. backward prop):

$$\hat{C}^i_\pi(t) = A^{i,0}_\pi e^{-M^0_\pi t} \qquad\qquad i = PP, PA$$

where $$A^{PP/PA}_\pi = \frac{\langle 0|\phi_{P/AP}|\pi\rangle\ \langle \pi|\phi^\dagger_P|0\rangle}{2M_\pi}$$

- The QED/scalar insertion correlators are:

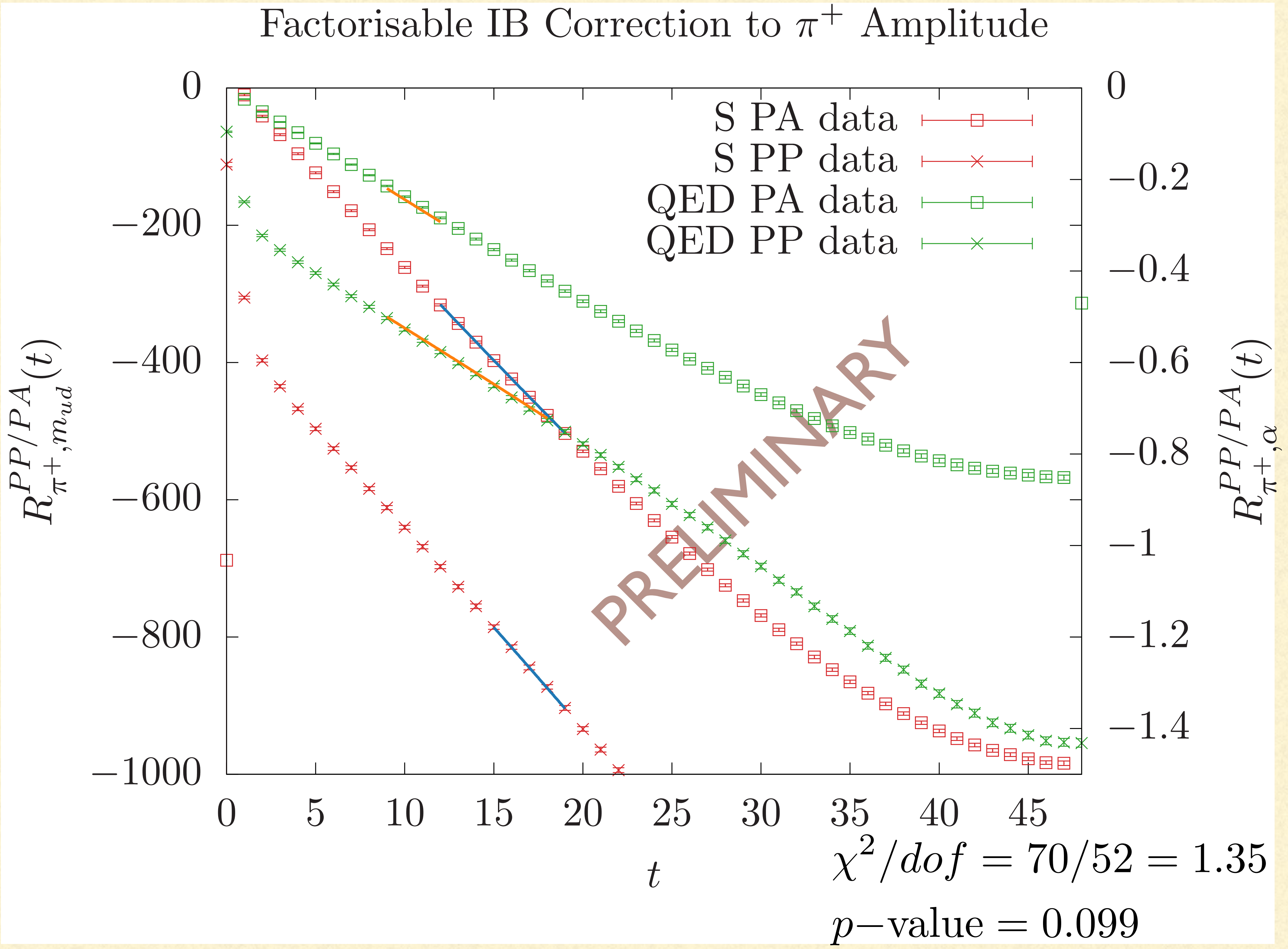
$$\partial_{g_j} C^i_\pi(t) = (\partial_{g_j} A^i_\pi + A^{i,0}_\pi \partial_{g_j} M_\pi t) e^{-(M^0_\pi + \Delta g_j \partial_{g_j} M_\pi)t}$$

