“Unitarity Triangle Measurements”

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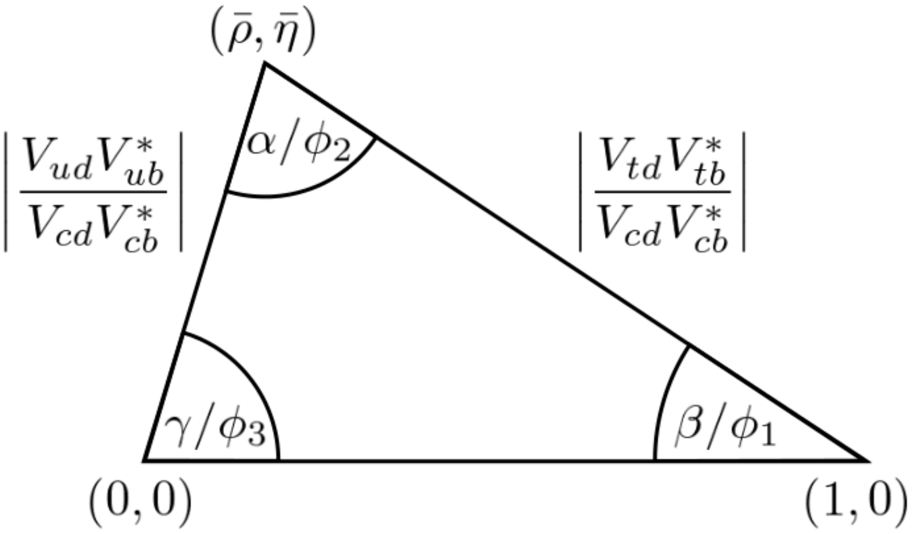
CKM Quark Mixing Matrix and the Unitarity Triangle

- Wolfenstein parametrization (expansion in powers of λ≈0.22) of the unitary CKM matrix:

$$\mathrm{V_{CKM}} = \begin{pmatrix} \mathrm{V_{ud}} & \mathrm{V_{us}} & \mathrm{V_{ub}} \\ \mathrm{V_{cd}} & \mathrm{V_{cs}} & \mathrm{V_{cb}} \\ \mathrm{V_{td}} & \mathrm{V_{ts}} & \mathrm{V_{tb}} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

3 real parameters (A, λ, ρ) + 1 complex phase (η) → irreducible phase causes CP violation

- Unitarity of matrix imposes 6 relations, e.g. $V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{ud}V_{ub}^* = 0$, which can be represented as triangles in the complex plane.



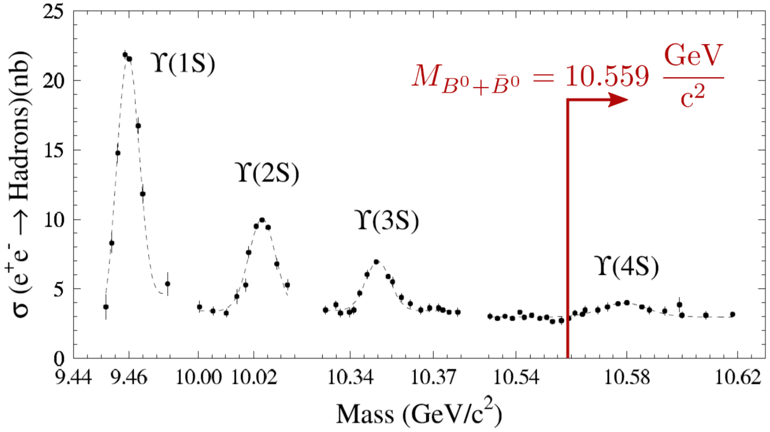
$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

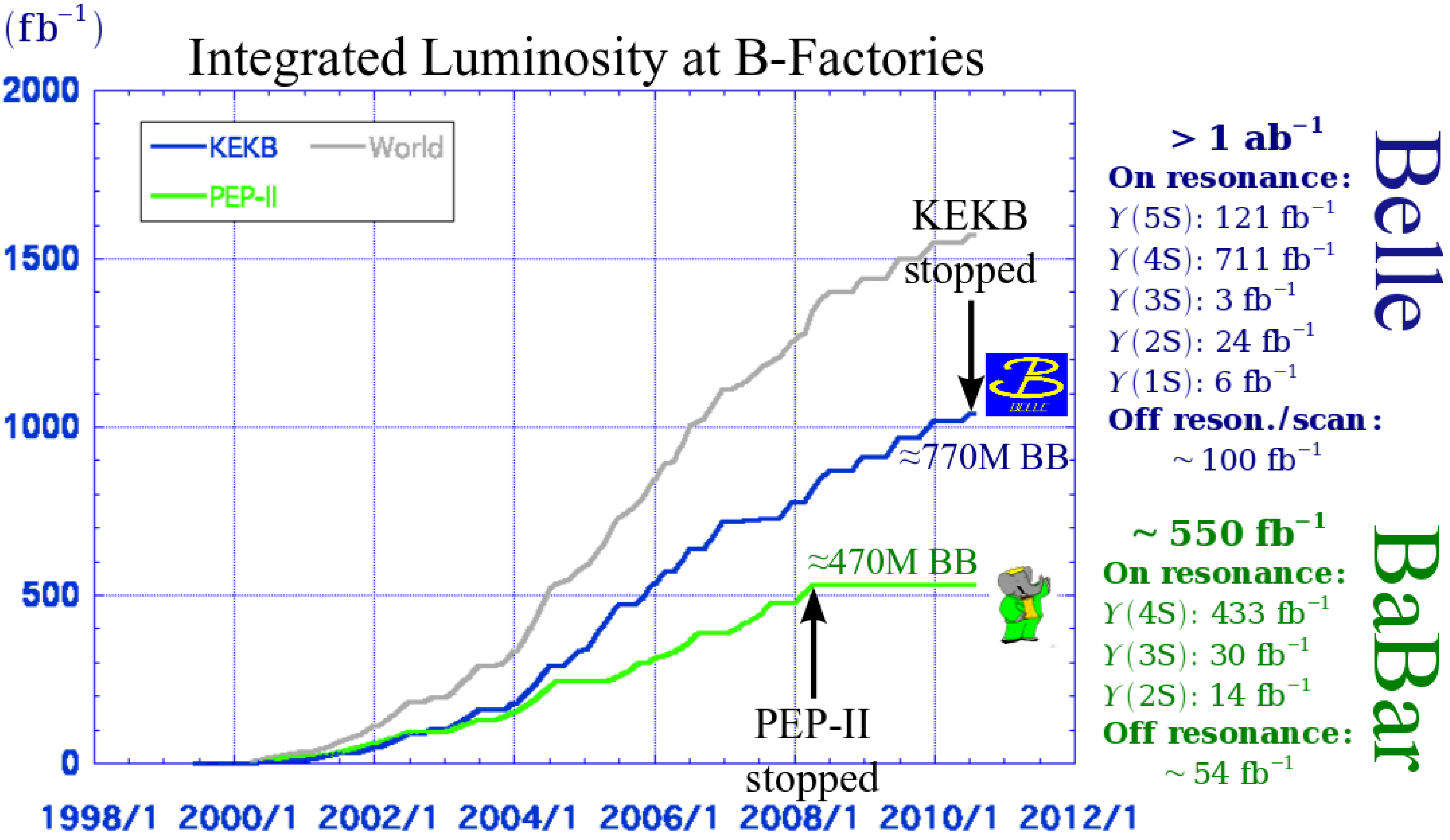
$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Experimental determination of the angles is closely related to measurements of CP asymmetries. This has been a key objective of the B-factories BaBar and Belle.

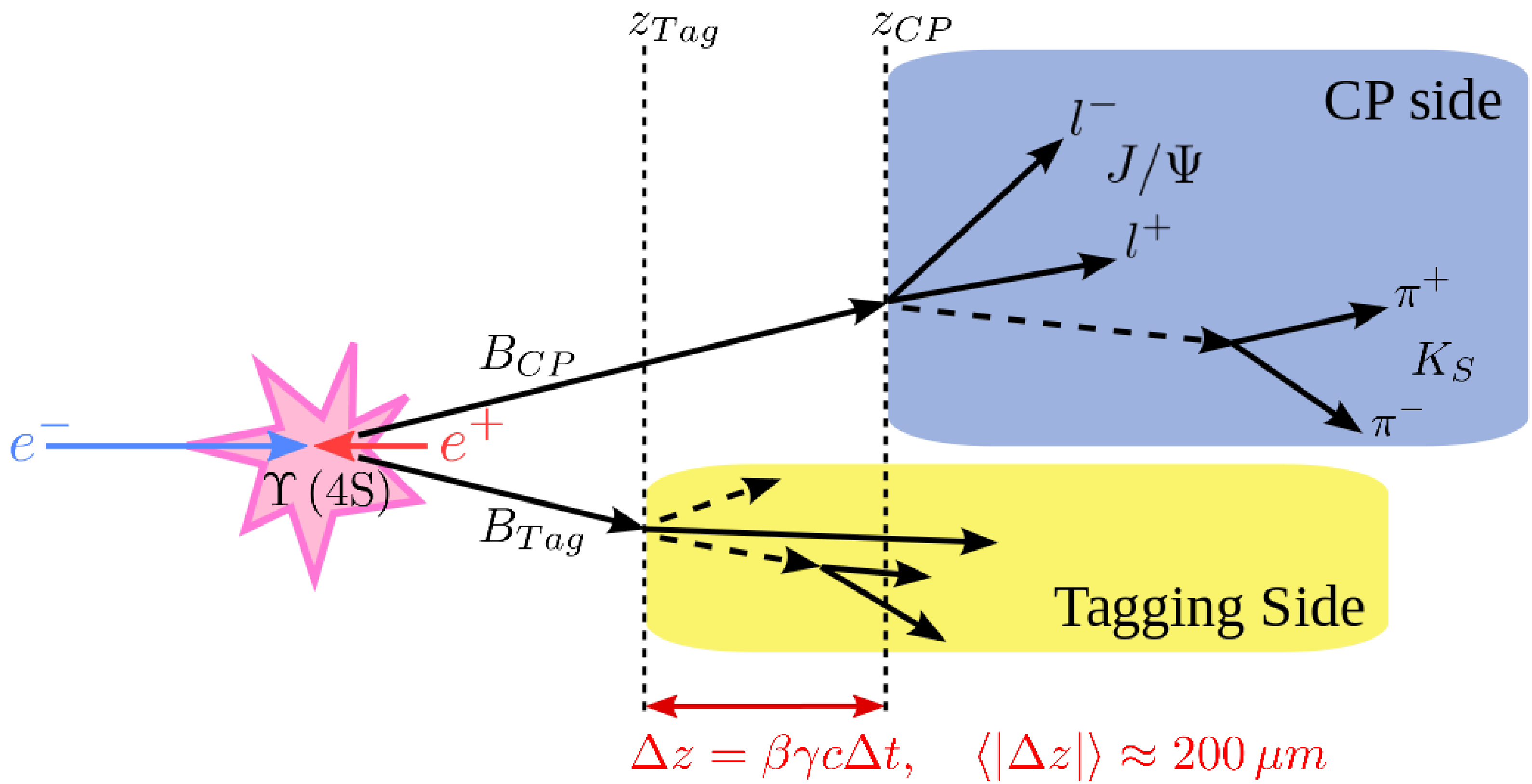
Data collected at the B-factories



- Belle set a world record in instantaneous luminosity of 2.11×10^{34} cm^{-2}s^{-1}
- BaBar and Belle have collected >1.5 ab^{-1} in 10 years
- This corresponds to ≈1200×10^6 $B\bar{B}$ pairs created and recorded on the Υ(4S)



