Theoretical Precision for Jet Substructure

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Reed College

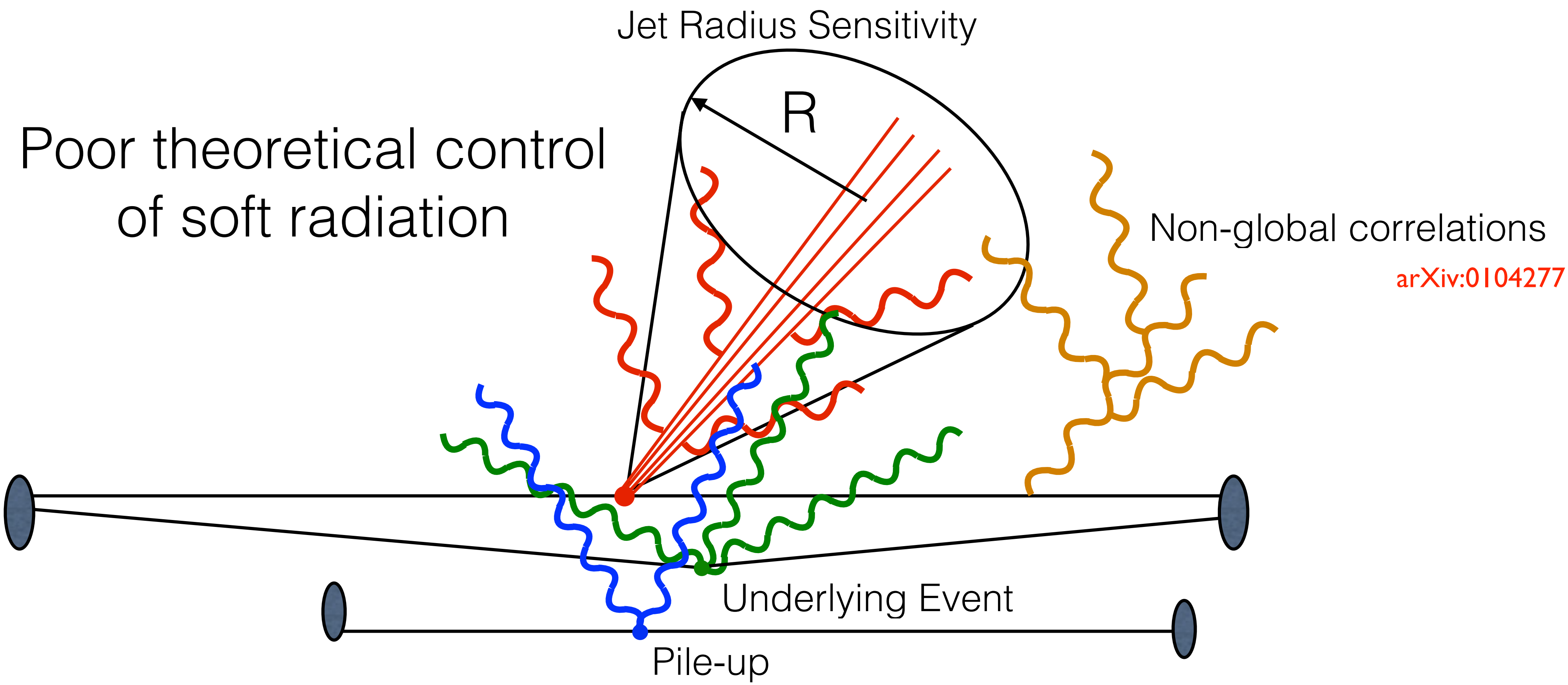
Jets and their substructure from LHC data, 2 June, 2021

Theoretical Motivation for Jet Grooming

Jets at the LHC are messy things!

Final-state radiation + Underlying Event
+ Pile-Up + Non-global effects + …

Poor theoretical control
of soft radiation



Theoretical Motivation for Jet Grooming

Jets at the LHC are messy things!

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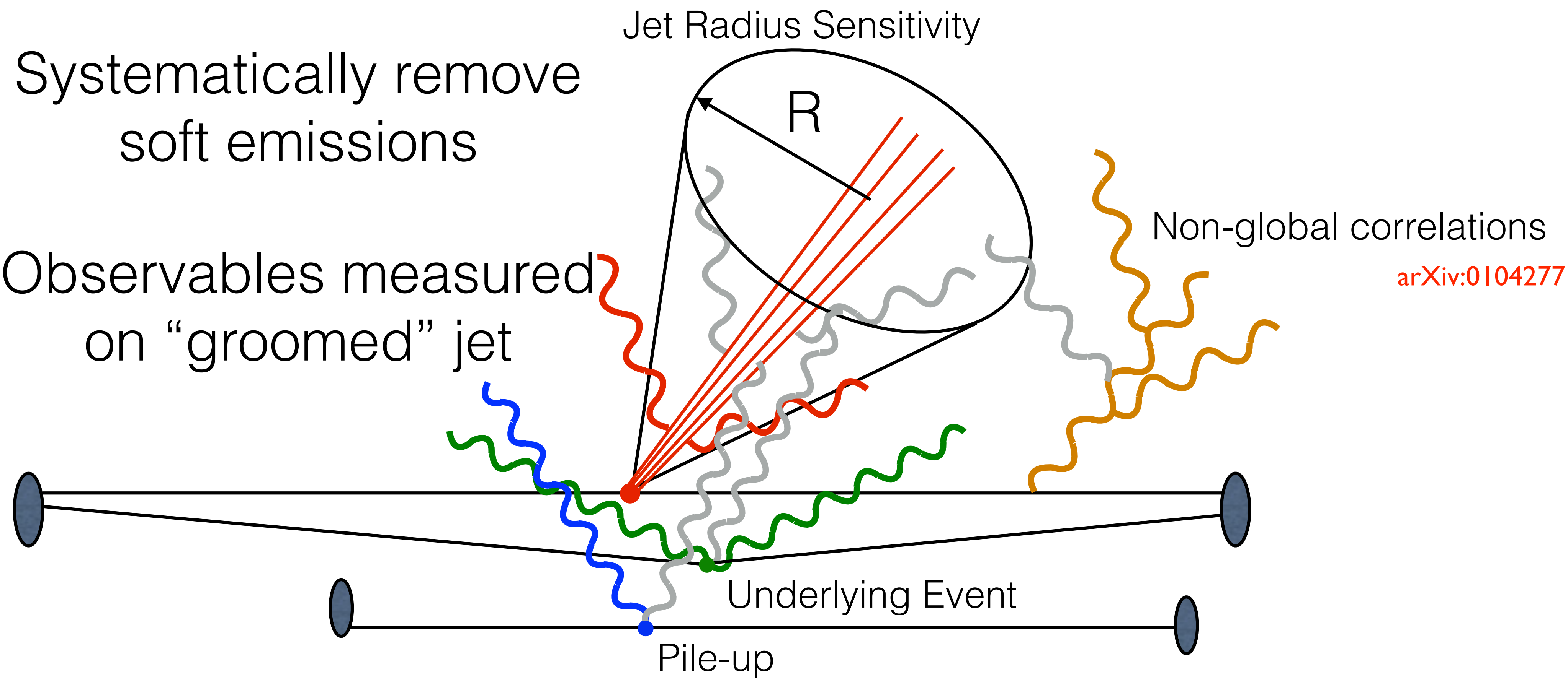
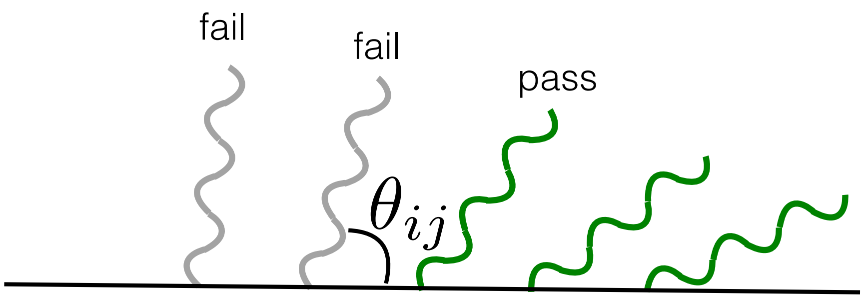
Systematically remove soft emissions

Observables measured on “groomed” jet



mMDT/soft drop Grooming



Angular-ordered emissions

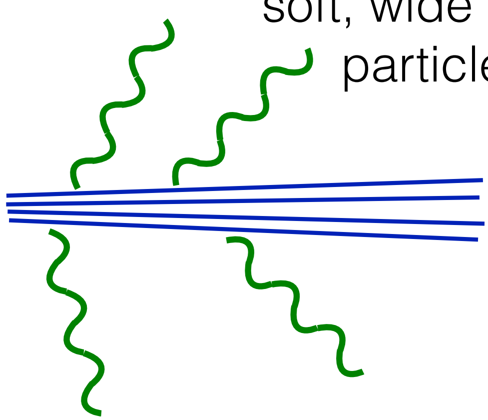
$$\frac{\min[p_{Ti}, p_{Tj}]}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left(\frac{\theta_{ij}}{R}\right)^{\beta}$$

arXiv:1307.0007, 1402.2657

β = 0: mMDT

Only jet groomer that removes non-global emissions!

