Theory Overview
of Semileptonic Decays

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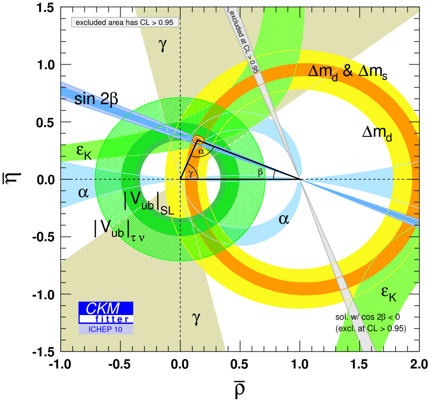
Contents

Introduction: Why is this of interest?

- Important Ingredient for the Unitarity Traingle



- Standard Fit of for the Unitarity Traingle
- "Unitarity Clock": $|V_{ub}/V_{cb}|$ Buras
- Relation between Kaon CP violation and the Unitarity Trangle

Theoretical Tools

- There is a large toolbox:
- For inclusive decays:
  - Heavy Quark Expansion: Local OPE
  - Heavy Quark Expansion: Shape functions and Soft Collinear Effective Theory (SCET)
- For exclusive decays:
  - Heavy Quark Effective Theory (HQET)
  - QCD (Light Cone) Sum Rules
  - Lattice
- Models are completely outdated!
- Precision Methods with controllable uncertainties

Heavy Quark Expansion

Heavy Quark Expansion = Operator Product Expansion
(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}_{eff}^\dagger(0) | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, \langle B(v) | T\{\mathcal{H}_{eff}(x) \mathcal{H}_{eff}^\dagger(0)\} | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v \cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}_{eff}^\dagger(0)\} | B(v) \rangle$$

- Last step: $b(x) = b_v(x) \exp(-im_v vx)$, corresponding to $p_b = m_b v + k$
  Expansion in the residual momentum k

- Perform an "OPE": m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\rightarrow$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\rightarrow$ Perturbation theory!
- Works also for differential rates!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$
\begin{aligned}
2M_H\mu_\pi^2 &= -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle \\
2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle
\end{aligned}
$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$
\begin{aligned}
2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\
2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle
\end{aligned}
$$

ρ_D: Darwin Term and ρ_{LS}: Spin-Orbit Term

- Γ_4 and Γ_5 have been computed Bigi, Uraltsev, Turczyk, TM, ...

Structure of the HQE

- Structure of the expansion (@ tree):

$$\begin{aligned} d\Gamma &= d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4 \\ &\quad + d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right) \\ &\quad + \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 \end{aligned}$$

- $d\Gamma_3 \propto \ln(m_c^2/m_b^2)$
- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, TM, ...
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
  Melnikov, Czarnecki, Pak
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
  Becher, Boos, Lunghi, Gambino
- In the pipeline:
  - Complete α_s/m_b^2, including the μ_G terms
  - More on the "Intrinsic charm" and "weak annihilation" contributions

A theo. uncertainty of 1% in $V_{cb,incl}$ looks plausible!

Modified Heavy Quark Expansion: $B \to X_u \ell \bar{\nu}$

- Problem: Cuts needed to suppress charmed decays
- Forces us into corners of phase space, where the usual OPE breaks down
- Expansion parameter $\Lambda_{\mathrm{QCD}}/(m_b - 2E_\ell)$
- Instead of HQE Parameters: Shape Functions $f(\omega)$

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

- Universal for all heavy-to-light decays
- Systematics: S$_{\text{oft}}$C$_{\text{ollinear}}$E$_{\text{ffective}}$T$_{\text{heory}}$ calculation
  - Several subleading shape functions
  - perturbative QCD corrections

Shape Functions

- Shape function vs. local OPE: Moment Expansion

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

- Perturbative “jetlike” contributions: Convolution

$$S(\omega, \mu) = \int dk \; C_0(\omega - k, \mu) f(k)$$

- Charged Lepton Energy Spectrum (H: hard QCD corrections)

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega \, \Theta(m_b(1-y) - \omega) H(\mu) S(\omega, \mu)$$

