Multipolar Components of Deflecting Field *Animated Discussion*

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Transverse Deflection, Some Basics

Transverse kick imparted to particle

$$p_\perp = \frac{q}{v}\int_0^L \left(\vec{E}_\perp + \vec{v}\times\vec{B}\right) dz$$

Expressing in terms of vector potential

$$p_\perp = q\int_0^L \left[-\left(\frac{1}{v}\frac{\partial \vec{A}_\perp}{\partial t} + \frac{\partial \vec{A}_\perp}{\partial z}\right) + \nabla_\perp A_z\right] dz$$

After boundary conditions & simplification ($v = c$)

$$\Delta p_\perp = \frac{q}{\omega}\int_0^L (-i)\nabla_\perp E_z \; dz$$

transverse gradient of longitudinal electric field.

TM, TE or TEM mode

Some Basics

Plane waves propagating down the waveguide of the form

$$\{\vec{E}, \vec{B}\} = \{\vec{E_0}, \vec{B_0}\}.e^{i(kz-\omega t)}$$

Differential form of wave eqn. inside a waveguide,

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left\{ \begin{array}{c} \vec{E} \\ \vec{B} \end{array} \right\} = 0$$

With boundary conditions

$$\hat{n} \times \vec{E} = 0, \quad \hat{n} \cdot \vec{B} = 0$$

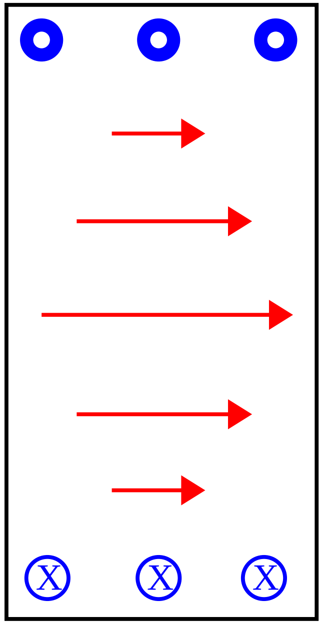
And perpendicular metal walls (standing waves):

$$z = 0 \quad z = L$$

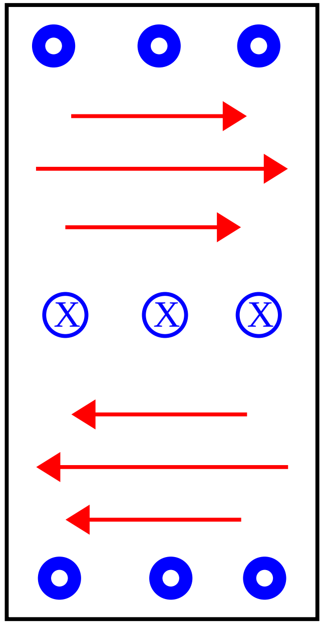
$$\begin{aligned} E_z &= \psi(x,y)\ \cos\left(\frac{p\pi z}{l}\right), \quad p = 0, 1 ... (\text{TM Modes}) \\ H_z &= \psi(x,y)\ \sin\left(\frac{p\pi z}{l}\right), \quad p = 1, 2 ... (\text{TE Modes}) \end{aligned}$$

