

Restricted Boltzmann Machines: theory and applications

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Focus of the talk:

- Generative models that are not so popular in particle physics.
- GANs, VAEs, etc. typically require a lot of fine tuning.
- Restricted Boltzmann Machines (RBMs):
 - are easy to train (old model \Rightarrow lots of training methods!),
 - can be used for efficient conditional sampling,
 - are theoretically well studied (known density, MCMC connection).

1. Restricted Boltzmann Machines (RBMs)
 - Model description
 - Training methods
2. RBMs for credit risk management
 - Problem description
 - Model training
 - Stress testing
3. RBMs for pharmaceutical product liability
 - Problem description
 - Learning patient features
 - Learning diagnostic/clinical features
 - Legal claims distribution
 - Evaluation of alternative policies

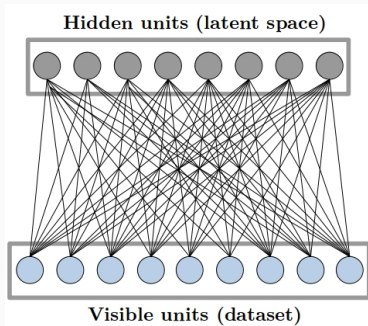
Restricted Boltzmann Machines (RBMs)

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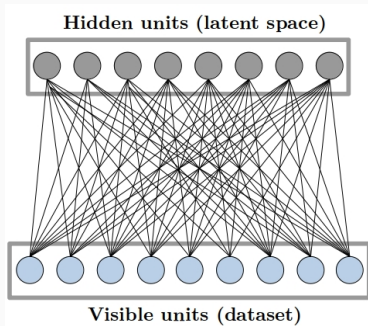
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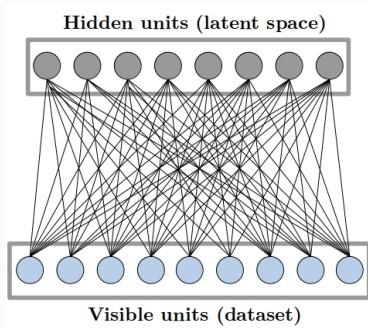
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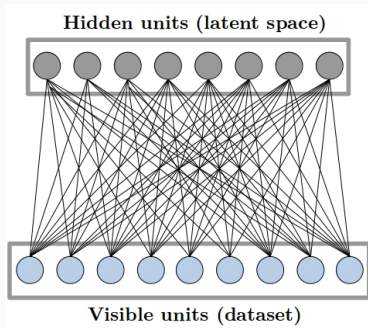
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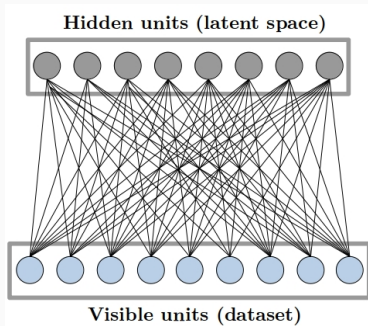
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- Hidden units are latent factors for the distribution of the visible units (non-linear version of Factor Analysis or PCA)

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Let's consider the case of binary units¹, i.e. $v \in \{0, 1\}^n$ and $h \in \{0, 1\}^m$.

¹Generalizations to real-valued units (both visible and hidden) exist.

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An RBM is parametrized using the following Gibbs measure:

$$p(v, h) = \frac{1}{Z} e^{-E(v, h)}$$

where

- Z is a normalization constant (partition function), such that

$$Z = \sum_{v \in \{0, 1\}^n} \sum_{h \in \{0, 1\}^m} e^{-E(v, h)}$$

- $E(v, h)$ is the energy function given by

$$E(v, h) = - \underbrace{\sum_{i=1}^n v_i b_i}_{\text{visible bias}} - \underbrace{\sum_{i=1}^m h_i c_i}_{\text{hidden bias}} - \underbrace{\sum_{i=1}^n \sum_{j=1}^m W_{ij} v_i h_j}_{\text{interaction term}}$$

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- Therefore computing the joint distribution $p(v, h)$ is intractable. Exact sampling from the model is not possible.
- Nevertheless the conditional distributions $p(v|h)$ and $p(h|v)$ are easy:

$$\mathbb{P}(V_i = 1|H = h) = \text{sigmoid} \left(\sum_{j=1}^m W_{ij} h_j + b_i \right)$$

$$\mathbb{P}(H_j = 1|V = v) = \text{sigmoid} \left(\sum_{i=1}^n W_{ij} v_i + c_j \right)$$

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Restricted Boltzmann Machines (RBMs): Training methods

We train an RBM using likelihood maximization via (stochastic) gradient ascent.