Principle of Time-dependent CP Violation Measurements



$\Delta z = \beta\gamma c\Delta t, \quad \langle|\Delta z|\rangle \approx 200\,\mu m$

- Time-dependent CP asymmetry:

$$A_{CP}(\Delta t) = \frac{\Gamma\left(\bar{B}^0(\Delta t) \to f_{CP}\right) - \Gamma\left(B^0(\Delta t) \to f_{CP}\right)}{\Gamma\left(\bar{B}^0(\Delta t) \to f_{CP}\right) + \Gamma\left(B^0(\Delta t) \to f_{CP}\right)} = \mathcal{S}_f \sin(\Delta m \Delta t) + \mathcal{A}_f \cos(\Delta m \Delta t)$$

$$\mathcal{S}_f = \frac{2\,\mathrm{Im}(\lambda_f)}{|\lambda_f^2| + 1} \qquad \mathcal{A}_f = \frac{|\lambda_f^2| - 1}{|\lambda_f^2| + 1} \qquad \lambda_f = \frac{q}{p}\frac{\bar{A}(f_{CP})}{A(f_{CP})}$$

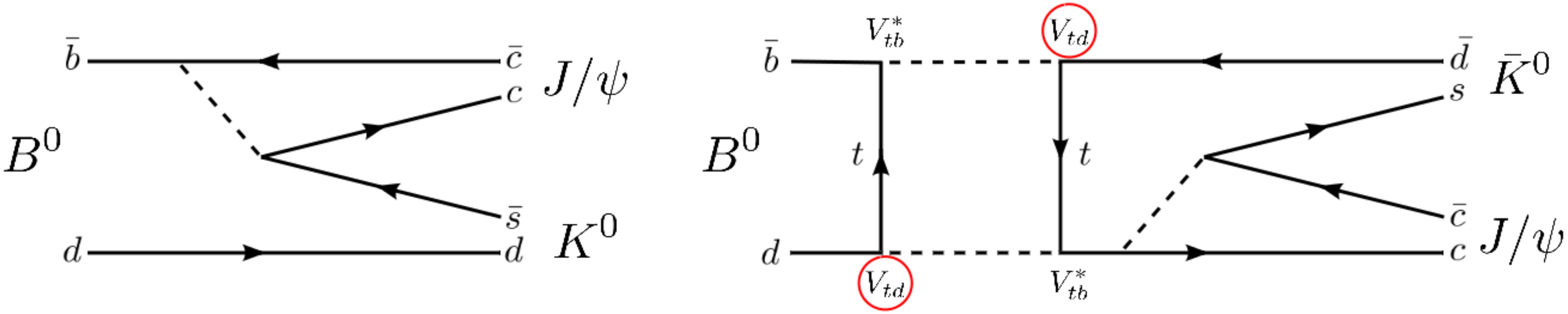
mixing-induced CPV

direct CPV

Measurement of Φ_1/β

- Φ_1 can in principle be obtained from decays involving $b \to c\bar{c}s$, $b \to c\bar{c}d$, $b \to c\bar{u}d$ or $b \to sq\bar{q}$.

- $B^0 \to J/\psi\, K^0$ is the "golden mode" for determination of Φ_1:



- Φ_1 determination from $B^0 \to J/\psi\, K^0$ decays is clean:
  - Decay is dominated by only one tree amplitude with real CKM elements, possible gluonic penguins have same weak phase and are small.
  - Mixing vertices V_{td} introduce phase, which leads to $\lambda_{J/\psi K^0} = \xi_f e^{-i2\phi_1}$

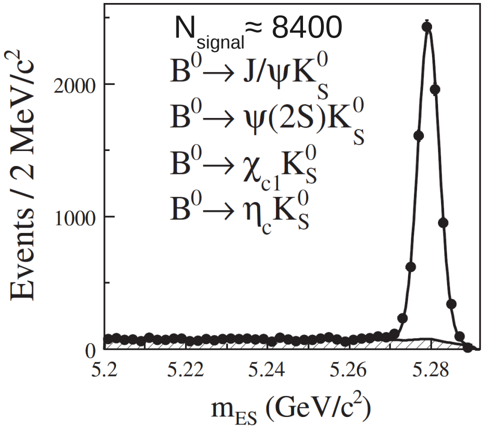
$$\longrightarrow \quad S_{J/\Psi K^0} = -\xi_f \sin(2\phi_1) \text{ and } A_{J/\psi K^0} = 0$$

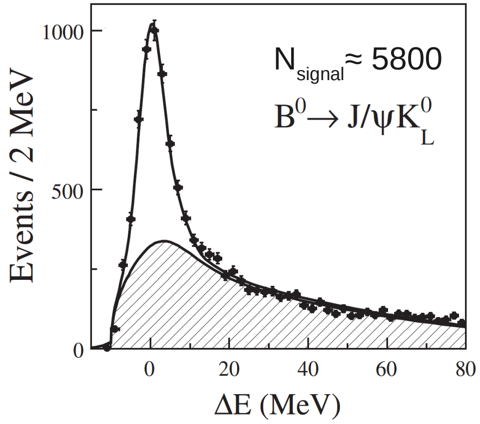
Φ_1 can be precisely determined from the time-dependent CP asymmetry

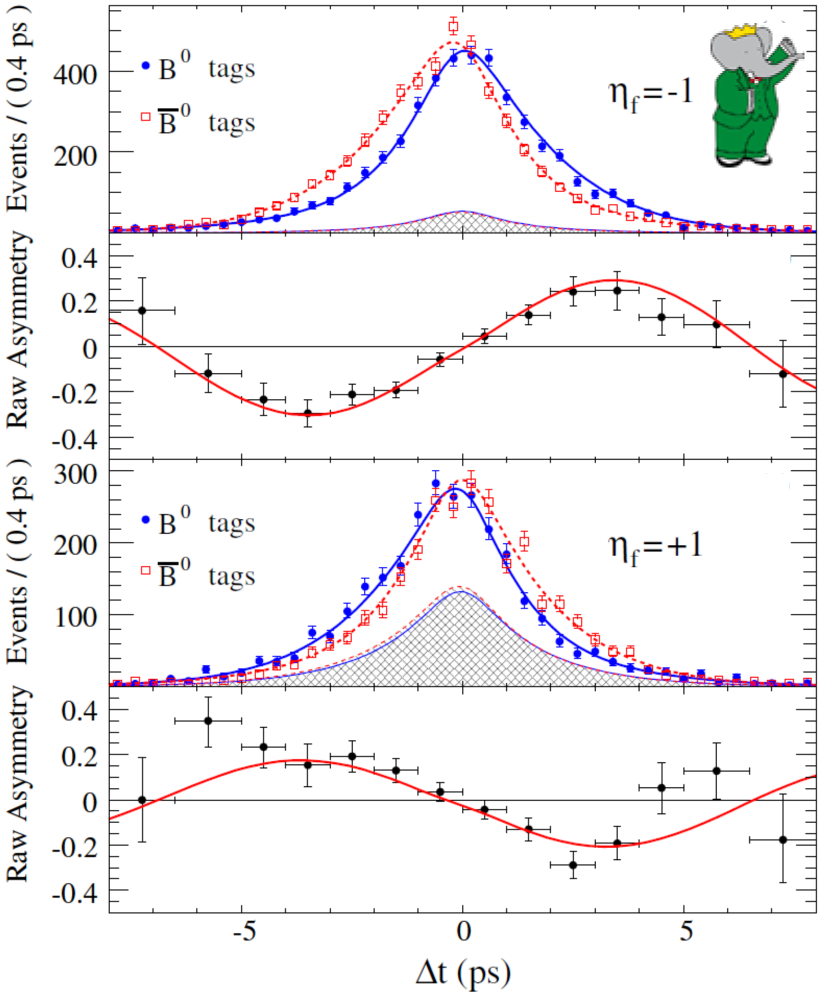
$$A_{CP_{J/\Psi K^0}}(\Delta t) = -\xi_f \sin(2\phi_1) \sin(\Delta m \Delta t)$$

Measurement of Φ_1/β

BaBar's last update on full dataset:







BaBar with 465×10⁶ BB:

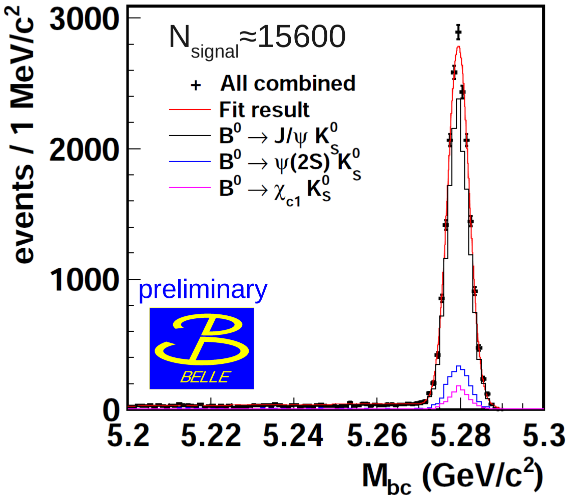
PRD **79**, 072009 (2009)

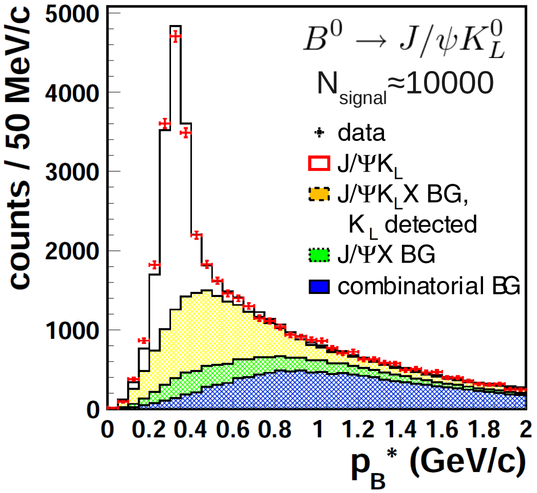
$$\mathcal{A} = -0.024 \pm 0.020\,(\text{stat}) \pm 0.016\,(\text{syst})$$

