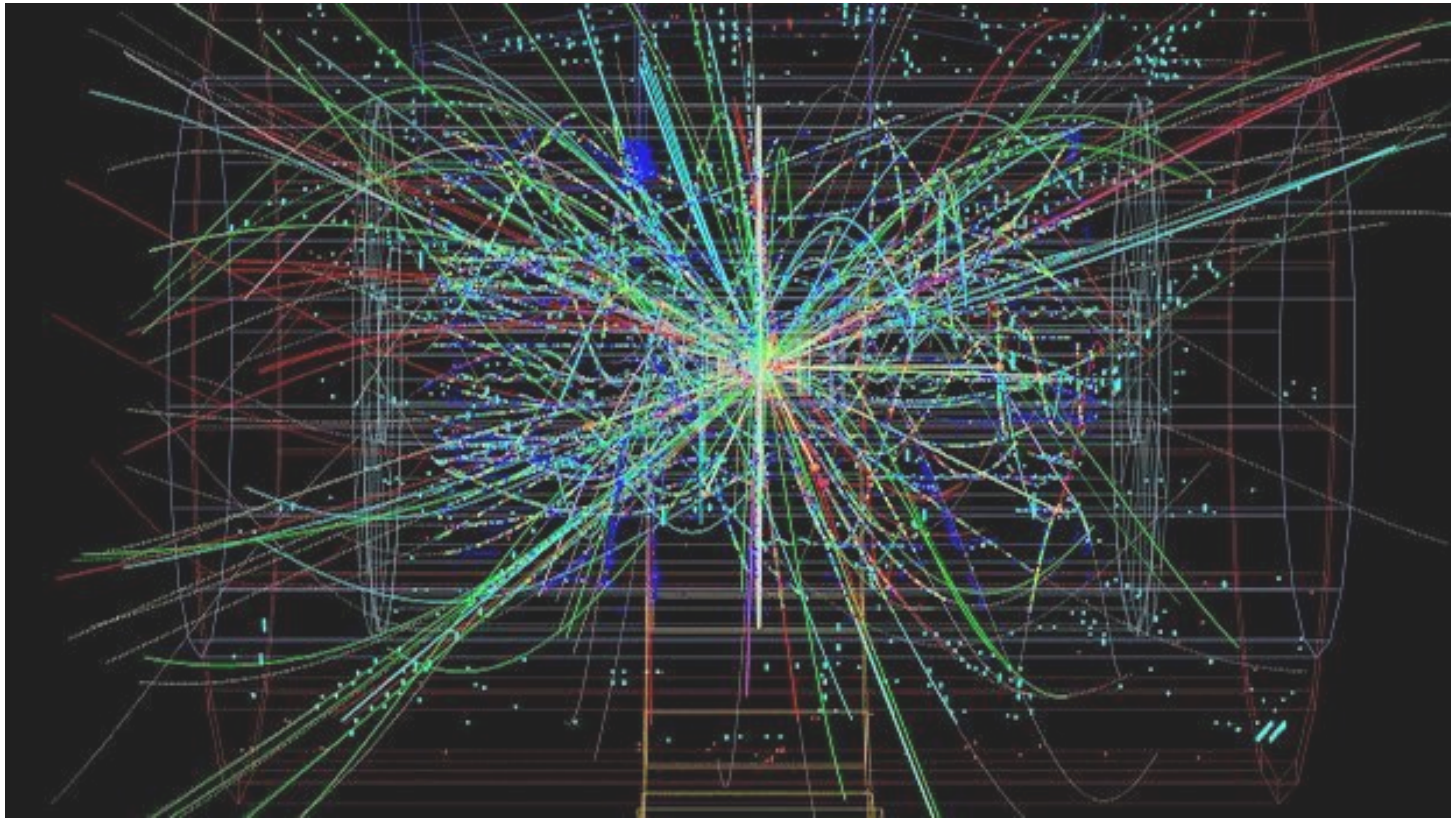
Track Functions at order α_s^2



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Outline

Introduction

Why study track functions?

What are track functions?

How to use track functions?

New in this talk

Evolution at order α_s^2

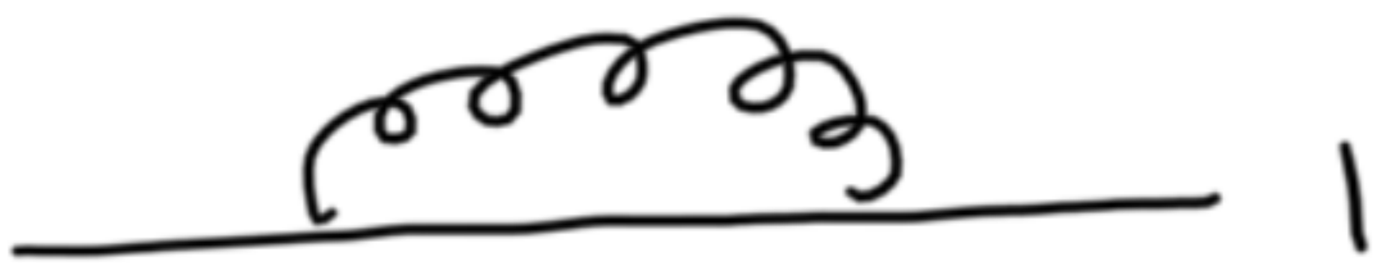
Two independent calculations

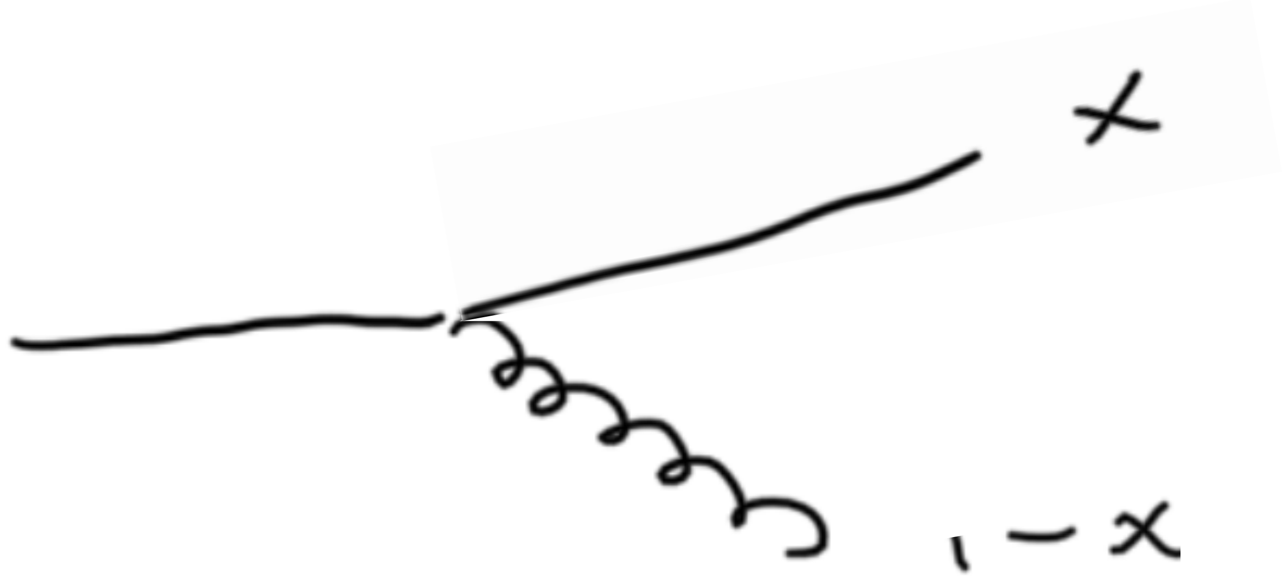
Summary & Outlook

Why study track functions?

