

# TESTING BELL INEQUALITIES AT THE LHC WITH TOP PAIRS

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**a LHC Top Work Group presentation**

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## Shift of emphasis from QM to QFT

QM

- Energy levels
- Measurements
- Non-commuting observables

QFT

- S-matrix
- Fixed representations
- Commuting observables

where is the **quantum** in QFT?

entanglement, Bell inequalities



framing Bell inequalities in a easy-to-use form

## CHSH inequality

Clauser-Horne-Shimony-Holt

probabilities

$$\mathcal{P}(\uparrow_{\hat{n}_i}; -)$$

spin of one quark up  
in the direction  $n_j$

$$\mathcal{P}(\uparrow_{\hat{n}_i}; \downarrow_{\hat{n}_j})$$

spin of one quark up in the direction  $n_i$   
other quark spin down in the direction  $n_j$

PRO

## CHSH inequality

stochastic variables

$$\mathcal{P}(\uparrow_{\hat{n}_1}; -) = \int d\lambda \, \eta(\lambda) \, \underline{p_\lambda(\uparrow_{\hat{n}_1}; -)}$$

$$\int d\lambda \, \eta(\lambda) = 1$$

$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) = \int d\lambda \, \eta(\lambda) \, \underline{p_\lambda(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2})}$$

$$p_\lambda(\uparrow_{\hat{n}}; \downarrow_{\hat{m}}) = p_\lambda(\uparrow_{\hat{n}}; -) p_\lambda(-; \downarrow_{\hat{m}})$$

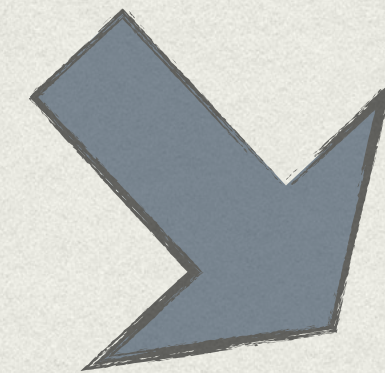
locality assumption

probability independence

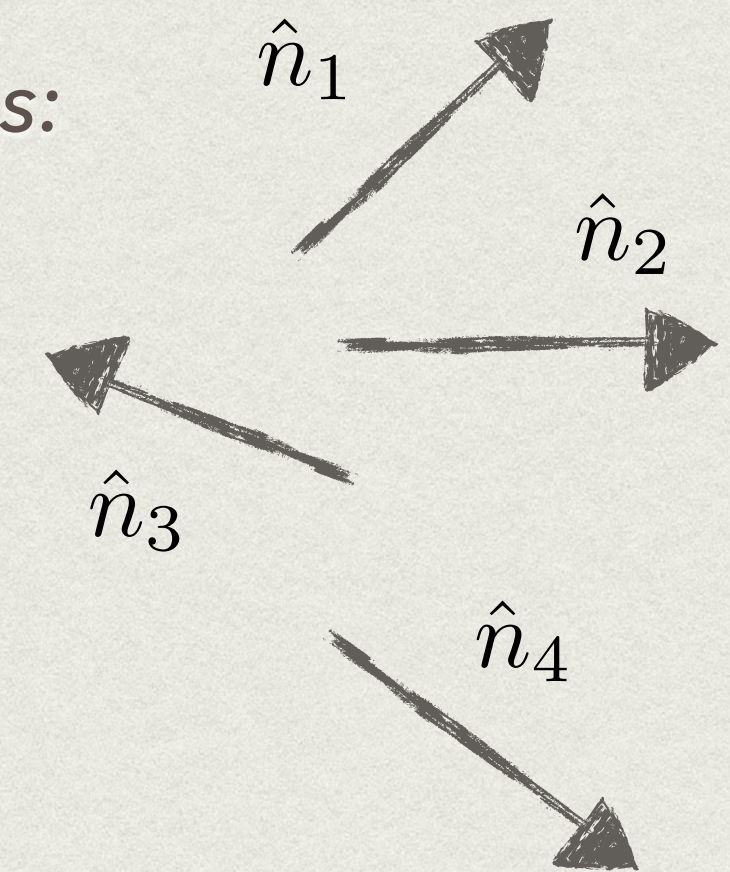
## CHSH inequality

any four non-negative numbers

$$x_1x_2 - x_1x_4 + x_3x_2 + x_3x_4 \leq x_3 + x_2$$



four directions:



$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_4}) \leq \mathcal{P}(\uparrow_{\hat{n}_3}; -) + \mathcal{P}(-; \uparrow_{\hat{n}_2})$$

## CHSH inequality

quantum state of two spin 1/2 particles

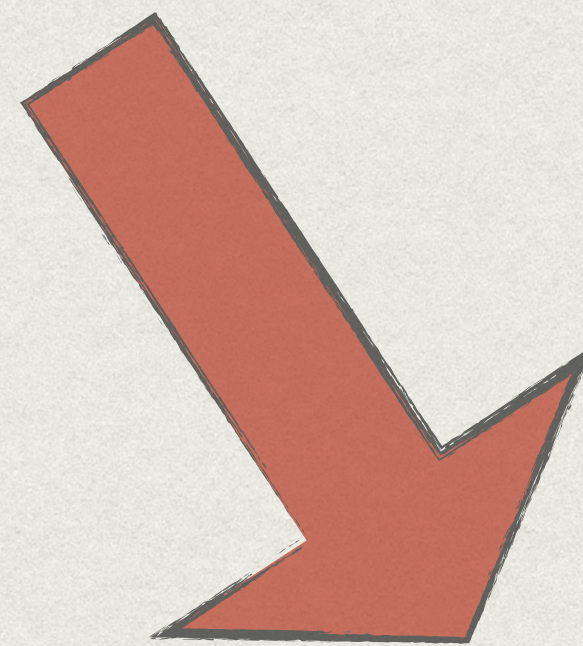
$$\rho = \frac{1}{4} \left[ 1 \otimes 1 + \sum_i A_i (\sigma_i \otimes 1) + \sum_j B_j (1 \otimes \sigma_j) + \sum_{ij} C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

take

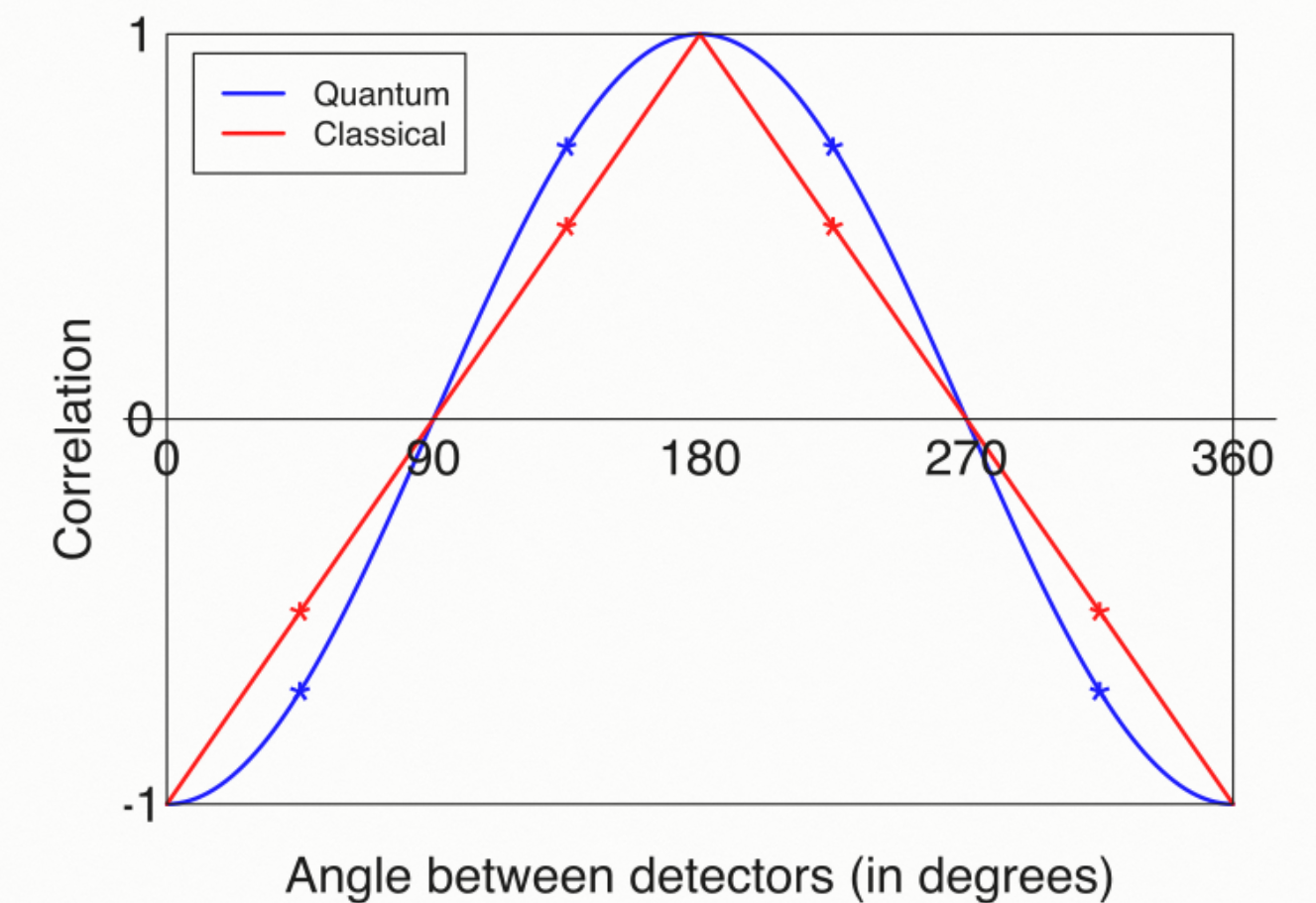
$$n_1 = \hat{z} \quad n_2 = \frac{-1}{\sqrt{2}}(\hat{z} + \hat{x}) \quad n_3 = \hat{x} \quad n_4 = \frac{1}{\sqrt{2}}(\hat{z} - \hat{x})$$