Relevant regime: $m_J^2 \ll z_{\text{cut}} p_T^2 \ll p_T^2$



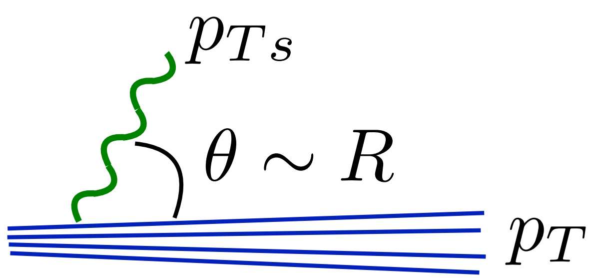
soft, wide angle particle *i*

1) $\dfrac{p_{Ti}}{p_T} \sim z_{\text{cut}} \longrightarrow m_J^2 \sim z_{\text{cut}} p_T^2$

2) $\dfrac{p_{Ti}}{p_T} \sim \dfrac{m_J^2}{p_T^2} \longrightarrow$ groomed away

Perturbative Lever Arm with Grooming

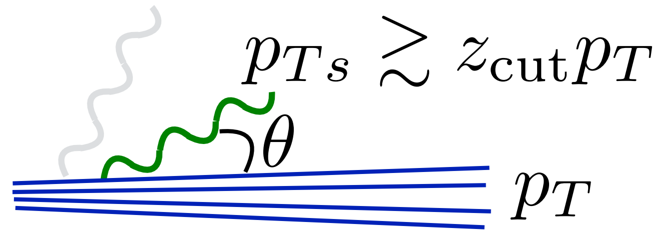
Ungroomed Jet Mass



$$p_{Ts} R \simeq \Lambda_{\rm QCD}$$

$$\frac{m_J^2}{p_T^2} \simeq \frac{p_{Ts}}{p_T} R^2 \simeq \frac{\Lambda_{\rm QCD} R}{p_T}$$

mMDT Groomed Jet Mass



$$p_{Ts} \theta \simeq \Lambda_{\rm QCD}$$

$$\frac{m_J^2}{p_T^2} \simeq \frac{p_{Ts}}{p_T} \theta^2 \simeq \frac{\Lambda_{\rm QCD}^2}{z_{\rm cut} p_T^2}$$

Parametrically larger perturbative regime with grooming!