Let θ be shorthand for one of the model's parameters (W, a, b), then the log-likelihood on a sample point is:

$$\log \mathcal{L}(\theta) = \log p(v) = \log \frac{1}{Z} \sum_h e^{-E(v,h)} = \log \sum_h e^{-E(v,h)} - \log \sum_{v,h} e^{-E(v,h)}.$$

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Its derivative w.r.t θ is given by:

$$\begin{aligned} \frac{\partial}{\partial \theta} \log \mathcal{L}(\theta) &= - \sum_h p(h|v) \frac{E(v,h)}{\partial \theta} + \sum_{v,h} p(v,h) \frac{\partial E(v,h)}{\partial \theta} \\ &\approx -\mathbb{E}_{\text{data}} \left[\frac{\partial E(v,h)}{\partial \theta} \right] + \mathbb{E}_{\text{model}} \left[\frac{\partial E(v,h)}{\partial \theta} \right] \end{aligned}$$

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Sampling from the model is intractable, therefore we need an approximation of the second term.

Restricted Boltzmann Machines (RBMs): Training methods

We can sample approximately from $p(v, h)$ by performing (block) Gibbs sampling:

1. Pick $v = v_0$ from dataset.
2. Sample alternately $h \sim p(h|V = v)$ and $v \sim p(v|H = h)$.
3. Repeat until Markov Chain thermalizes and you obtain $(v, h) \sim p(v, h)$.

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Main problems:

- Thermalization may take many sampling steps ($\geq 10^4$ for exact iid sampling).
- Equivalently, the chain may be slow-mixing.

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- Other methods: Pseudo-likelihood, ratio-matching, denoising score-matching.

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Advantages of the model:

- Fast and easy training (e.g. compared to adversarial learning).
- Conditional sampling is a built-in feature!

To sample (v, h) given $v_i = x$:

1. Pick $v = v_0$ from dataset (or random).
2. Sample $h \sim p(h|V = v)$, sample $v \sim p(v|H = h)$ and fix $v_i = x$.
3. Repeat until Markov Chain thermalizes and you obtain $(v, h) \sim p(v, h|v_i = x)$.

RBM for credit risk management

Joint work with Giuseppe Genovese⁶ and Ashkan Nikeghbali^{7 8}.

⁶University of Basel, Department of Mathematics and Computer Science

⁷UZH, Mathematics Institute

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Background information:

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Goal:

- Model the joint distribution of default probabilities and macroeconomic factors using RBMs.

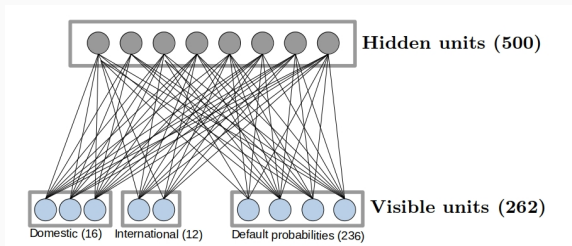
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Goal:

- Model the joint distribution of default probabilities and macroeconomic factors using RBMs.
- Perform portfolio stress-testing (e.g. how do portfolio losses change if unemployment increases?).

RBM for credit risk: Model training

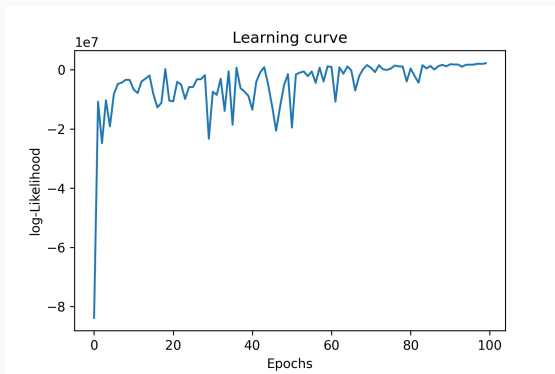


- Data.
 - Daily estimated 1-year default probabilities⁹ from January 2000 to March 2020 of 236 top listed US firms.
 - Quarterly macroeconomic variables¹⁰ (domestic and international).
- Training: hidden units: 500 (5-fold cross-validated), epochs: 10000, method: Stochastic Maximum Likelihood (100 Gibbs steps).