Pill-Box Cavity

TM010



TM110



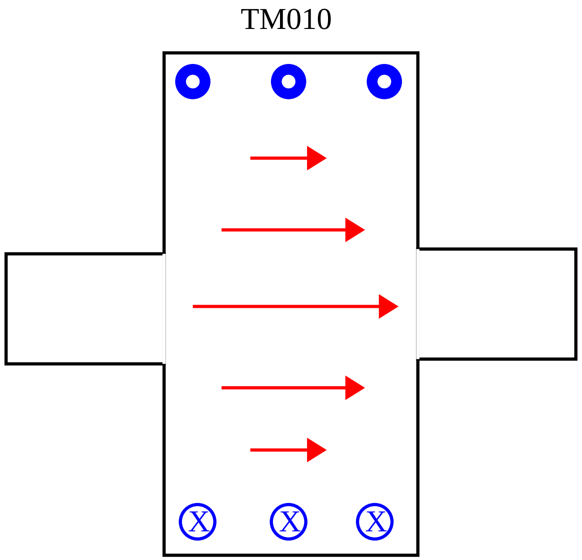
$$E_z = E_0 J_0(k_r r) e^{-i\omega t}$$

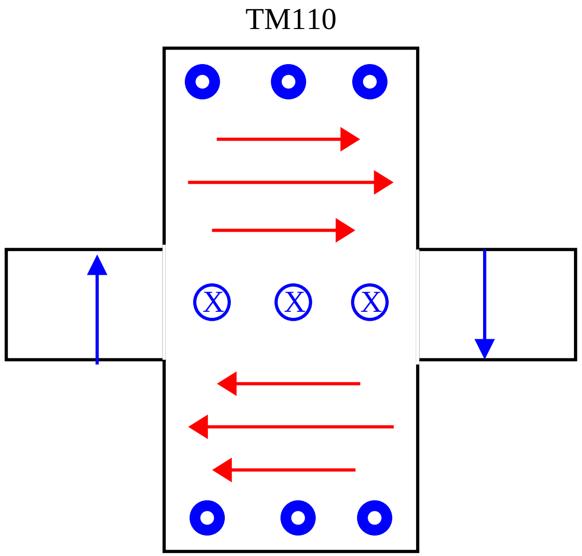
$$H_\phi = -\frac{i}{\mu_0 c} E_0 J_1(k_r r) e^{-i\omega t}$$

$$E_z = E_0 J_1(k_r r) \cos(\phi) \cos(k_z z) e^{-i\omega t}$$

$$H_\phi = -i\frac{\omega}{c^2 k_r} E_0 J_1'(k_r r) \cos(\phi) \cos(k_z z) e^{-i\omega t}$$

Pill-Box + Beam Pipes





For $m \neq 0$, both TE & TM modes exist (E_z, H_z are finite)

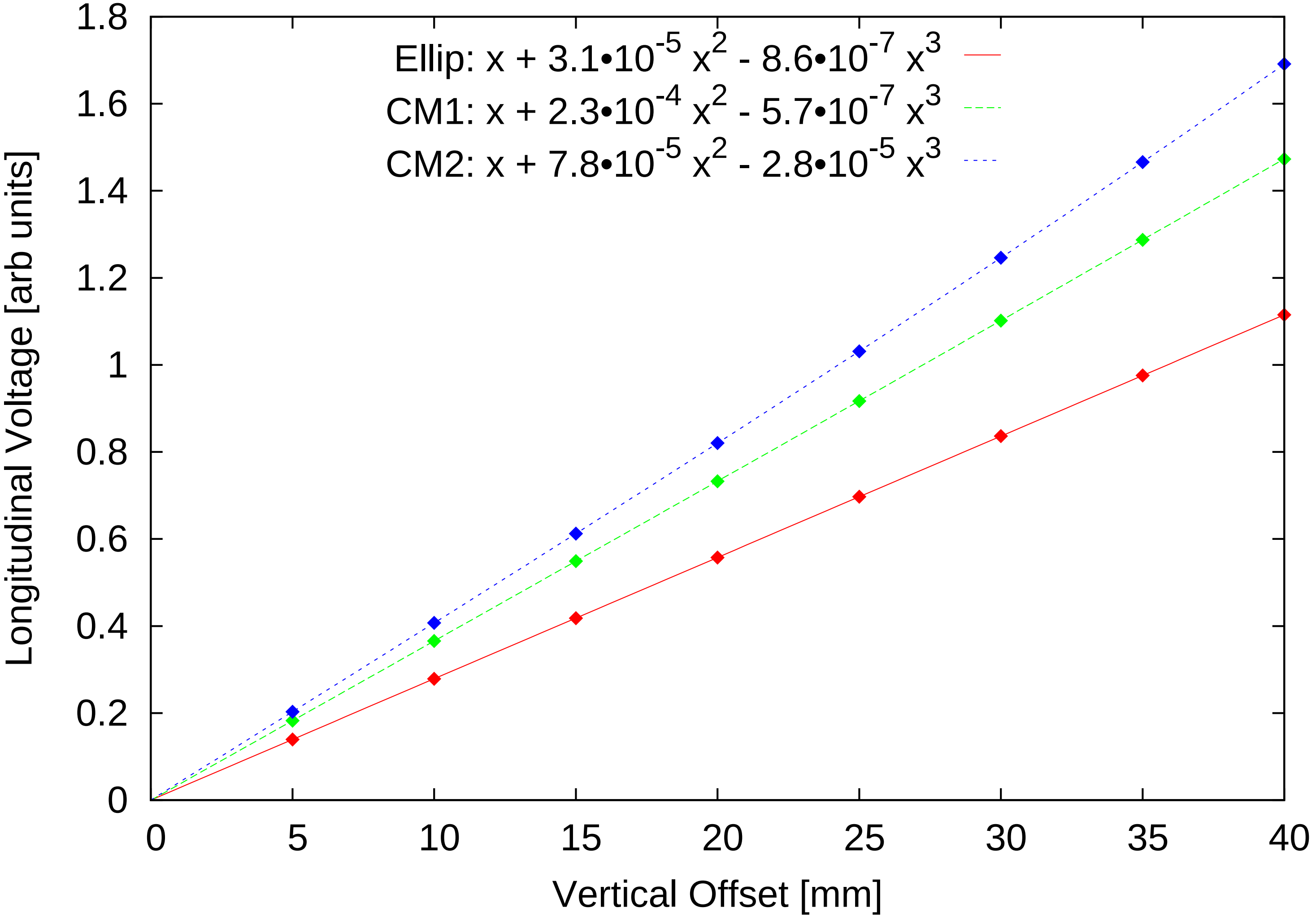
Beam tubes create a "Hybrid" mode, where fields vary as[1]:

$$E_z = E_0 \left(\frac{r}{b}\right)^n \cos(n\phi) e^{-i\omega t}$$

$$H_\phi = \frac{-i}{\mu_0 ckr} E_0 \left(\frac{r}{b}\right)^n \left[m - \frac{k^2 r^2}{m+1}\right] \cos(n\phi) e^{-i\omega t}$$

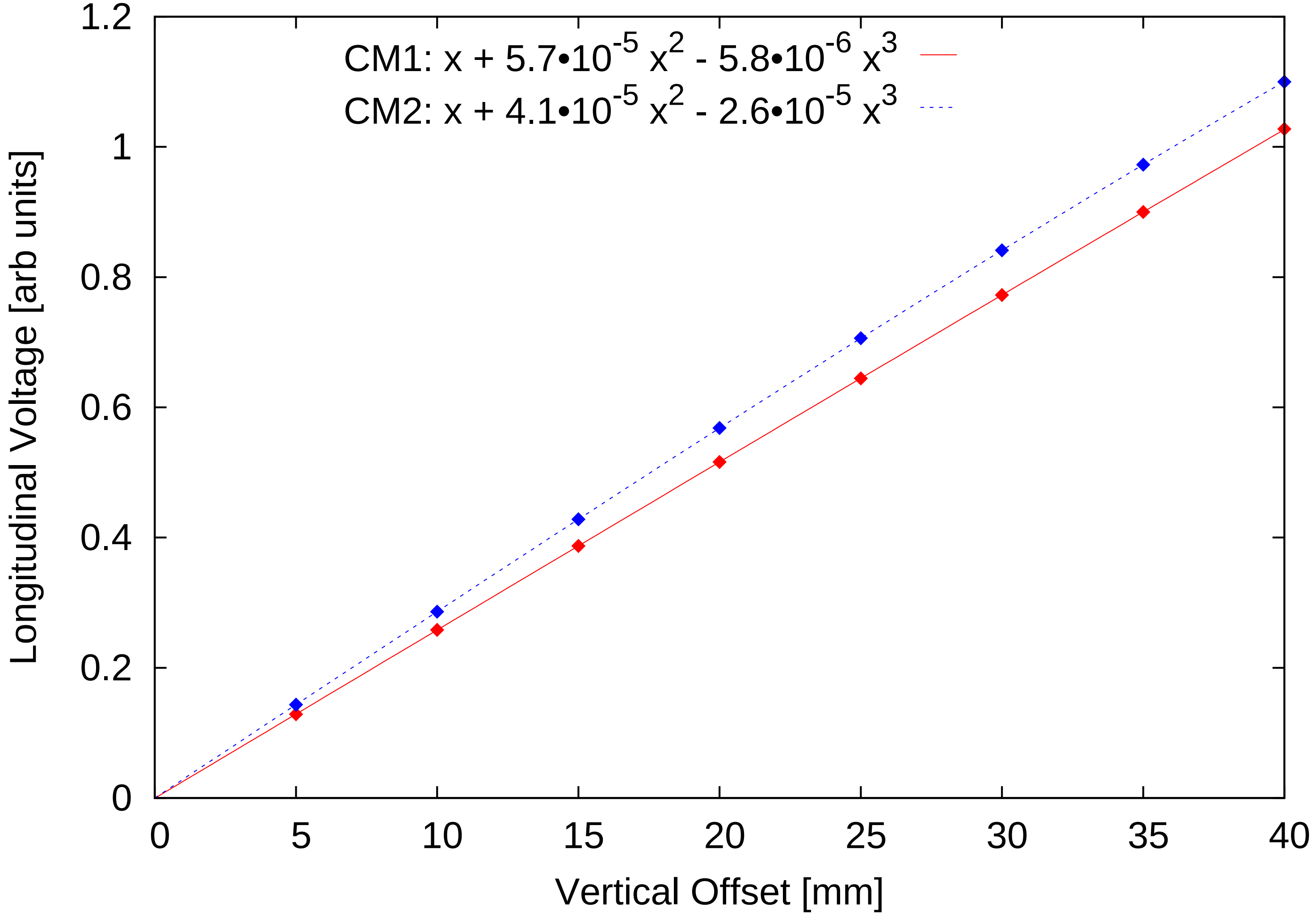
[1]: B. Zotter, K. Bane, PEP-Note 308 (1979)

$V_\perp$ (X_{off}, Num Integration)



Note: $\sim$ 1 million mesh cells used

$V_\perp$ (R_{off} Num Integration)



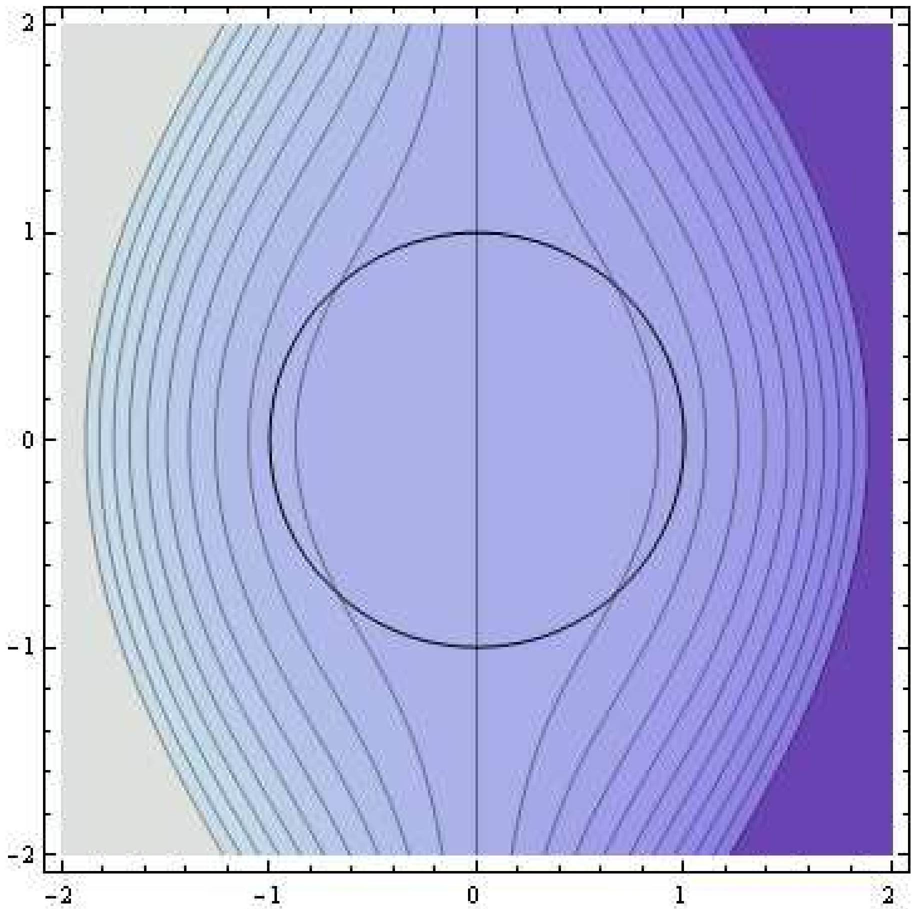
Note: $\sim$ 1 million mesh cells used

Erk's Simple Explanation

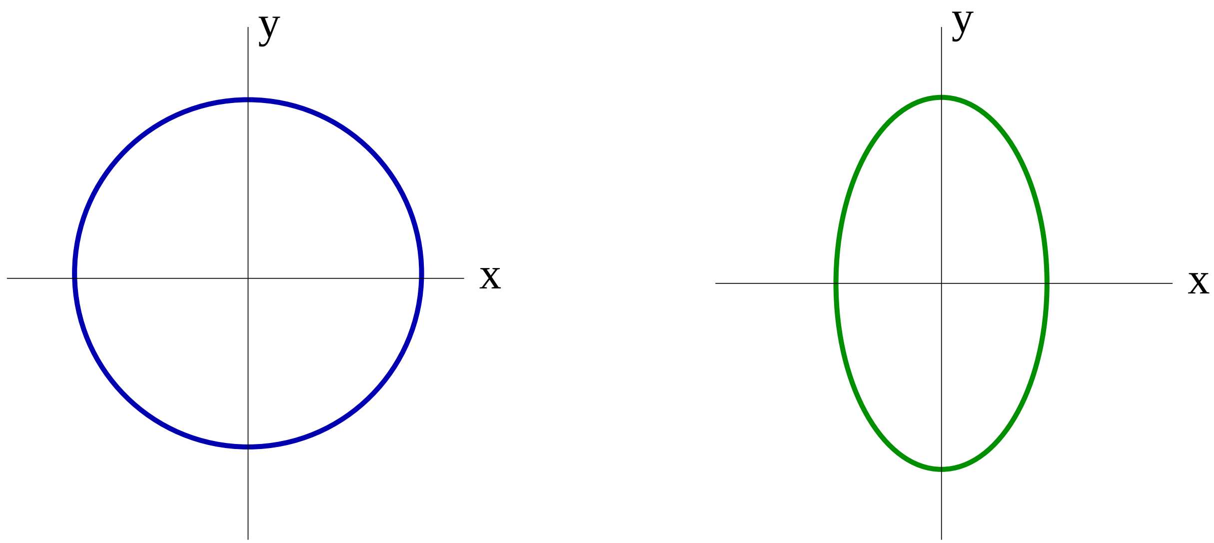
Series expand Bessel Function $J_n(x)$:

$$J_1(x) = x - 1.5 \times 10^{-3}\mathrm{x}^3 + 7.5 \times 10^{-7}\mathrm{x}^5 + O[\mathrm{x}]^7$$

Countour plot of $AJ_n(r)\sin(n\phi) - x$, circle at 25 mm



Elliptical Cross Section



$\psi(x, y, z) \rightarrow \psi(\xi, \eta, z)$

$$x = h \cos(\xi) \cos(\eta) \quad ; \quad y = h \sin(\xi) \sin(\eta)$$

$$\begin{aligned} E_z &= Ce_m(\xi, q_{mn})\ ce_m(\eta, q_{mn})\ \cos(k_z z)\ e^{-\omega t} \\ H_\xi &= \frac{i\omega\epsilon_0}{D_{mn} d} Ce_m(\xi, q_{mn})\ ce'_m(\eta, q_{mn})\ \cos(k_z z)\ e^{-\omega t} \end{aligned}$$

where ce_m & Ce_m are even and modified Matheiu functions of 1^{st} kind.

Ref: J Yang, K Chen, Particle Accelerators, Vol 35, 1991.

Some Comparisons

Reference radius for multipole components, 25 mm

Element	Aper [mm]	I [kA]	Kick [μrad]	$\{b_2,\ b_3\}$ [10^{-4} Units]