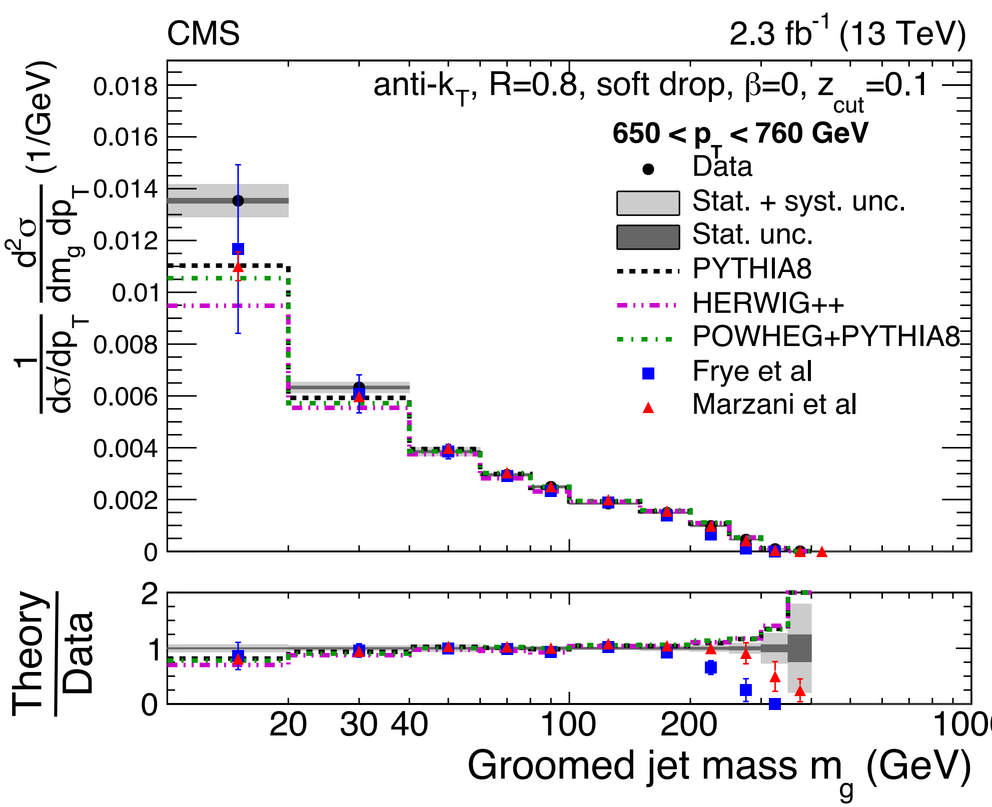
Increase of ~100x for TeV-scale jets

New Precision Measurements

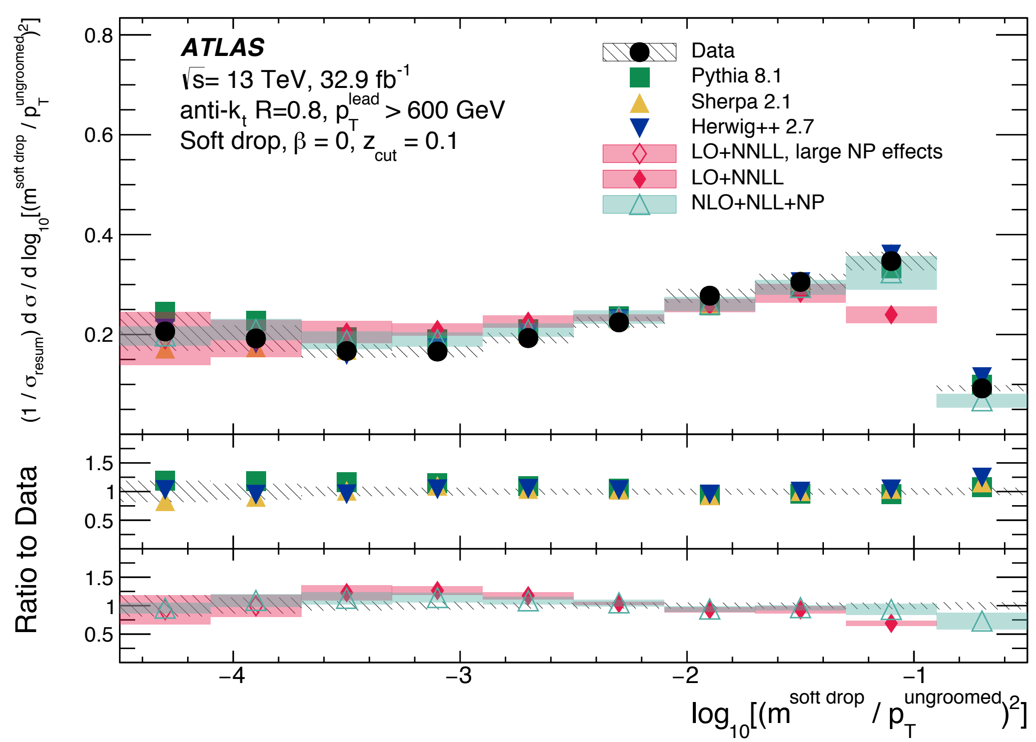
Unprecedented probe of jets at LHC

Sensitivity to masses over 2+ decades

≤10% experimental uncertainty



arXiv:1807.05974



arXiv:1711.08341

New Precision Measurements

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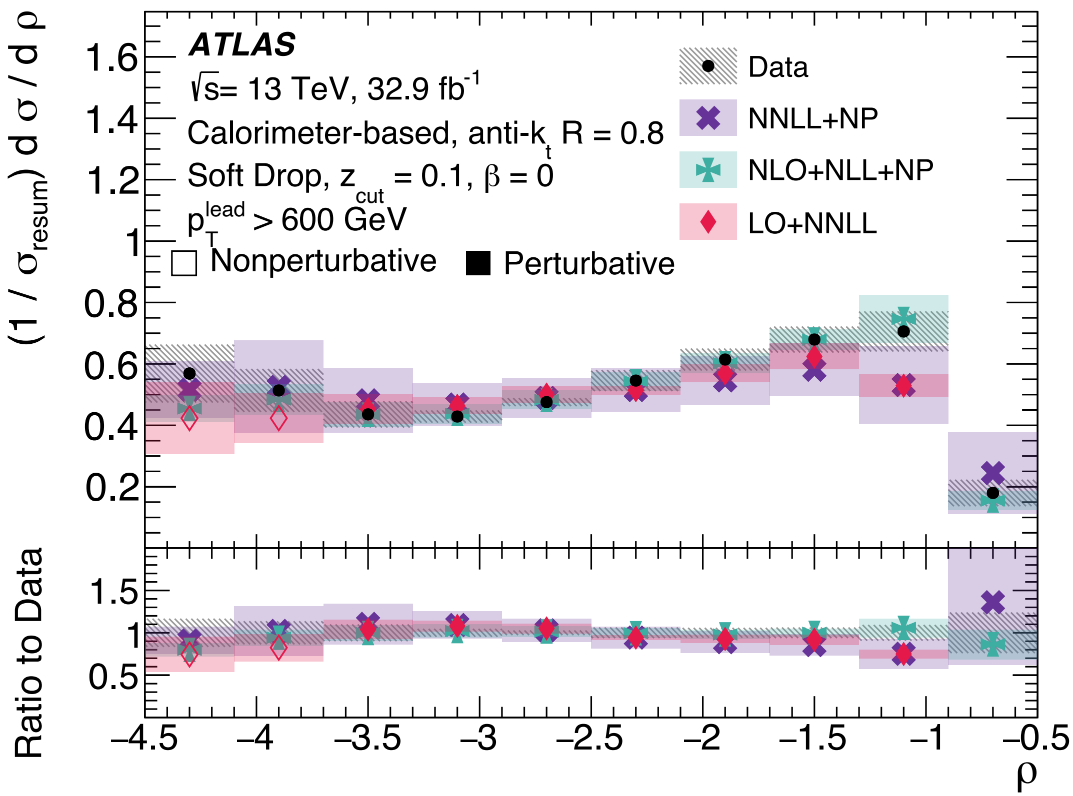
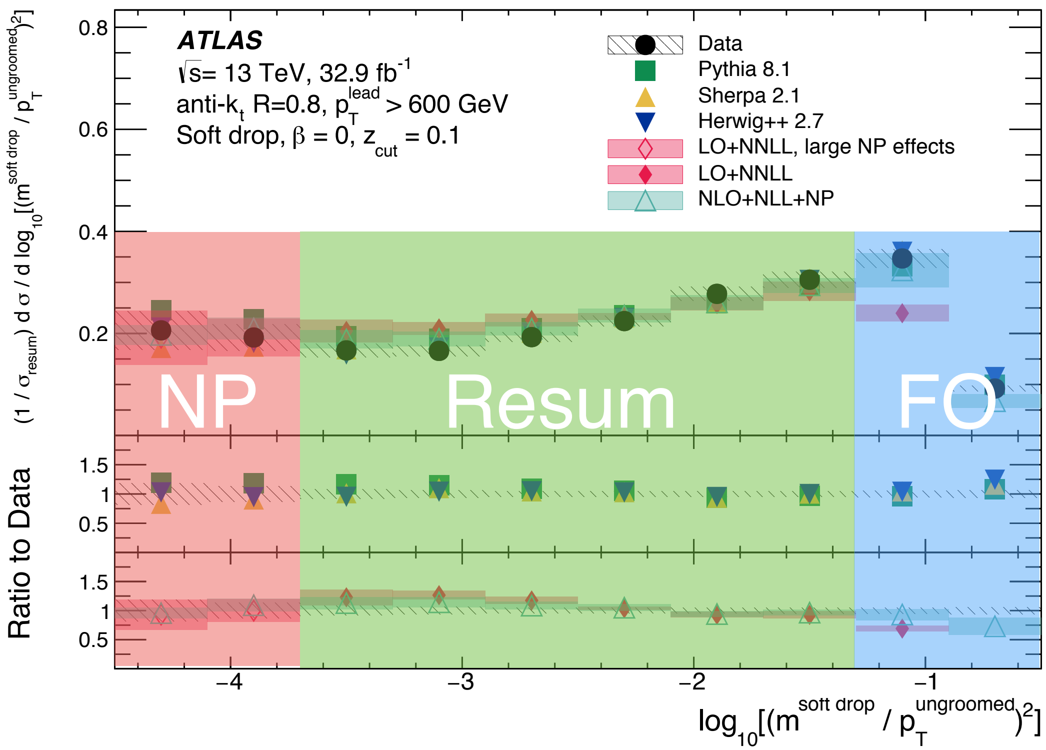


Experimental plot with only data and analytic predictions!

arXiv:1912.09837

Perturbative Lever Arm with Grooming



ATLAS jet mass data

Regions:

$\rho > z_{cut}$ (fixed-order)

$\rho < z_{cut}$ (resummed)

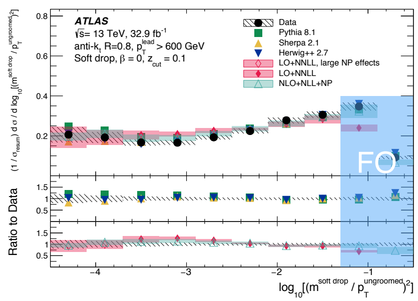
$\rho < z_{cut}(\Lambda_{QCD}/z_{cut}\, p_T)^2$ (non-pert)

$\rho = (m_J/p_T)^2$

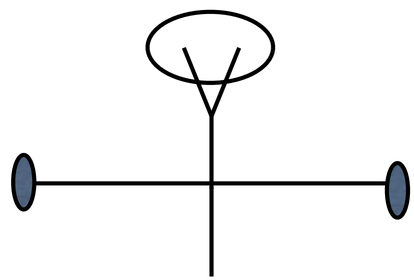
Potential α_s fits over dynamic range of ~100x!