for maximal entanglement  $C_{ij} = -\delta_{ij}$



$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| = \boxed{2\sqrt{2}}$$

violation



Wikipedia

## Tests of violation of Bell inequalities

### entangled photons

Freedman&Clauser 1972  
Fry&Thomson, 1976  
Aspect group, 1982  
Zeilinger group, 1998

### Kaon system

Benatti&Floresanini, 1998  
Bertlmann et al., 2001

### other HEP systems

Positronium,  
Charmonium,  
neutrino oscillations

direct CP violation = Bell inequalities violation

# from the CHSH inequality to a single observable

Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Rev. Mod. Phys. 81, 865-942 (2009)

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

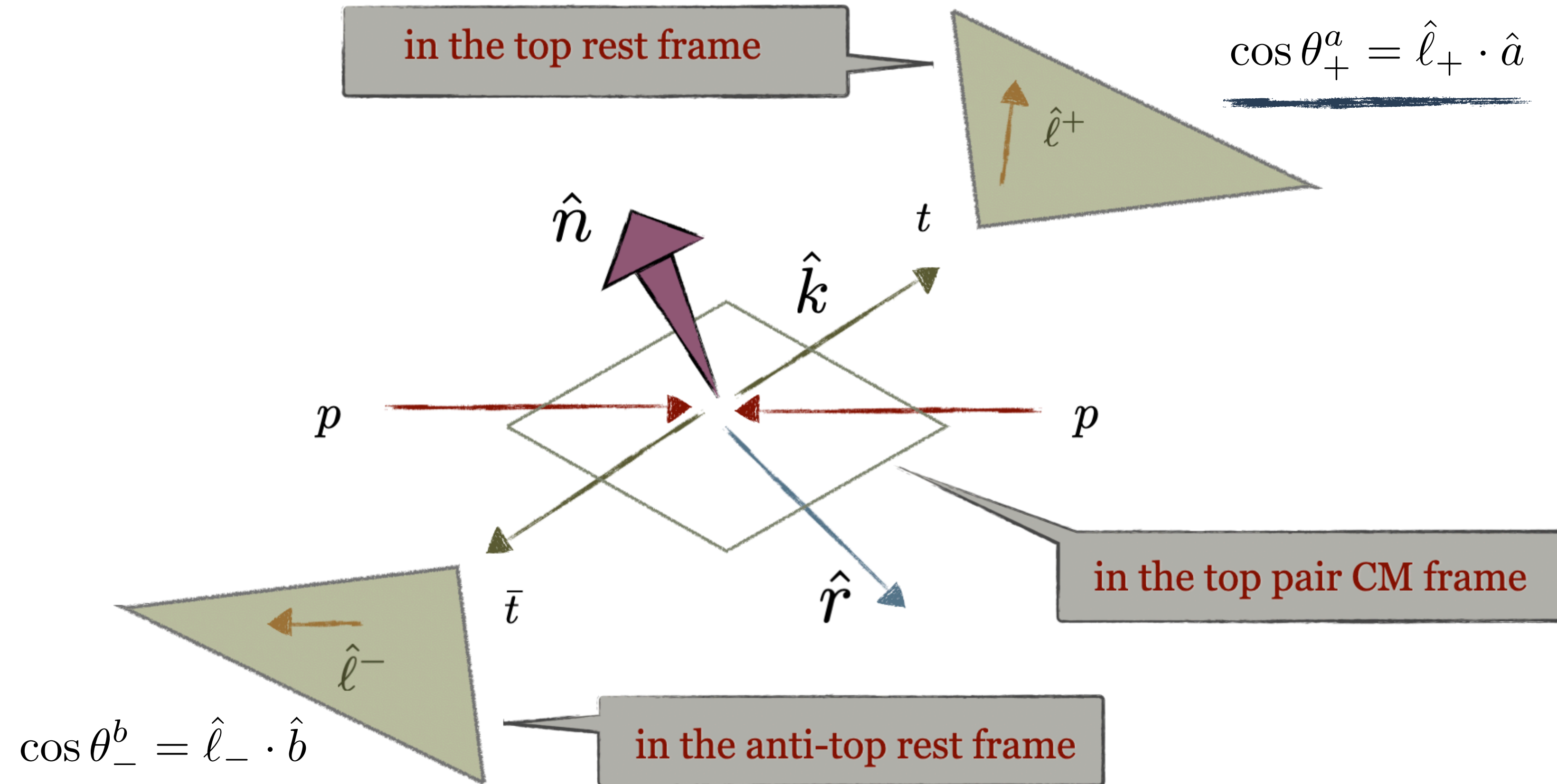
$$M = C^T C$$

$$m_1 \geq m_2 \geq m_3$$

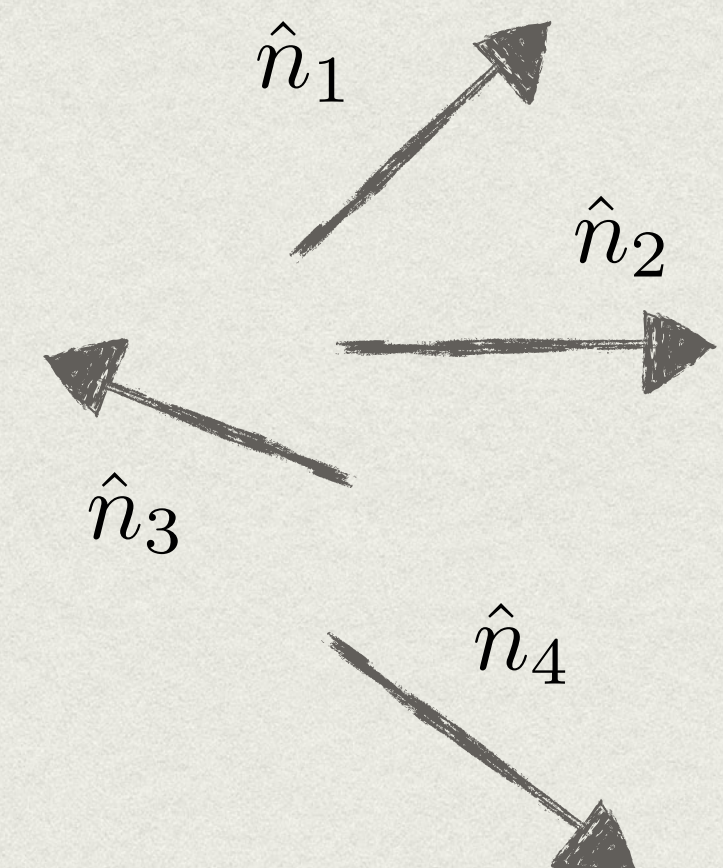

$$m_1 + m_2 \leq 1$$

# Implementing at the LHC

$$pp \rightarrow t + \bar{t} \rightarrow \ell^{\pm} \ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$



$$\begin{aligned} \hat{r} &= \frac{1}{r}(\hat{p} - y\hat{k}) \\ y &= \hat{p} \cdot \hat{k} \\ r &= \sqrt{1 - y^2} \\ \hat{n} &= \frac{1}{r}(\hat{p} \times \hat{k}) \end{aligned}$$





