
Multipolar Components
of Deflecting Field

Animated Discussion

R. Calaga, E. Jensen

LHC-CC10, December 15-17, 2010

Transverse Deflection, Some Basics

Transverse kick imparted to particle

$$p_{\perp} = \frac{q}{v} \int_0^L \left(\vec{E}_{\perp} + \vec{v} \times \vec{B} \right) dz$$

Expressing in terms of vector potential

$$p_{\perp} = q \int_0^L \left[- \left(\frac{1}{v} \frac{\partial \vec{A}_{\perp}}{\partial t} + \frac{\partial \vec{A}_{\perp}}{\partial z} \right) + \nabla_{\perp} A_z \right] dz$$

After boundary conditions & simplification ($v = c$)

$$\Delta p_{\perp} = \frac{q}{\omega} \int_0^L (-i) \nabla_{\perp} E_z dz$$

transverse gradient of longitudinal electric field.

TM, TE or TEM mode

Some Basics

Plane waves propagating down the waveguide of the form

$$\{\vec{E}, \vec{B}\} = \{\vec{E}_0, \vec{B}_0\} \cdot e^{i(kz - \omega t)}$$

Differential form of wave eqn. inside a waveguide,

$$\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} = 0$$

With boundary conditions

$$\hat{n} \times \vec{E} = 0, \quad \hat{n} \cdot \vec{B} = 0$$

And perpendicular metal walls (standing waves):

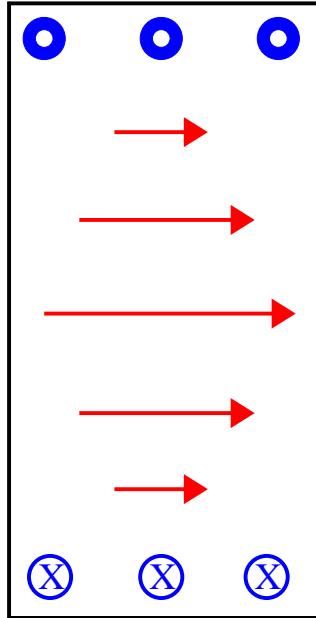
$$z = 0 \quad z = L$$

$$E_z = \psi(x, y) \cos\left(\frac{p\pi z}{l}\right), \quad p = 0, 1, \dots \text{(TM Modes)}$$

$$H_z = \psi(x, y) \sin\left(\frac{p\pi z}{l}\right), \quad p = 1, 2, \dots \text{(TE Modes)}$$