Track-based measurements:

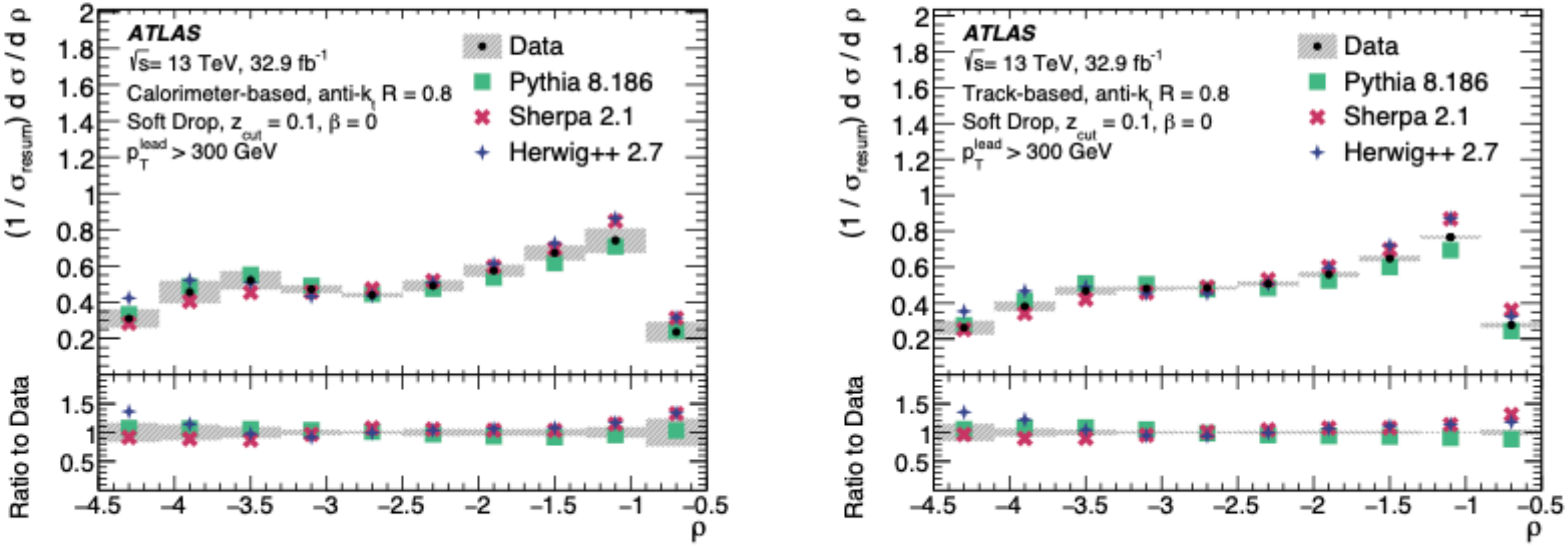
- Superior angular resolution
- Remove pile-up effects
- IR unsafe: the measurement includes just a subset of the final state particles. → Use track functions





Why study track functions?

’20 ATLAS Collaboration



(a) $\beta = 0$, calorimeter-based

(b) $\beta = 0$, track-based

Why study track functions?

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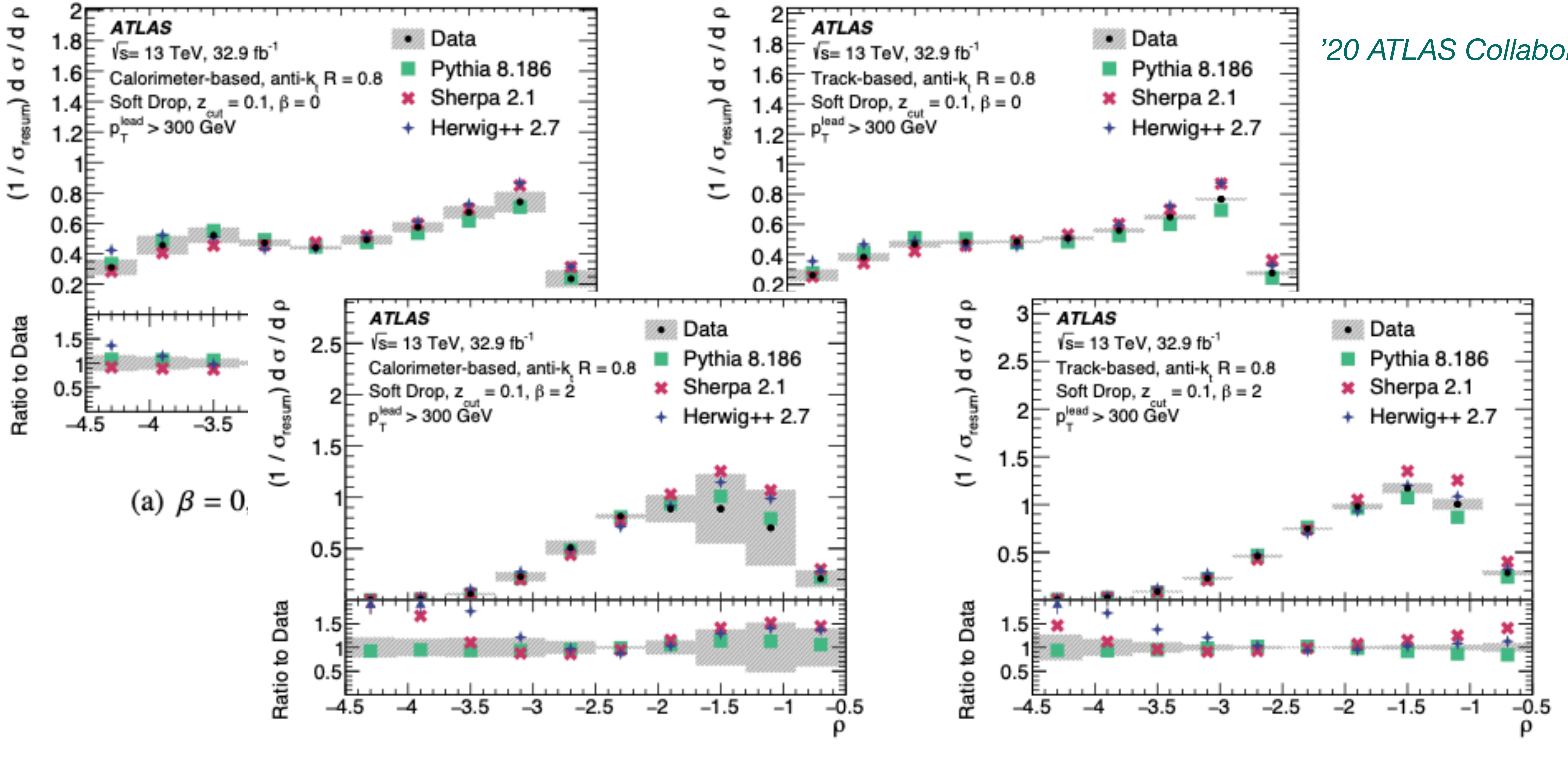

(a) $\beta = 0,$

(e) $\beta = 2$, calorimeter-based

(f) $\beta = 2$, track-based

Why study track functions?