$$g_j = \{\alpha, m_u, m_d, m_s\}$$

- Then, the ratios give:

$$R^i_{\pi,g_j}(t) \equiv \frac{\partial_{g_j} C^i_\pi(t)}{\hat{C}^i_\pi(t)} = \boxed{\frac{\partial_{g_j} A^i_\pi}{A^{i,0}_\pi}} - \boxed{\partial_{g_j} M_\pi} t$$

Simultaneous fit → corrections to f_π



NON-FACTORISABLE AMPLITUDE CORRECTION

- Spectral representation of new correlation function (excl. backward prop and excited states):

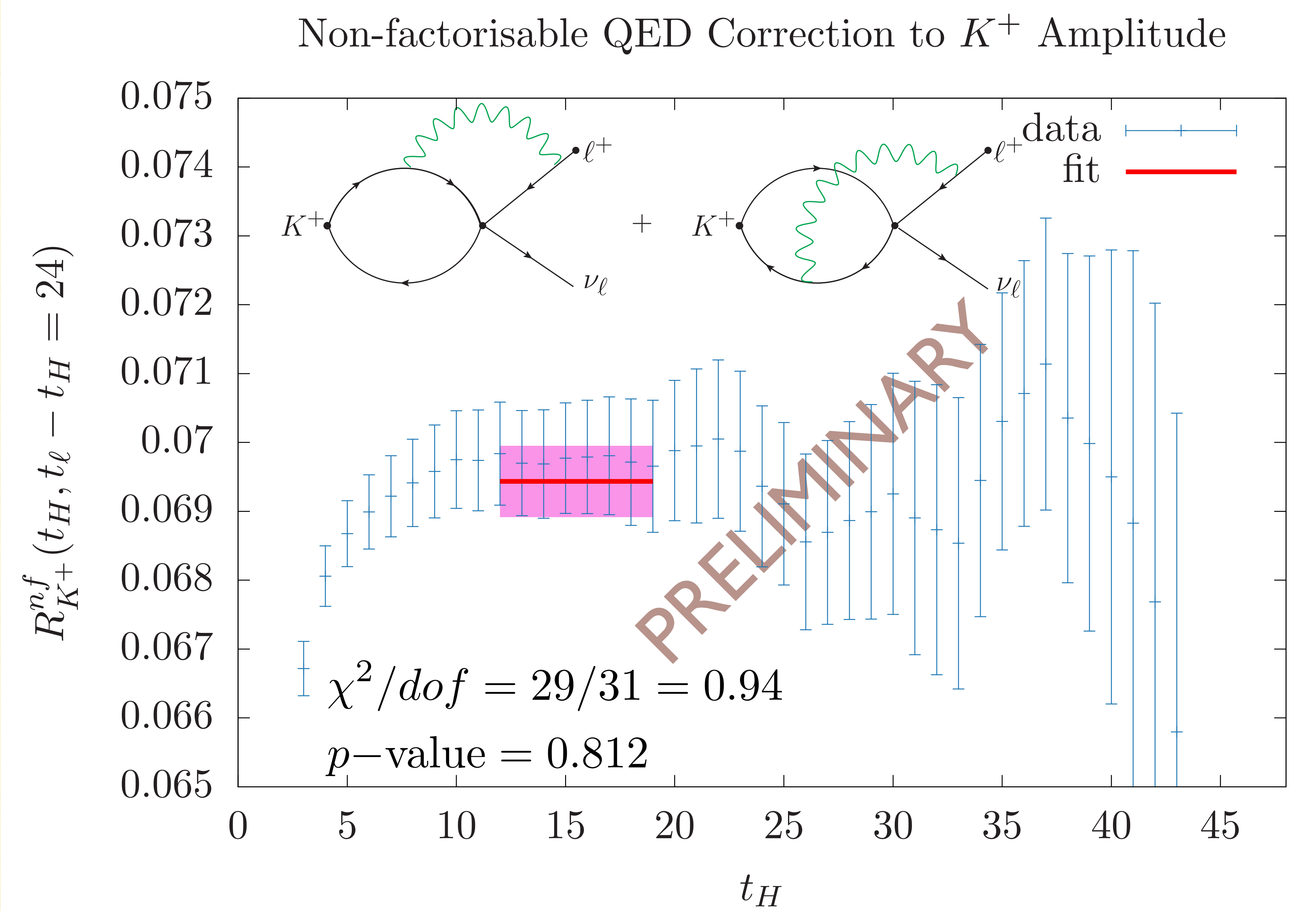
$$\partial_\alpha C_P^{nf}(t_H, t_\ell - t_H)_{\varepsilon\beta} = \frac{\langle P|\phi_P^\dagger|0\rangle}{4E_\ell M_P^0}\left[\partial_\alpha \mathcal{A}_P^{nf}(-\not{p}_\ell - im_\ell)\right]_{\varepsilon\beta} e^{-M_P^0 t_H} e^{-E_\ell(t_\ell - t_H)}$$