Perturbative Lever Arm with Grooming

Fixed-Order Corrections



$\rho > z_{cut}$

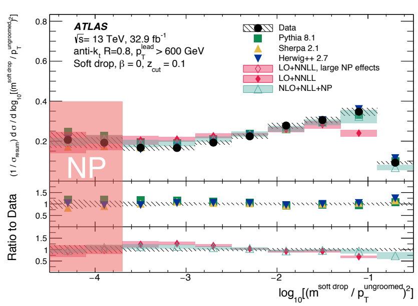


$m_J > 0$: 2 → 3 process

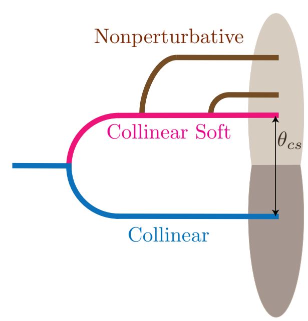
Lots of recent progress at NNLO

arXiv:1712.02229,1712.03946,1805.09182, 1809.09067,1811.11699,1812.04586,1812.11160, 1904.00945,1905.03733,1906.11862,…

Non-perturbative Corrections



$\rho < z_{cut}(\Lambda_{QCD}/z_{cut}\,p_T)^2$



Multiple moments encode NP corrections

$$\frac{d\sigma_\kappa^{\rm had}}{dm_J^2} = \frac{d\hat{\sigma}_\kappa}{dm_J^2} - \frac{Q\Omega_{1\kappa}^{\circ\!\circ}}{m_J^2}\frac{d\Delta\hat{\sigma}_\kappa^{\circ\!\circ}}{dm_J^2} + \frac{Q\Upsilon_1^\kappa(\beta)}{m_J^2}\frac{d\Delta\hat{\sigma}_\kappa^{\circledcirc}}{dm_J^2}$$

arXiv:1906.11843

also arXiv:1307.0007, 1712.05105

This Talk

Fixed-Order Corrections

$m_J > 0$: 2 → 3 process

Lots of recent progress at NNLO

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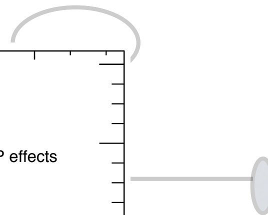
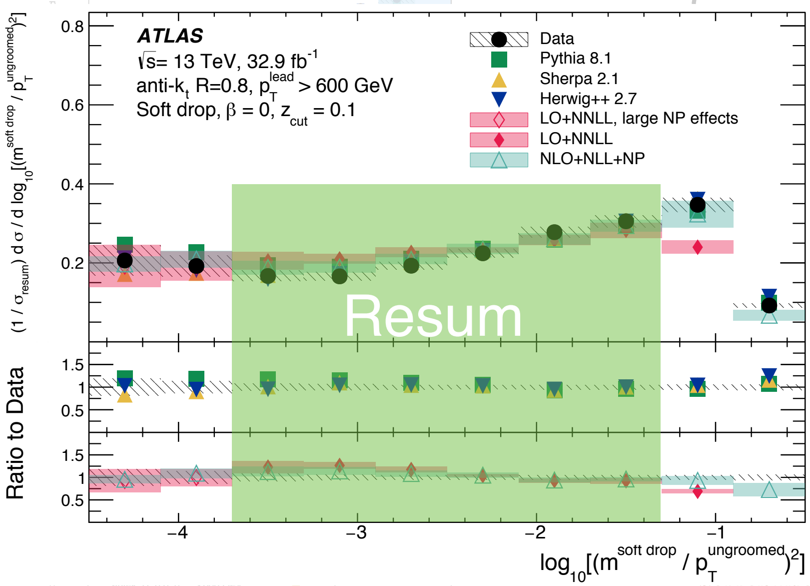
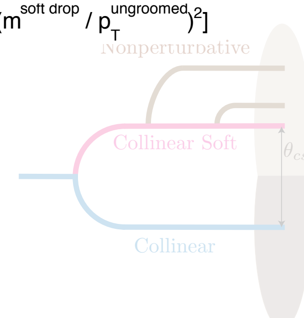




Bulk of distribution: Resummation Regime

...ative Corrections



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$\rho < z_{cut}(\Lambda_{QCD}/z_{cut}\, p_T)^2$

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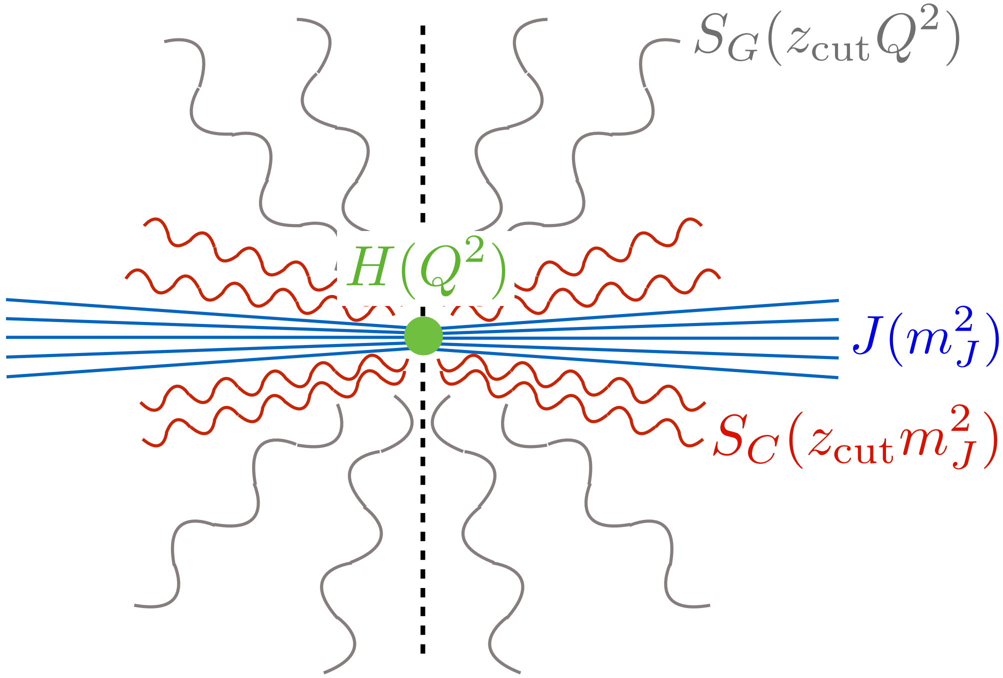
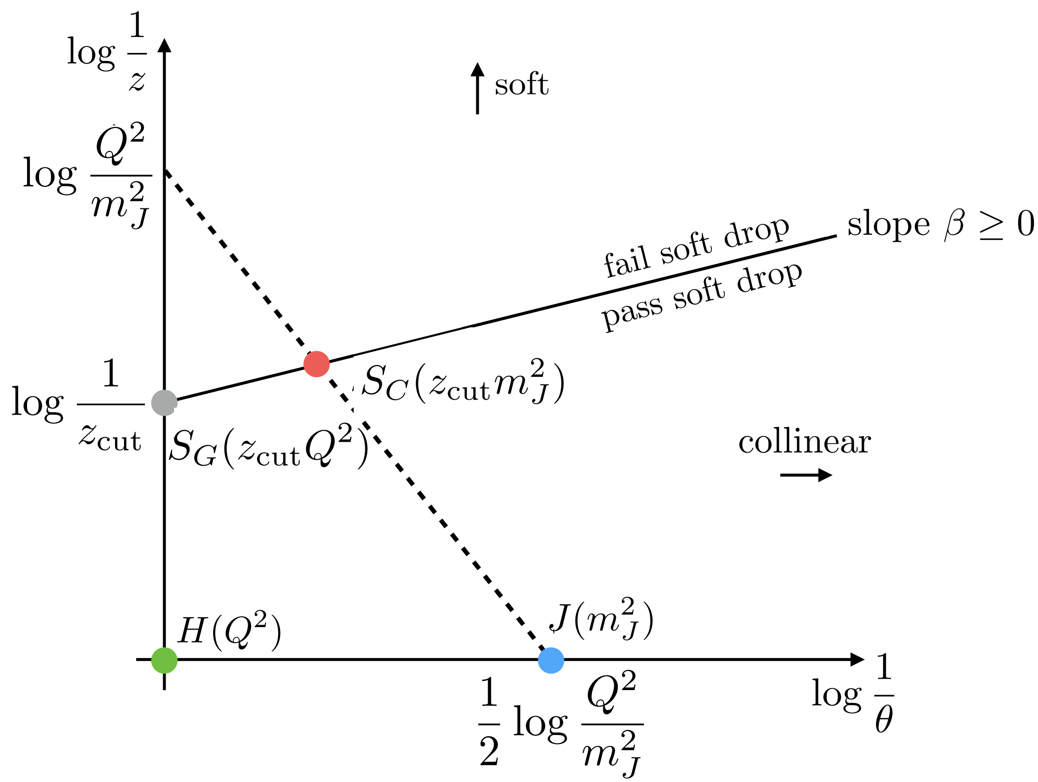
Factorization Theorem for Jet Grooming