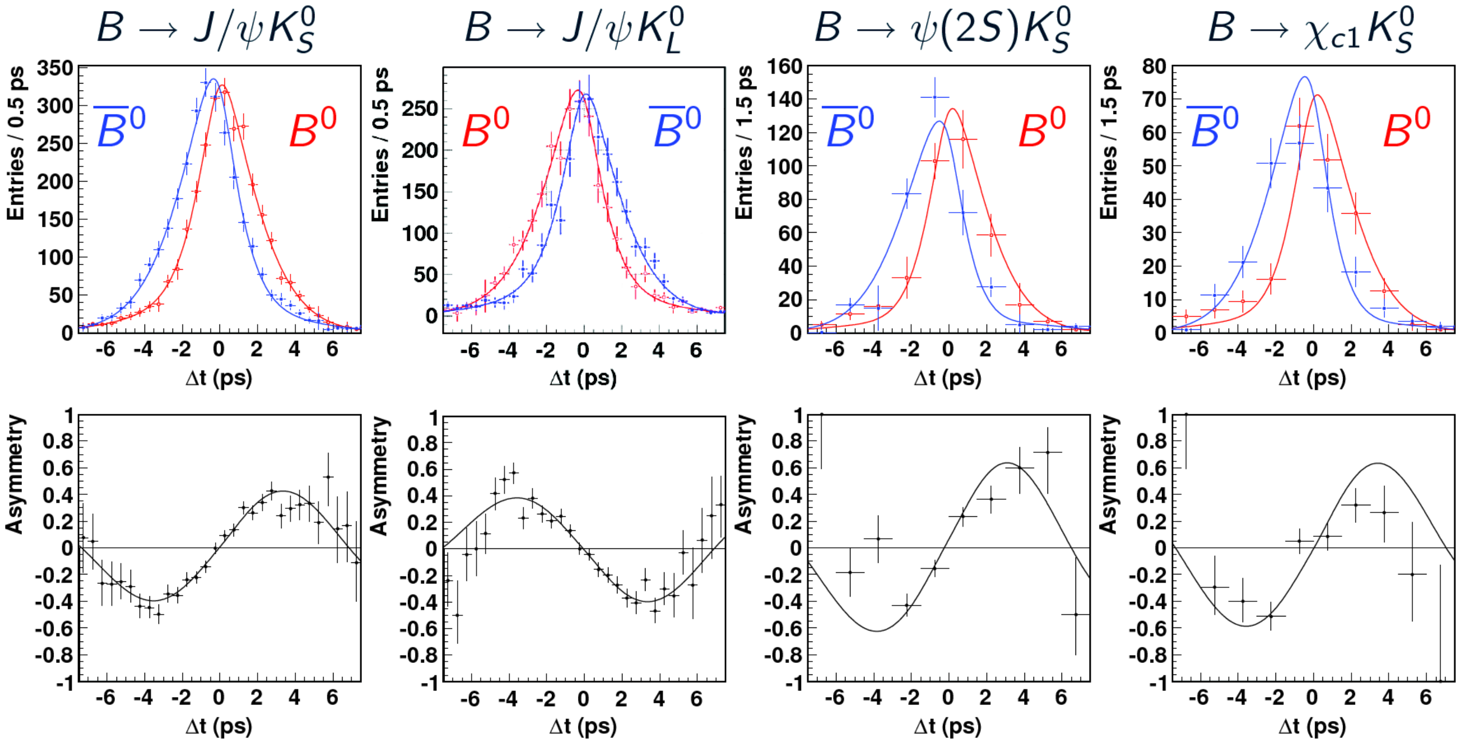
$$\sin(2\phi_1) = 0.687 \pm 0.028 \pm 0.012$$

Measurement of Φ_1/β

Belle's update on full dataset (preliminary):







Belle with 772×10^{6} BB:

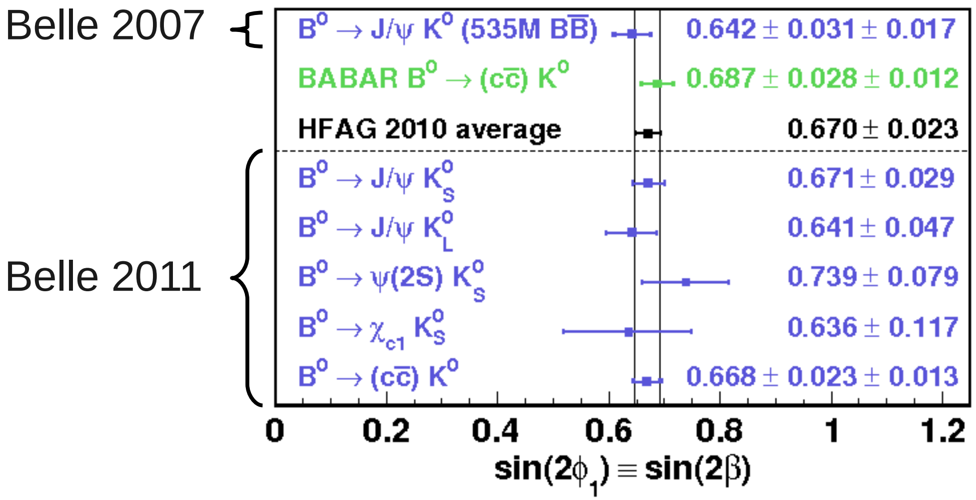
$$\mathcal{A} = 0.007 \pm 0.016\,(\mathrm{stat}) \pm 0.013\,(\mathrm{syst})$$

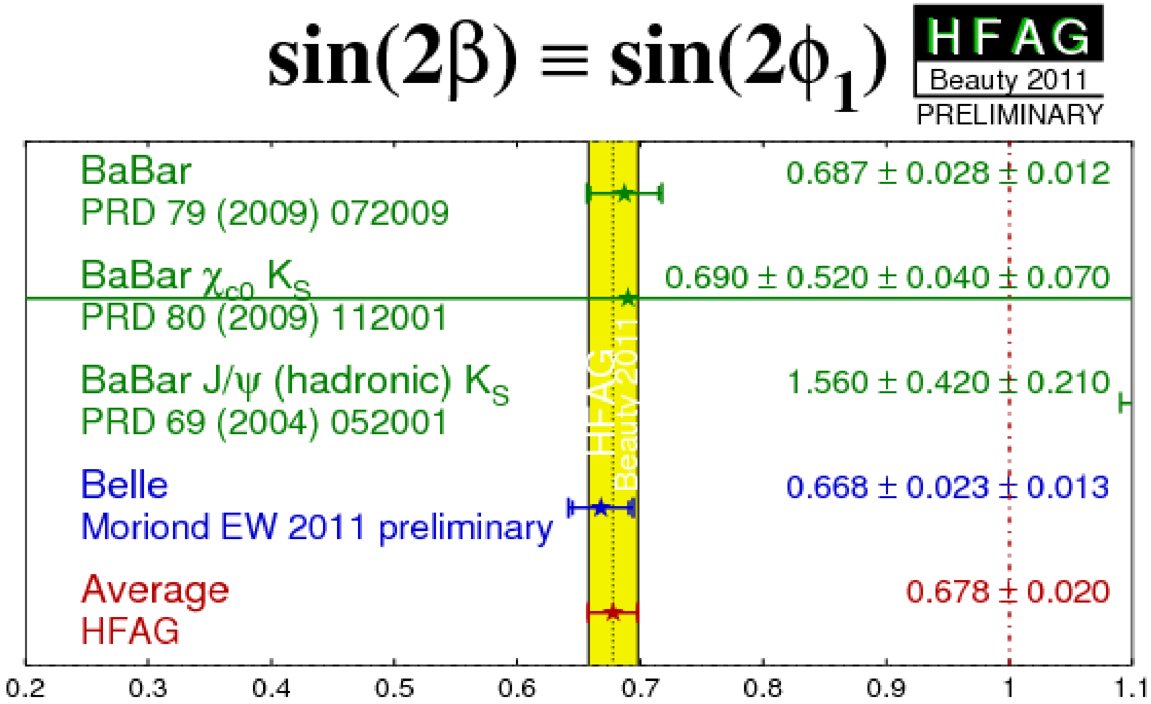
$$\sin(2\phi_1) = 0.668 \pm 0.023 \qquad \pm 0.013$$

preliminary

Current world's most precise measurement of Φ_1

Measurement of Φ_1/β



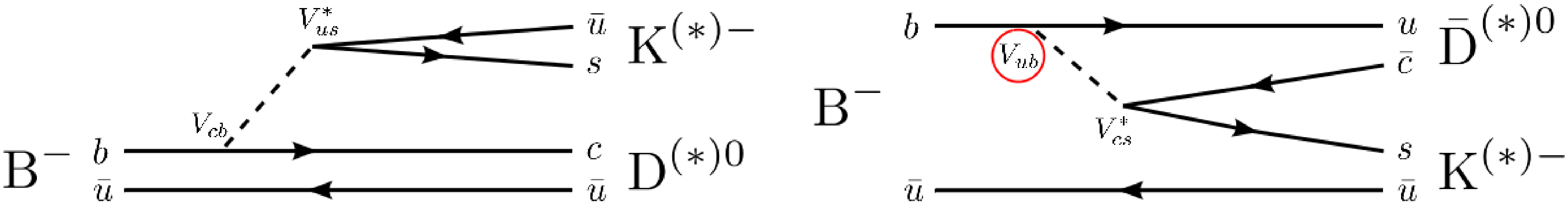


Indirect estimation: $\sin\left(2\phi_1\right) = 0.830^{+0.013}_{-0.034}$ (global fit, CKMfitter ICHEP10)

Tension of 2.9σ in global CKM fit is expected to reduce slightly due to new Belle result.