- The ratio of correlator traces give:

$$R_P^{nf}(t_H, t_\ell - t_H) = \frac{\mathrm{Tr}\left[\not{p}_\nu \partial_\alpha C_P^{nf}(t_H, t_\ell - t_H)\gamma_L^0\right]}{\mathrm{Tr}\left[\not{p}_\nu C_{P,0}^{nf}(t_H, t_\ell - t_H)\gamma_L^0\right]},$$

$$\underset{t_\ell \gg t_H \gg t_{src}}{=} \boxed{\frac{\mathrm{Tr}\left[\not{p}_\nu \partial_\alpha \mathcal{A}_P^{nf}(-\not{p}_\ell + im_\ell)\gamma_L^0\right]}{M_P^0 f_P \,\mathrm{Tr}\left[\not{p}_\nu \gamma_L^0(-\not{p}_\ell + im_\ell)\gamma_L^0\right]}}$$

Non-factorisable amplitude correction



$\frac{V_{us}}{V_{ud}}$ FROM PHYSICAL POINT CALCULATION

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \underbrace{\boxed{\frac{\Gamma(K^+ \to \mu^+\nu_{\mu^+}[\gamma])}{\Gamma(\pi^+ \to \mu^+\nu_{\mu^+}[\gamma])} \frac{M_{K^+}^3}{M_{\pi^+}^3} \frac{(M_{\pi^+}^2 - m_{\mu^+}^2)^2}{(M_{K^+}^2 - m_{\mu^+}^2)^2}}}_{\text{Experiment}} \underbrace{\boxed{\frac{(f_\pi/f_K)^2}{1+(\delta R_K - \delta R_\pi)}}}_{\text{Theory}}$$

- PDG → inputs for inclusive rates and masses
- Simultaneous fits of lattice correlators → for calculating $\left(1+(\delta R_K - \delta R_\pi)\right) f_K^2/f_\pi^2$
- All lattice inputs available to calculate V_{us}/V_{ud} at physical point
- Full systematics error budget on $\delta R_K - \delta R_\pi$, includes…

  1. EM quenching
  2. $\mathcal{O}(a^2)$ cut-off effects
  3. Fit systematics
  4. $\mathcal{O}(L^{-2})$ FVE

See N. Hermansson-Truedsson's talk.

- Progress: 2nd independent analysis under way

CONCLUSION & OUTLOOK

- IB effects are necessary to improve precision on light CKM matrix elements
- Lattice calculation setup for physical point calculation
- All ingredients available for calculating $\delta R_{K\pi}$ and $\frac{V_{us}}{V_{ud}}$ - 2nd independent analysis under way

---

- Renormalise H_W to obtain V_{us} and V_{ud}individually
- Unquenched calculation
- Lattice calculation of real photon emission diagram
- CKM matrix element from semi-leptonic decays, e.g $K^{\pm} \to \pi^0 \ell^{\pm} \nu_{\ell}$

THANK YOU FOR YOUR ATTENTION