$$m_J^2 \ll z_{\text{cut}} Q^2 \ll Q^2$$





$H(Q^2)$ e+e- → qq virtuals

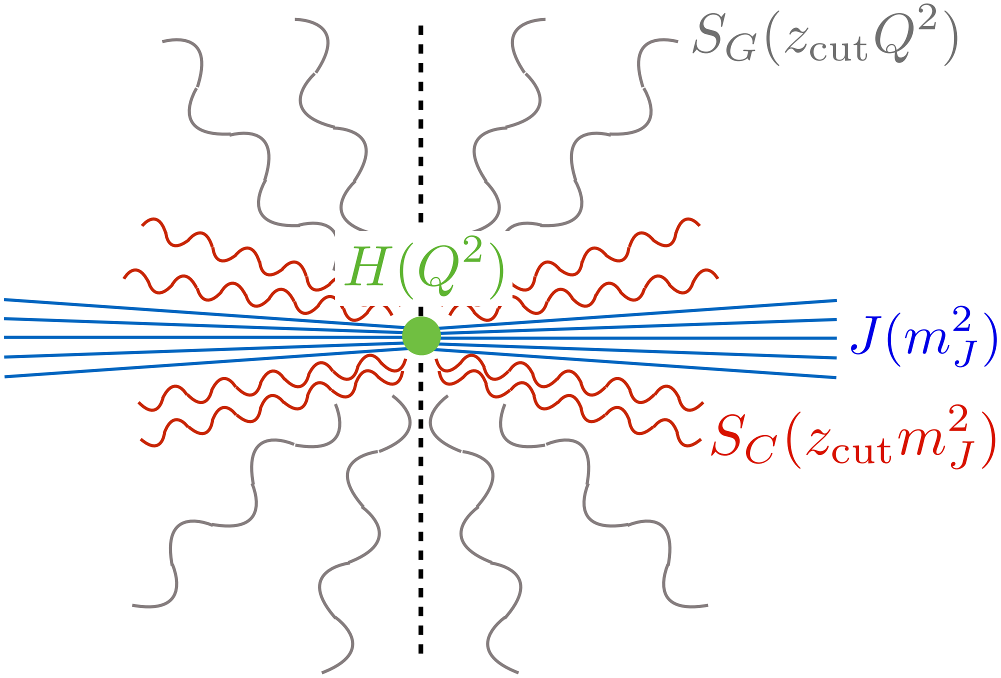
$J(m_J^2)$ ungroomed collinears

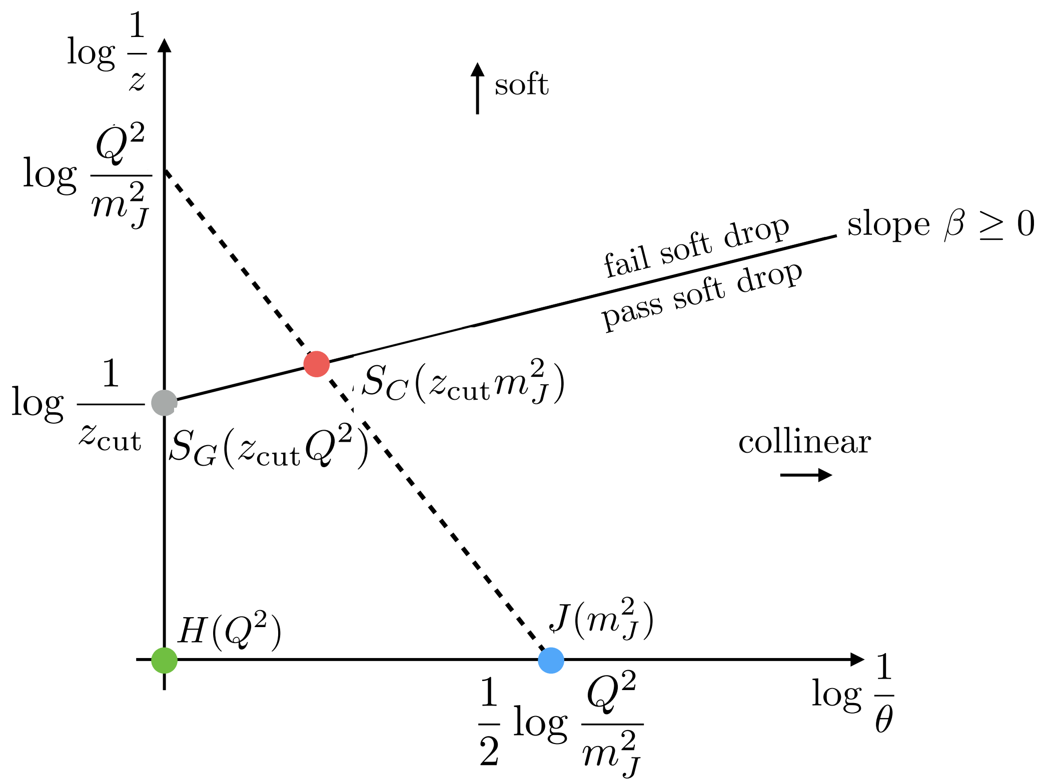
$S_G(z_{\text{cut}} Q^2)$ groomed away softs

