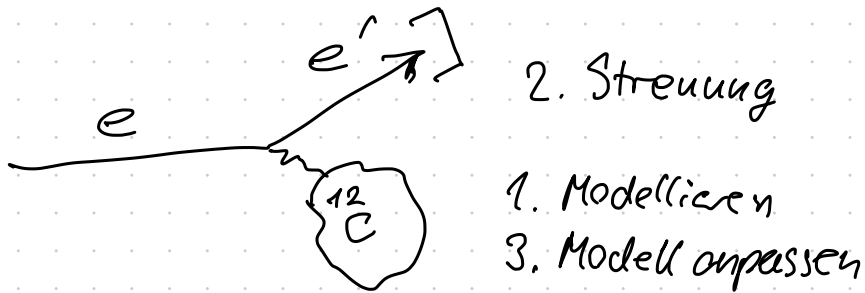


September 1. Tag Strahlzeit

Idee: der rote Faden von Modell
bis zur Zählrate im Detektor



Modell des Kohlenstoffkerns

Anfang der Kernphysik: Massen der Atomkerne

$$m = Z \cdot m_p + N \cdot m_n - \frac{B}{c^2}$$

$$A = Z + N$$

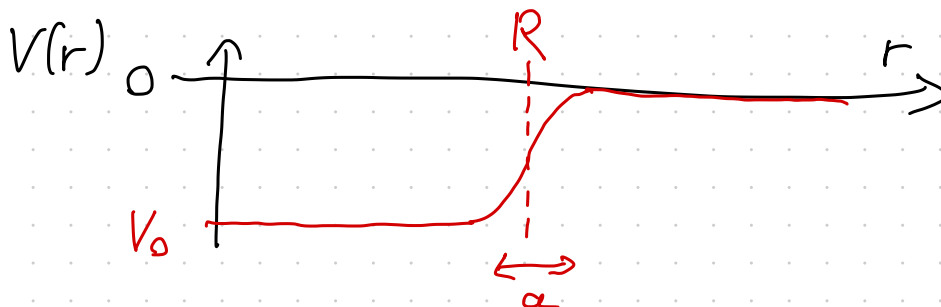
Tröpfchenmodell

$$B = \underbrace{a_v A}_{\text{Volumen}} - \underbrace{a_o A^{2/3}}_{\text{Oberfläche}} - \underbrace{a_c Z^2 A^{-1/3}}_{\text{Coulomb}} + \dots$$

Quantenmechanisch: Nukleonen im mittleren Potential

Potentialmodell:

- suche Potential
- löse Schrödingergleichung
- vergleiche mit Messung



empirisches Potential

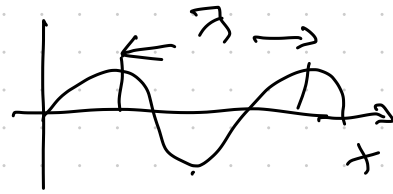
$$V(r) = \frac{V_0}{1 + e^{\frac{r-R}{a}}}$$

Woods-Saxon

V_0 : Tiefe des Potentials

R : Radius

a : Randunschärfe



1. Löse Schrödingergleichung

$$\Psi(t, \vec{x}) = \Psi_0 e^{-i(\omega t - \vec{k} \cdot \vec{x})}$$

$$\left. \begin{array}{l} \text{M. Planck} \quad E = h \cdot \nu \quad \Rightarrow \quad \omega = \frac{E}{\hbar} \\ \text{L. de Broglie} \quad \lambda = \frac{h}{p} \quad \Rightarrow \quad \vec{k} = \frac{\vec{p}}{\hbar} \end{array} \right\} \Psi = \Psi_0 e^{-\frac{i}{\hbar}(Et - \vec{p} \cdot \vec{x})}$$

$$\frac{\partial}{\partial t} \Psi(\vec{x}, t) = -\frac{i}{\hbar} E \Psi(\vec{x}, t) \Rightarrow E \cdot \Psi = i\hbar \frac{\partial}{\partial t} \Psi$$

$$\vec{\nabla} \Psi(\vec{x}, t) = \frac{i}{\hbar} \vec{p} \Psi(\vec{x}, t) \Rightarrow \vec{p} \Psi = -i\hbar \vec{\nabla} \Psi$$

$$E = \frac{p^2}{2m} + V$$

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{x}, t) = \left(-\hbar^2 \frac{\Delta}{2m} \Psi(\vec{x}, t) + V(\vec{x}) \cdot \Psi(\vec{x}, t) \right)$$

$$\Psi(\vec{x}, t) \equiv e^{i\omega t} \cdot \frac{u(r)}{r} Y_{l,m}(\theta, \varphi)$$