Pill-Box Cavity

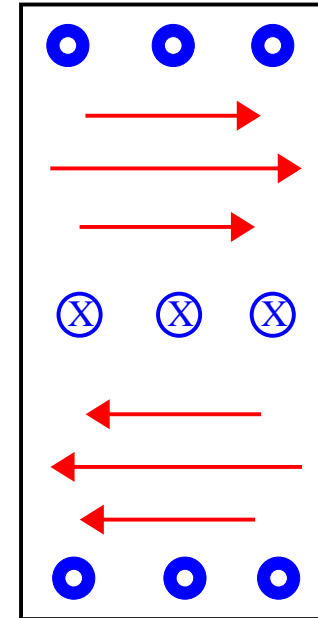
TM010



$$E_z = E_0 J_0(k_r r) e^{-i\omega t}$$

$$H_\phi = -\frac{i}{\mu_0 c} E_0 J_1(k_r r) e^{-i\omega t}$$

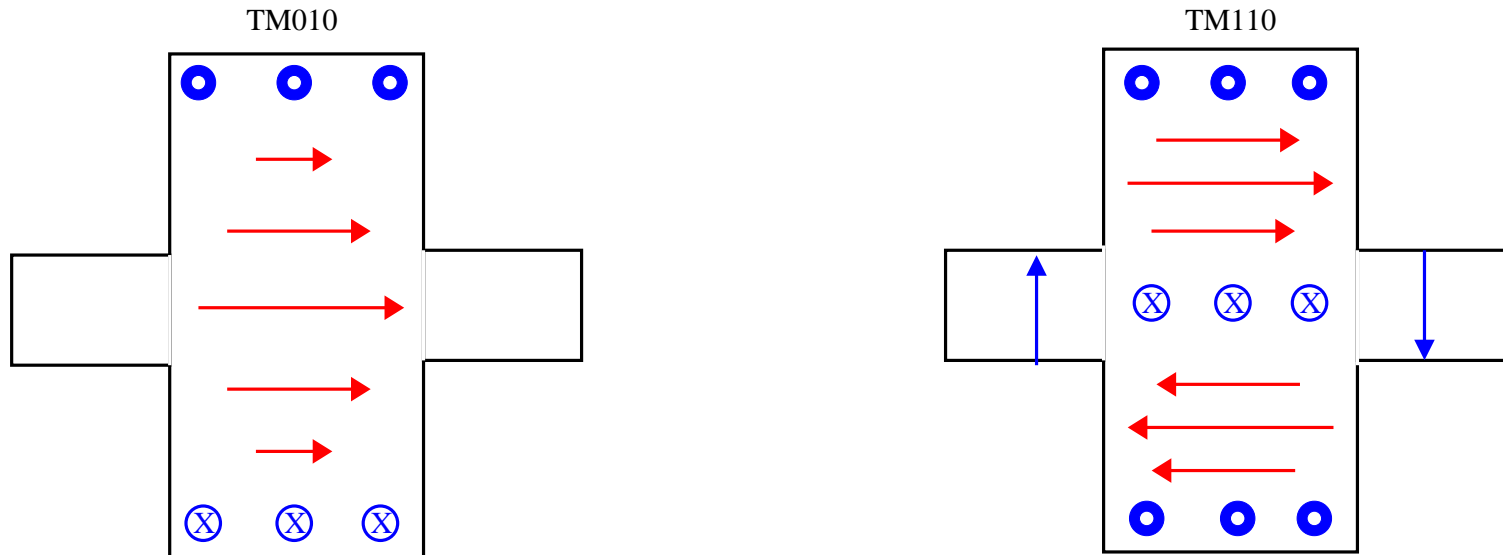
TM110



$$E_z = E_0 J_1(k_r r) \cos(\phi) \cos(k_z z) e^{-i\omega t}$$