Measurement of Φ_3/γ

- Φ_3 is accessible by utilizing the interference of b → c and b → u transitions in $B^{\pm} \rightarrow D^{(*)}K^{(*)\pm}$



$$\boxed{A_1 \sim V_{cb}V_{us}^* \sim A\lambda^3}$$

$$\boxed{A_2 \sim V_{ub}V_{cs}^* \sim A\lambda^3 (\rho - i\eta) \sim e^{-i\phi_3}}$$

- Interference, if D^0 and $\bar{D}^0$ decay into common final state: $|\tilde{D}\rangle = |D^0\rangle + r_B e^{i(\pm\phi_3+\delta_B)}|\bar{D}^0\rangle$

- Weak phase difference Φ_3, strong phase difference δ_B and amplitude ratio

$$r_B = \left|\frac{A\left(B^- \rightarrow \bar{D}^0 K^-\right)}{A\left(B^- \rightarrow D^0 K^-\right)}\right| = \left|\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*}\right| \times [\text{color supp}] \approx 0.1$$

Advantage:
- only tree, no penguin contributions – can provide SM anchor point

Drawback:
- small branching fractions due to Cabibbo- and color-suppression
- small r_B

Measurement of Φ_3/γ

Three methods and their observables that can be used to determine Φ_3 in $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$:

GLW: Gronau, Wyler, London PLB **253**, 483 (1991) PLB **265**, 172 (1991)

Cabibbo-suppressed D decays to CP eigenstates, e.g. $D \rightarrow K^+K^-$ (CP-even) and $D \rightarrow K_S\pi^0$ (CP-odd)

$$A_{CP\pm} = \frac{2r_B \sin\delta_B \sin\phi_3}{1 + r_B^2 + 2r_B + \cos\delta_B \cos\phi_3}$$

direct CP asymmetry

$$R_{CP\pm} = 1 + r_B^2 + 2r_B + \cos\delta_B \cos\phi_3$$

ratio charged averaged decay rates

ADS: Atwood, Dunietz, Soni PRL **78**, 3257 (1997) PRD **63**, 036005 (2001)

Doubly Cabibbo-suppressed (e.g. $D^0 \rightarrow K^+\pi^-$) and Cabibbo-favored ($\overline{D}^0 \rightarrow K^+\pi^-$) D decays

$$\mathcal{A}_{DK} = 2r_B r_D \sin(\delta_B + \delta_D) \frac{\sin(\phi_3)}{\mathcal{R}_{DK}}$$

direct CP asymmetry

$$\mathcal{R}_{DK} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\phi_3)$$

ratio suppressed to favored decay

GGSZ: Giri, Grossman, Soffer, Zupan PRD **68**, 054018 (2003)

Dalitz analysis of multi-body D decays into self-conjugated states (e.g. $D \rightarrow K_S\pi^+\pi^-$)

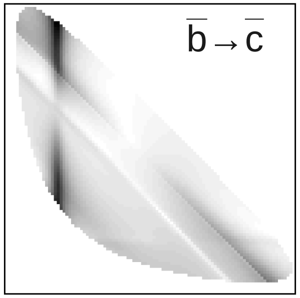
Interference $M_\pm(m_\pm^2, m_\mp^2) = f_D(m_\pm^2, m_\mp^2) + r_B e^{i(\pm\phi_3 + \delta_B)} f_D(m_\mp^2, m_\pm^2)$ varying over Dalitz plot

Measurement of Φ_3/γ

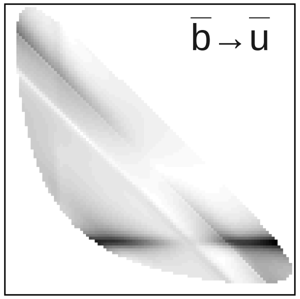
GGSZ method [PRD **68**, 054018 (2003)]:

- Reconstruct D in self-conjugated state like $D \to K_S\pi^+\pi^-$

$$\left|M_{B^+\to DK^+}\left(m^2_{K_S\pi^+}, m^2_{K_S\pi^-}\right)\right|^2 = \Bigg|$$

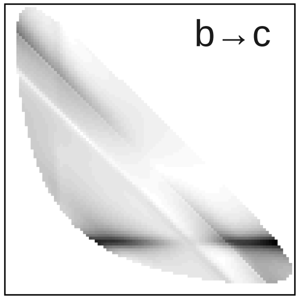


$$+\, r_B e^{i(+\phi_3+\delta_B)}$$

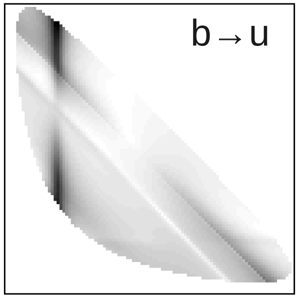


$$\Bigg|^2$$

$$\left|M_{B^-\to DK^-}\left(m^2_{K_S\pi^+}, m^2_{K_S\pi^-}\right)\right|^2 = \Bigg|$$



$$+\, r_B e^{i(-\phi_3+\delta_B)}$$



$$\Bigg|^2$$

- Advantage: large interferences can occur in some regions of the Dalitz plot

- In previous analyses amplitudes f_D have been extracted by fitting tagged D^0 decays using model assumptions involving several two-body intermediate states (Isobar model, K-matrix model). The systematic uncertainty to Φ_3 associated to modeling may be up to ≈10°.

Measurement of Φ_3/γ

GGSZ method - model-dependent analysis at BaBar:

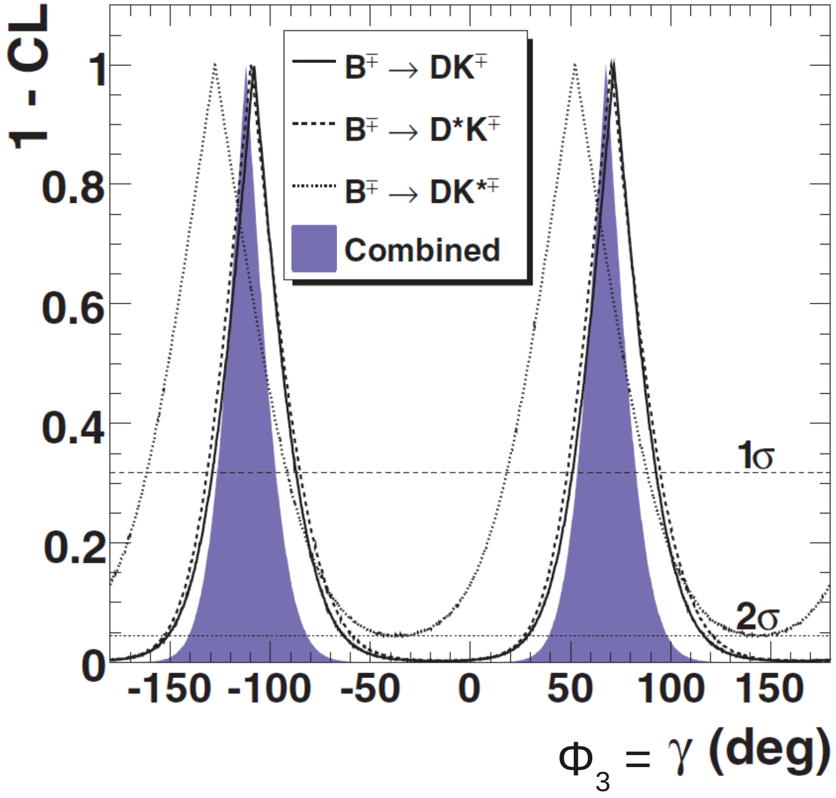
- BaBar uses $D \to K_S \pi^+ \pi^-$ and $D \to K_S K^+ K^-$ in:

$$B^\pm \to DK^\pm$$

$$B^\pm \to D^* K^\pm, \qquad D^* \to D\pi^0,\ D\gamma$$

$$B^\pm \to DK^{*\pm}, \quad K^{*\pm} \to K_S \pi^\pm$$

- Modelling of f_D applies K-matrix formalism (amongst others).



BaBar with 468×10^6 BB:

PRL **105**, 121801 (2010)

$$\phi_3 = (68 \ {}^{+15}_{-14}(\text{stat}) \ \pm 4(\text{syst}) \ \pm 3(\text{model}))^\circ$$

$$r_B = (9.6 \ \pm 2.9 \ \pm 0.5 \ \pm 0.4)\%$$

$$r_B^* = (13.3 \ {}^{+4.2}_{-3.9} \ \pm 1.3 \ \pm 0.3)\%$$

$$\kappa r_s = (14.9 \ {}^{+6.6}_{-6.2} \ \pm 2.6 \ \pm 0.6)\%$$

$$\delta_B = (119 \ {}^{+19}_{-20} \ \pm 3 \ \pm 3)^\circ$$

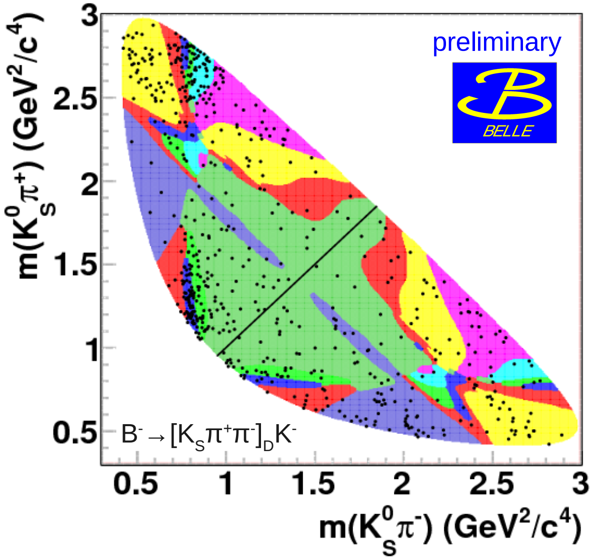
$$\delta_B^* = (-82 \ \pm 21 \ \pm 5 \ \pm 3)^\circ$$

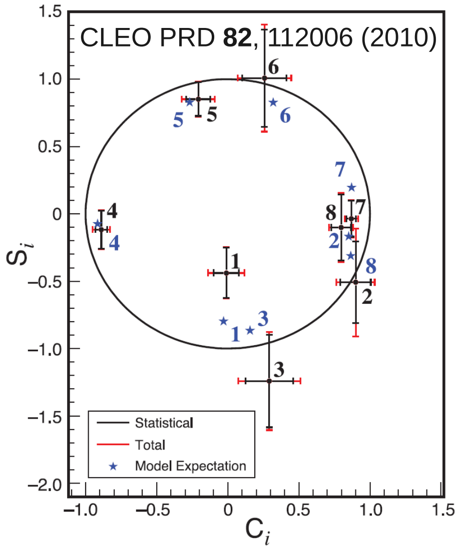
$$\delta_s = (111 \ \pm 32 \ \pm 11 \ \pm 3)^\circ$$

Measurement of Φ_3/γ

GGSZ method - new model-independent approach at Belle [EPJC **55**, 51 (2008)]:

- bin Dalitz plot and use in each bin strong phase difference obtained in measurements on quantum-correlated D^0 decays in $\psi(3770) \rightarrow D\overline{D}$ by CLEO