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9.3 Comparison of track-based and calorimeter-based measurements

On a jet-by-jet basis, the value of the all-particles and charged-particles jet substructure observables are largely uncorrelated. However, due to isospin symmetry, the probability distributions for all-particles and charged-particles distributions are nearly identical. This is studied by comparing the unfolded distributions for the cluster-based and track-based measurements, which are shown in ... which includes both jets in the dijet ... high ...

... and track-based ... for the track-based observables are significantly ... for the calorimeter-based observables, particularly for higher values of β, where more soft radiation is included within the jet. However, since no track-based calculations exist at the present time, calorimeter-based measurements are still useful for precision QCD studies.

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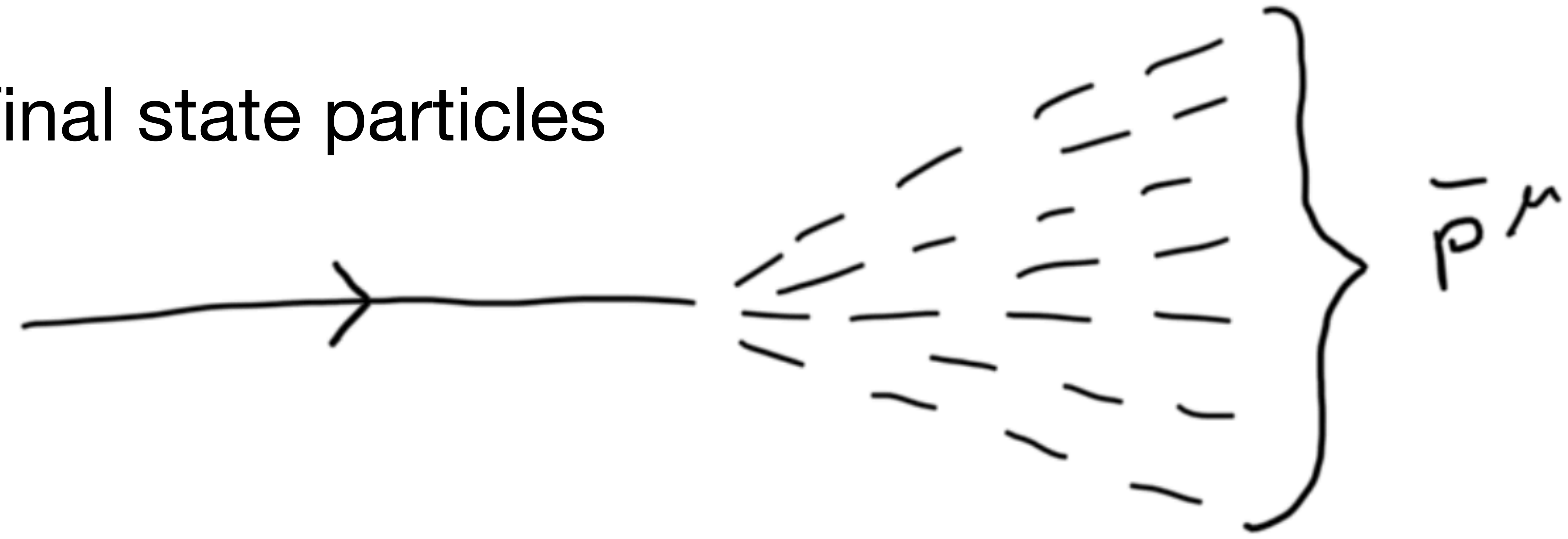
Until now !

What are track functions?

Track functions describes the momentum fraction *x* of initial parton *i* that is converted into tracks, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{QCD})$

- Independent of the hard process
- Non-perturbative objects that can describe hadronization
- Normalized: $\int T_i(x) dx = 1$
- Can be used for any subset of the final state particles

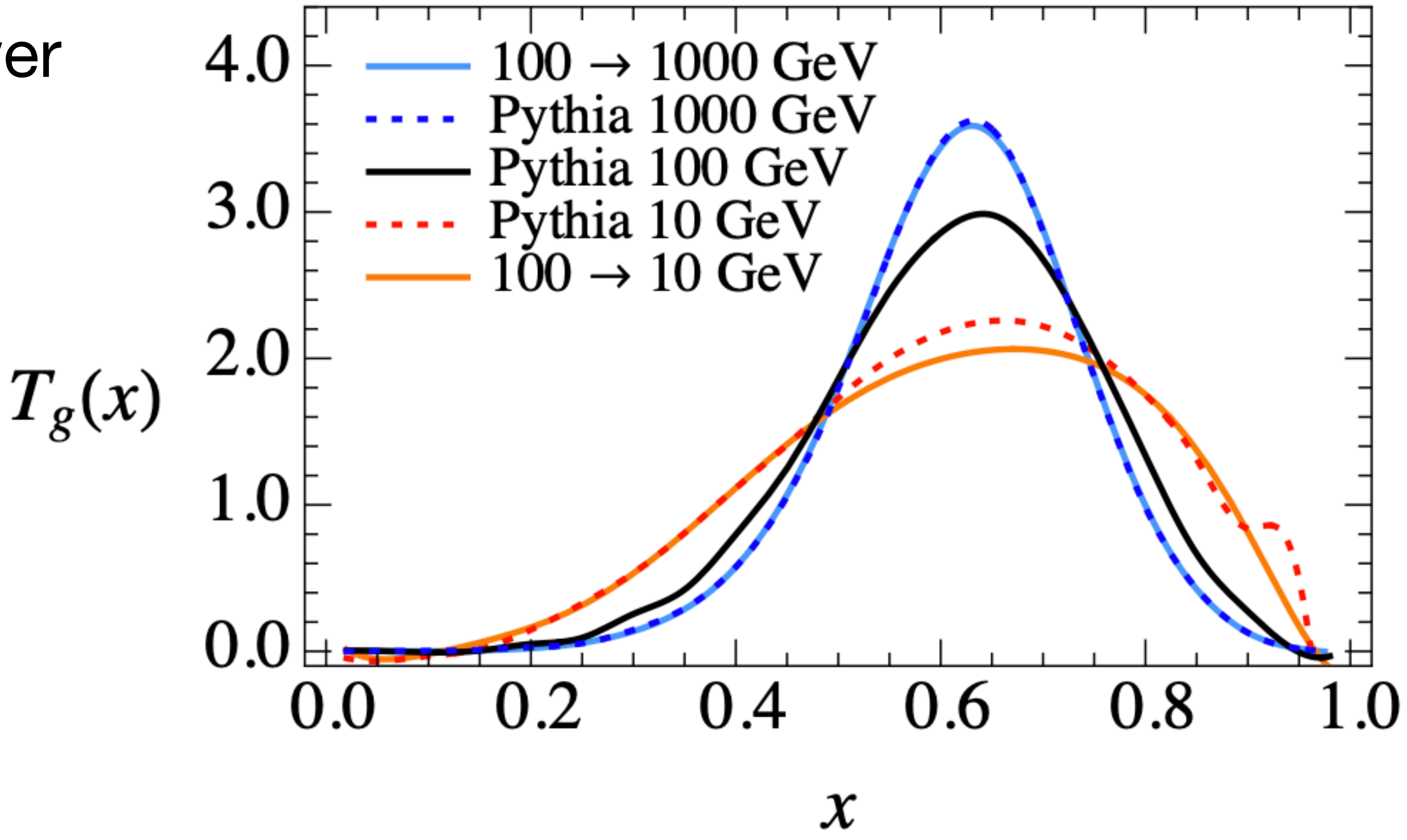


What are track functions?