## Event generation

**MadGraph5** (NNPDF23)  
**DELPHES** (fast simulation  
ATLAS detector)

exactly two opposite sign leptons (e,mu) of different flavor

- at least 2 anti-k<sub>t</sub> jets with R=0.4
- at least 1 b-tagged jet
- $p_T > 25 \text{ GeV}$   $|\eta| < 2.5$  jets
- $p_T > 20 \text{ GeV}$   $|\eta| < 2.47$  leptons
- neutrino weighting technique (top quark momenta)

## Implementing at the LHC

W. Bernreuther, D. Heisler and Z. G. Si, JHEP 12, 026 (2015)  
Y. Afik and J. R. M. de Nova, [[arXiv:2003.02280](#) [quant-ph]].

$$pp \rightarrow t + \bar{t} \rightarrow \ell^{\pm} \ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\xi_{ab} = \cos \theta_+^a \cos \theta_-^b$$

3 x 3 matrix

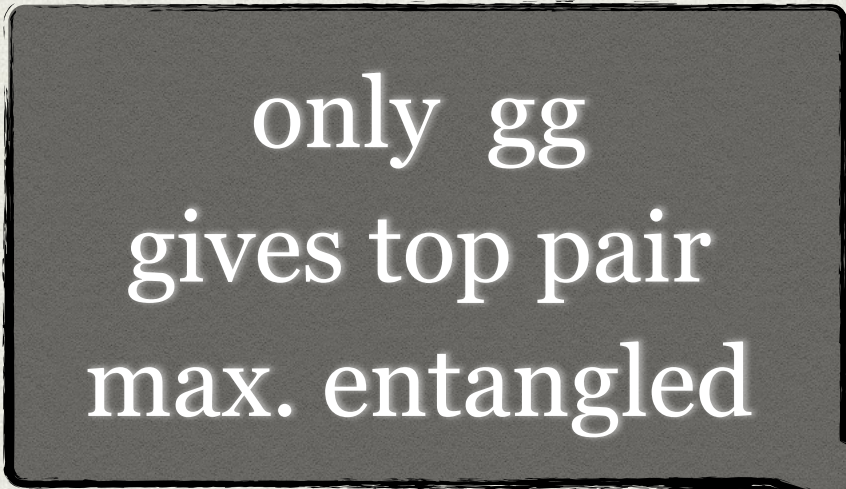
$$C_{ab} [\sigma(m_{t\bar{t}}, \cos \Theta)] = -9 \frac{1}{\sigma} \int d\xi_{ab} \frac{d\sigma}{d\xi_{ab}} \xi_{ab}$$

diagonalization for each value  
of invariant mass and scattering angle

	label	$\hat{a}$	$\hat{b}$
transverse	n	$\text{sign}(y_p) \hat{\mathbf{n}}_p$	$-\text{sign}(y_p) \hat{\mathbf{n}}_p$
r axis	r	$\text{sign}(y_p) \hat{\mathbf{r}}_p$	$-\text{sign}(y_p) \hat{\mathbf{r}}_p$
helicity	k	$\hat{\mathbf{k}}$	$-\hat{\mathbf{k}}$

$$m_1 + m_2 > 1$$

analysis





## Results

bins

$$\frac{2\Theta}{\pi} \gtrsim 0.7 \quad m_{t\bar{t}} \gtrsim 0.9 \text{ TeV}$$

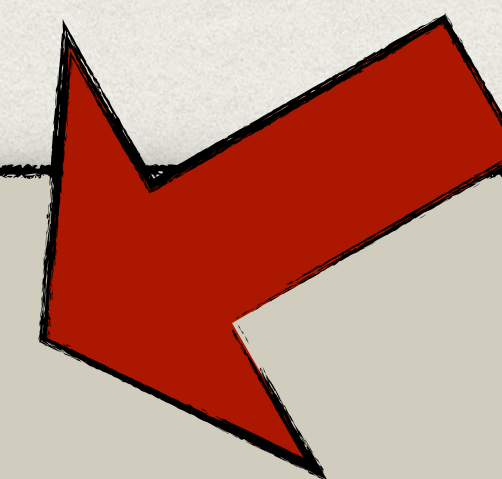
1.1	1.3	1.6
1.0	1.2	1.4
0.9	1.1	1.2

null hypothesis:  $m_1 + m_2 \leq 1$

Hypothesis test

$$\chi^2 = \sum_i \frac{(1 - m_1^i - m_2^i)^2}{s_i^2}$$

violation: **98% CL** w/ Run II data (139 fb<sup>-1</sup>)  
**99.99% CL** with Run III