$Y_{l,m}$	$l=0$	$l=1$
$m=-1$		$\sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\varphi}$
$m=0$	$1/\sqrt{4\pi}$	$\sqrt{\frac{3}{4\pi}} \cos\theta$
$+1$		$-\sqrt{\frac{3}{8\pi}} \sin\theta e^{+i\varphi}$

$$\Rightarrow u''(r) + \left(\frac{2m}{\hbar^2} (E - V(r)) - \frac{l(l+1)}{r^2} \right) u(r) = 0$$

$$u(0) = 0, \quad u(\infty) = 0$$

numerische Lösung

1-dim, 2. Ordnung \Rightarrow 2-dim, 1. Ordnung

$$w = \begin{pmatrix} u \\ u' \end{pmatrix} \quad w' = \begin{pmatrix} u' \\ u'' \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{2m}{\hbar^2}(E-V) - \frac{l(l+1)}{r^2} & 0 \end{pmatrix}}_A \begin{pmatrix} u \\ u' \end{pmatrix}$$

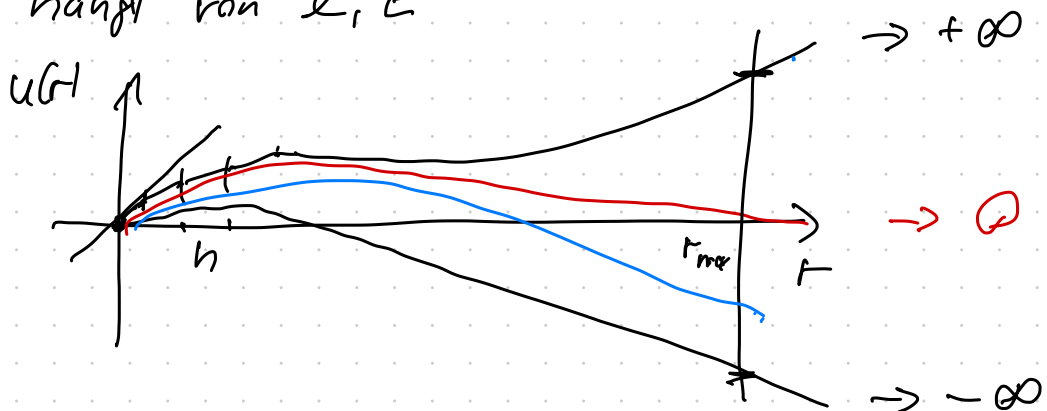
$\Rightarrow w' = A \cdot w$

numerisch:

$$w' = \lim_{\varepsilon \rightarrow 0} \frac{A \cdot w(r+\varepsilon) - A w(r)}{\varepsilon}$$

$$w_{n+1} = w_n + h \cdot w_n' = \begin{pmatrix} u \\ u' \end{pmatrix} + h \cdot A \begin{pmatrix} u \\ u' \end{pmatrix}$$

A hängt von l, E



Bisektionsverfahren

starte E_1, E_2 mit $u(r_{max}, E_1) > 0$
 $u(r_{max}, E_2) < 0$

$$E_{neu} = \frac{E_1 + E_2}{2}$$

wenn $u(r_{max}, E_{neu}) > 0$ $E_1 = E_{neu}$

sonst $E_2 = E_{neu}$

Anschließend

$$\psi(\vec{x}) = \frac{u(r)}{r} Y_{lm}(\theta, \varphi)$$

$$\Rightarrow \text{normieren auf } |\psi(\vec{x})|^2 \Rightarrow \int |\psi(\vec{x})|^2 dx = \int \left(\frac{u(r)}{r}\right)^2 dr = 1$$

Spin-Bahn-Kopplung

$$V(r) = V_{ws}(r) + V_{ls}(r) \cdot \langle \vec{l} \cdot \vec{s} \rangle$$

$$(\vec{l} + \vec{s})^2 = \vec{l}^2 + 2\vec{l} \cdot \vec{s} + \vec{s}^2 \Rightarrow$$

$$\vec{l} \cdot \vec{s} = \frac{1}{2} (\underbrace{(\vec{l} + \vec{s})^2}_{\vec{j}} - \vec{l}^2 - \vec{s}^2)$$