$$H_\phi = -i \frac{\omega}{c^2 k_r} E_0 J_1'(k_r r) \cos(\phi) \cos(k_z z) e^{-i\omega t}$$

Pill-Box + Beam Pipes



For $m \neq 0$, both TE & TM modes exist (E_z , H_z are finite)

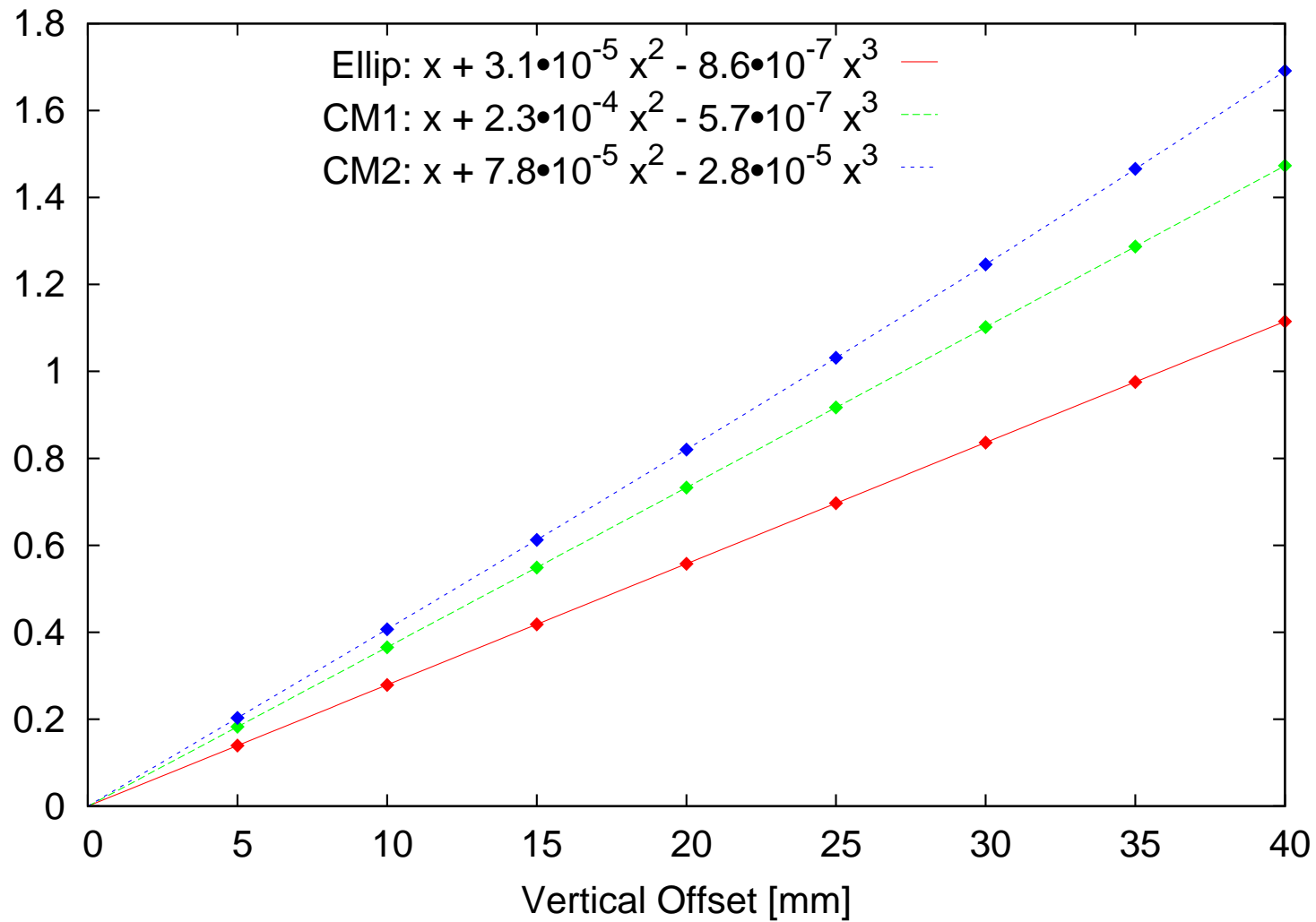
Beam tubes create a "Hybrid" mode, where fields vary as¹:

$$E_z = E_0 \left(\frac{r}{b}\right)^n \cos(n\phi) e^{-i\omega t}$$

$$H_\phi = \frac{-i}{\mu_0 c k r} E_0 \left(\frac{r}{b}\right)^n \left[m - \frac{k^2 r^2}{m+1} \right] \cos(n\phi) e^{-i\omega t}$$

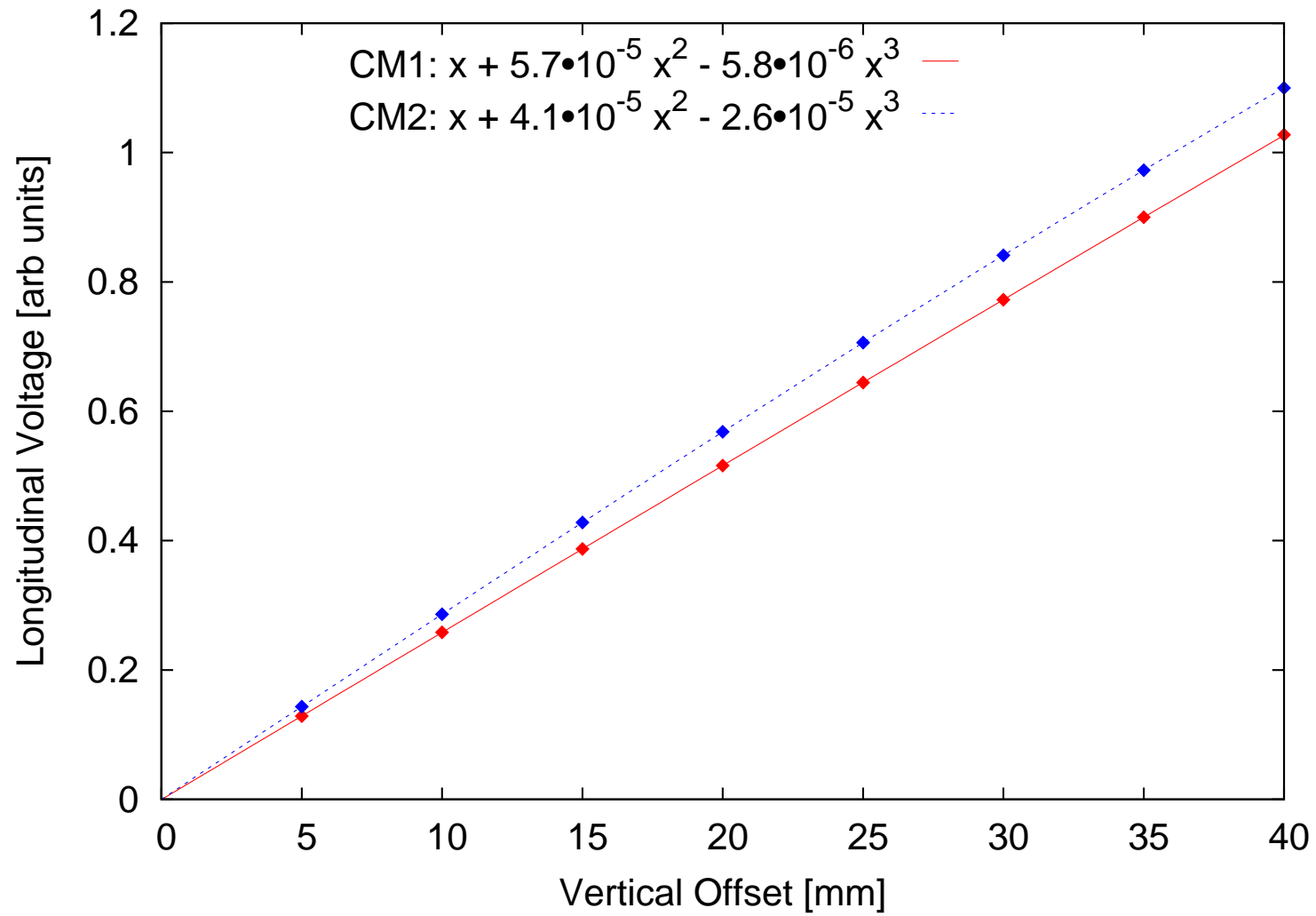
[1]: B. Zotter, K. Bane, PEP-Note 308 (1979)