$S_C(z_{\text{cut}} m_J^2)$ groomed collinear-softs

Factorization Theorem for Jet Grooming

$$m_J^2 \ll z_{\rm cut} Q^2 \ll Q^2$$



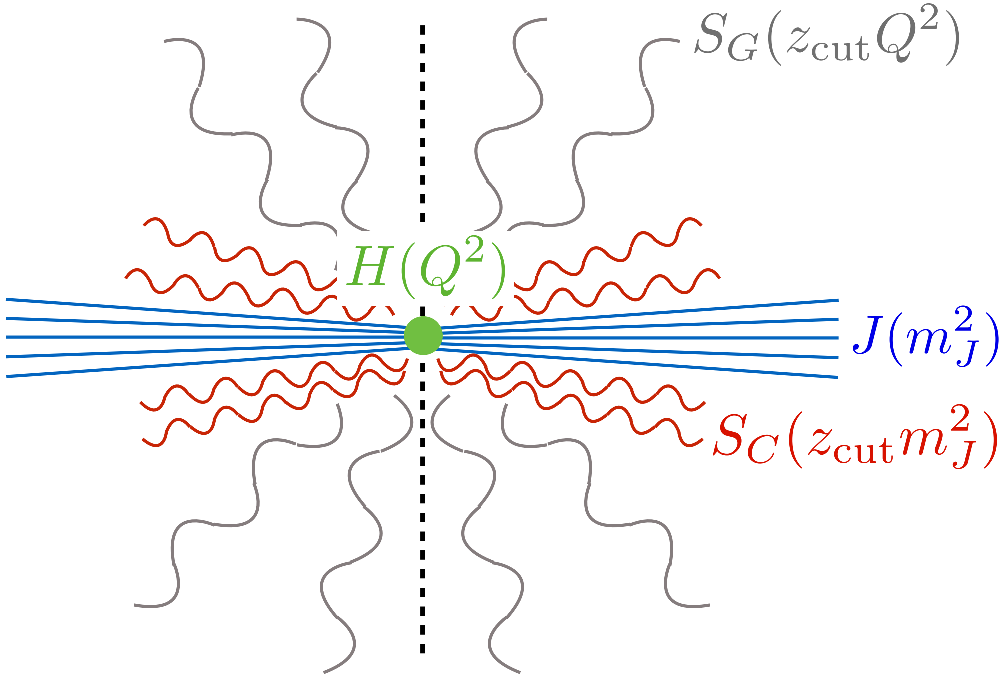


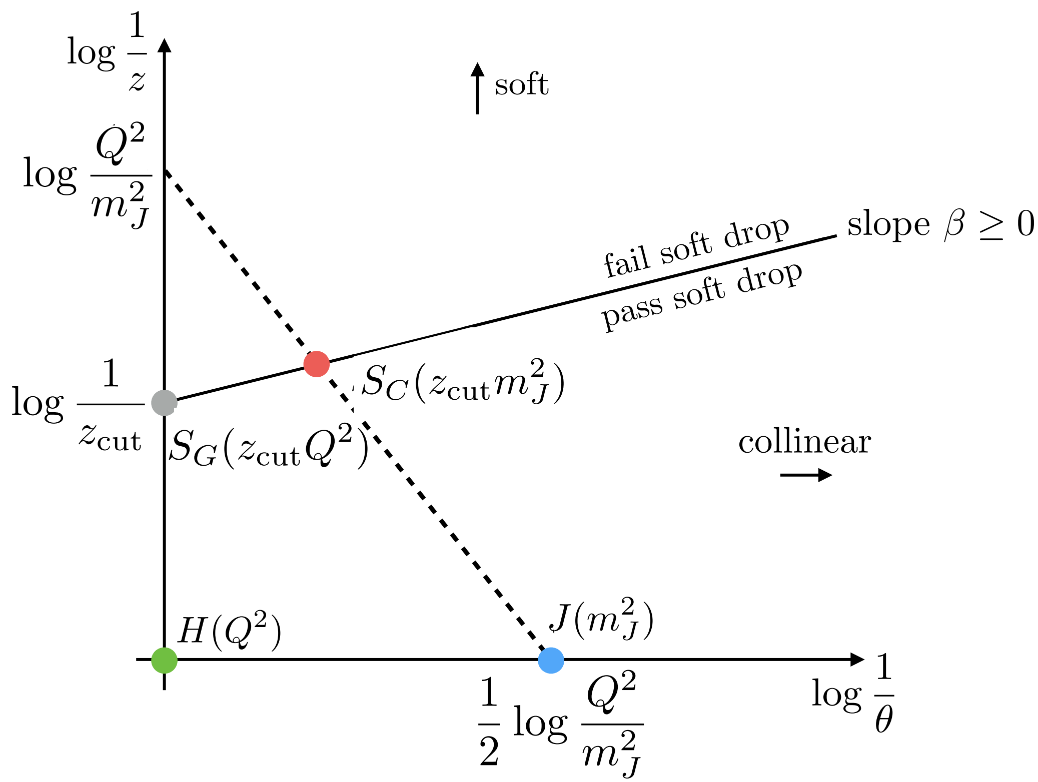
Leading-Power Factorization Theorem:

$$\frac{d^2\sigma}{dm_{J,L}^2\, dm_{J,R}^2} = H(Q^2) S_G(z_{\rm cut}Q^2) \left[S_C(z_{\rm cut}m_{J,L}^2) J(m_{J,L}^2)\right] \left[S_C(z_{\rm cut}m_{J,R}^2) J(m_{J,R}^2)\right]$$

Factorization Theorem for Jet Grooming

$$m_J^2 \ll z_{\text{cut}} Q^2 \ll Q^2$$

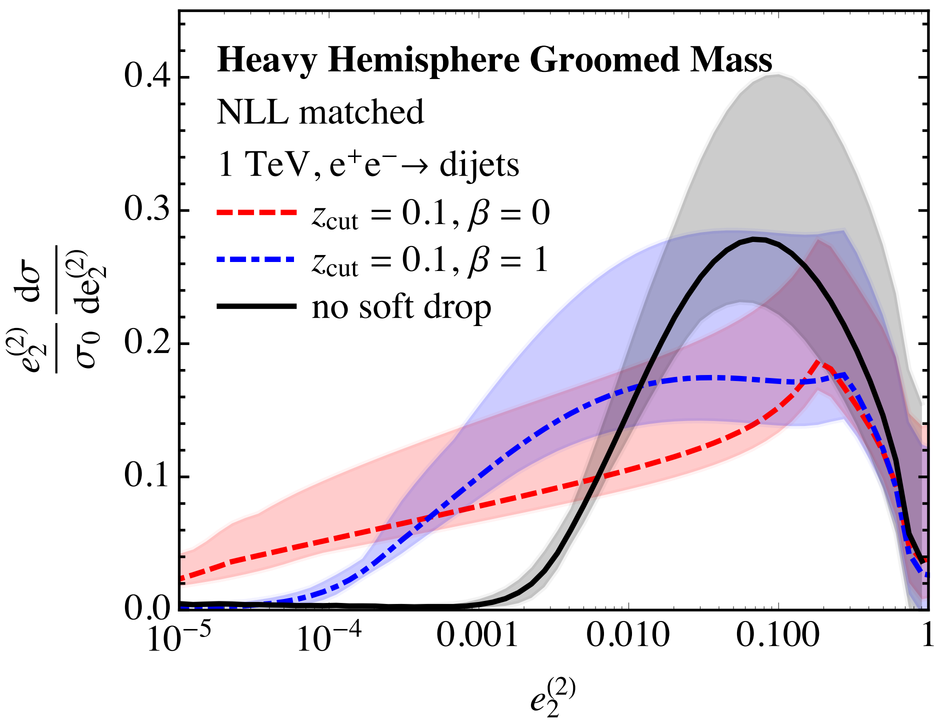


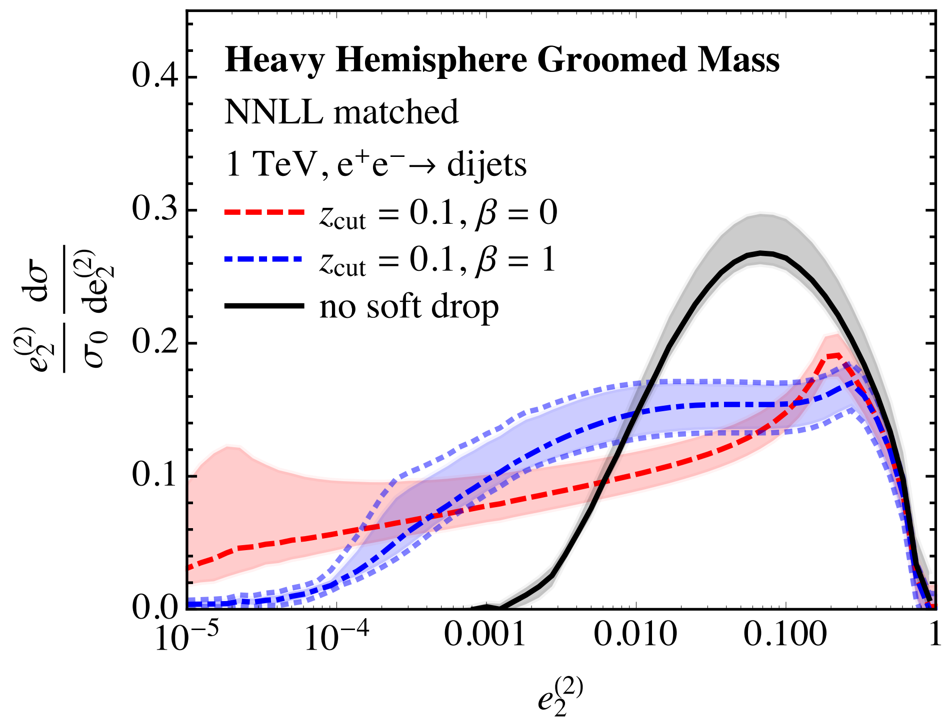


Resummation by Renormalization Group Running

$$\mu\frac{\partial F}{\partial \mu} = \left(\Gamma_{\text{cusp}}\log\frac{\mu}{\mu_F} + \gamma_F\right) F$$

Factorization Theorem for Jet Grooming





NLL+LO and NNLL+NLO with different levels of grooming

Significant reduction of scale uncertainties

Dramatically increased dynamic range

arXiv:1603.09338, 2007.10355

Going Beyond NNLL

Ingredients necessary for resummation:

	Γ_{cusp}	γ_F	β	c_F	Matching
LL	α_s	-	α_s	-	-
NLL	α_s^2	α_s	α_s^2	-	α_s
NNLL	α_s^3	α_s^2	α_s^3	α_s	α_s^2
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

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NLL	α_s^2	α_s	α_s^2	-	α_s
NNLL	α_s^3	α_s^2	α_s^3	α_s	α_s^2
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

↑ Can match to NNLO

Going Beyond NNLL

Ingredients necessary for resummation:

	$\Gamma_{\rm cusp}$	γ_F	β	c_F	Matching
LL	$\alpha_{\rm s}$	-	$\alpha_{\rm s}$	-	-
NLL	$\alpha_{\rm s}^2$	$\alpha_{\rm s}$	$\alpha_{\rm s}^2$	-	$\alpha_{\rm s}$
NNLL	$\alpha_{\rm s}^3$	$\alpha_{\rm s}^2$	$\alpha_{\rm s}^3$	$\alpha_{\rm s}$	$\alpha_{\rm s}^2$
NNNLL	$\alpha_{\rm s}^4$	$\alpha_{\rm s}^3$	$\alpha_{\rm s}^4$	$\alpha_{\rm s}^2$	$\alpha_{\rm s}^3$

↑

Now analytically known at four-loops

arXiv:9401214, 1610.07477, 1612.04389, 1604.03126, 1707.08315, 1805.09638, 1901.03693, 1901.02898, 1902.08208, 1902.05076, 1911.10174, 2002.04617

Going Beyond NNLL

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LL	α_s	-	α_s	-	-
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NNLL	α_s^3	α_s^2	α_s^3	α_s	α_s^2
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

↑ (β at NNNLL)

Known for a long time

arXiv:9701390

Going Beyond NNLL

Ingredients necessary for resummation:

	Γ_{cusp}	γ_F	β	c_F	Matching
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NLL	α_s^2	α_s	α_s^2	-	α_s
NNLL	α_s^3	α_s^2	α_s^3	α_s	α_s^2
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

$$\frac{d\sigma}{dm^2} \sim H(Q^2) S(z_{\mathrm{cut}} Q^2) J(m^2) S_c(z_{\mathrm{cut}} m^2)$$

Two-loop constants and three-loop anomalous dimensions known

arXiv:0507039, 0507061, 0902.3519, 1001.2887, 1004.3653

Going Beyond NNLL

Ingredients necessary for resummation:

	Γ_{cusp}	γ_F	β	c_F	Matching
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NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

$$\frac{d\sigma}{dm^2} \sim H(Q^2)S(z_{\text{cut}}Q^2)J(m^2)S_c(z_{\text{cut}}m^2)$$

Same jet function as for thrust, *C*-parameter

Two-loop constants and three-loop anomalous dimensions known

arXiv:0803.0342, 0607228

Going Beyond NNLL

Ingredients necessary for resummation:

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NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

$$\frac{d\sigma}{dm^2} \sim H(Q^2) S(z_{\text{cut}} Q^2) J(m^2) S_c(z_{\text{cut}} m^2)$$

Two-loop constants recently calculated

arXiv:1805.12414, 1812.08690, 2004.08396

Three-loop anomalous dimension recently extracted with MCCSM at NNLO

arXiv:1603.08927, 1606.03453, 1708.04093, 1807.11472, 2002.05730

Going Beyond NNLL

Ingredients necessary for resummation:

	Γ_{cusp}	γ_F	β	c_F	Matching
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NLL	α_s^2	α_s	α_s^2	-	α_s
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NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

$$\frac{d\sigma}{dm^2} \sim H(Q^2)S(z_{\text{cut}}Q^2)J(m^2)S_c(z_{\text{cut}}m^2)$$

Two-loop constants recently extracted from EVENT2

arXiv:9605323, 2002.05730

Three-loop anomalous dimension constrained by

$$0 = \gamma_H + \gamma_S + 2\gamma_J + 2\gamma_{S_c}$$

Going Beyond NNLL

Ingredients necessary for resummation:

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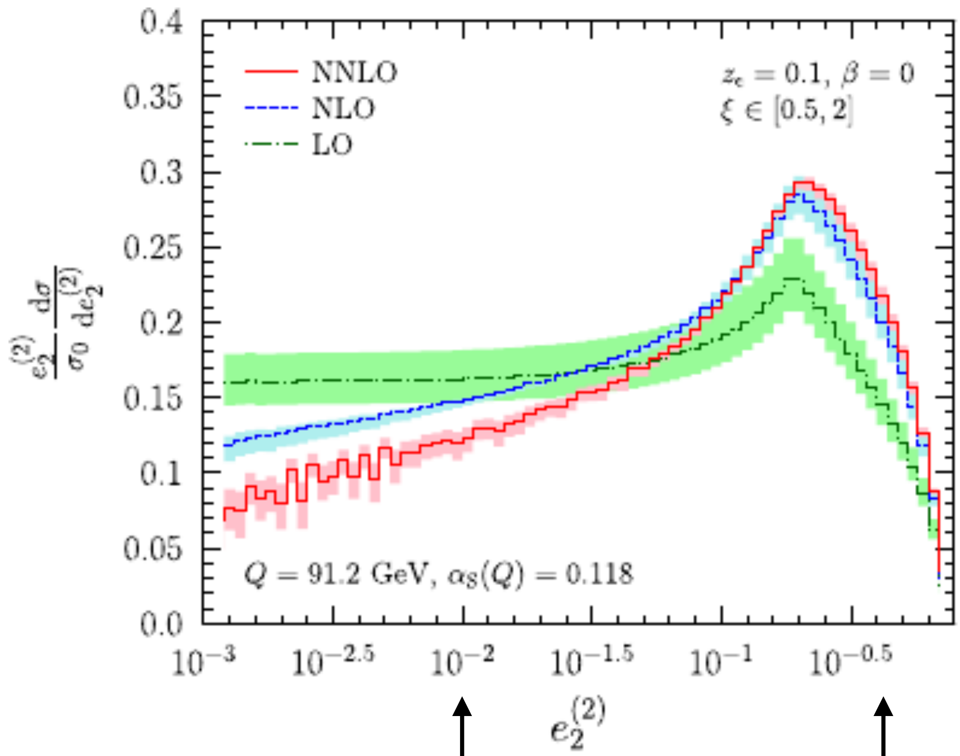
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2	α_s^3

$$\frac{d\sigma}{dm^2} \sim H(Q^2)S(z_{\text{cut}}Q^2)J(m^2)S_c(z_{\text{cut}}m^2)$$

All ingredients for NNNLL resummation known!