⁹Estimation via vanilla Merton model, similarly to Bloomberg's DRISKTM and Moody's EDFTM.

¹⁰For a complete list see the Federal Reserve 2020Q4 stress testing documentation.

RBM for credit risk: Model training



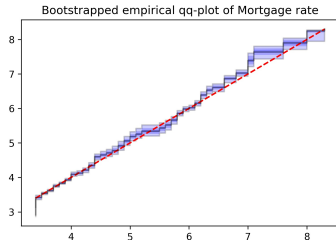
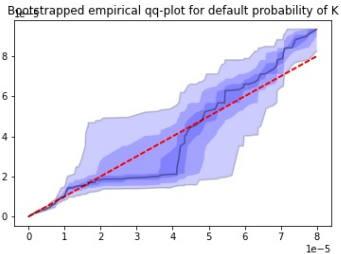
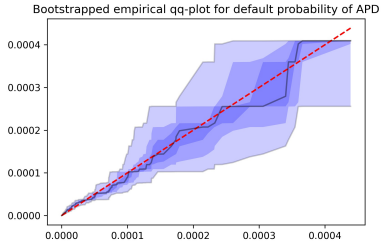
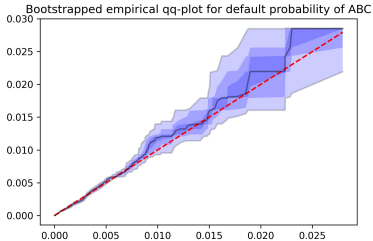
The log-likelihood is intractable (especially at training time!).

Fast ways to monitor learning:

- Log-likelihood estimation via KDE from a model's sample.
- Annealed Importance Sampling for approximation of partition function¹¹.

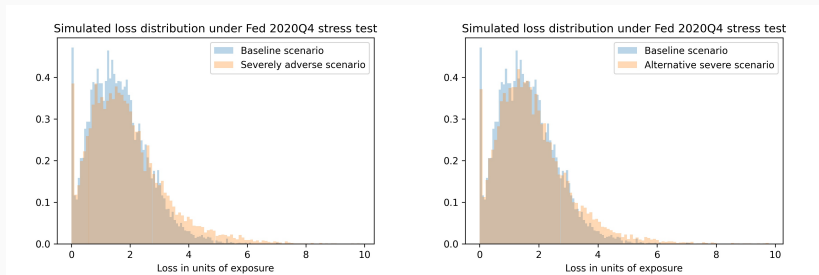
¹¹See Salakhutdinov, Murray, *On the quantitative analysis of Deep Belief Networks*, 2008

RBM for credit risk: Model training



The model has successfully learned the **joint** probability distribution.

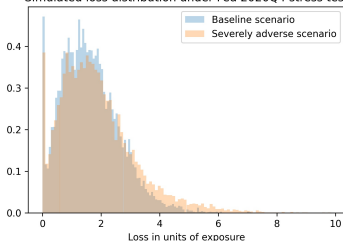
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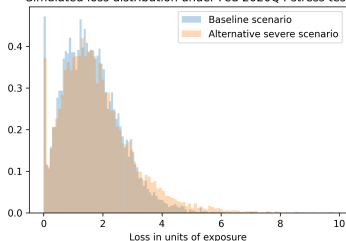
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RBM for credit risk: Stress testing

Simulated loss distribution under Fed 2020Q4 stress test



Simulated loss distribution under Fed 2020Q4 stress test



- We can implement Federal Reserve 2020Q4 stress-test by sampling conditionally on their projected macroeconomic variables and see how they affect the total losses distribution.
- We can compute risk measures (capital requirements) under different scenarios:
 - Value at Risk (95%)
baseline (3.38), alternative severe (4.06, \uparrow 20.1%), severely adverse (4.33, \uparrow 28.1%)
 - Expected Shortfall (95%)
baseline (4.02), alternative severe (5.15, \uparrow 28.1%), severely adverse (5.37, \uparrow 33.6%)

RBM for pharmaceutical product liability

Joint work with Nicola Serra¹², Giuseppe Genovese¹³, and Ashkan Nikeghbali¹⁴ ¹⁵.

¹²UZH, Physics Institute

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Background information:

- Trastuzumab is a very effective medicine used to treat a specific kind of very aggressive breast cancer (HER2-positive¹⁶).