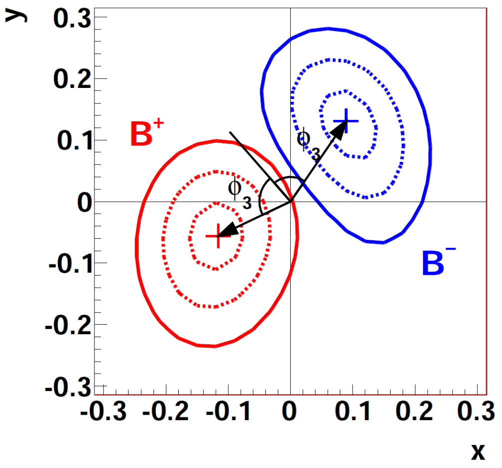




$$N_i^{\pm} = h_B\left[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}\,(x_+ c_i + y_+ s_i)\right]$$

$$x_{\pm} = r_B\cos(\delta_B \pm \phi_3)\,, \quad y_{\pm} = r_B\sin(\delta_B \pm \phi_3)$$

$$c_i = \langle\cos(\delta_D)\rangle\,, \quad s_i = \langle\sin(\delta_D)\rangle$$



Belle with 772×10^6 $B\overline{B}$:

$$\phi_3 = (77.3 \quad {}^{+15.1}_{-14.9}(\text{stat}) \quad \pm4.2(\text{syst}) \quad \pm4.3(c_i,\, s_i \text{ prec.}))^\circ$$

$$r_B = 0.145 \quad \pm0.030 \quad \pm0.011 \quad \pm0.011$$

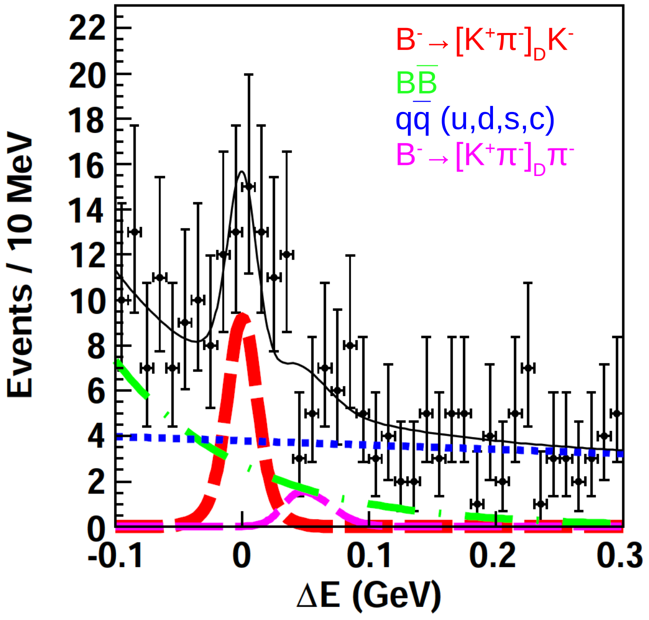
$$\delta_B = (129.9 \quad \pm15.0 \quad \pm3.9 \quad \pm4.7)^\circ$$

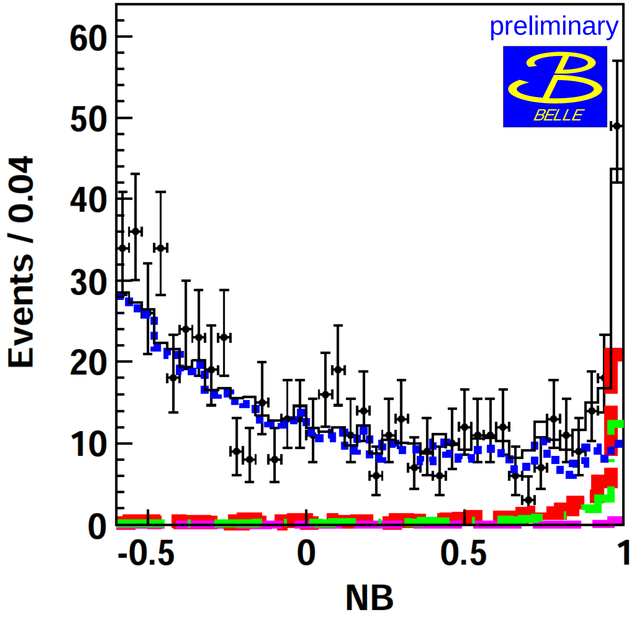
preliminary

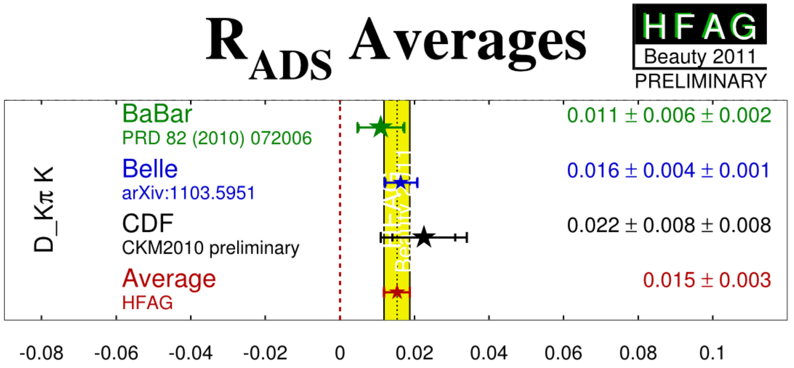
Measurement of Φ_3/γ – ADS Method

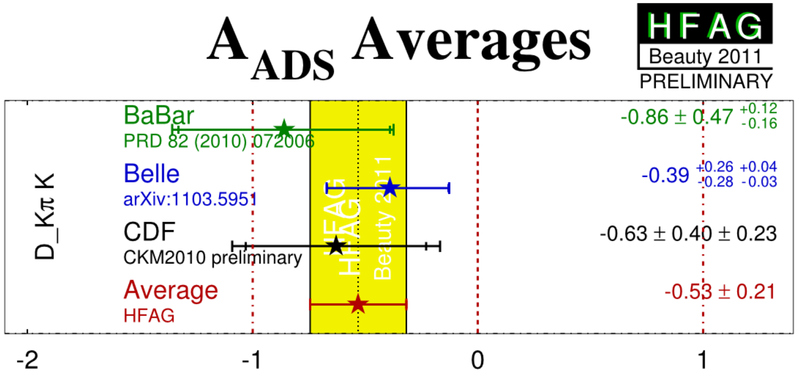
ADS method - new Belle measurement:

- First evidence for the suppressed decay $B^- \to DK^-$ with $D \to K^+\pi^-$ (significance of 4.1σ)
- Improved continuum suppression ($e^+e^- \to q\bar{q}$ (q=u,d,s,c)) utilizing neural networks (NB)









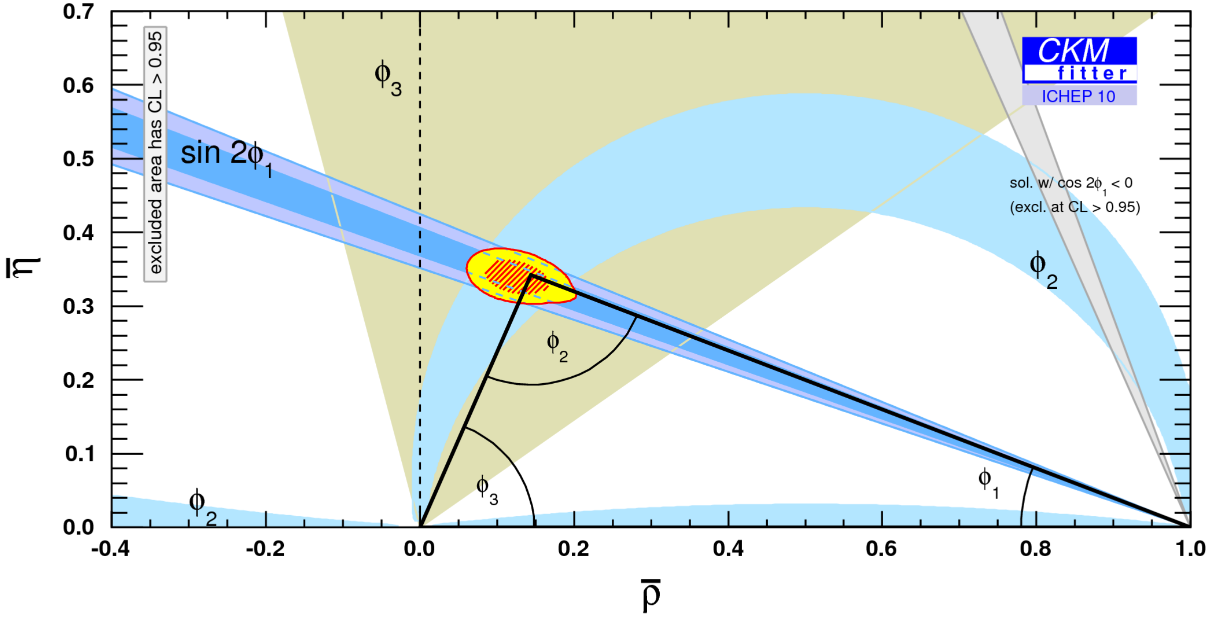
Belle with 772×10^6 $B\bar{B}$: arXiv:1103.5951

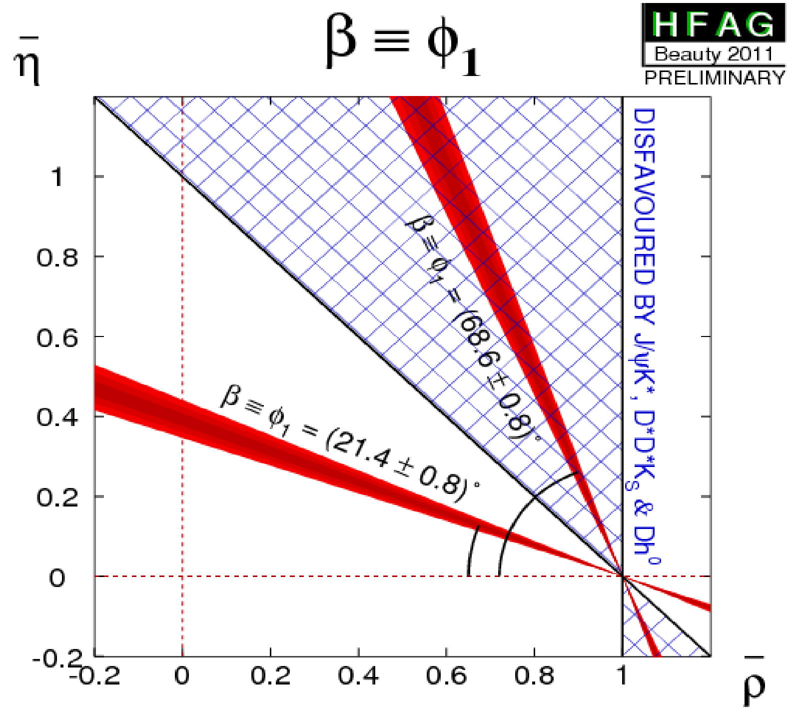
$$\mathcal{R}_{DK} = \left(1.63\ {}^{+0.44}_{-0.41}\ (\text{stat})\ {}^{+0.07}_{-0.13}\ (\text{syst})\right) \times 10^{-2}$$

$$\mathcal{A}_{DK} = -0.39\ {}^{+0.26}_{-0.28}\ {}^{+0.04}_{-0.03}$$

preliminary

Summary





	Overview of results of a few analyses			
Angle	BaBar		Belle	
ϕ_1 / β	$(21.7 \pm 1.2)^\circ$	$[B^0 \to c\bar{c}K^{(*)0}]$	$(21.0 \pm 1.0)^\circ$	$[B^0 \to c\bar{c}K^0]$
ϕ_2 / α	$71^\circ < \phi_2 < 109^\circ$ $(87\ {}^{+45}_{-13})^\circ$ $(92.4\ {}^{+6.0}_{-6.5})^\circ$	$[B \to \pi\pi]$ $[B \to \rho\pi]$ $[B \to \rho\rho]$	$(97 \pm 11)^\circ$ $68^\circ < \phi_2 < 95^\circ$ $(91.7 \pm 14.9)^\circ$	$[B \to \pi\pi]$ $[B \to \rho\pi]$ $[B \to \rho\rho]$
ϕ_3 / γ	$(68\ {}^{+15}_{-14} \pm 4 \pm 3)^\circ$	$[B^\pm \to D^{(*)}K^{(*)\pm}]$ [model-dep. GGSZ]	$(78\ {}^{+11}_{-12} \pm 4 \pm 9)^\circ$ $(77.3\ {}^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$	$[B^\pm \to D^{(*)}K^\pm]$ [model-dep. GGSZ] $[B^\pm \to DK^\pm]$ [model-indep. GGSZ]

Supplementary Slides

Measurement of Φ_1/β

- Belle's update on full dataset (preliminary):

systematic uncertainties	ΔS	ΔA
Vertexing	$^{+0.008}_{-0.009}$	± 0.008
Flavor tagging	$^{+0.004}_{-0.003}$	± 0.003
Resolution function	± 0.007	± 0.001
Physics parameters	± 0.001	< 0.001
Fit bias	± 0.004	± 0.005
$J/\psi K^0_S$ signal fraction	± 0.002	± 0.001
$J/\psi K^0_L$ signal fraction	± 0.004	$^{+0.000}_{-0.002}$
$\psi(2S) K^0_S$ signal fraction	< 0.001	< 0.001
$\chi_{c1} K^0_S$ signal fraction	< 0.001	< 0.001
Background Δt	± 0.001	< 0.001
Tag-side interference	± 0.001	± 0.008
Total	± 0.013	± 0.013

significant improvement in systematic error compared to Belle's last update on $\sin(2\Phi_1)$

(mainly due to vertexing and resolution functions)

Belle with 772×10^6 BB:

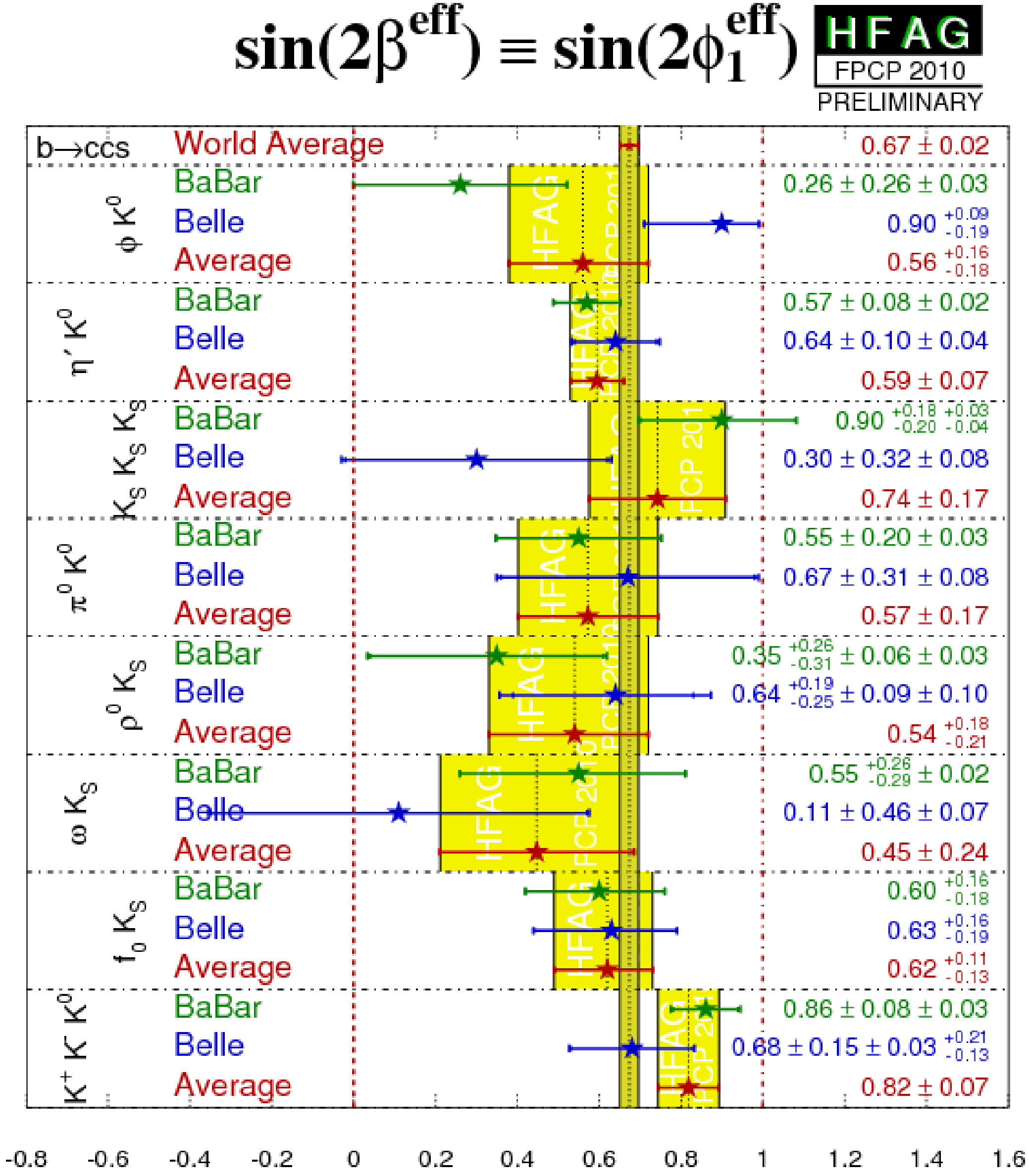
$$\mathcal{A} = 0.007 \pm 0.016\,(\mathrm{stat}) \pm 0.013\,(\mathrm{syst})$$

$$\sin(2\phi_1) = 0.668 \pm 0.023 \pm 0.013$$

preliminary

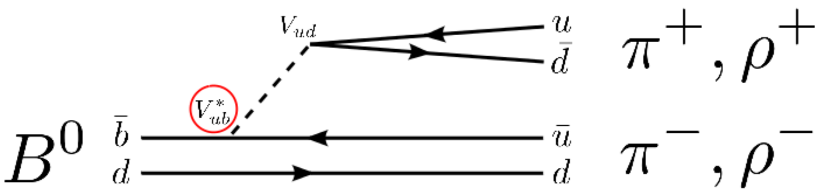
Measurement of Φ_1/β

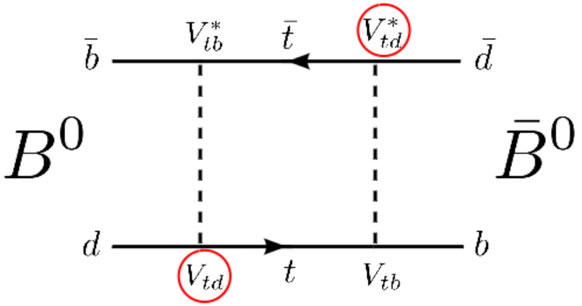
Overview of measurements of $\sin(2\Phi_1^{\mathrm{eff}})$ in $b \to q\bar{q}s$ penguin transitions



Measurement of Φ_2/α

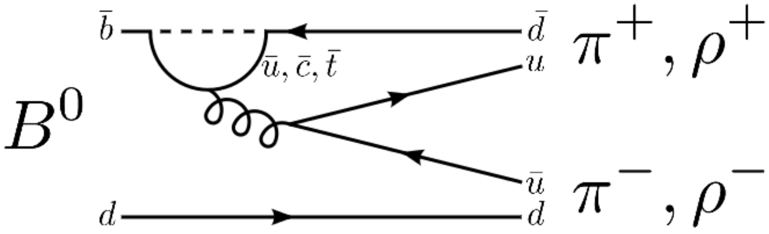
- Φ_2 is directly accessible only by measurements of time-dependent CPV in $b \to u\bar{u}d$ transitions like in $B \to \pi\pi$, $\rho\pi$, $\rho\rho$ or $a_1(1260)\pi$.





$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = e^{-i2\phi_1}e^{-i2\phi_3} = e^{i2\phi_2} \longrightarrow \mathcal{A}_f = 0 \quad \text{and} \quad \mathcal{S}_f = \sin(2\phi_2)$$

- Problem: $b \to d$ penguins have different weak phases and magnitudes of same order in λ

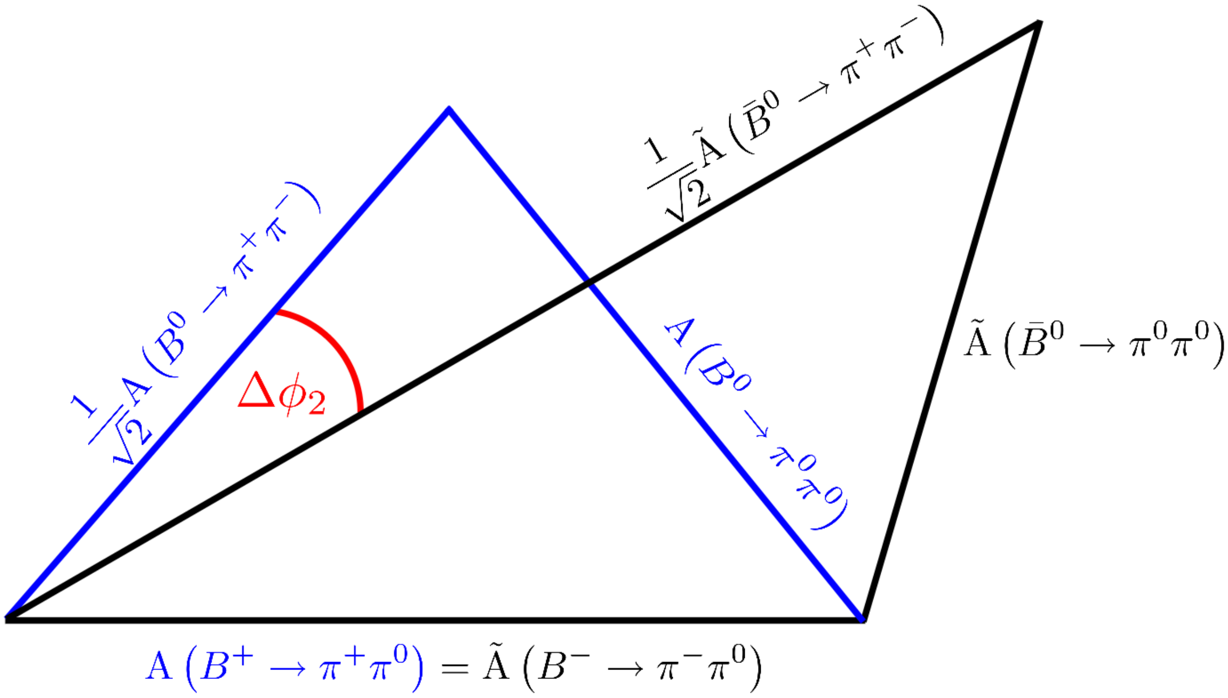


- This results in a "penguin pollution" with a shifted $\Phi_{2,\text{eff}} = \Phi_2 + \Delta\Phi_2$:

$$\mathcal{A}_f = -\sin(2\delta) \quad \text{and} \quad \mathcal{S}_f = \sqrt{1-\mathcal{A}_f^2}\,\sin(2\phi_{2,\text{eff}})$$