systematic uncertainties (e.g. from unfolding) not included

Bell inequalities can be tested with LHC data already acquired

it is an important test never performed at these energies

it consists in straightforward physical analysis of just one observable

*in case someone asks*



$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( \left| +\frac{1}{2} \right\rangle_1 \left| -\frac{1}{2} \right\rangle_2 - \left| -\frac{1}{2} \right\rangle_1 \left| +\frac{1}{2} \right\rangle_2 \right)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( \left| +\frac{1}{2} \right\rangle_1 \left| -\frac{1}{2} \right\rangle_2 - \left| -\frac{1}{2} \right\rangle_1 \left| +\frac{1}{2} \right\rangle_2 \right)$$



$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( \left| +\frac{1}{2} \right\rangle_1 \left| -\frac{1}{2} \right\rangle_2 - \left| -\frac{1}{2} \right\rangle_1 \left| +\frac{1}{2} \right\rangle_2 \right)$$

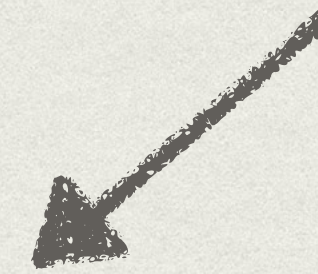
advantage

the study of the spin correlation matrix  $C$ ,  
without the need of any a priori commitment  
about efficiencies of detectors

loophole

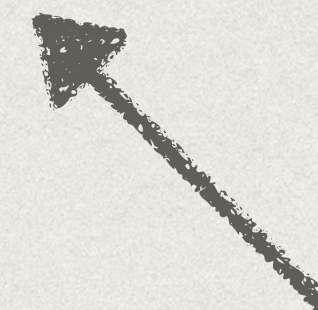
a deterministic theory that predetermines  
the statistics of future spin measurements  
at the moment of the quark-antiquark production

s-channel, spin quarks both 1/2



at threshold:  $q\bar{q}$  maximally correlated but separable  
 $gg$  anti-correlated and entangled


large  $m_{t\bar{t}}$  both maximally entangled



higher angular momentum states

# Measurement of the top quark polarization and $t\bar{t}$ spin correlations using dilepton final states in proton-proton collisions at $\sqrt{s}=13$ TeV

A. M. Sirunyan *et al.*\*  
(CMS Collaboration)

 (Received 8 July 2019; published 8 October 2019)

Measurements of the top quark polarization and top quark pair ( $t\bar{t}$ ) spin correlations are presented using events containing two oppositely charged leptons ( $e^+e^-$ ,  $e^\pm\mu^\mp$ , or  $\mu^+\mu^-$ ) produced in proton-proton collisions at a center-of-mass energy of 13 TeV. The data were recorded by the CMS experiment at the LHC in 2016 and correspond to an integrated luminosity of  $35.9 \text{ fb}^{-1}$ . A set of parton-level normalized differential cross sections, sensitive to each of the independent coefficients of the spin-dependent parts of the  $t\bar{t}$  production density matrix, is measured for the first time at 13 TeV. The measured distributions and extracted coefficients are compared with standard model predictions from simulations at next-to-leading-order (NLO) accuracy in quantum chromodynamics (QCD), and from NLO QCD calculations including electroweak corrections. All measurements are found to be consistent with the expectations of the standard model. The normalized differential cross sections are used in fits to constrain the anomalous chromomagnetic and chromoelectric dipole moments of the top quark to  $-0.24 < C_{tG}/\Lambda^2 < 0.07 \text{ TeV}^{-2}$  and  $-0.33 < C'_{tG}/\Lambda^2 < 0.20 \text{ TeV}^{-2}$ , respectively, at the 95% confidence level.

DOI: [10.1103/PhysRevD.100.072002](https://doi.org/10.1103/PhysRevD.100.072002)

## $t\bar{t}$ Spin Density-Matrix Measurement Analysis overview and unfolding

Théo Megy, on behalf of the analysis team

Laboratoire de Physique de Clermont  
07.10.2020