MBX ($D_{1,3}$)	74	5.75	500	-0.19 $\pm$ 0.24 -3.1 $\pm$ 0.98
MBRB/C ($D_{2,4}$)	69	6.05	500	-0.18 $\pm$ 0.1 -0.5 $\pm$ 0.4
MCBC/Y	56/70	0.1	80	No Spec
MCBCH/V	56/70	0.1	80	No Spec
CRAB.[L,R].[1,5]	84	-	1.5-6	35 $\pm$ 32 231 $\pm$ 231

$$V_c = \frac{cE_0 \tan(\theta_c/2)}{\omega_0\sqrt{\beta_c\beta^*}} \quad ; \quad \Delta x'_c = \frac{qV_c}{E_0}\sin\left(\frac{\omega_0 z}{c} - \phi\right)$$

- Integrated kick of crabs x5-50 smaller than corrector & dipoles
- b_2, b_3 to be checked for numerical artifacts, optimization possible if needed (No spec for correctors)

Discussion

- Field linear for cavity with beam pipes
  - Superposition of TM & TE due to addition of beam pipes
  - Numerical integration confirms this for arbitrary cavity

- Kick from crabs x100 or even smaller than insertion magnets

- Field non-linearity
  - 3-100 units of b_2 and 2-500 units of b_3 (numerical errors ?)
  - What is the specification ?
  - Additional optimization possible to linearize more if needed

- Remember, no specification for orbit correctors
  - Kick from corrector magnets atleast x40 more than crabs