Measurement of Φ_2/α

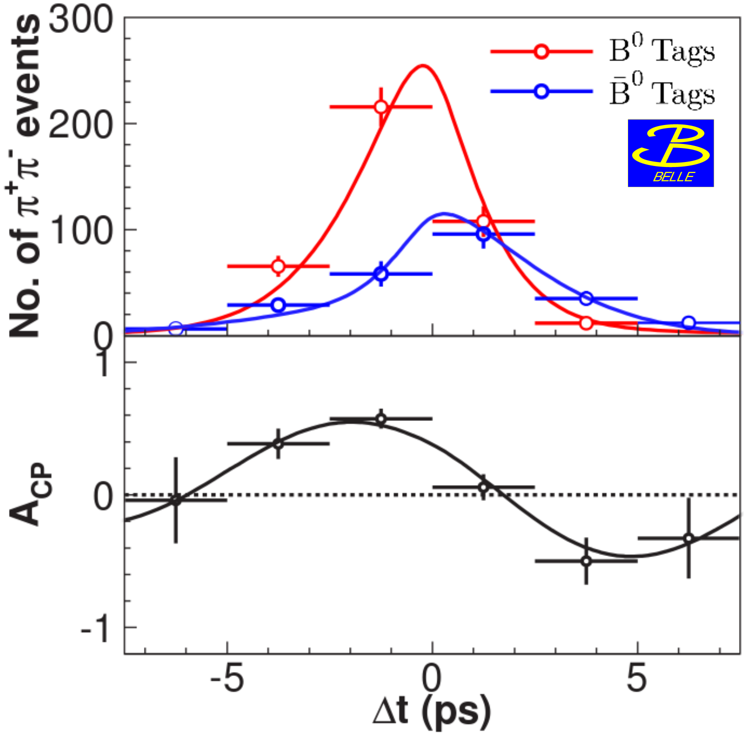
- Φ_2 in $B^0 \rightarrow \pi^+\pi^-$ and $\rho^+\rho^-$ can be extracted from $\Phi_{2,\,\mathrm{eff}} = \Phi_2 + \Delta\Phi_2$ by disentangling tree and penguin contributions using isospin relations: [Gronau, London: PRD **65**, 3381 (1990)]
  - $I(\pi^+) = (1, +1)$ $\quad$ $I(\pi\pi) = 0$ (tree, penguin) ∨ $I(\pi\pi) = 2$ (tree)
    $I(\pi^-) = (1, -1)$
  - $B^\pm \rightarrow \pi^\pm\pi^0$ can have only I=2, it arises only from tree diagram.

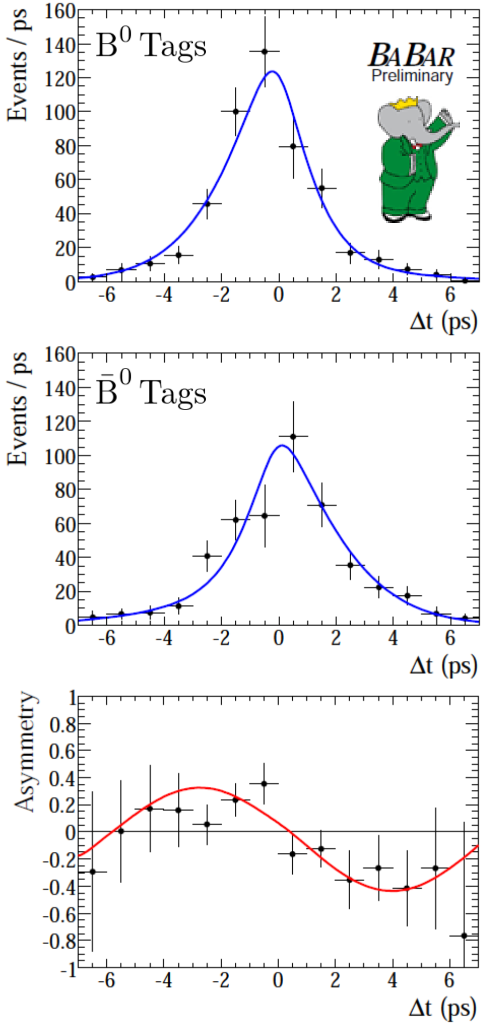


Expansion of amplitudes for $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \pi^0\pi^0$ and $B^+ \rightarrow \pi^+\pi^0$ and their charge-conj. in terms of I=0 and I=2 leads to relations of 2 complex triangles sharing same base.

Measurement of Φ_2/α

Measurements of $B^0 \rightarrow \pi^+\pi^-$:





Belle with 535×10^6 $B\bar{B}$: PRL **98**, 211801 (2007)

$$\mathcal{A}_{\pi^+\pi^-} = 0.55 \pm 0.08\,(\mathrm{stat}) \pm 0.05\,(\mathrm{syst})$$
$$\mathcal{S}_{\pi^+\pi^-} = -0.61 \pm 0.10 \pm 0.04$$

BaBar with 465×10^6 $B\bar{B}$: arXiv:0807.4226

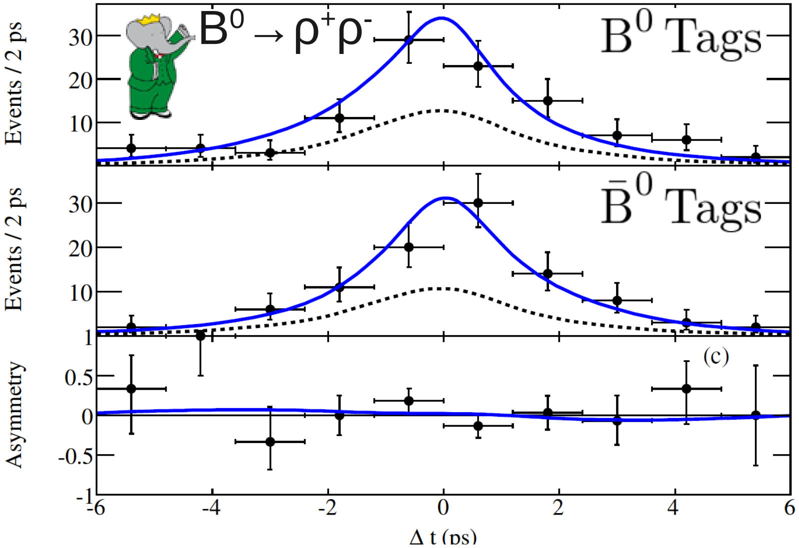
$$\mathcal{A}_{\pi^+\pi^-} = 0.25 \pm 0.08\,(\mathrm{stat}) \pm 0.02\,(\mathrm{syst})$$
$$\mathcal{S}_{\pi^+\pi^-} = -0.68 \pm 0.10 \pm 0.03$$

preliminary

$$\mathcal{A}_f \neq 0 \Rightarrow \mathcal{S}_f = \sqrt{1-\mathcal{A}_f^2}\,\sin\left(2\phi_{2,\mathrm{eff}}\right)$$

Measurement of Φ_2/α

Measurement of Φ_2 in B → ρρ:



BaBar with 384×10^6 $B\bar{B}$:

PRD **76**, 052007 (2007)

$$\mathcal{B}\left(B^0 \rightarrow \rho^+\rho^-\right) = (25.5 \pm 2.1\ (\text{stat})\ {}^{+3.6}_{-3.9}\ (\text{syst})) \times 10^{-6}$$

$$f_{\text{long}} = 0.992 \pm 0.024\ {}^{+0.026}_{-0.013}$$

$$\mathcal{A}_{\rho^+\rho^-,\text{long}} = -0.01 \pm 0.015 \pm 0.06$$

$$\mathcal{S}_{\rho^+\rho^-,\text{long}} = -0.17 \pm 0.020\ {}^{+0.005}_{-0.006}$$

BaBar with 465×10^6 $B\bar{B}$:

PRD **78**, 071104 (2008)