- Track functions have already been studied at order α_s *'13 Chang, Procura, Thaler, Waalewijn*
- Evolution consistent with parton shower
- At higher orders the evolution becomes increasingly non-linear

$$\mu \frac{\mu}{d\mu} T_q \supset \alpha_s^2 T_q T_g T_g$$

- Goal: Find the full evolution at α_s^2



How to use track functions?

The cross section for an IRC safe observable ‘e’ measured using partons:

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta(e - \hat{e}(\{p_i^\mu\}))$$

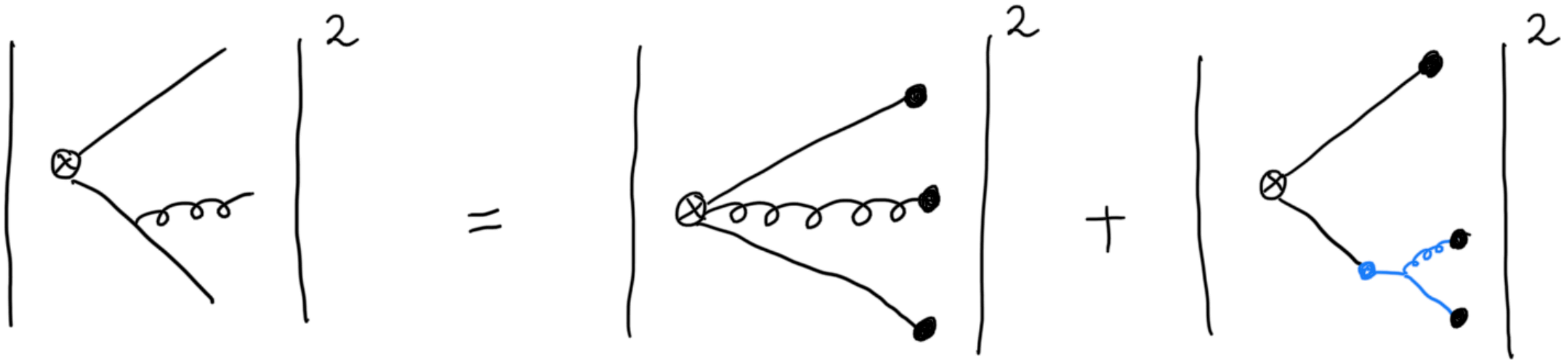
The cross section for the same observable measured using only tracks:

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i T_i(x_i) \delta(\bar{e} - \hat{e}(\{x_i p_i^\mu\})$$

- IR divergences are subtracted in $\bar{\sigma}$ (absorbed in T_i)

How to use track functions?

- LO track function
- NLO track function



$$\sigma_3 = \bar{\sigma}_3 \otimes T_q^{(0)} \otimes T_q^{(0)} \otimes T_g^{(0)} + \bar{\sigma}_2 \otimes T_q^{(1)} \otimes T_q^{(0)}$$

Note that $\sigma_2 = \bar{\sigma}_2$

Track function evolution

Two independent calculations to extract the track function evolution at α_s^2

'19 Moult, Zhu, Dixon
'20 Chen, Moult, Zhang, Zhu

EEC cross section — Methods agree — Jet function

- Correlation between energy deposits
- Tracking can easily be incorporated with moments of Track functions.

$$E_i \to \int dx_i \, x_i T_i(x_i) E_i = T_i^{(1)} E_i$$

- Direct calculation of the track jet function $\mathcal{G}$

'14 Ritzmann, Waalewijn

- Matching the IR-poles gives the track function evolution at α_s^2

$$\mathcal{G}_i^{(2)} = T_i^{(2)} + \sum J_{i\to jk}^{(1)} \otimes [T_j^{(1)} T_k^{(0)}] + \sum_{j,k} J_{i\to jk\ell}^{(2)} \otimes [T_j^{(0)} T_k^{(0)} T_\ell^{(0)}]$$

Track function evolution

$$\frac{d}{d\ln\mu^2}T_g[1,\mu] = \gamma_{gg}[1,\mu]T_g[1,\mu] + \sum_i \gamma_{qg}[1,\mu](T_{q_i}[1,\mu] + T_{\bar{q}_i}[1,\mu])\,,$$

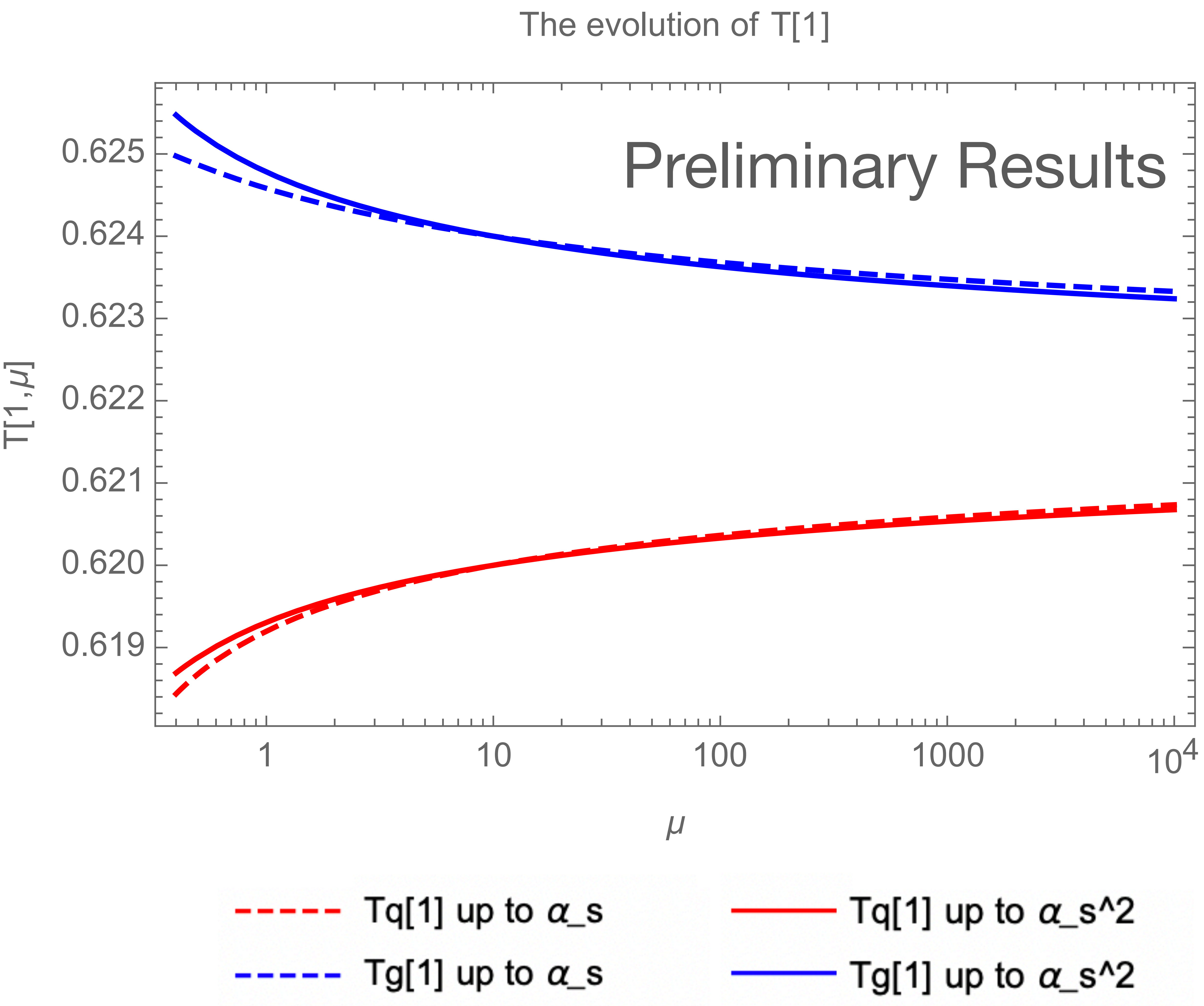
$$\frac{d}{d\ln\mu^2}T_g[2,\mu] = \gamma_{gg}^{(1)}[2,\mu]T_g[2,\mu] + \left(\frac{\alpha_s}{4\pi}\right)^2\left[C_A^2\left(-8\zeta_3 + \frac{26}{45}\pi^2 + \frac{2158}{675}\right) - \frac{4}{9}C_A n_f T_F\right] T_g[1,\mu]T_g[1,\mu] + \ldots$$