V_{\perp} (X_{Off} , Num Integration)



Note: \sim 1 million mesh cells used

V_{\perp} (R_{off} Num Integration)



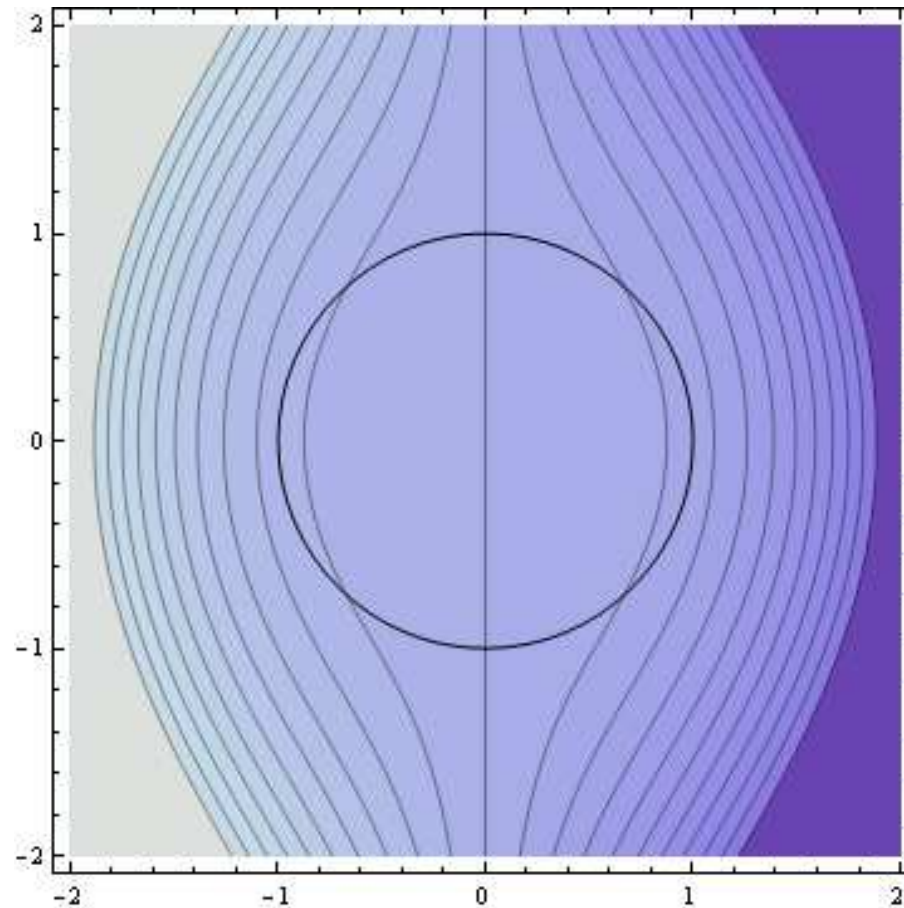
Note: \sim 1 million mesh cells used

Erk's Simple Explanation

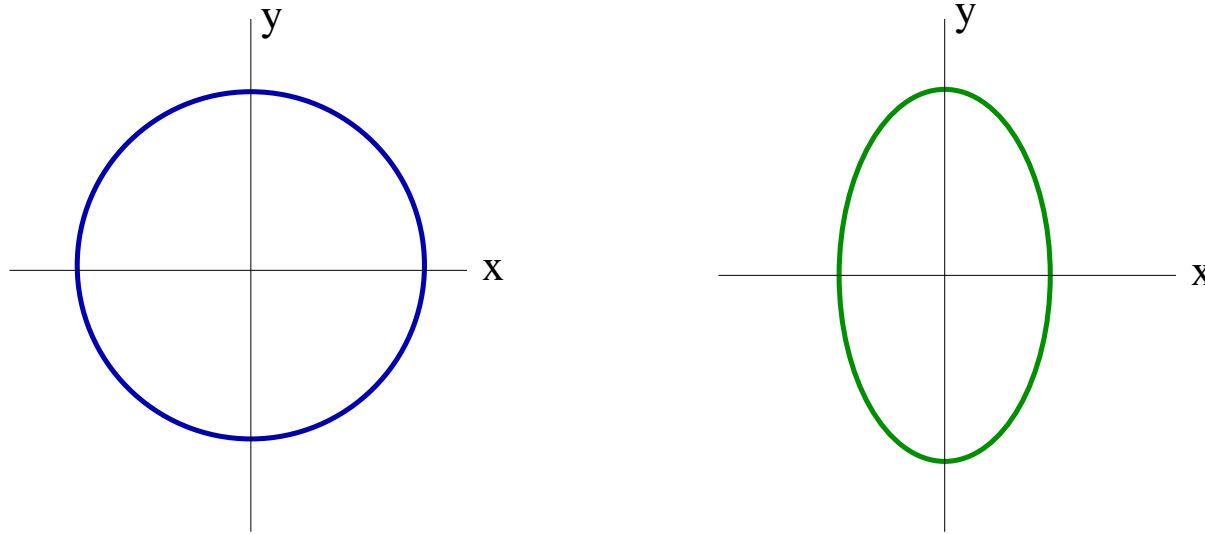
Series expand Bessel Function $J_n(x)$:

$$J_1(x) = x - 1.5 \times 10^{-3}x^3 + 7.5 \times 10^{-7}x^5 + O[x]^7$$

Countour plot of $AJ_n(r) \sin(n\phi) - x$, circle at 25 mm



Elliptical Cross Section



$$\psi(x, y, z) \rightarrow \psi(\xi, \eta, z)$$

$$x = h \cos(\xi) \cos(\eta) \quad ; \quad y = h \sin(\xi) \sin(\eta)$$

$$E_z = C e_m(\xi, q_{mn}) c e_m(\eta, q_{mn}) \cos(k_z z) e^{-\omega t}$$
$$H_\xi = \frac{i\omega\epsilon_0}{D_{mnd}} C e_m(\xi, q_{mn}) c e'_m(\eta, q_{mn}) \cos(k_z z) e^{-\omega t}$$

where $c e_m$ & $C e_m$ are even and modified Matheiu functions of 1st kind.

Ref: J Yang, K Chen, Particle Accelerators, Vol 35, 1991.

Some Comparisons

Reference radius for multipole components, 25 mm

Element	Aper [mm]	I [kA]	Kick [μ rad]	$\{b_2, b_3\}$ [10^{-4} Units]
MBX ($D_{1,3}$)	74	5.75	500	-0.19 ± 0.24 -3.1 ± 0.98
MBRB/C ($D_{2,4}$)	69	6.05	500	-0.18 ± 0.1 -0.5 ± 0.4
MCBC/Y	56/70	0.1	80	No Spec
MCBCH/V	56/70	0.1	80	No Spec
CRAB.[L,R].[1,5]	84	-	1.5-6	3.2 ± 13.4 -0.1 ± 0.3

$$V_c = \frac{cE_0 \tan(\theta_c/2)}{\omega_0 \sqrt{\beta_c \beta^*}} \quad ; \quad \Delta x'_c = \frac{qV_c}{E_0} \sin\left(\frac{\omega_0 z}{c} - \phi\right)$$

- Integrated kick of crabs x5-50 smaller than corrector & dipoles
- b_2, b_3 values for crabs revised with azimuth averaging (A. Grudiev)

Discussion

- Field linear for cavity with beam pipes
 - Superposition of TM & TE due to addition of beam pipes
 - Numerical integration confirms this for arbitrary cavity
- Kick from crabs x100 or even smaller than insertion magnets
- Field non-linearity
 - 3 ± 13 units of b_2 and 0.1 ± 0.3 units of b_3 (using azimuth averaging)
 - What is the specification ?
 - Additional optimization possible to linearize more if needed
- Remember, no specification for orbit correctors
 - Kick from corrector magnets atleast x40 more than crabs