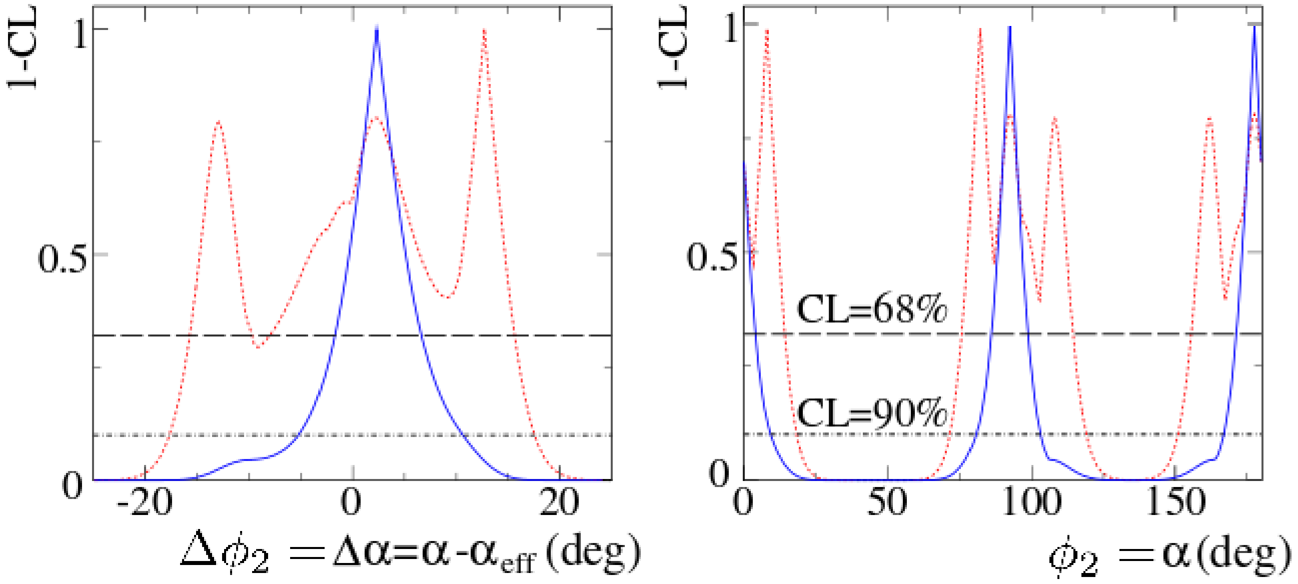
$$\mathcal{B}\left(B^0 \rightarrow \rho^0\rho^0\right) = (0.92 \pm 0.32\ (\text{stat}) \pm 0.14\ (\text{syst})) \times 10^{-6}$$

$$\mathcal{A}_{\rho^0\rho^0,\text{long}} = -0.2 \pm 0.8 \pm 0.3$$

$$\mathcal{S}_{\rho^0\rho^0,\text{long}} = 0.3 \pm 0.7 \pm 0.2$$

PRL **102**, 141802 (2009)

$$\mathcal{B}\left(B^+ \rightarrow \rho^+\rho^0\right) = (23.7 \pm 1.4\ (\text{stat}) \pm 1.4\ (\text{syst})) \times 10^{-6}$$



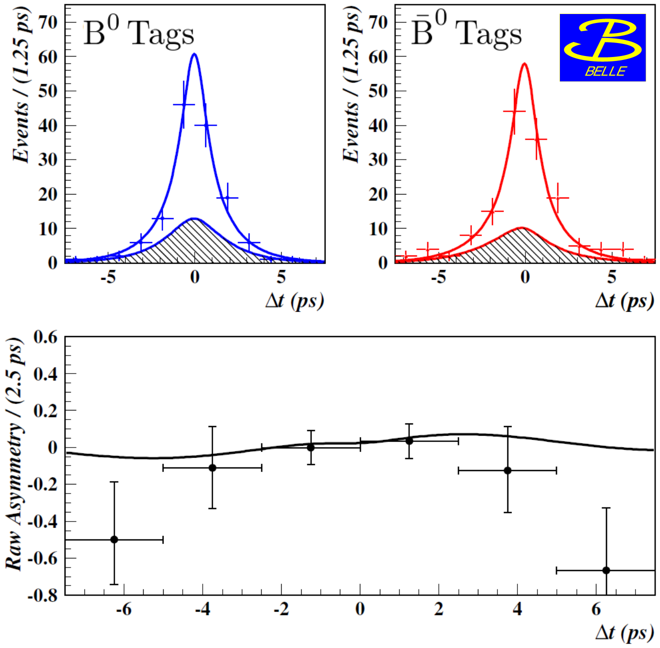
B → ρρ is mostly long. polarized and has an almost flat isospin triangle

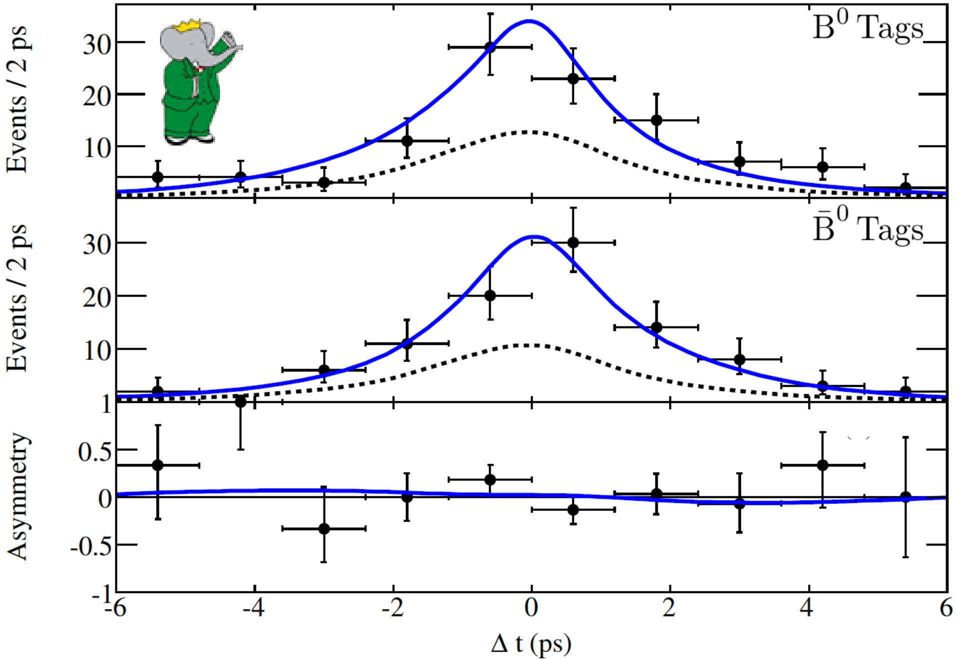
$$\phi_2 = \left(92.4^{+6.0}_{-6.5}\right)^\circ$$

at 68% C.L. interval

Measurement of Φ_2/α

Measurements of $B^0 \to \rho^+\rho^-$:





PRD **76**, 011104 (2007)

Belle with 535×10⁶ $B\bar{B}$:

$$\mathcal{A}_{\rho^+\rho^-,L} = 0.16 \pm 0.21\,(\text{stat}) \pm 0.08\,(\text{syst})$$
$$\mathcal{S}_{\rho^+\rho^-,L} = 0.19 \pm 0.30 \pm 0.08$$

PRD **76**, 052007 (2007)

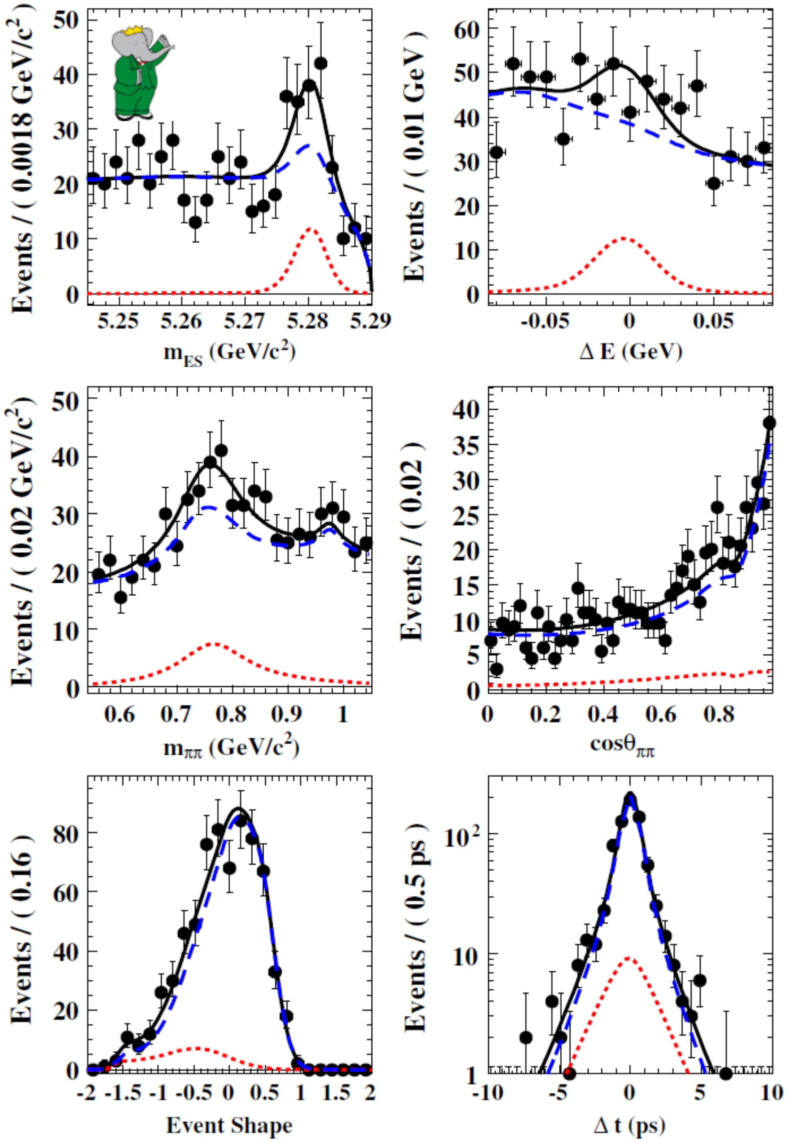
BaBar with 384×10⁶ $B\bar{B}$:

$$f_L = 0.992 \pm 0.024\,(\text{stat})\,^{+0.026}_{-0.013}\,(\text{syst})$$
$$\mathcal{A}_{\rho^+\rho^-,L} = -0.01 \pm 0.015 \pm 0.06$$
$$\mathcal{S}_{\rho^+\rho^-,L} = -0.17 \pm 0.20\,^{+0.05}_{-0.06}$$

Latest results of $B^+ \to \rho^+\rho^0$ [PRL **102**, 141802 (2009)] and $B^0 \to \rho^0\rho^0$ [PRD **78**, 071104 (2008)] from BaBar suggest an almost flat isospin triangle in $B \to \rho\rho$.

Measurement of Φ_2/α

Measurements of $B^0 \to \rho^0\rho^0$:



┈┈┈ projections of $B^0 \to \rho^0\rho^0$ PDF component

BaBar could see a signal of $B^0 \to \rho^0\rho^0$ with 3.1σ significance:

BaBar with 465×10^6 $B\overline{B}$:

PRD **78**, 071104 (2008)

$$\mathcal{B}\left(B^0 \to \rho^0\rho^0\right) = (\;0.92 \pm 0.32\ (\text{stat}) \pm 0.14\ (\text{syst})) \times 10^{-6}$$

$$\mathcal{A}_{\rho^0\rho^0,\text{long}} = -0.2 \pm 0.8 \pm 0.3$$

$$\mathcal{S}_{\rho^0\rho^0,\text{long}} = 0.3 \pm 0.7 \pm 0.2$$

Belle hasn't found evidence for $B^0 \to \rho^0\rho^0$ yet:

Belle with 657×10^6 $B\overline{B}$:

PRD **78**, 111102 (2008)

$$\mathcal{B}\left(B^0 \to \rho^0\rho^0\right) < 1.0 \times 10^{-6} \quad \text{at } 90\%\ \text{CL}$$