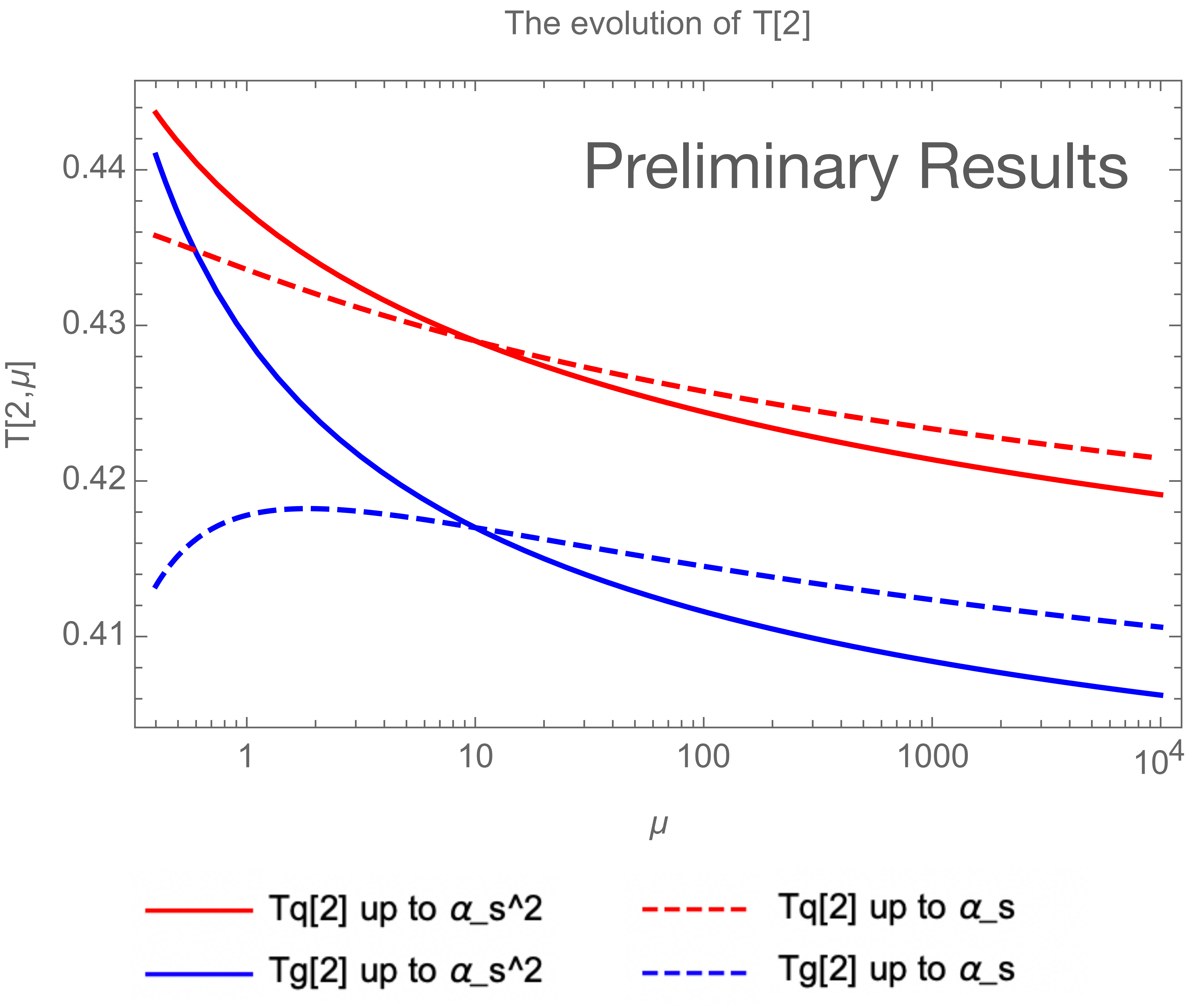
$$\frac{d}{d\ln\mu^2}T_g[3,\mu] = \gamma_{gg}^{(1)}[3,\mu]T_g[3,\mu] + \left(\frac{\alpha_s}{4\pi}\right)^2\left[C_A^2\left(24\zeta_3 - \frac{278}{15}\pi^2 + \frac{767263}{4500}\right) - \frac{2}{3}C_A n_f T_F\right] T_g[2,\mu]T_g[1,\mu]$$

$$+ \left(\frac{\alpha_s}{4\pi}\right)^2\left[C_A T_f\left(\frac{23051}{1125} - \frac{28}{15}\pi^2\right) - C_F T_f \frac{28}{15}\right] T_g[1,\mu]\sum_i T_{q_i}[1,\mu]T_{\bar{q}_i}[1,\mu] + \ldots\,.$$