¹⁶HER2-positive cancer is receptive to the human epidermal growth factor. Trastuzumab downregulates it, thus reducing cancer growth.

Background information:

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Goal:

- Learn joint distribution of patient features (e.g. age, tumor status, survival) and clinical/diagnostic features (HER2+, tests, cardiotoxicity).

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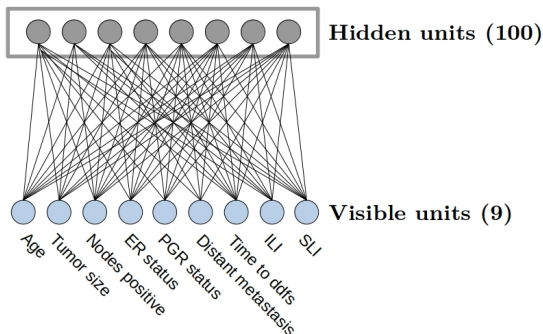
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- Model financial impact of product liability claims (legal claims due to lack of therapeutic success, serious side effects, diagnostic failure).
- Test alternative treatments and diagnostic procedures.

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RBM for pharma: Learning patient features

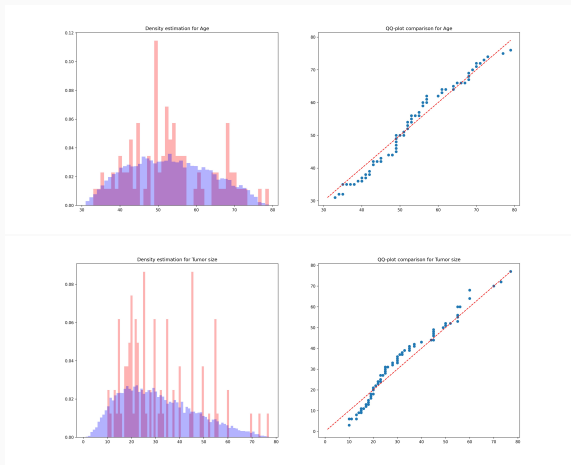


- Data: GEO2R dataset¹⁷ with patient features from 94 HER2+ breast cancer patients¹⁸.
- Training: hidden units: 100 (5-fold cross-validated), epochs: 10000, method: Stochastic Maximum Likelihood (100 Gibbs steps).

¹⁷The National Center for Biotechnology Information (NCBI) provides public access to Gene Expression Omnibus (GEO) dataset. This dataset contains gene profiling of HER2+ breast cancer patients treated with Trastuzumab.

¹⁸Larger datasets require long authorization procedures, in the following we will use the RBM to generate a bigger synthetic dataset on which to test our methodology.

RBM for pharma: Learning patient features



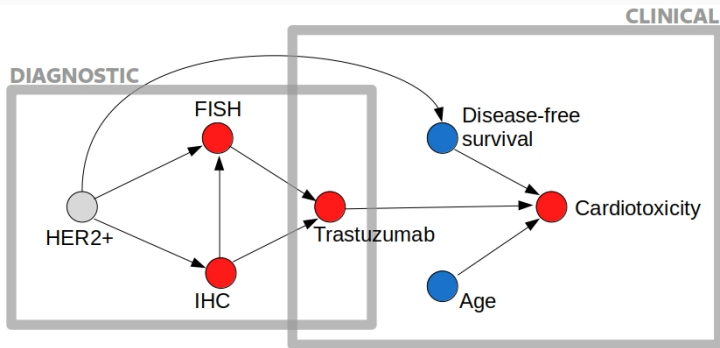
- Due to small sample size, the RBM smoothens the empirical distribution to avoid overfitting and generalize well.

RBM for pharma: Learning diagnostic/clinical features

- We are also interested in diagnostic and clinical features not present in the GEO2R dataset.