High-Precision Predictions

NNLO Prediction at the Z-pole



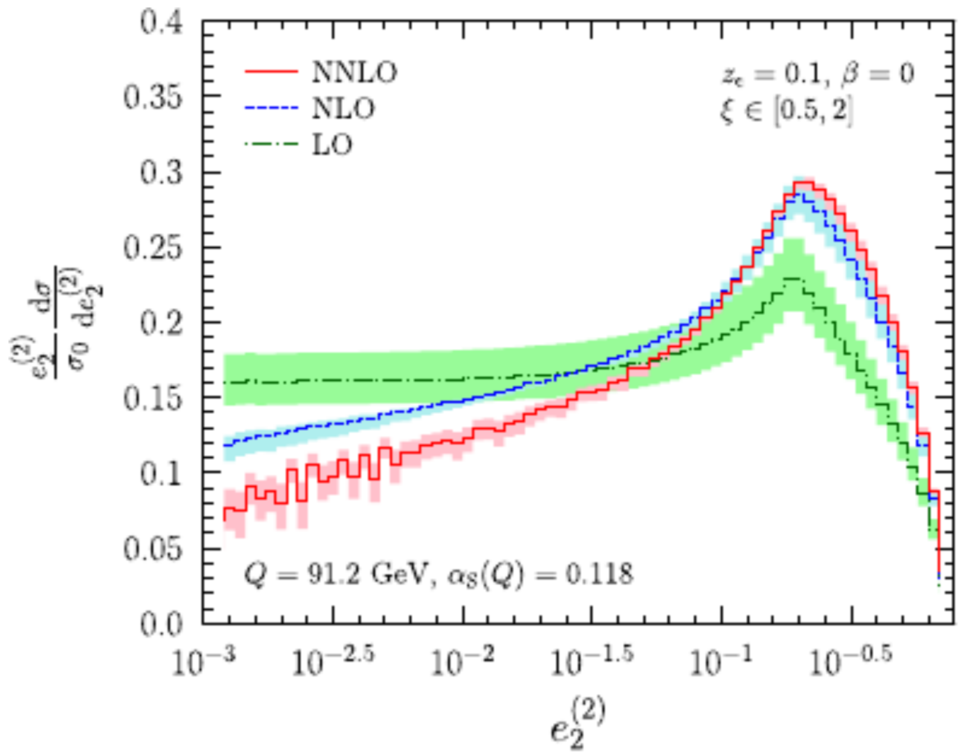
Plot from MCCSM
arXiv:1807.11472

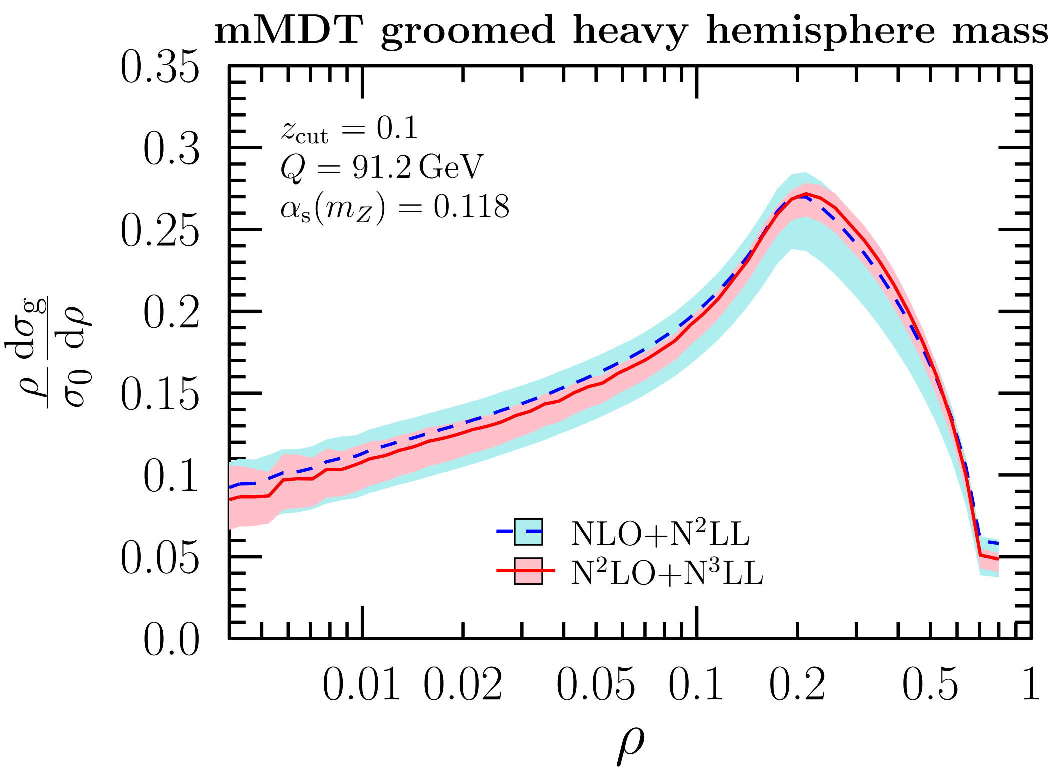
Poor convergence for small ρ

Good convergence for large ρ

High-Precision Predictions

NNNLL+NNLO Prediction at the *Z*-pole





Good convergence observed for **all** ρ!

Going to pp Collisions

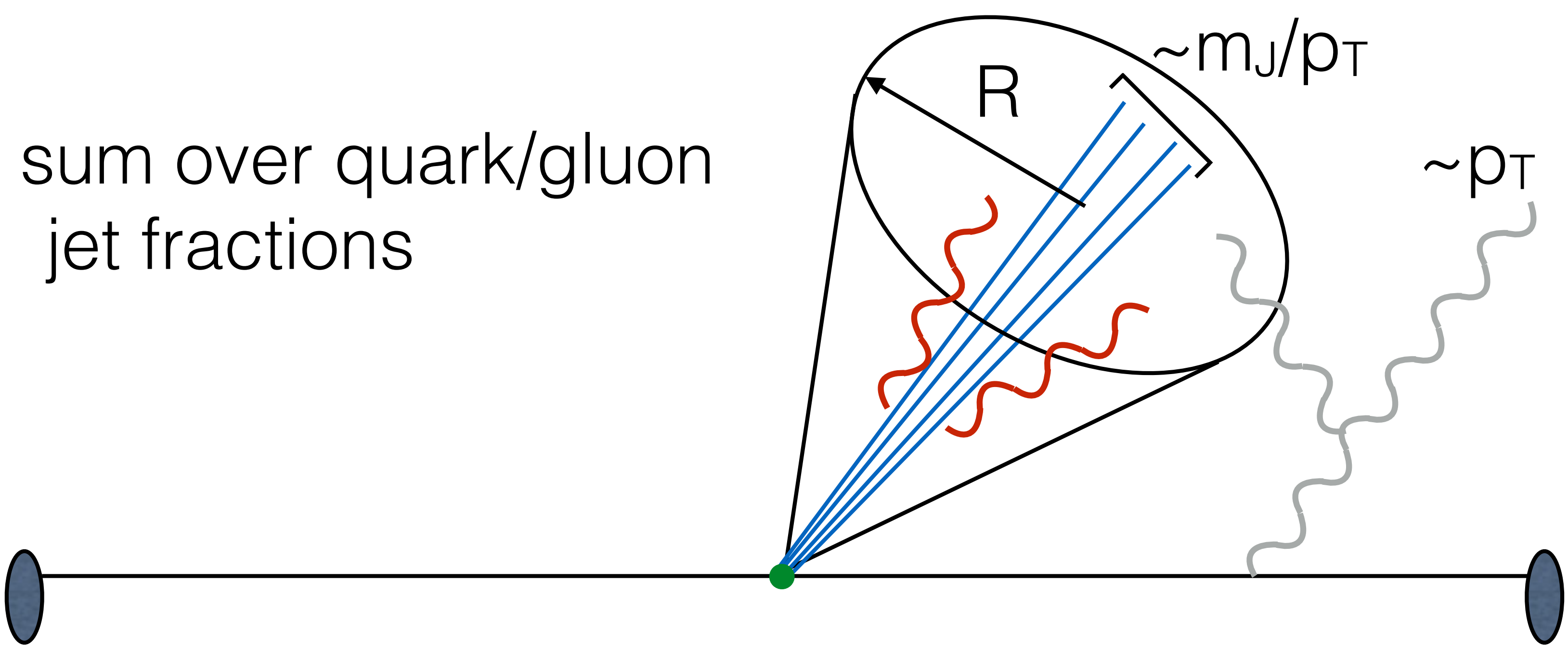
Not quite a factorization theorem

$$\frac{d\sigma}{d\rho} = \sum_{i\in q,g} \mathcal{N}_i(p_T, y, z_{\rm cut}, R)\, J_i(\rho) \otimes S_{c,i}(\rho, z_{\rm cut})$$

Universal

Depends on all scales except groomed mass

Need to sum over quark/gluon jet fractions



Conclusions

Grooming solves experimental contamination issues in jets

Grooming solves theoretical non-global issues in jets

Factorization theorem enables arbitrary logarithmic accuracy

NNNLL resummation for mMDT grooming possible
with (N)NLO codes

Future Directions

Can Legacy LEP data be used for α_s fit?

Can LHC Jet data be used for α_s fit?

Progress toward analytic calculation of anomalous dimensions/constants?

Resummation in $\rho \sim z_{cut}$ regime and non-global effects?

NNNLL resummation for general soft drop grooming?