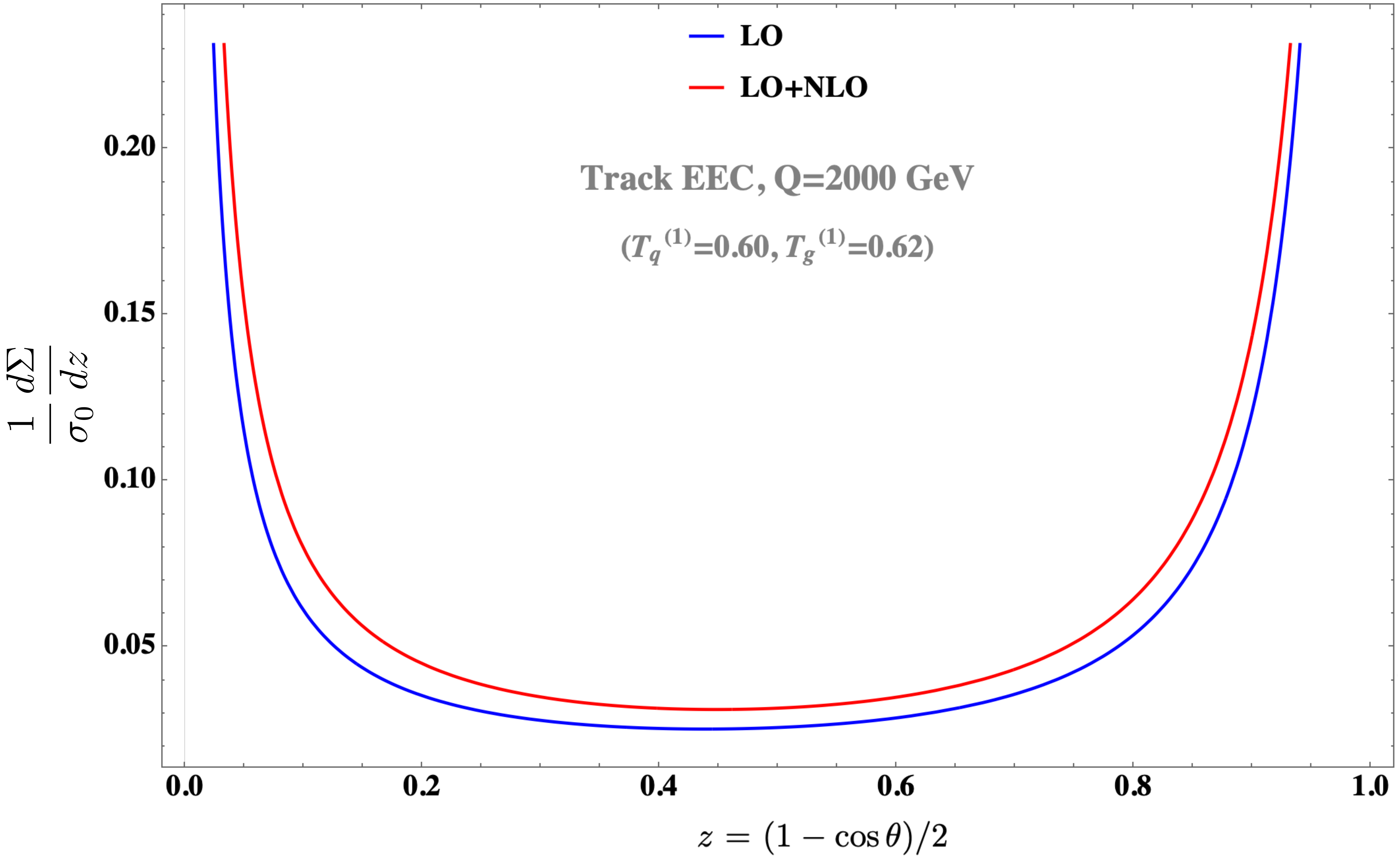
- For the first moment the evolution is the same as fragmentation functions up to all orders in perturbation theory.
- At LO the evolution is the same as for fragmentation functions
- The evolution of the track function up to third moment can be expressed in terms of moments of splitting functions.

Track function at order α_s^2

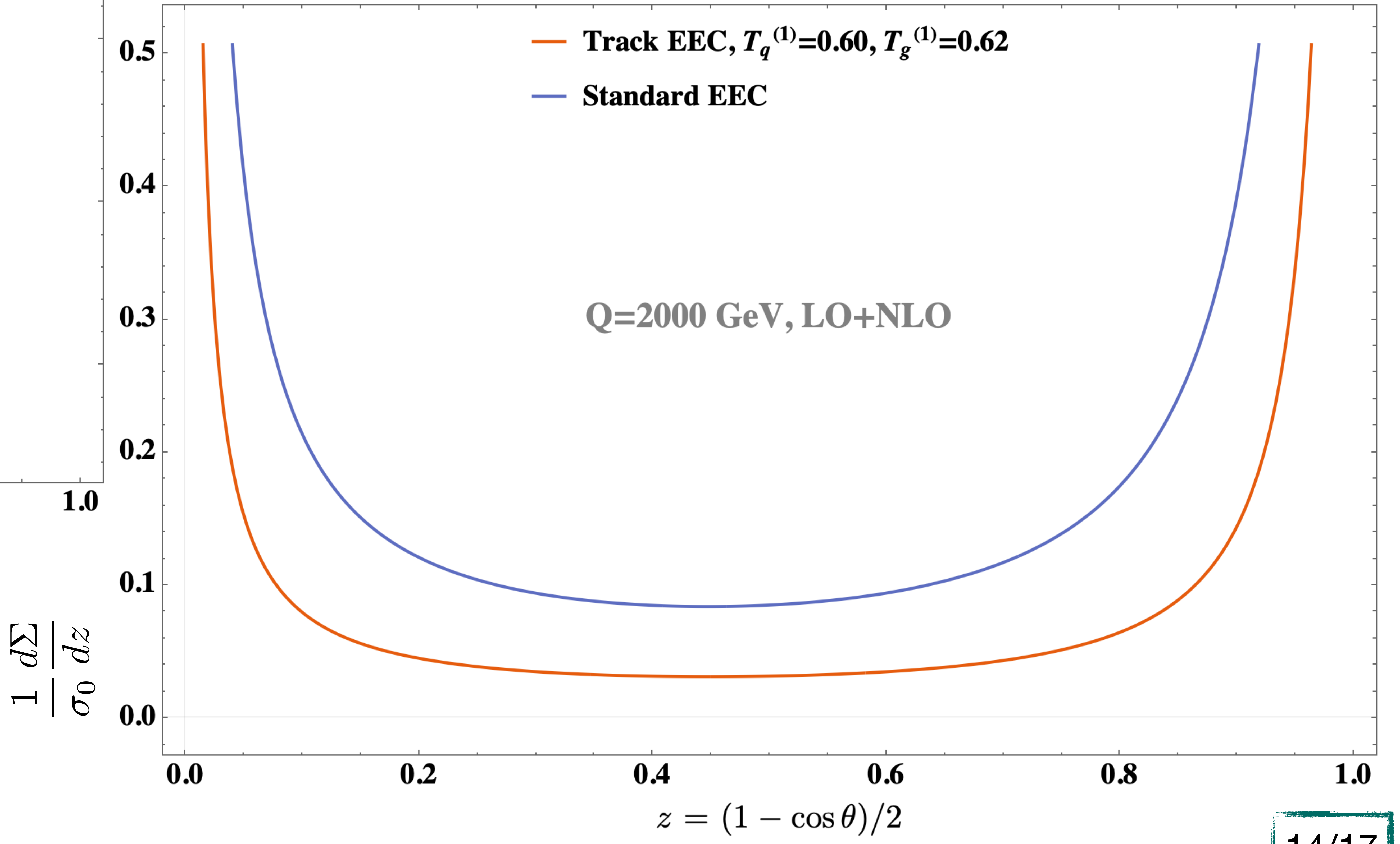




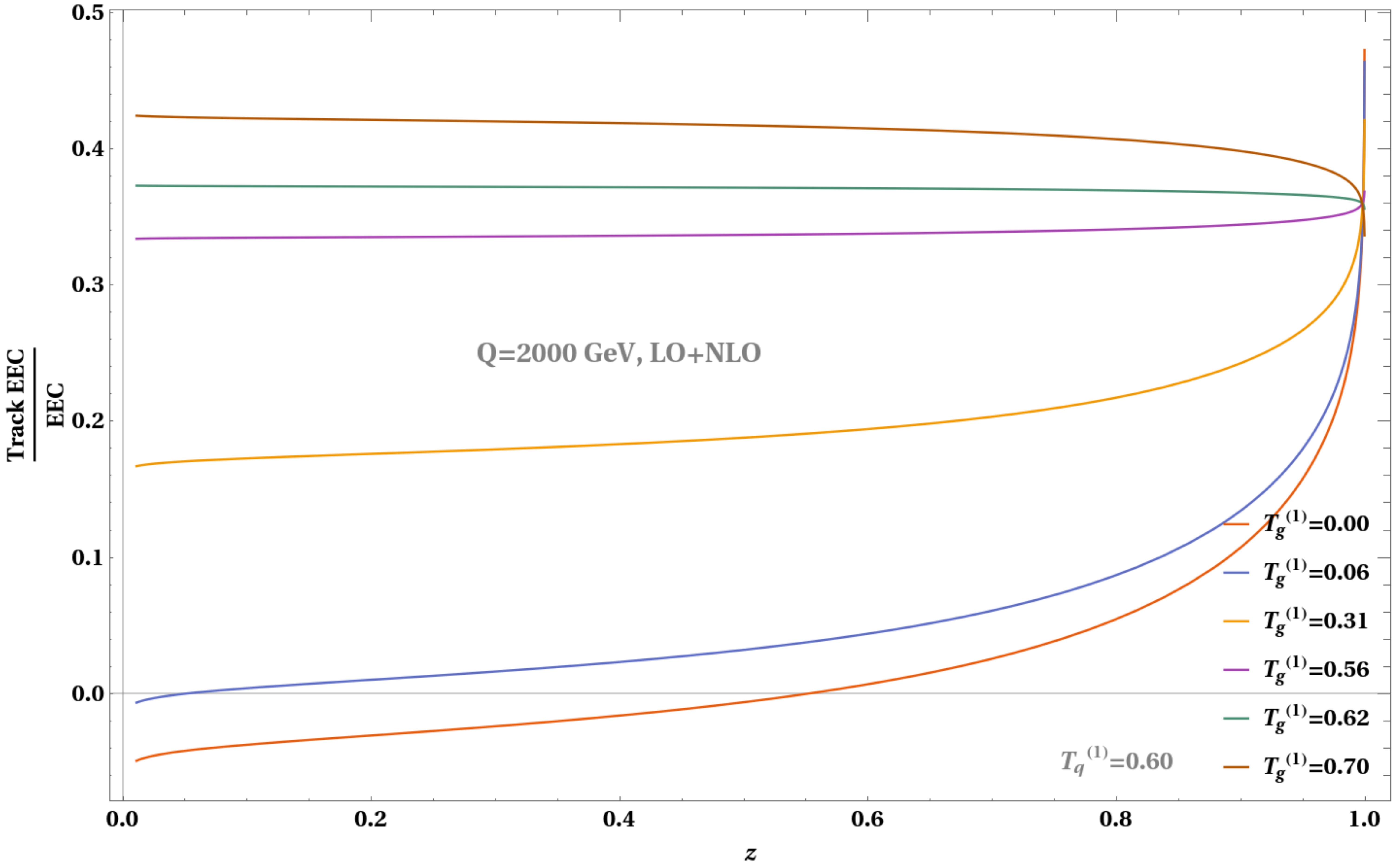
Predictions for track EEC at order α_s^2



- First track-based event shape at NLO



Predictions for track EEC at order α_s^2

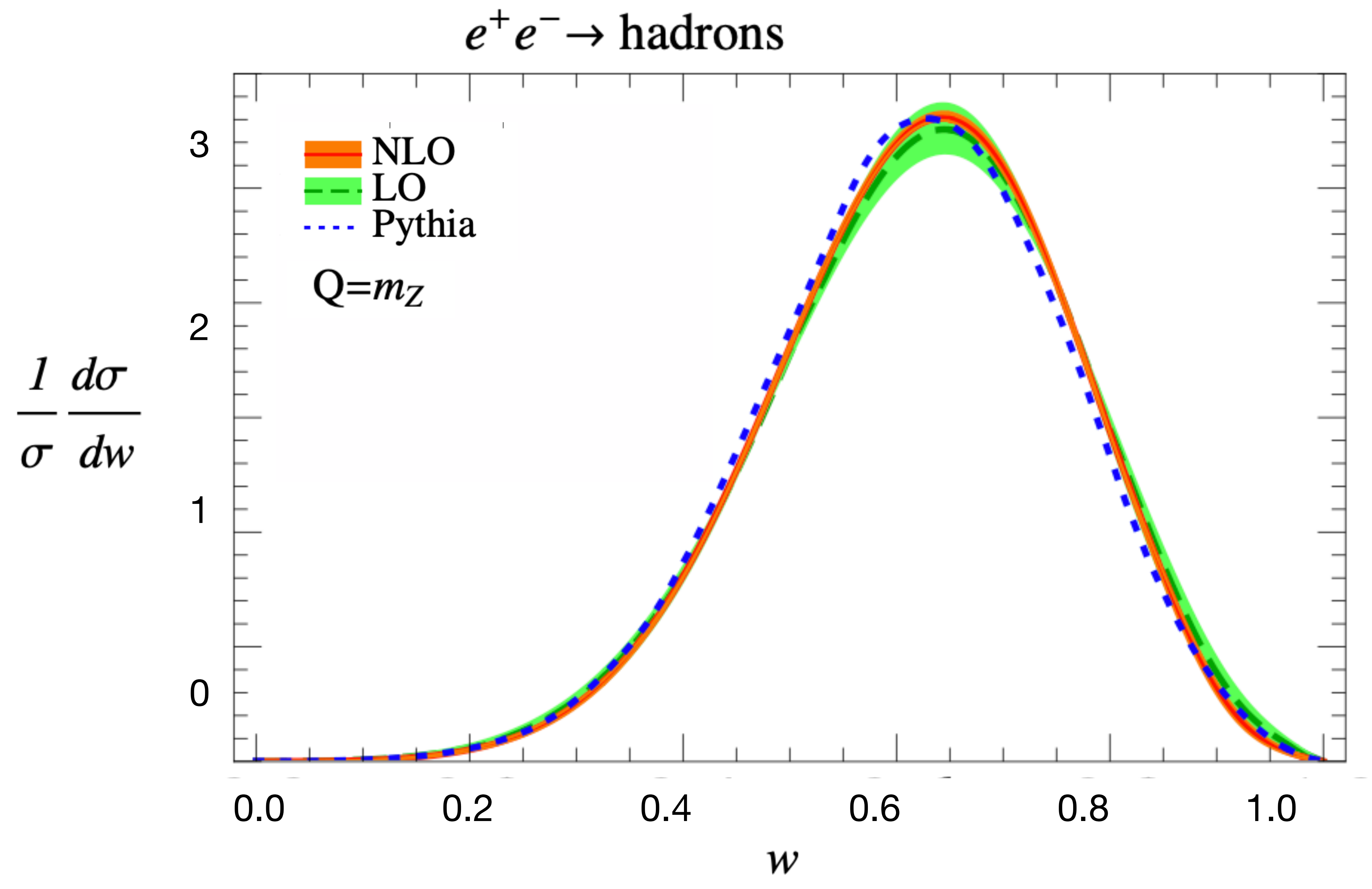


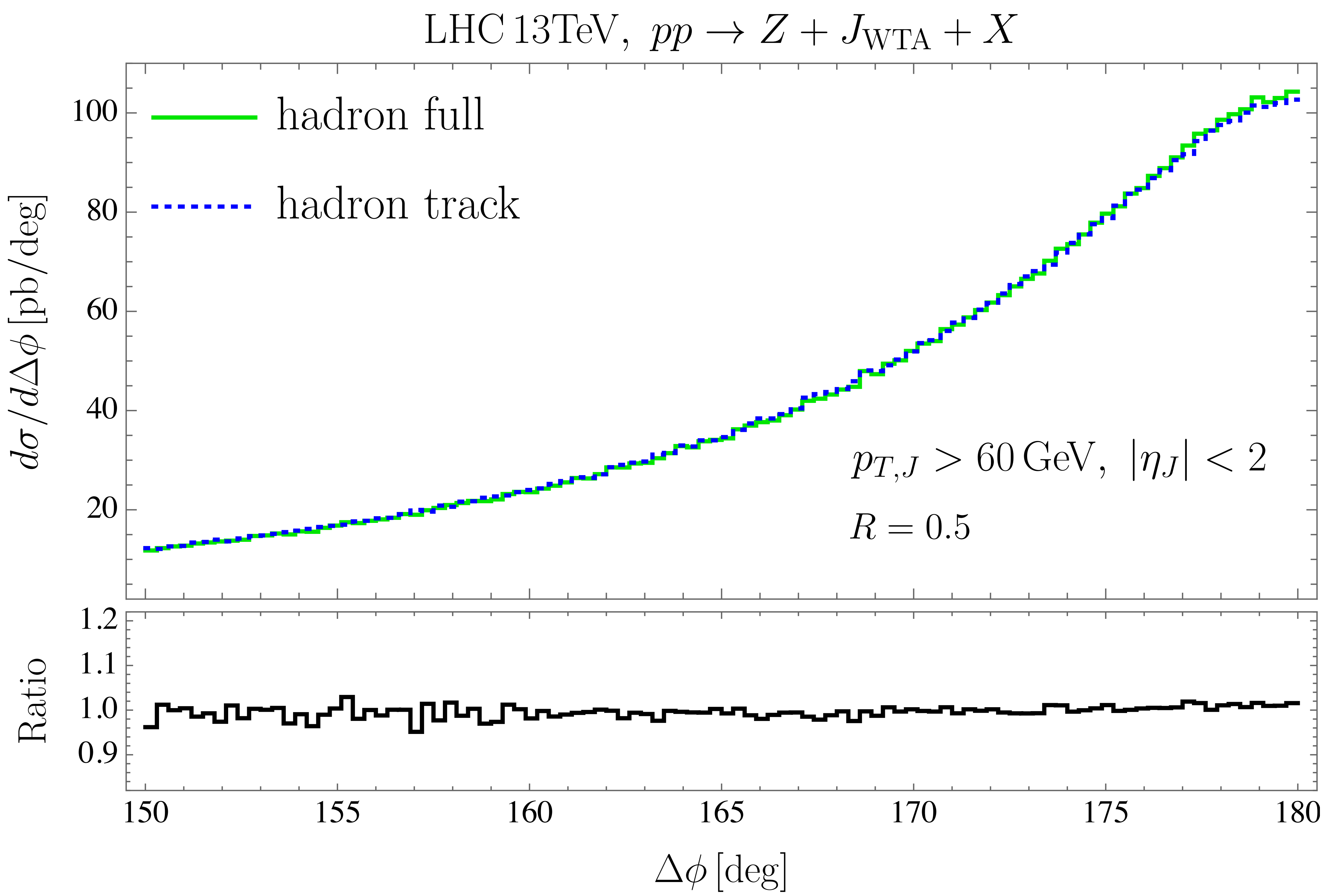
- The shape of this ratio contains information about the track functions.
- Negative ratio is not a physical choice of the track function values

Track function in observables

'13 Chang, Procura, Thaler, Waalewijn

*'21 Chien, Rahn, **S2V2**, Shao, Waalewijn, Wu*





- Energy fraction w of charged particles
- Azimuthal angle in V + jet with WTA axis
- Very sensitive to the track function
- (Almost) insensitive to track functions

Summary

- Track functions can be used to calculate track-based observables
  - → Superior angular resolution
  - → Removes pile-up
- Evolution for moments of track functions extended to α_s^2
  - → Higher precision
  - → Strong check on formalism

Outlook

- Resummation for track-based observables (EEC type)
  - → We are able to calculate the jet constants at α_s^2 for track EEC
- Can be used for any subset of final state hadrons

Thank you!

Backup: T from EEC

EEC measurement: $$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

Track EEC measurement:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}z} &= \sum_N \int \mathrm{d}\Pi_N \, \bar{\sigma}_N \left[\sum_{i\neq j} \frac{E_i E_j}{Q^2} \delta\Big(z - \frac{1-\cos\chi_{ij}}{2}\Big) T_i[1,\mu] T_j[1,\mu] + \sum_i \frac{E_i^2}{Q^2} \delta(z) T_i[2,\mu]\right] \\ &= \sum_N \int \mathrm{d}\Pi_N \, \sigma_N \left[\sum_{i\neq j} \frac{E_i E_j}{Q^2} \delta\Big(z - \frac{1-\cos\chi_{ij}}{2}\Big) T_{i,\mathrm{bare}}[1,\mu] T_{j,\mathrm{bare}}[1,\mu] + \sum_i \frac{E_i^2}{Q^2} \delta(z) T_{i,\mathrm{bare}}[2,\mu]\right]\end{aligned}$$

Requiring the poles to cancel fixes $T_i[2,\mu]$

Illustration at order α_s

$$\frac{1}{\sigma_0} \frac{\mathrm{d}\sigma}{\mathrm{d}z} = T_q[2,\mu]\, \delta(z) \left(\frac{1}{2} + \frac{\alpha_s C_F}{4\pi} \frac{25}{12} \frac{1}{\epsilon}\right) + \dots$$

$$\rightarrow \quad T^{(1)}_{q,\mathrm{bare}}[2] = \frac{\alpha_s C_F}{4\pi} \left(-\frac{25}{12}\right) \frac{1}{\epsilon} T_q^{(0)}[2,\mu] + \dots$$

Backup: Jet Consants EEC

Extracting jet constants from the factorisation of the EEC measurement

$$\Sigma(z, \ln \frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}(\ln \frac{z x^2 Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)\,.$$

where $\Sigma\left(z, \ln \frac{Q^2}{\mu^2}, \mu\right) \equiv \frac{1}{\sigma_0} \int_0^z dz' \frac{d\sigma}{dz}\left(z', \ln \frac{Q^2}{\mu^2}, \mu\right)$

Backup: Track functions

$$T^{(1)}_{i,\mathrm{bare}}(x) = \frac{1}{2}\sum_{j,k}\int \mathrm{d}z \Big[\frac{\alpha_s(\mu)}{2\pi}\Big(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\Big)P_{i\to jk}(z)\Big] \times \int \mathrm{d}x_1\,\mathrm{d}x_2\,T^{(0)}_j(x_1,\mu)T^{(0)}_k(x_2,\mu) \times \delta\big[x - zx_1 - (1-z)x_2\big]\,,$$

Expected due to scaleless integrals beyond LO

Backup: Plot TEEC

$$\left(\frac{d\sigma}{dz}\right)_{\rm tr} = \sum_{i\neq j} T_i^{(1)}(\mu) T_j^{(1)}(\mu) \left\{ \int \frac{E_i E_j}{Q^2} |\mathcal{M}|^2 \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right) d\mathrm{LIPS}_n \right\}_{\rm partonic, subtracted} + \sum_{k=1}^{n} T_k^{(2)}(\mu) \left\{ \int \frac{E_k^2}{Q^2} |\mathcal{M}|^2 \delta(z)\, d\mathrm{LIPS}_n \right\}_{\rm partonic, subtracted}, \quad (5)$$