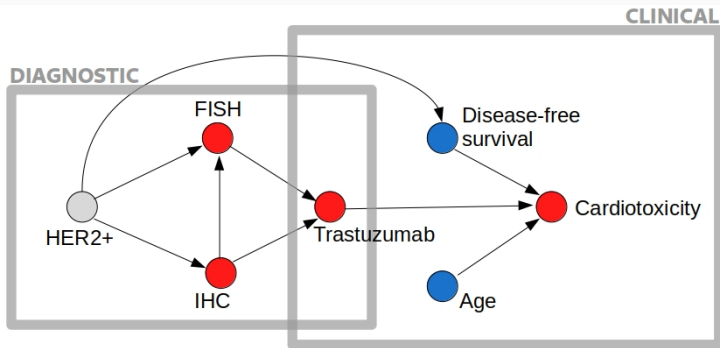
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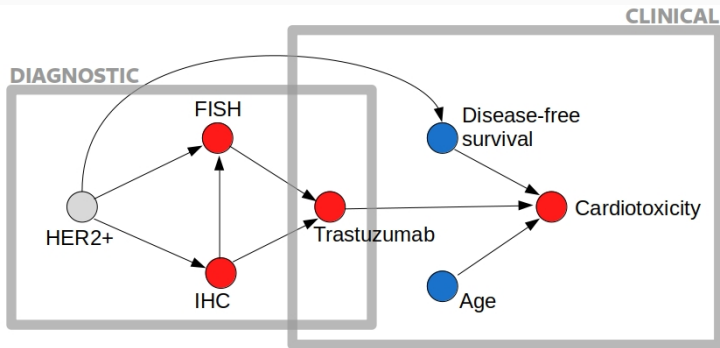
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RBM for pharma: Learning diagnostic/clinical features

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- Given the value of **observed** nodes, we can infer the value of **unobserved** and unobservable nodes by sampling from the Bayesian network.
- We obtain a synthetically generated sample (n=10000) from our RBM and extend it to include diagnostic/clinical variables using the Bayesian Network.

We can use our model to answer basic queries:

- Frequency of Type I error (false positive) of current diagnostic strategy: 6.31%.
- Primary cardiotoxicity¹⁹ is approx. 4.5 times more likely in 80-year-olds than 40-year-olds.
- IHC is 53% more likely than FISH to result in false positives.

What's the financial impact due to legal claims?

¹⁹Congestive Heart Failure or any cardiac event which may lead to hospitalization.

The connection between diagnostic/clinical variables and the size of legal claims might be given, for example, by the following educated guess:

Metastasis	Cardiotoxicity	Multiplier	Claim probability
Yes	Primary	1.50	50%
Yes	Secondary	1.25	35%
Yes	None	1.00	25%
No	Primary	1.00	15%
No	Secondary	0.20	5%
No	None	0.00	0%

- The expected claim size is:

$$[\text{Median claim size} = 250'000 \text{ USD}] \times [\text{Multiplier}] \times [\text{Claim probability}]$$

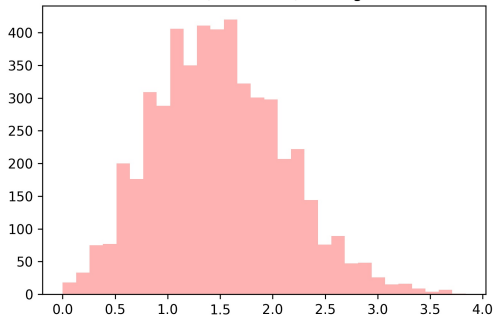
RBM for pharma: Legal claims distribution

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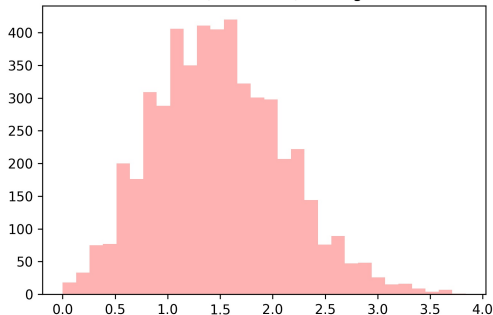
Total claims distribution (in USD mil) for original GEO2R dataset



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Total claims distribution (in USD mil) for original GEO2R dataset



- We can compute risk management metrics (VaR, ES, spectral measures).
- $\text{VaR}(95\%) = 2.56$ USD mil, $\text{VaR}(99\%) = 3.10$ USD mil.
- $\text{ES}(95\%) = 2.88$ USD mil, $\text{ES}(99\%) = 3.60$ USD mil.