Approaches

- Obtaining the Shape functions:
  - From Comparison with $B \to X_s \gamma$
  - From the knowledge of (a few) moments
  - From modeling
- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
- QCD inspired:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti et al.)
- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

Theo. uncertainty in $V_{ub,incl}$ is still (7 ... 10) %

Exclusive $b \to c$ Decays

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] + (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\rm QED}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\rm QED} = 1.007$

$B \to D^{(*)}$ Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.908 \pm 0.016}$$

$$\boxed{\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016}$$

F(1): upd. from CKM2010 , G(1): A. Kronfeld et al. 2005

$B \to D^{(*)}$ Form Factors: Non-Lattice Results

- $B \to D^*$ Form Factor:
  - Based on Zero Recoil Sum Rules (Uraltsev, also Ligeti et al.)
  - Including full α_s and up to $1/m_b^5$

$$\boxed{\mathcal{F}(1) = 0.86 \pm 0.04}$$ (Gambino, Uraltsev, M (2010))

- $B \to D$ Form Factor:
  - Based on the "BPS limit" $\mu_\pi^2 = \mu_G^2$

$$\boxed{\mathcal{G}(1) = 1.04 \pm 0.02}$$ (Uraltsev)

The tension between $V_{cb,incl}$ and $V_{cb,excl}$ is about to disappear!

Exclusive Decays: $b \to u$

- Focus on $B \to \pi \ell \bar{\nu}_\ell$
- Hadronic Matrix Element:

$$\langle \pi(p) | \bar{u} \gamma_\mu b | B(p+q) \rangle = f^+_{B\pi}(q^2)(2p+q)_\mu + \text{Terms with } f^0_{B\pi}(q^2)$$

- Differential rate:

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

- Measurements are quickly improving
- Shape constrained by analyticity
- Need the normalization $f^+_{B\pi}(0)$
- In case $\ell = \tau$ also $f_0(q^2)$ s needed!

Tools: Form Factor Parametrizations

- Becirevic Kaidalov Parametrization

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}$$

- z parametrization (Arnesen et al., Boyd, Grinstein, Lebed)

$$P(t)\phi(t, t_0)f_+(t) = \sum_{k=0}^{\infty} a_k(t_0) z^k(t, t_0)$$

with

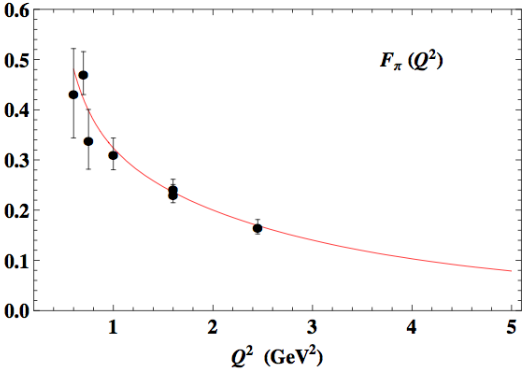
$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \qquad t_+ = (m_B + m_\pi)^2$$

Status of LCSR calculation

- latest update: A. Khodjamirian, TM., N. Offen, Y-M. Wang 2011
- LCQCDSR Calculation of

$$\Delta\zeta\,(0, q^2_{max}) = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int\limits_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B \to \pi \ell \nu_\ell)}{dq^2},$$

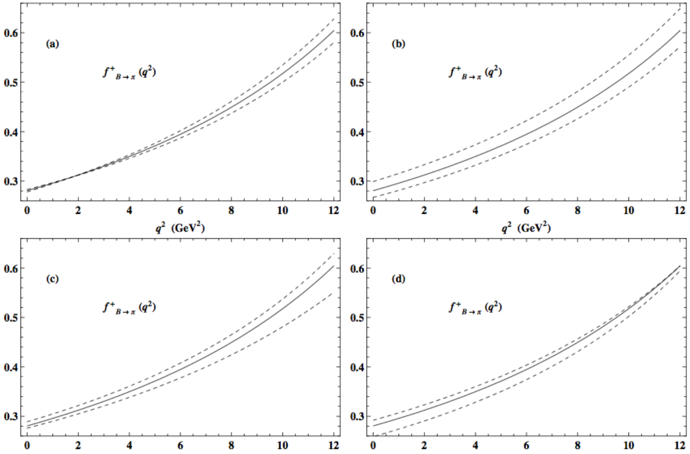
- ... including
  - Full $\mathcal{O}(\alpha_s)$ QCD corrections
  - Subleading twists
  - a_2 and a_4 corrections to the pion DA, fitted from the electromagnetic pion form factor



The pion e.m. form factor calculated from LCSR [16, 17] as a function of Gegenbauer moments $a_2^\pi(1\ GeV)$ and $a_4^\pi(1\ GeV)$ and fitted (solid) to the experimental data points taken from from [18].

$$a_2^\pi(1\text{ GeV }) = 0.17 \pm 0.08, \quad a_4^\pi(1\text{ GeV }) = 0.06 \pm 0.10$$

LCQCDSR Result for the from factor, $0 \leq q^2 \leq 12$ GeV^2

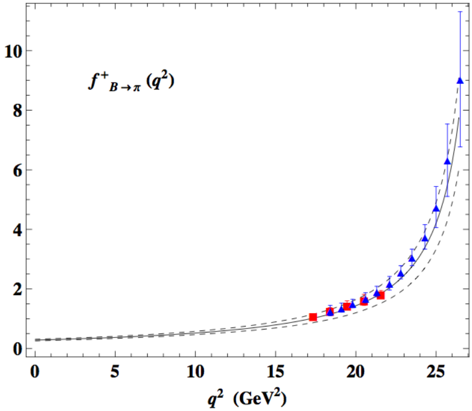


(a): a_2^π, a_4^π, (b): μ_π, (c): μ, (d): M^2, s_0^B

Linking high q^2 with low q^2

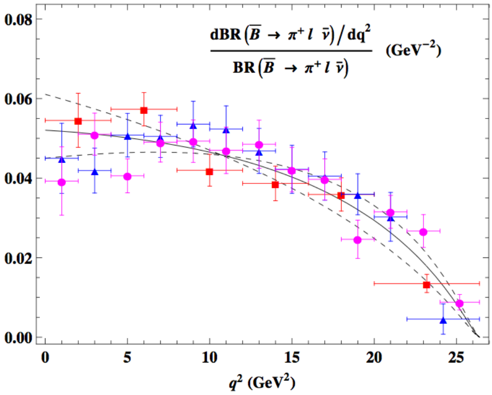
- LCQCDSR are limited to “small” values of q^2
- Complementary to lattice calculations
- We have QCD based calculations / estimates of the from factors f_+ and f_0 in the full kinematic region
- Uncertainties become controllable and are already quite small !
- May become the most accurate way to determine V_{ub}

Linking high q^2 with low q^2: z parametrization



The vector form factor $f^+_{B\pi}(q^2)$ calculated from LCSR and fitted to the BCL parameterization (solid) with uncertainties (dashed), compared with the HPQCD [4] (triangles) and FNAL/MILC [5] (squares) results.

Theory vs. Experiment



(colour online) The normalized q^2-distribution in $B \to \pi l \nu$ obtained from LCSR and extrapolated with the z-series parameterization (central input- solid, uncertainties -dashed). The experimental data points are from BABAR: (red) squares [1], (blue) triangles [2] and Belle [3]: (magenta) full circles.

Value of V_{ub} from this work:

$$|V_{ub}| = (3.50^{+0.38}_{-0.33}\big|_{th.} \pm 0.11\big|_{exp.}) \times 10^{-3}$$

Lattice $\otimes$ LCQCDSR has reached 10% th. uncertainty in $V_{ub,excl}$!

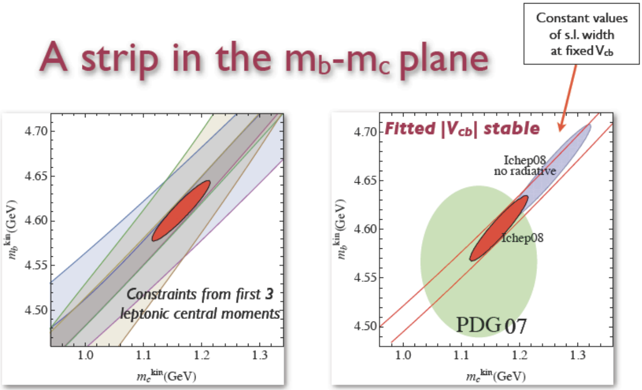
Inputs for the standard OPE

- Two, in principle equivalent schemes:
  kinetic scheme and 1S scheme

	Kinetic scheme	1S scheme
$O(1)$	m_b, m_c	m_b
$O(1/m^2_b)$	μ^2_π, μ^2_G	λ_1, λ_2
$O(1/m^3_b)$	ρ_D, ρ_{LS}	ρ_1, τ_{1-3}

- Parameters are determined from the spectra: hadronic invariant mass, charged lepton and hadronic energy

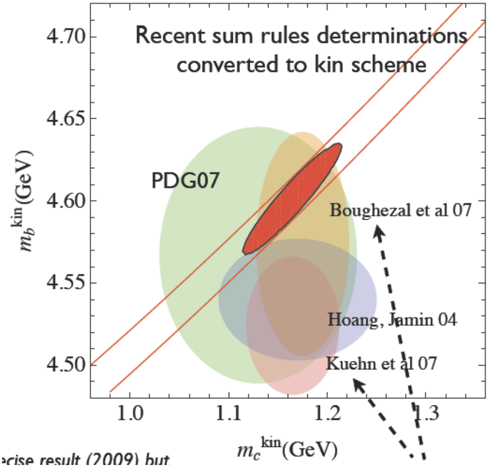
Quark Masses



(Plot from Paolo Gambino)

The semileptonic moments identify only a strip $m_b - 0.6 m_c$ in the m_b, m_c plane!

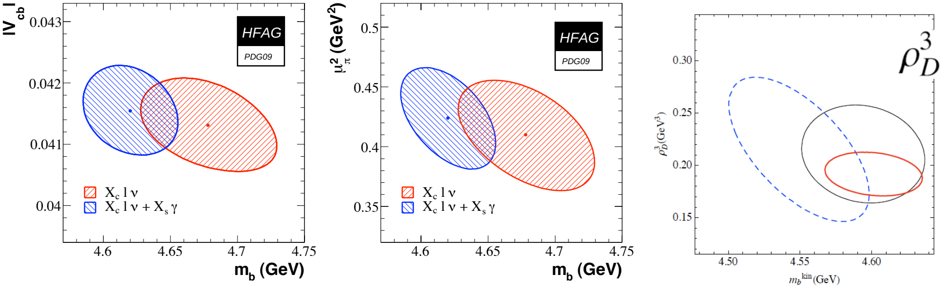
Recent Mass Determinations



(Plot from Paolo Gambino)

HQE Parameters

- These may be determined from the spectra
  - Hadronic invariant mass
  - Charged Lepton Energy
  - Hadronic Energy
- Moments in terms of a $1/m_b$ expansion
- Higher Moments $\Leftrightarrow$ Higher Dimensional Operators
- Lepton-Energy Cut dependence of the moments can be reliably calculated
- Determinaton of m_b, m_c μ_π^2, μ_G^2, ... from data



(Plots from P. Gambino and C. Schwanda)

- Reasonably good determination of the mass and the HQE parameters (up to $1/m_b^3$)

Shape Functions

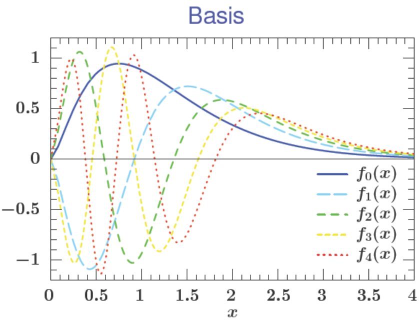
- Shape functions can be extracted from $B \to X_s\gamma$
- Several sub-leading shape functions !
- Attempt for a systematic fit: SIMBA

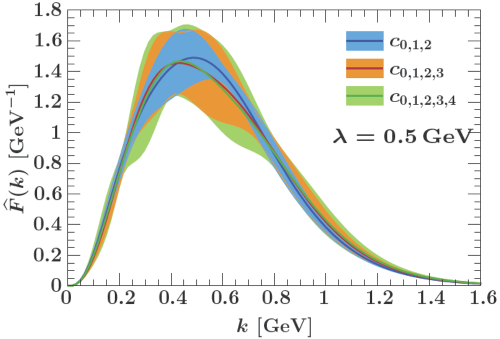
(Tackmann, Tackmann, Lacker, Liegti, Stewart ...)

  - Systematic expansion in terms of basis functions

$$F(\lambda x) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n(x)\right]^2 \qquad \int dk\, F(k) = 1 = \sum_{n=0}^{\infty} c_n^2$$

  - Reduce the cut dependences





(Plots from K. Tackmann)

- Chose different bases:
  Check for basis independence

The Role of $B \to \tau \bar{\nu}$

- $B \to \tau \bar{\nu}$ depends crucially on f_B

$$\mathcal{B}(B^- \to \tau \bar{\nu}_\tau) = \frac{G_F^2}{8\pi} |V_{ub}|^2 m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 \tau_{B^-}$$

- The extracted V_{ub} value is quite large ...
- However, if the data are right, QCD (or the SM) must have a problem: Define

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta \mathcal{B}_{B \to \pi \ell \nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B \to \tau \nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}}\right)$$

$$= \frac{\Delta \zeta(q_1^2, q_2^2)}{(G_F^2/8\pi) m_\tau^2 m_B (1 - m_\tau^2/m_B^2)^2 f_B^2}$$

Exp.	$\Delta\mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \to \tau\nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.32 ± 0.03 [1] $0.33 \pm 0.03 \pm 0.03$ [2]	1.76 ± 0.49 [36, 37]	$0.20^{+0.08}_{-0.05}$
Belle	0.398 ± 0.03 [3]	$1.54^{+0.38+0.29}_{-0.37-0.31}$ [38]	$0.28^{+0.13}_{-0.07}$
QCD	$\Delta\zeta(\text{ps}^{-1})$ [Ref.]	$f_B(\text{MeV})$ [Ref.]	$R_{s/l}$
HPQCD	2.02 ± 0.55 [4]	190 ± 13 [34]	0.52 ± 0.16
FNAL/MILC	$2.21^{+0.47}_{-0.42}$ [5]	212 ± 9 [35]	0.46 ± 0.10

$R_{s/l}$ for the region $16\ \text{GeV}^2 < q^2 < 26.4\ \text{GeV}^2$

Exp.	$\Delta\mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \to \tau\nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.88 ± 0.06 [1] $0.84 \pm 0.03 \pm 0.04$ [2]	1.76 ± 0.49 [36, 37]	$0.52^{+0.20}_{-0.12}$
QCD	$\Delta\zeta$ [Ref.]	f_B(MeV) [Ref.]	$R_{s/l}$
LCSR/QCDSR	$4.59^{+1.00}_{-0.85}$ [this work]	210 ± 19 [41]	$0.97^{+0.28}_{-0.24}$

$R_{s/l}$ for the region $0\ \mathrm{GeV}^2 < q^2 < 12.0\ \mathrm{GeV}^2$

Some clarification is needed here ...

Conclusions and Outlook

- V_{cb} is in good shape
  - OPE in inclusive method very well understood
  - Excl. vs. Incl. tension is becomming smaller
  - Calculation of $\alpha_s \mu_G^2$ terms
- The tension in inc. vs. excl. V_{ub} stays with us ...
  - Form factors are pretty well constrained
    smaller uncertainties in V_{ub} from $B \to \pi \ell \bar{\nu}$
  - Scrutinize both inclusive and exclusive methods
  - It is not yet time to speculate about new physics in $b \to u$ semileptonics ...
- What is going on in $B \to \tau \bar{\nu}$?