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{1}{2} (j(j+1) - l(l+1) - s(s+1))$$

$$l=0 \Rightarrow 0$$

$$l=1, j = l + \frac{1}{2} \Rightarrow \frac{1}{2} \left(\frac{3}{2} \left(\frac{3}{2} + 1 \right) - 2 - \frac{3}{4} \right) = \frac{1}{2}$$

$$l - \frac{1}{2} = -1$$

$$\Delta E_{ls} = \frac{2l+1}{2} \langle V_{ls}(r) \rangle$$

$$V_{ls}(r) \sim \frac{1}{r} \frac{dV(r)}{dr}$$

Für Protonen zusätzlich Coulomb-Potential

homogene Kugel

$$V_C(r) = \alpha(Z-1) \begin{cases} 1/r & \text{für } r > R \\ \frac{3}{2R} - \frac{r^2}{2R^3} & r \leq R \end{cases}$$

$$E_s, E_p, \gamma$$

$$E_p^n = m_{12C} - m_{11C} - m_n = 18.7 \text{ MeV}$$

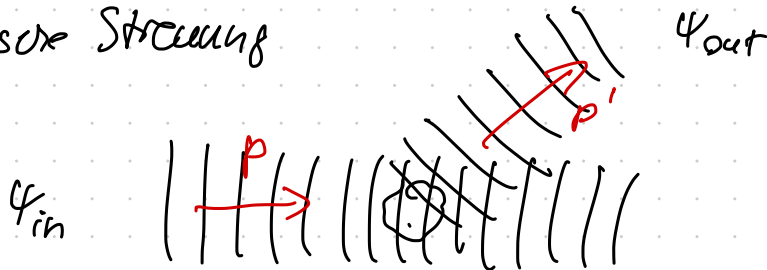
$$E_p^p = m_{12C} - m_{11B} - m_p = 16.0 \text{ MeV}$$

Ladungsverteilung $|4|^2$ im Ortsraum

Nur Protonen, 2 in s-, 4 in p-Schale

$$\rho(r) = \frac{e}{6} \left(2 \frac{u_s^2(r)}{r^2} + 4 \frac{u_p^2(r)}{r^2} \right)$$

elastische Streuung



ebene Welle

$$\psi_{in} = \frac{1}{\sqrt{v}} e^{i\vec{p} \cdot \vec{x}}$$
$$\psi_{out} = \frac{1}{\sqrt{v}} e^{i\vec{p}' \cdot \vec{x}}$$

Störungstheorie

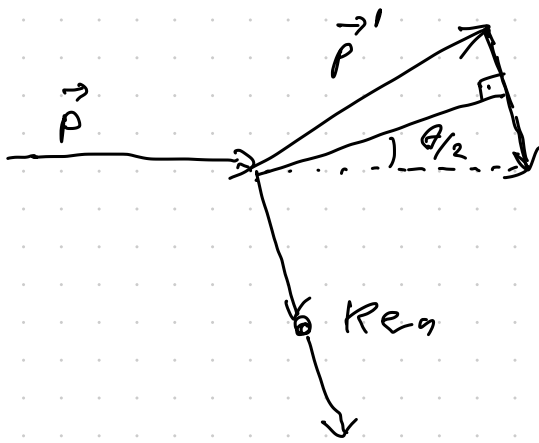
$$\frac{d\sigma}{d\Omega} = \frac{v^2 E^2}{(2\pi)^2} \left| \langle \psi_{out} | H_{int} | \psi_{in} \rangle \right|^2$$

$$H_{int} = e \phi(\vec{x})$$

↑ Potential

$$\langle \psi_{out} | H_{int} | \psi_{in} \rangle = \frac{e}{v} \int e^{-i\vec{p}' \cdot \vec{x}} \phi(\vec{x}) e^{i\vec{p} \cdot \vec{x}} d^3x$$
$$= \frac{e}{v} \int \phi(\vec{k}) e^{i\vec{q} \cdot \vec{x}} d^3x$$