RBM for pharma: Evaluation of alternative policies

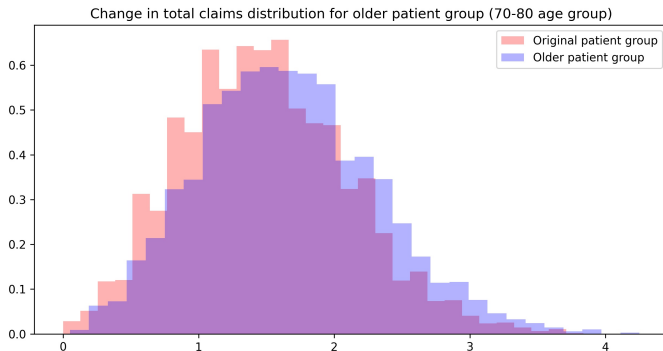
- We can estimate conditional distributions and evaluate **how clinical changes affect the distribution of losses**.

RBM for pharma: Evaluation of alternative policies

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- What is the **effect of patient age on the total claims** distribution? We sample from the model, given that all patients are in the 70-80 age group.

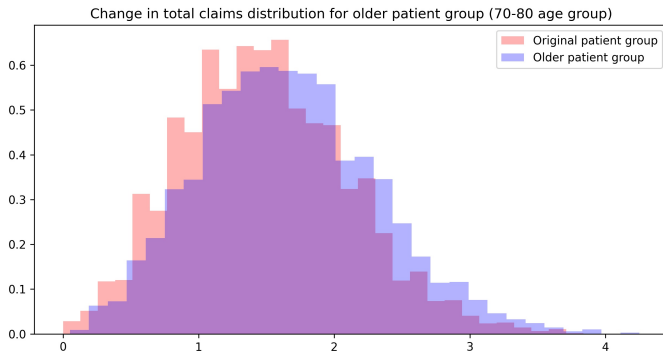
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RBM for pharma: Evaluation of alternative policies

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- $\text{VaR}(95\%) = 2.80 \text{ USD mil}$ ($\uparrow 9.27\%$)
- $\text{ES}(95\%) = 3.13 \text{ USD mil}$ ($\uparrow 8.47\%$)

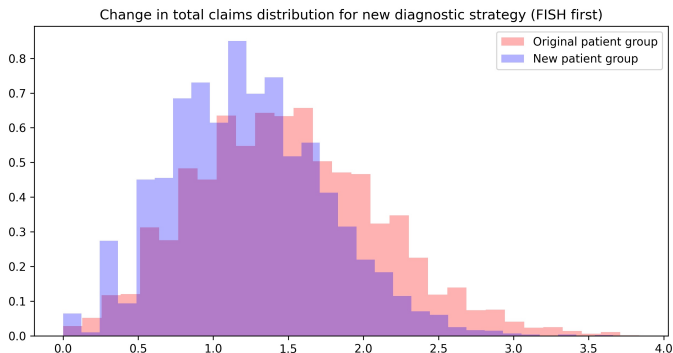
- We can evaluate the **effect of alternative diagnostic strategies on the total claims distribution.**

RBM for pharma: Evaluation of alternative policies

- We can evaluate the **effect of alternative diagnostic strategies on the total claims** distribution.
- We sample from the model, assuming an alternative diagnostic procedure (for example: always use FISH test first).

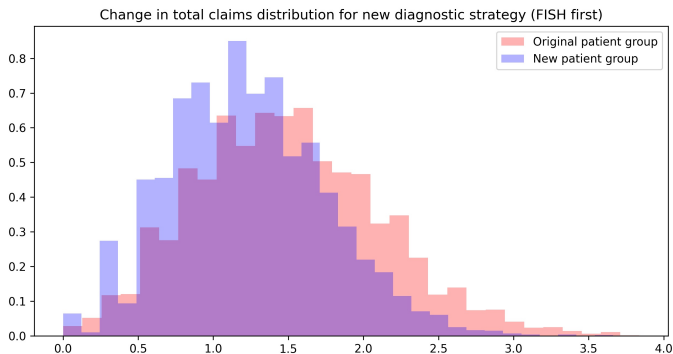
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- $\text{VaR}(95\%) = 2.13 \text{ USD mil}$ ($\downarrow 17.1\%$)
- $\text{ES}(95\%) = 2.39 \text{ USD mil}$ ($\downarrow 17.2\%$)

This methodology allows us to:

- Augment and extend existing datasets.
- Combine peer-reviewed research and ML.
- Generate synthetic datasets exhibiting complex non-linear dependencies.
- Estimate quantitatively how different diagnostic/clinical strategies can impact financial losses due to liability claims.

- Results of the applications are preliminary: papers are still in progress.
- RBMs are easy to train and easy to deploy.
- Conditional sampling is efficient and very useful for scenario generation.

Thanks for your attention!