mit $\vec{q} = \vec{p} - \vec{p}'$
Impulsübertrag



$$|\vec{q}| = 2|p| \sin \frac{\theta}{2}$$

Partielle Integration

$$\rho(x) = \Delta \phi(x)$$

Green'sches Theorem

$$\int (u \Delta v - v \Delta u) d^3x = 0$$

$$\Delta e^{i\vec{q}\vec{x}} = -|\vec{q}|^2 e^{i\vec{q}\vec{x}}$$

$$\Rightarrow \frac{e}{v} \int \phi(x) e^{i\vec{q}\vec{x}} d^3x = \frac{-e}{v|\vec{q}|^2} \int \Delta \phi e^{i\vec{q}\vec{x}} d^3x$$

$$= \langle \psi_f | H_{\text{int}} | \psi_{\text{in}} \rangle$$

Def. Formfaktor $F(\vec{q}) = \int \rho(x) e^{i\vec{q}\vec{x}} d^3x$

$$\frac{d\sigma}{d\Omega} = \frac{v^2 E^2}{(2\pi)^4} |\langle |H| \rangle|^2$$

$$= \frac{E^2 z^2 e^4}{(2\pi)^2 |\vec{q}|^4} |F(\vec{q})|^2$$

$$= \underbrace{\frac{z^2 \alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Rutherford}} \cdot |F(\vec{q})|^2$$

$$\frac{e^2}{4\pi} = \alpha = \frac{1}{137}$$

"Rutherford"-Wirkungsquerschnitt

genauer: "Mott" - WQ

$$\left(\frac{d\delta}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\delta}{d\Omega}\right)_{\text{Rutherford}} \cdot \underbrace{\cos^2 \frac{\theta}{2}}_{\text{Spin}} \cdot \underbrace{\frac{E'}{E}}_{\text{Rückstoß}}$$

Unser Modell

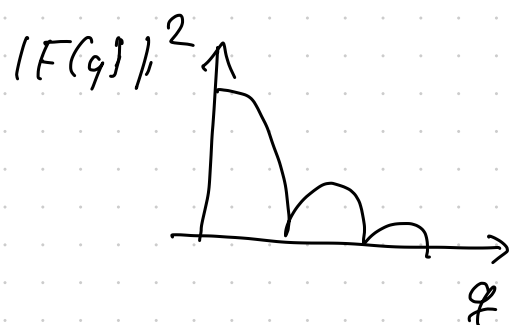
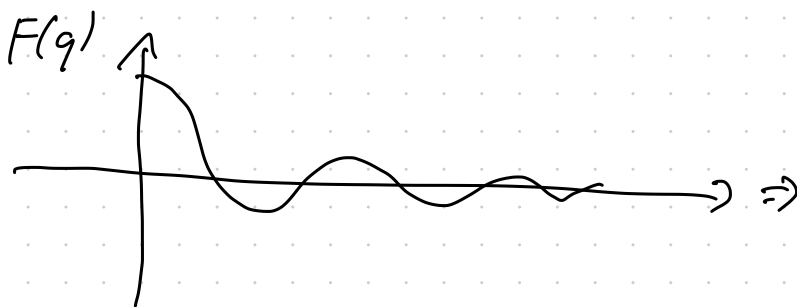
$$F(q) = \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} e^{iqr \cos \theta} \rho(r) r^2 dr \sin \theta d\theta d\varphi$$

$$= 2\pi \int_0^{\infty} \underbrace{\int_0^{\pi} e^{iqr \cos \theta} \sin \theta d\theta}_{\text{}} \rho(r) r^2 dr$$

$$\sin \theta d\theta = dz$$

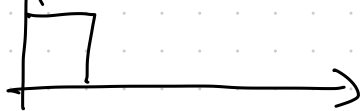
$$\int_{-1}^1 e^{iqr z} dz = \frac{1}{iqr} (e^{iqr} - e^{-iqr}) = \frac{2}{qr} \sin(qr)$$

$$F(q) = 4\pi \int_0^{\infty} \frac{\sin(qr)}{qr} \rho(r) r^2 dr = \frac{4\pi}{q} \int_0^{\infty} \rho(r) \sin(qr) r dr$$



Worauf sind wir sensitiv

$\rho(r)$



$$\rho(r) = \begin{cases} \text{const} & r \leq R \\ 0 & \text{sonst} \end{cases}$$

$$F(q) = \frac{3}{(qR)^2} \left(\frac{\sin qR}{qR} - \cos qR \right)$$

\Rightarrow 1. Nullstelle liefert uns Radius

\Rightarrow Verhältnis der Maxima \Rightarrow Randunschärfe

Wie bestimmt man $\Psi(E, \vec{p})$

Idee:

