

# Jets and their substructure from LHC data

June 2nd, 2021

## Soft drop momentum sharing fraction $z_g$ beyond LL

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arXiv: 2106.this\_week



UNIVERSITY  
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# Outline

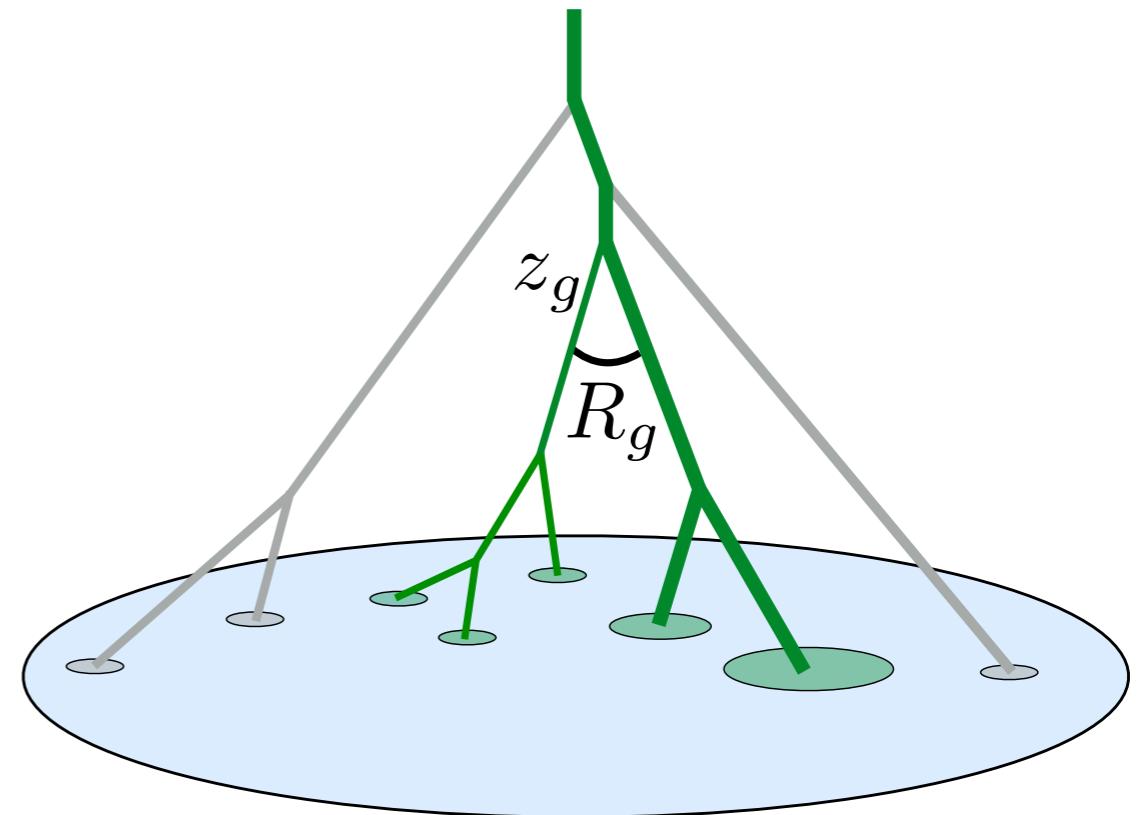
- Introduction

- Soft Drop,  $z_g$ , and  $R_g$
- Why  $z_g$ ?
- Sudakov Safety

- $z_g$  at NLL' accuracy

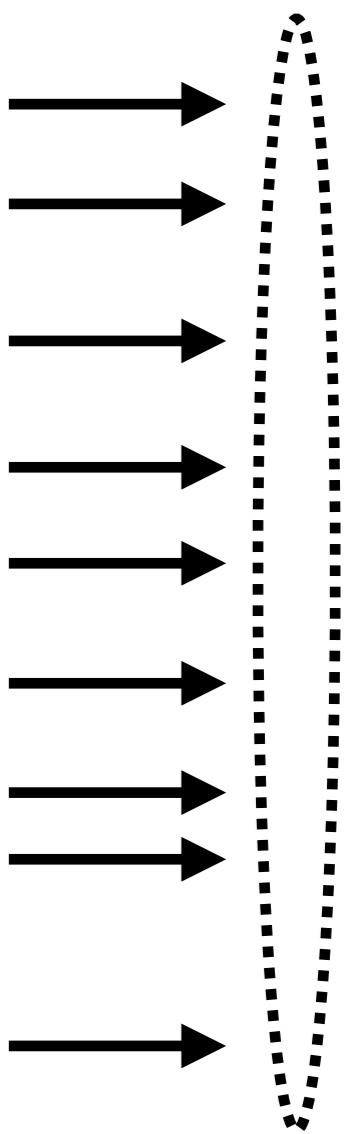
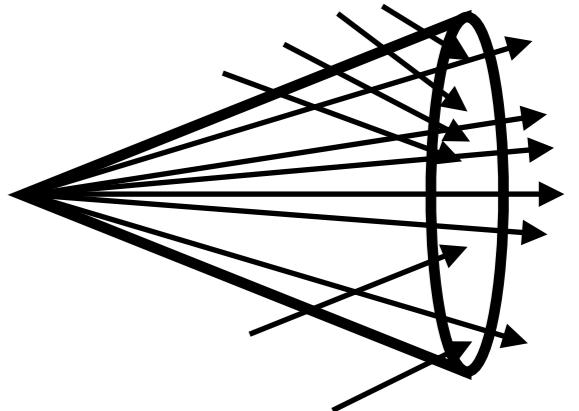
- Jet production
- Fixed-order computation
- Refactorization and resummation
- Non-global logarithms
- Results

- Conclusions



# Soft Drop, $z_g$ , and $R_g$

- Use C/A to obtain angular-ordered tree

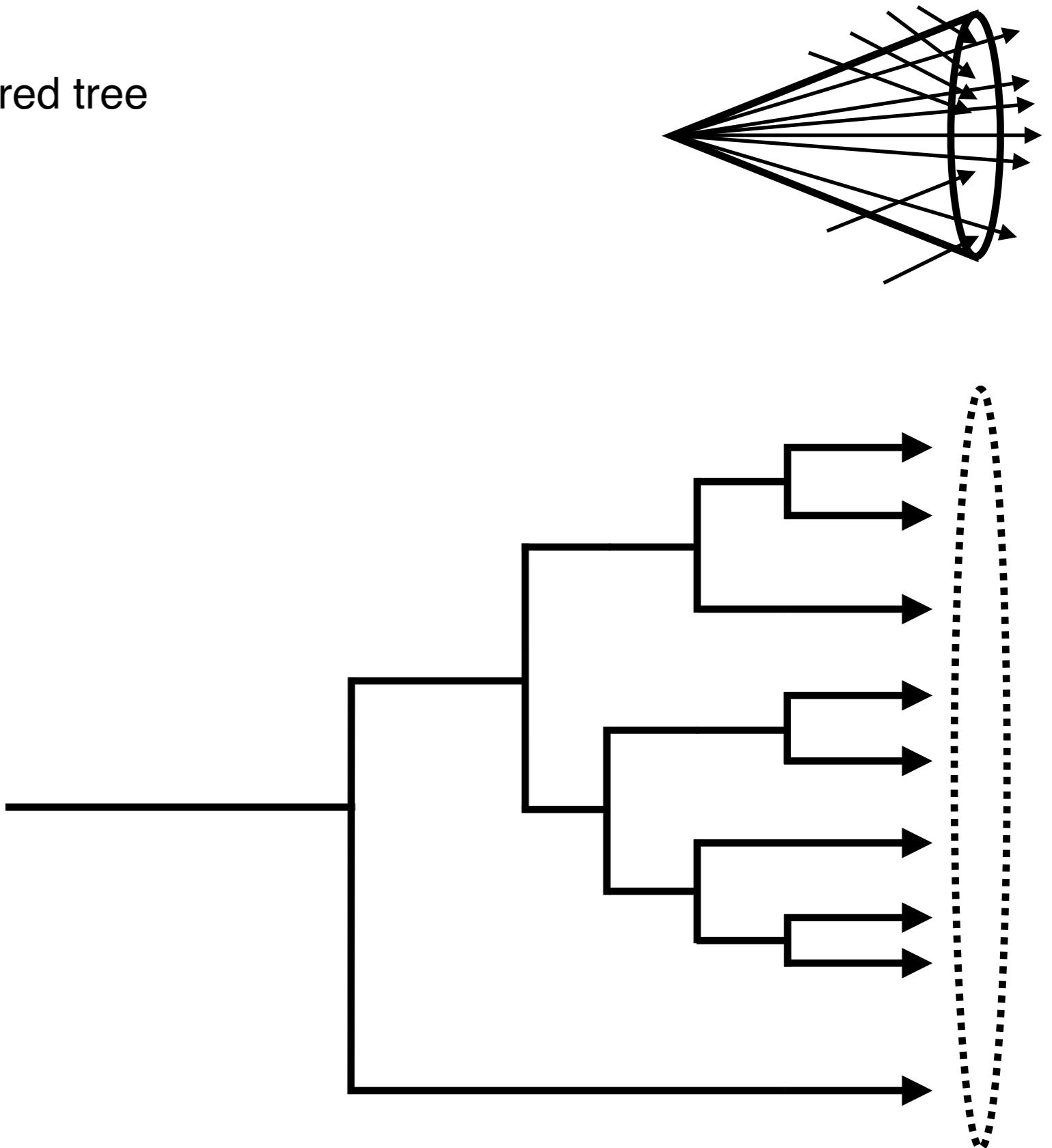


Larkoski, Marzani, Soyez, Thaler '14

Larkoski, Marzani, Thaler '15

# Soft Drop, $z_g$ , and $R_g$

- Use C/A to obtain angular-ordered tree

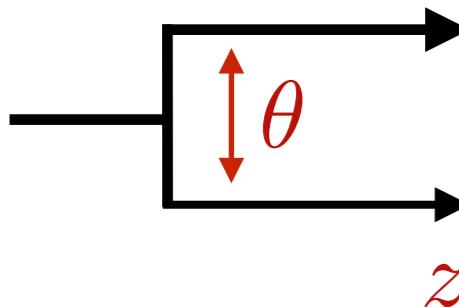


Larkoski, Marzani, Soyez, Thaler '14

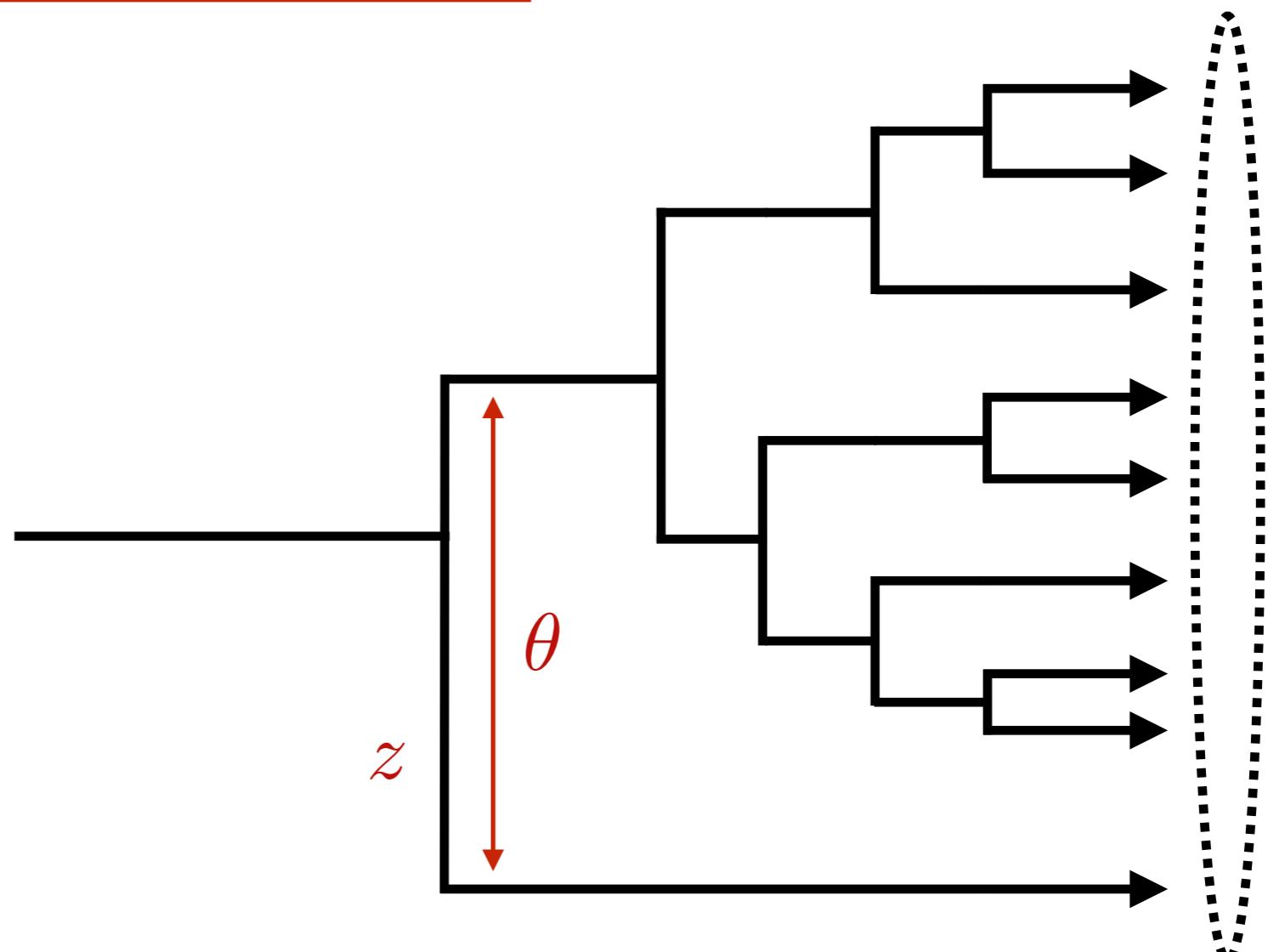
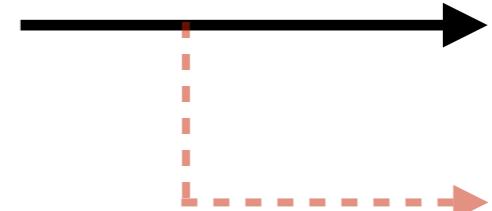
Larkoski, Marzani, Thaler '15

# Soft Drop, $z_g$ , and $R_g$

- SD criterium:



$$z < z_{\text{cut}} \left( \frac{\theta}{R} \right)^\beta$$

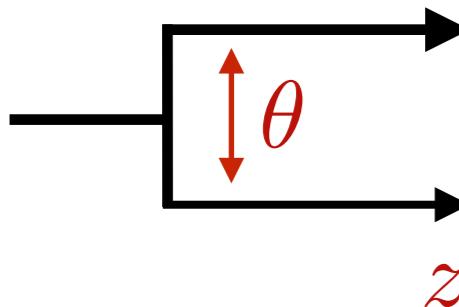


Larkoski, Marzani, Soyez, Thaler '14

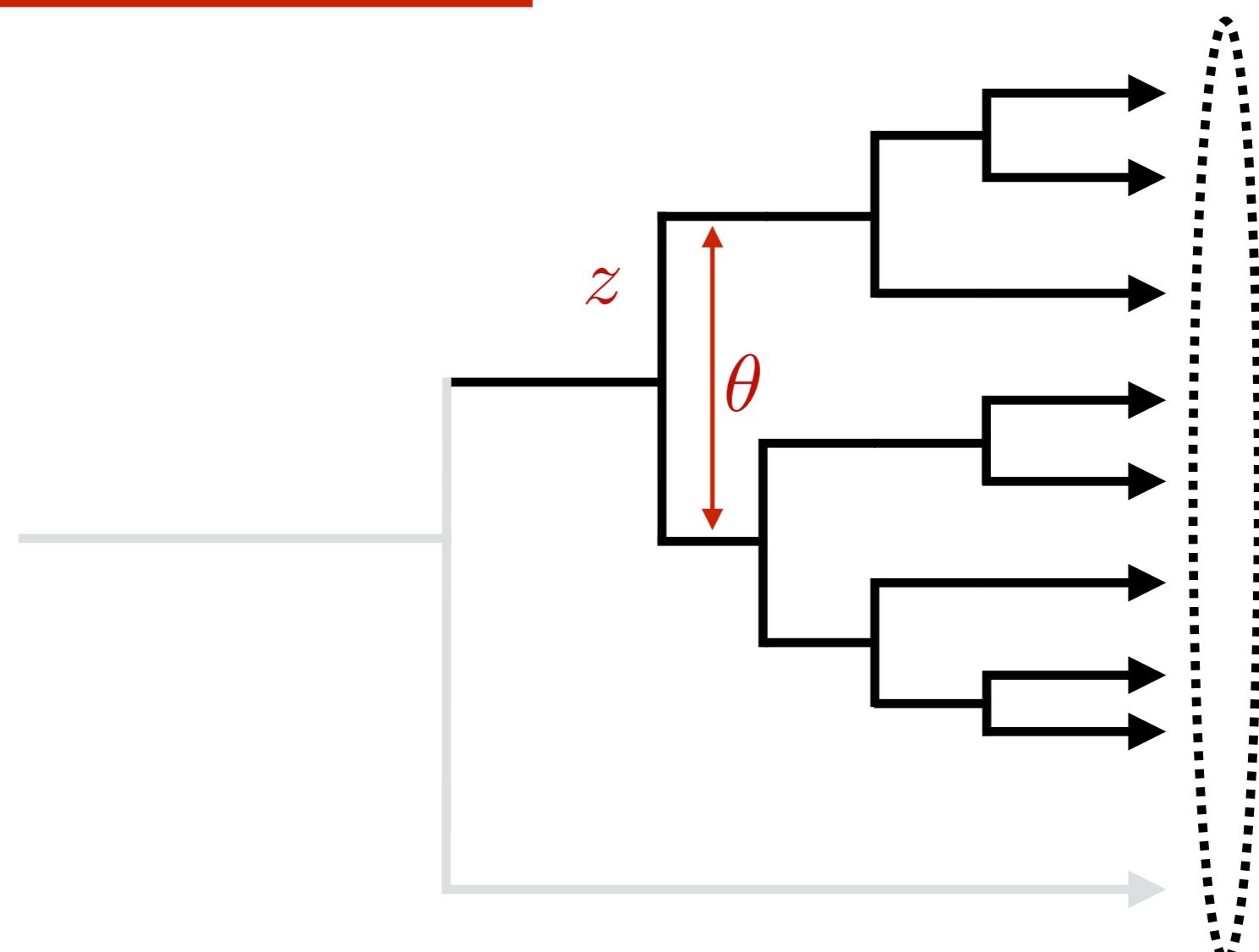
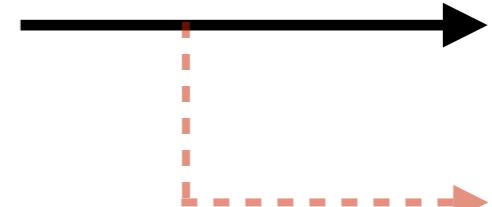
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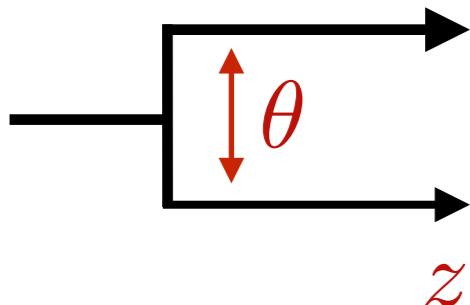


Larkoski, Marzani, Soyez, Thaler '14

Larkoski, Marzani, Thaler '15

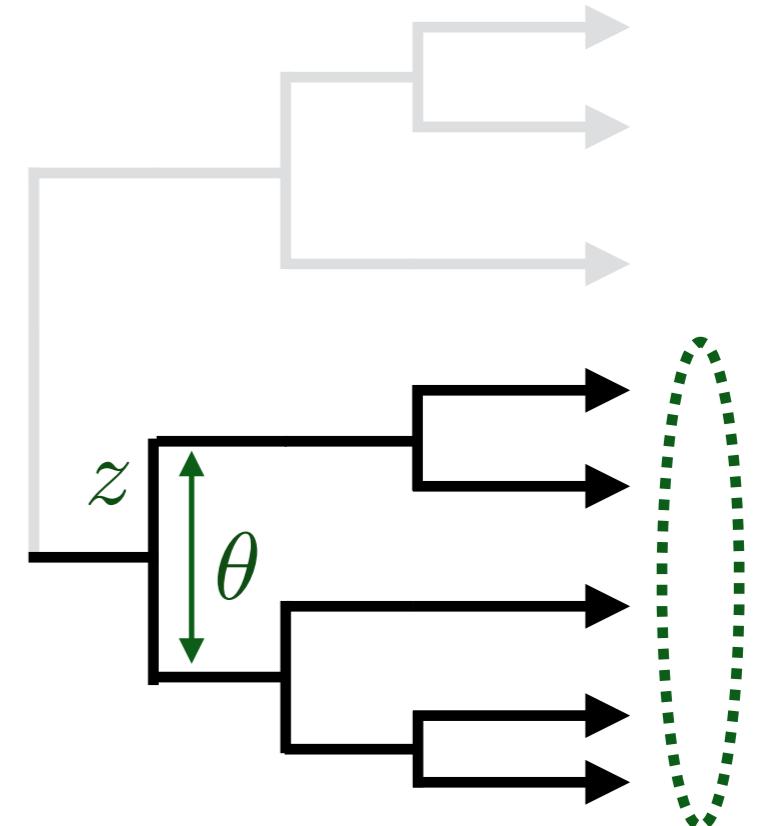
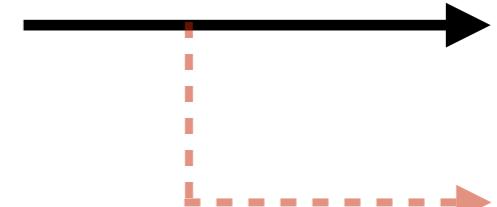
# Soft Drop, $z_g$ , and $R_g$

- SD criterium:



$$z > z_{\text{cut}} \left( \frac{\theta}{R} \right)^\beta$$

**Stop**

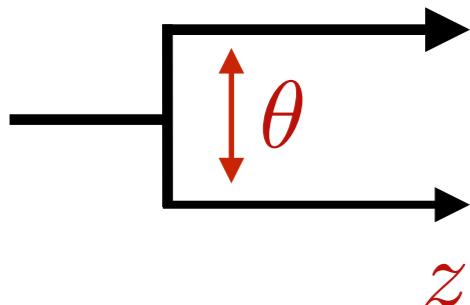


Larkoski, Marzani, Soyez, Thaler '14

Larkoski, Marzani, Thaler '15

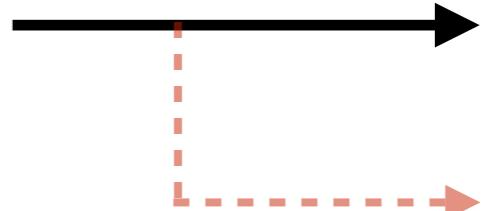
# Soft Drop, $z_g$ , and $R_g$

- SD criterium:



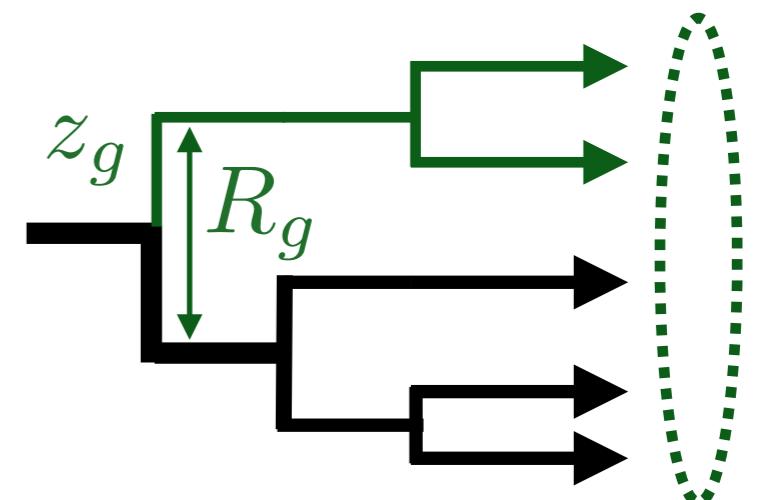
$$z > z_{\text{cut}} \left( \frac{\theta}{R} \right)^\beta$$

Stop



$z_g \equiv z$  of softer branch

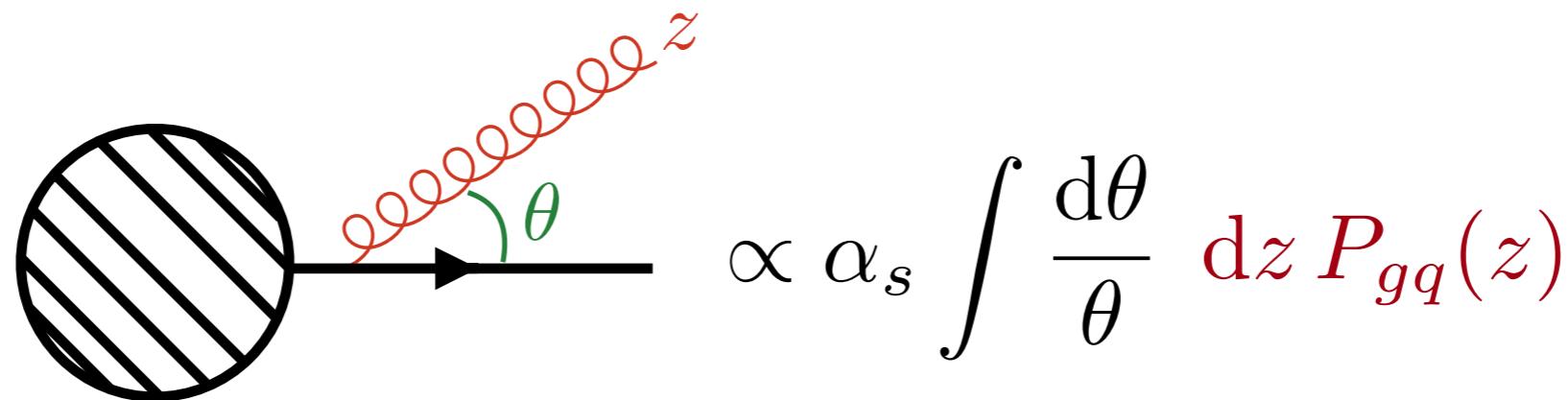
$R_g \equiv \theta$  between branches



Larkoski, Marzani, Soyez, Thaler '14

Larkoski, Marzani, Thaler '15

# Why $z_g$ ?



- Most direct measurement of the QCD splitting functions
- Measured at ATLAS, CMS, ALICE, STAR
- LL accuracy: Measurement probes color of parton initiating jet
- NLL' accuracy: Probes color and spin
- Heavy ion - probes hard-collinear splittings in Quark-Gluon Plasma

# Sudakov safety

- For  $\beta < 0$ ,  $z_g$  is infrared and collinear safe (IRC safe)



- For  $\beta \geq 0$ ,  $z_g$  is IRC unsafe



Larkoski, Marzani, Thaler '15

$$z_g = 0$$



$$z_g \neq 0$$

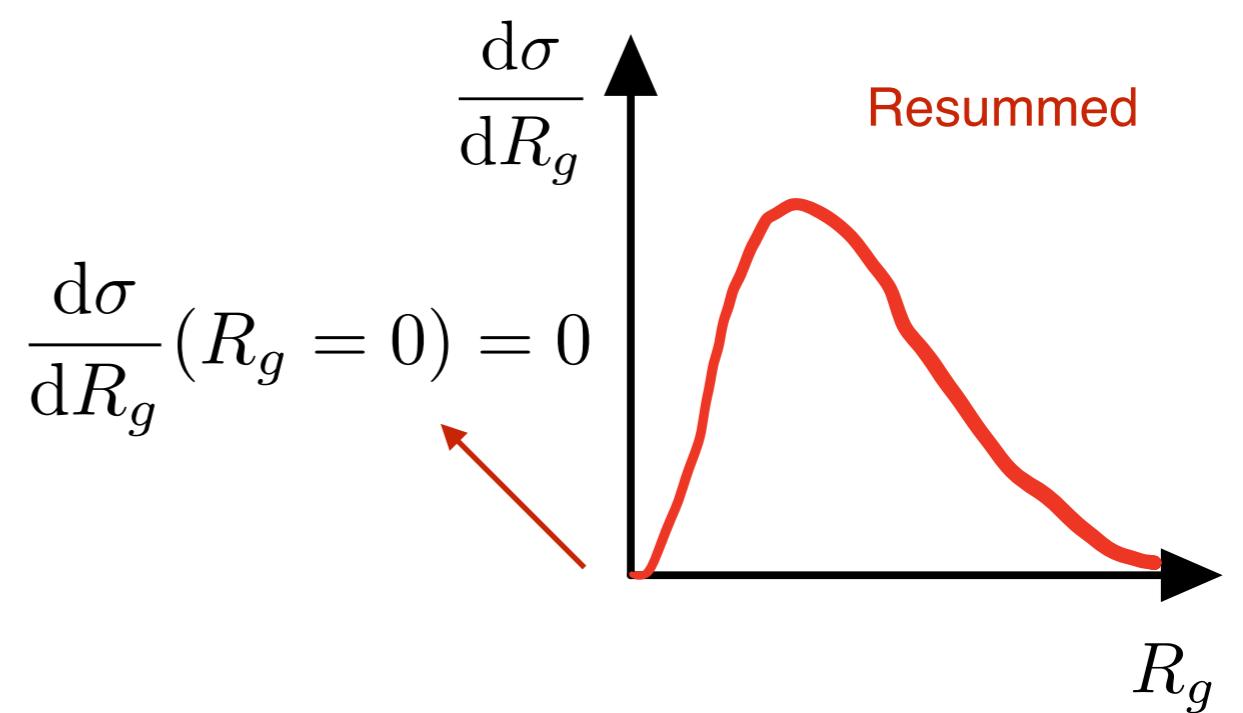


- What to do?

1) Measure  $z_g$  and  $R_g$

2) Resum  $R_g$ , hence avoiding collinear configuration

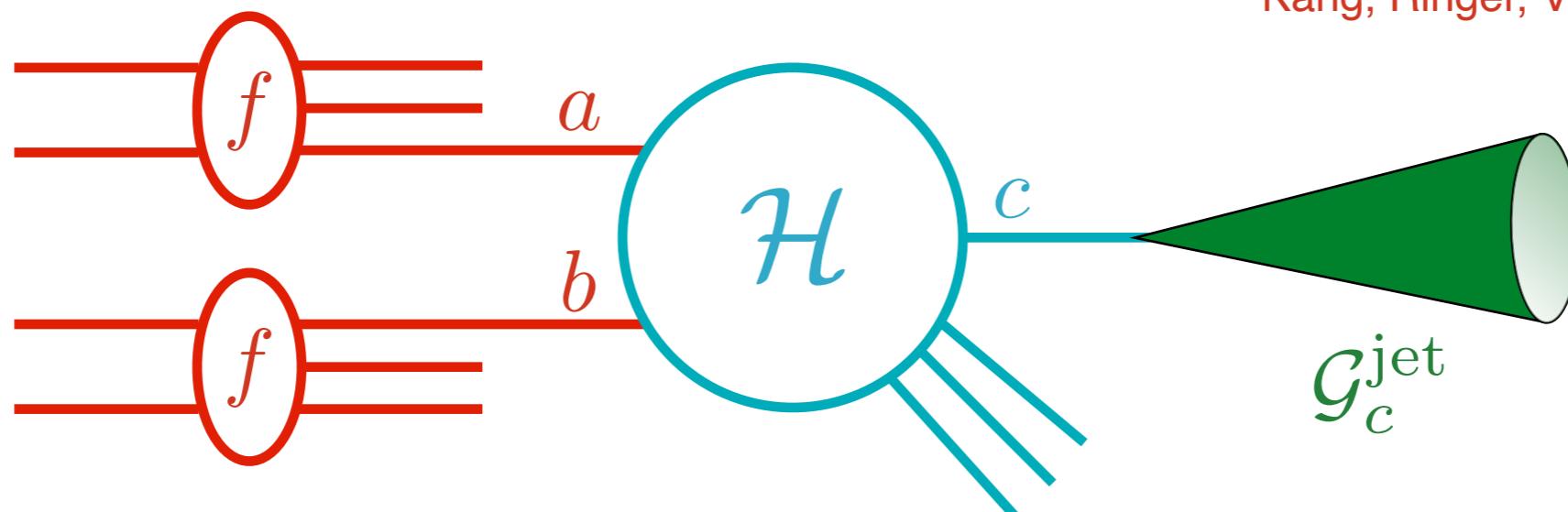
3) Integrate out  $R_g$ , obtaining  $\frac{d\sigma}{dz_g}$



# Jet production

- Collinear factorization

Dasgupta, Dreyer, Salam, Soyez '14  
Kaufmann, Mukherjee, Vogelsang '15  
Kang, Ringer, Vitev '16



$$\frac{d\sigma}{dp_T d\eta dz_g} = f_a \otimes f_b \otimes \mathcal{H}_{ab}^c \otimes \mathcal{G}_c^{\text{jet}} [1 + \mathcal{O}(R^2)]$$

- Schematically:

$$\mathcal{G}_c^{\text{jet}}(z_g) = \sum_d J_{cd} \tilde{\mathcal{G}}_d(z_g)$$

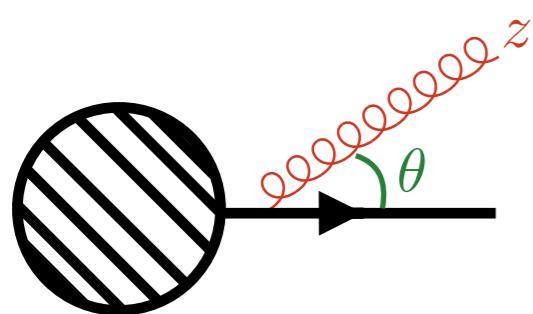
talks to hard function

Cal, Ringer, Waalewijn '19

Sensitive to measurement

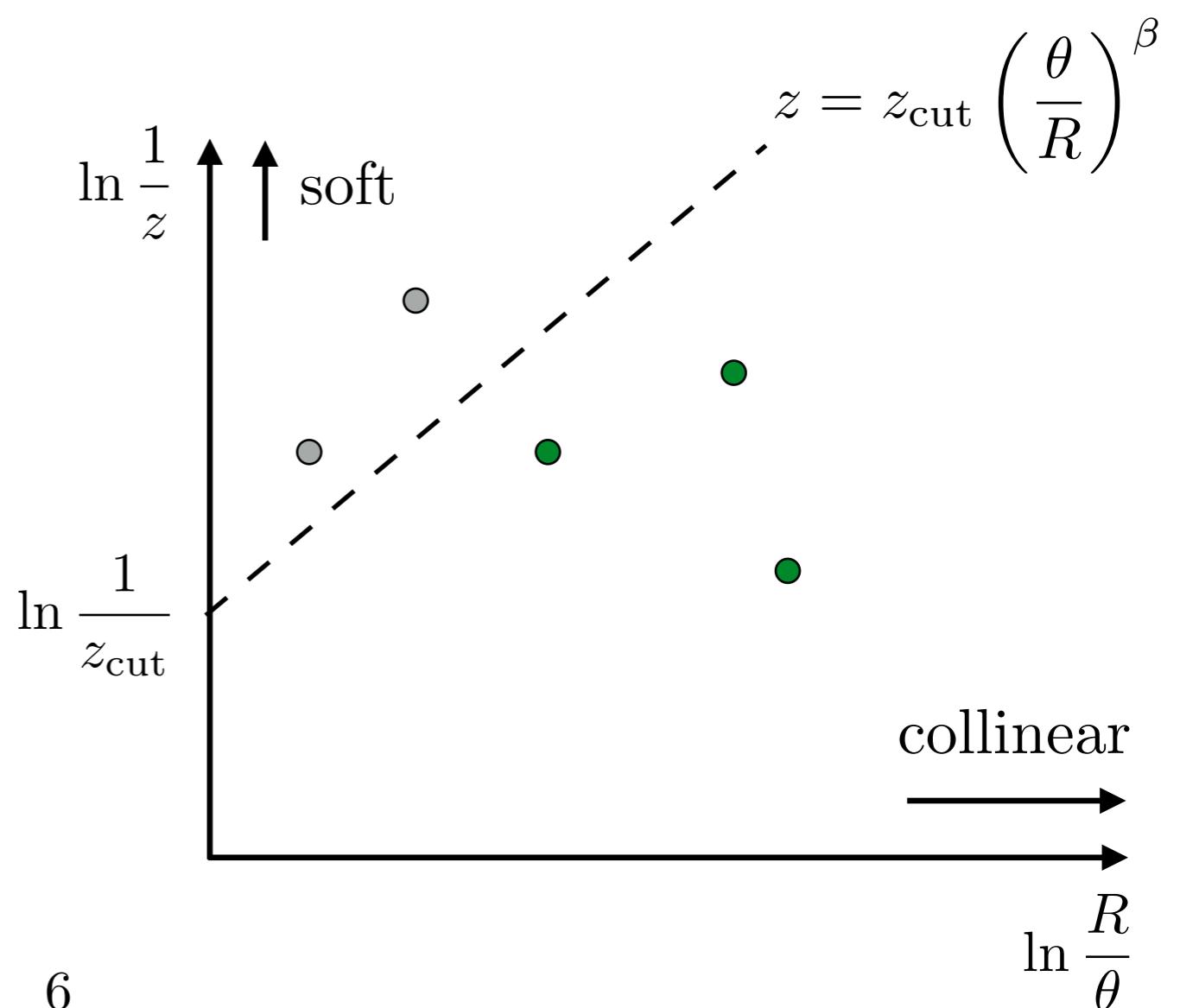
# $z_g$ at LL

- LL accuracy:
  - First computed by Larkoski, Marzani, Thaler 15'
  - Dominated by one soft-collinear emission



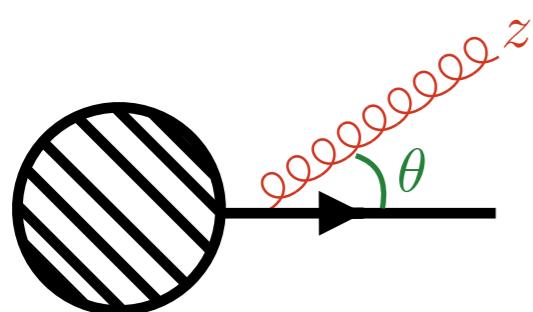
Soft and collinear limit:  $d\text{Prob}(z, \theta) = \frac{2\alpha_s C_i}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$

- Lund plane:

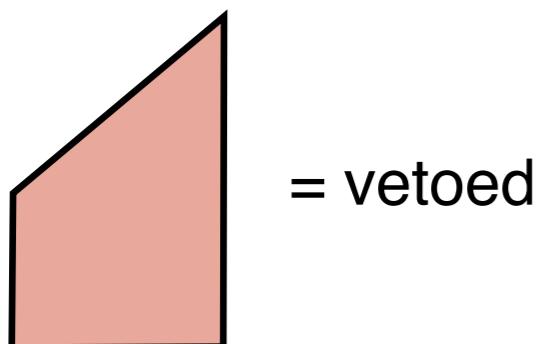


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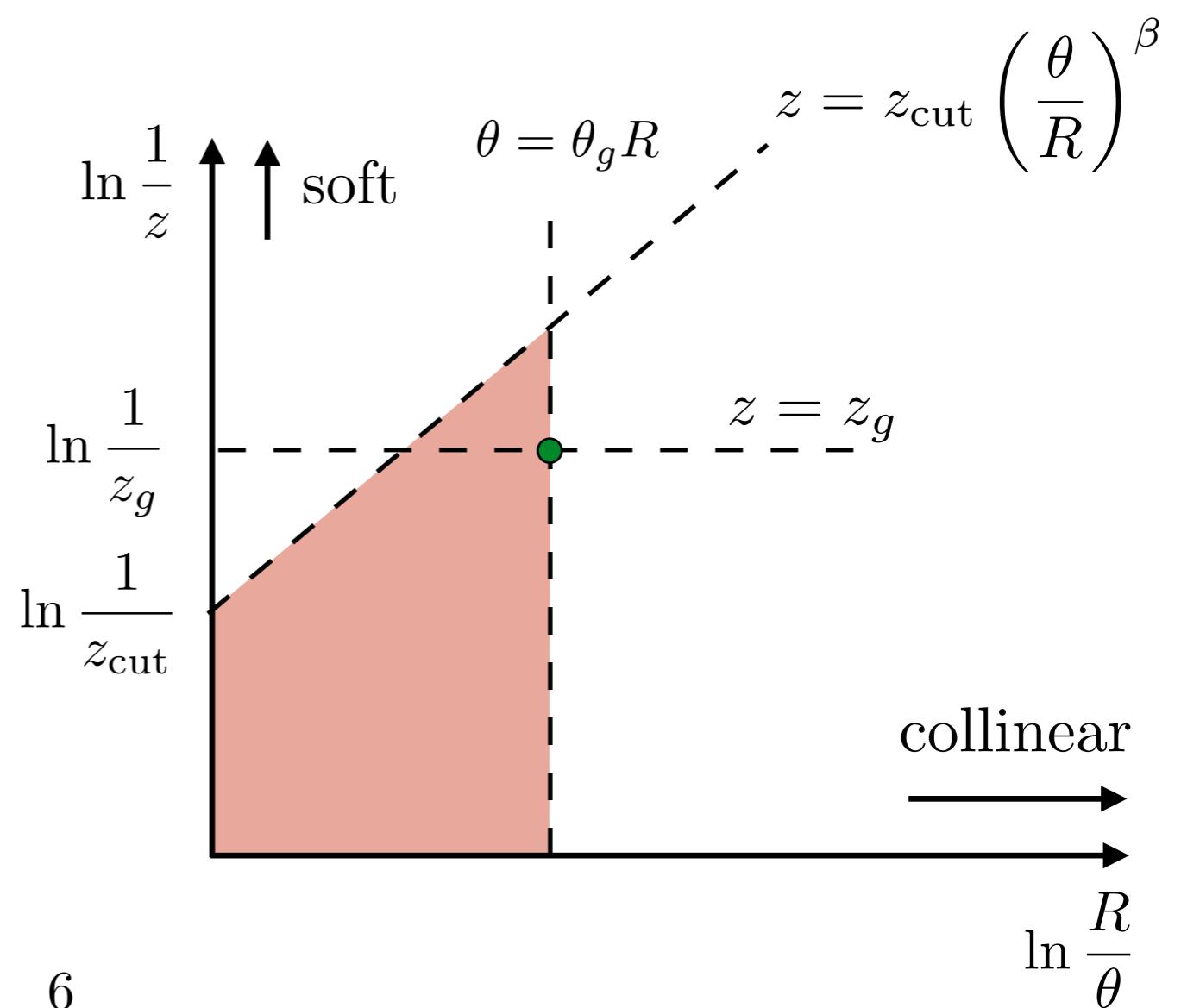


- Lund plane:



$$\theta_g \equiv \frac{R_g}{R}$$

Soft and collinear limit:  $d\text{Prob}(z, \theta) = \frac{2\alpha_s C_i}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$



# $z_g$ at LL

- LL result:

$$\frac{d\sigma}{dz_g d\theta_g} = \frac{2\alpha_s C_i}{\pi} \text{ (green circle)} \times \exp \left[ -\frac{2\alpha_s C_i}{\pi} \text{ (red triangle)} \right] \Theta(1/2 > z_g > z_{\text{cut}} \theta_g^\beta)$$

$$\theta_g \equiv \frac{R_g}{R}$$

$$\frac{d\sigma}{dz_g d\theta_g} = \frac{2\alpha_s C_i}{\pi} \frac{1}{z_g \theta_g} \exp \left[ -\frac{\alpha_s C_i}{\pi} (\beta \ln^2 \theta_g + 2 \ln z_{\text{cut}} \ln \theta_g) \right] \Theta(1/2 > z_g > z_{\text{cut}} \theta_g^\beta)$$

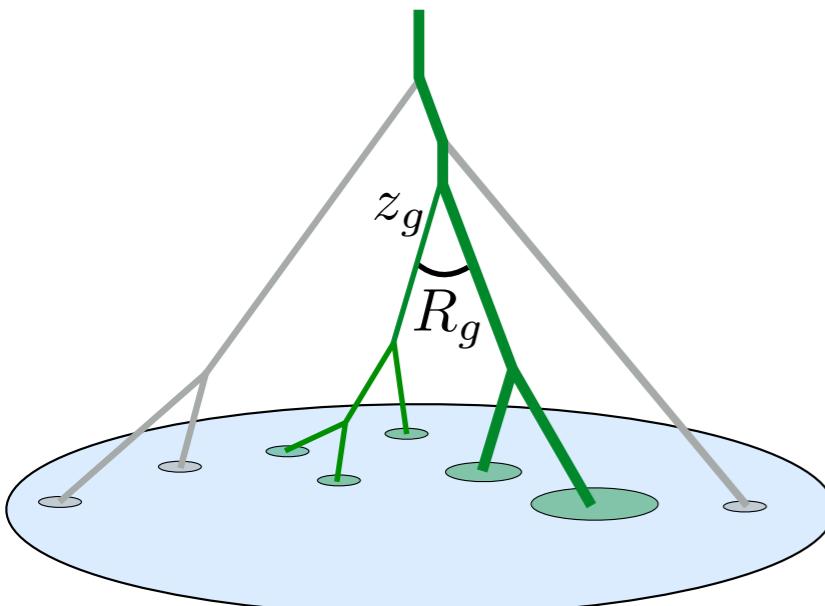
- Integrating over  $\theta_g$  agrees with **Larkoski, Marzani, Thaler 15'**

- LL result above:
  - Probes color factor  $C_i$
  - Probes singular part of splitting function  $P_i \sim \frac{1}{z}$
  - Fixed coupling
- Can we probe the **full** splitting function?
- Can we probe the **spin** of the initiating parton? }  $\implies$  Need higher accuracy

# $z_g$ at NLL'

- First step: fixed order calculation

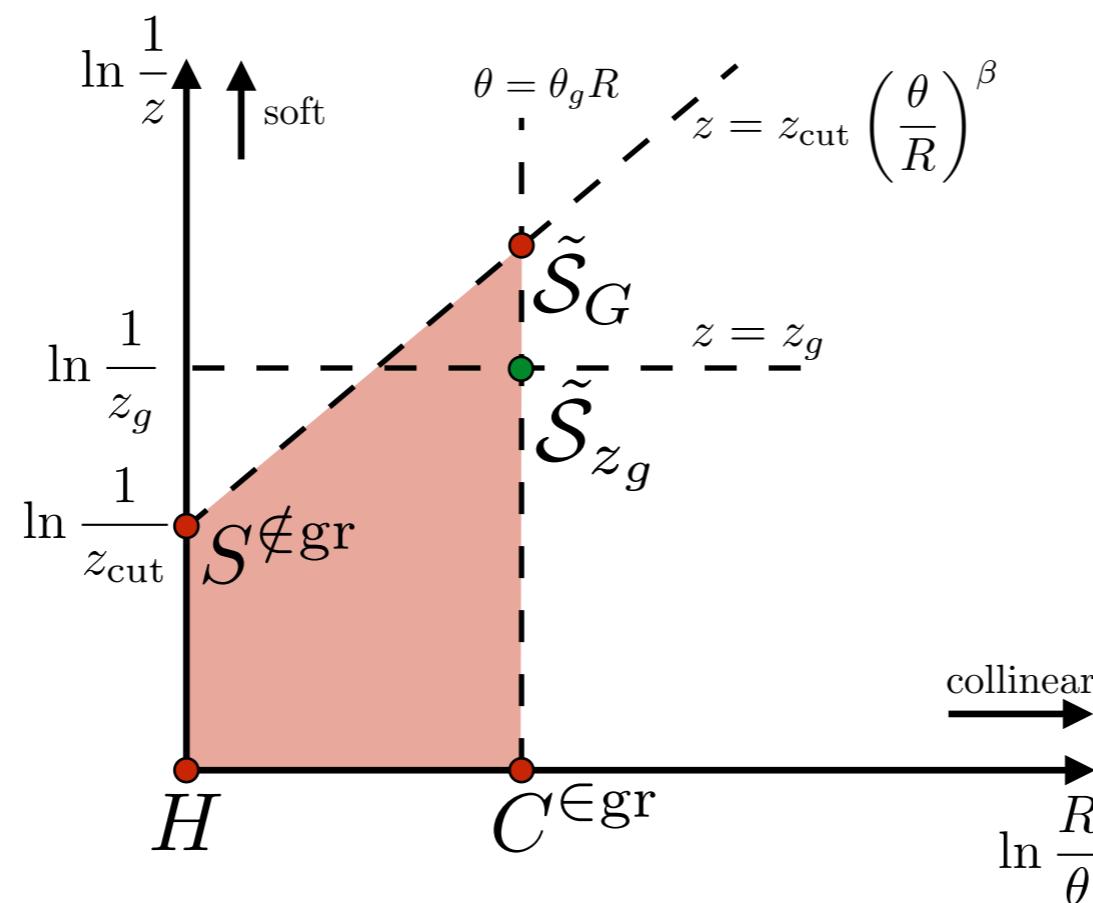
$$\tilde{\mathcal{G}}_q^{(1)} = \Theta(1/2 > z_g > z_{\text{cut}}) \Theta(\theta_g < 1) \frac{\alpha_s}{\pi} \frac{1}{\theta_g} [P_{qq}(z_g) + P_{gq}(z_g)]$$



- Probes **full** splitting function
  - Cannot integrate out  $\theta_g$  for  $\beta \geq 0$ , it's IRC unsafe
- ↓
- Joint resummation to NLL' accuracy in  $\theta_g$  and  $z_g$
  - Match to FO in order to probe non-singular
  - Integrate out  $\theta_g$

- How to achieve NLL'? → Factorization theorem in **Soft Collinear Effective Theory (SCET)**

# $z_g$ at NLL'



$$\theta_g \equiv \frac{R_g}{R}$$

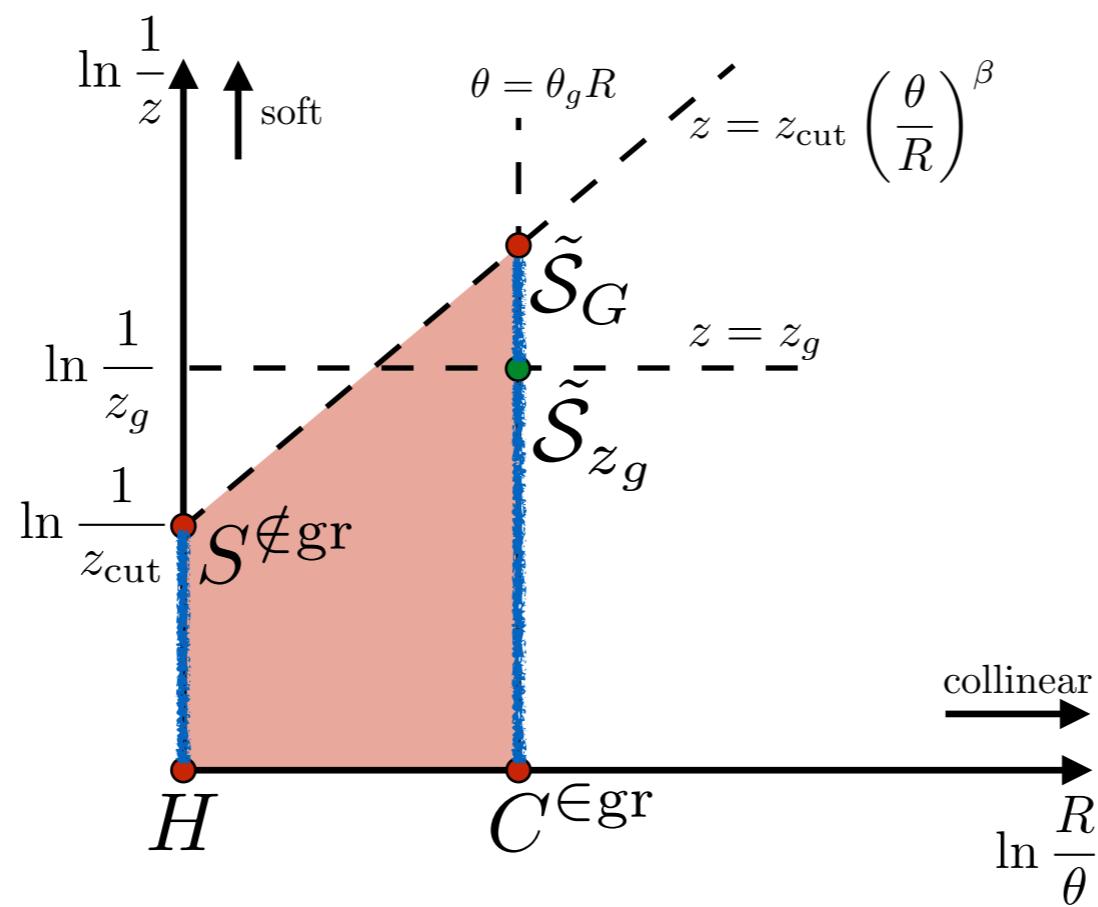
$$\tilde{\mathcal{G}}_i = \Theta(1/2 > z_g > z_{\text{cut}} \theta_g^\beta) \tilde{H}_i C_i^{\infty \text{gr}} S \notin \text{gr} \tilde{\mathcal{S}}_G \times \frac{d}{dz_g} \frac{d}{d\theta_g} \tilde{\mathcal{S}}_{z_g}$$

↓       $\theta_g$  cross section at NLL'      ↓  
Kang, Lee, Liu, Neill, Ringer 19'      sets  $z_g$  and  $\theta_g$

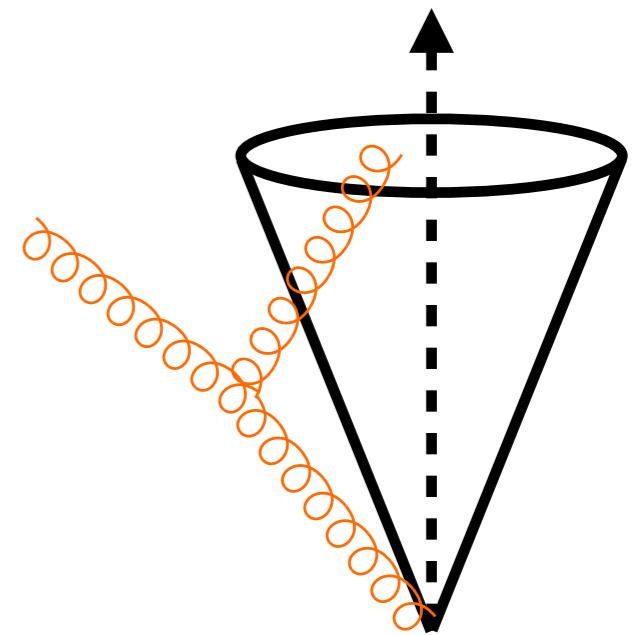
- Only RG consistent expression possible

See also Cal, Lee, Ringer, Waalewijn '20 for factorization of energy drop

# $z_g$ at NLL'



$$\theta_g \equiv \frac{R_g}{R}$$



- Additional complication: Non Global Logarithms (NGLs) can also set  $\theta_g$  and  $z_g$

Dasgupta, Salam '01'

- NGL Numerical effects are small  $\sim 1\%$

Banfi, Marchesini, Smye '02

Schwartz, Zhu '14

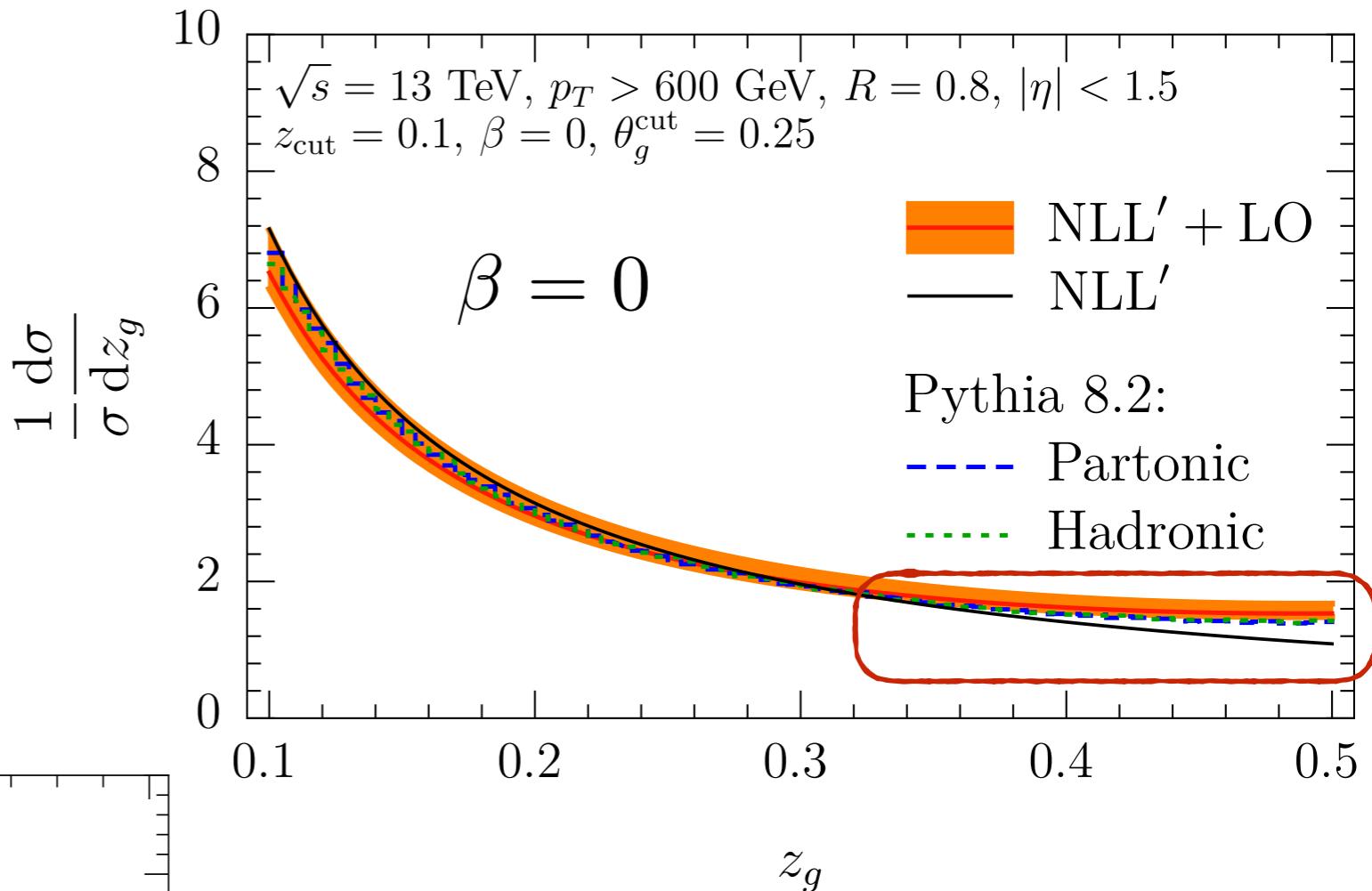
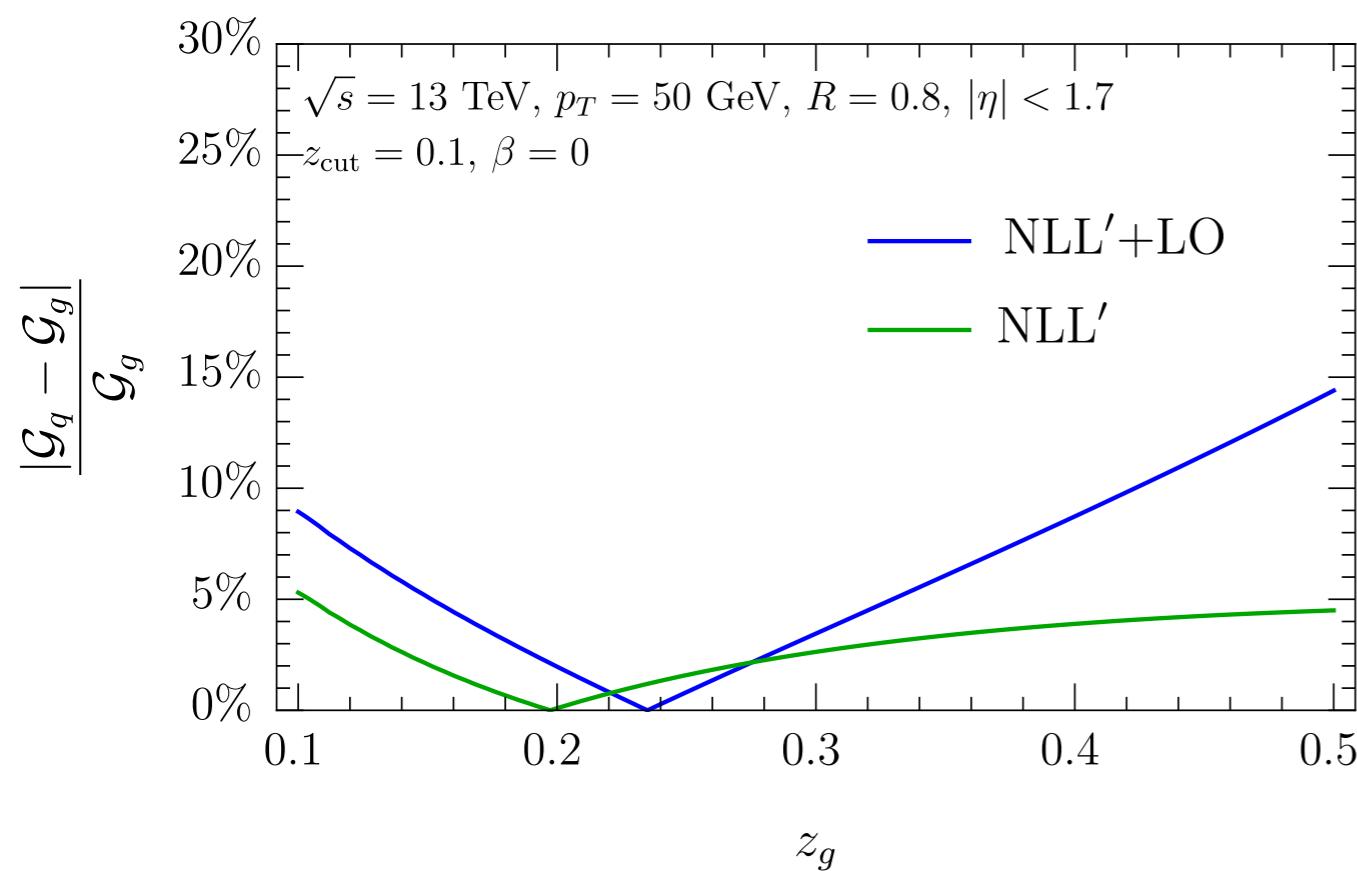
$$\tilde{\mathcal{G}}_i = \Theta(1/2 > z_g > z_{\text{cut}} \theta_g^\beta) \tilde{H}_i C_i^{\in \text{gr}} S^{\notin \text{gr}} \tilde{\mathcal{S}}_G S^{\text{NG}}(z_{\text{cut}})$$

$$\times \left[ \frac{d}{dz_g} \frac{d}{d\theta_g} \tilde{\mathcal{S}}_{z_g} + \tilde{\mathcal{S}}_1'^{\text{NG}}(z_g) + \tilde{\mathcal{S}}_2'^{\text{NG}}(z_{\text{cut}} \theta_g^\beta / z_g) \right]$$

# Results

- Comparison to Pythia

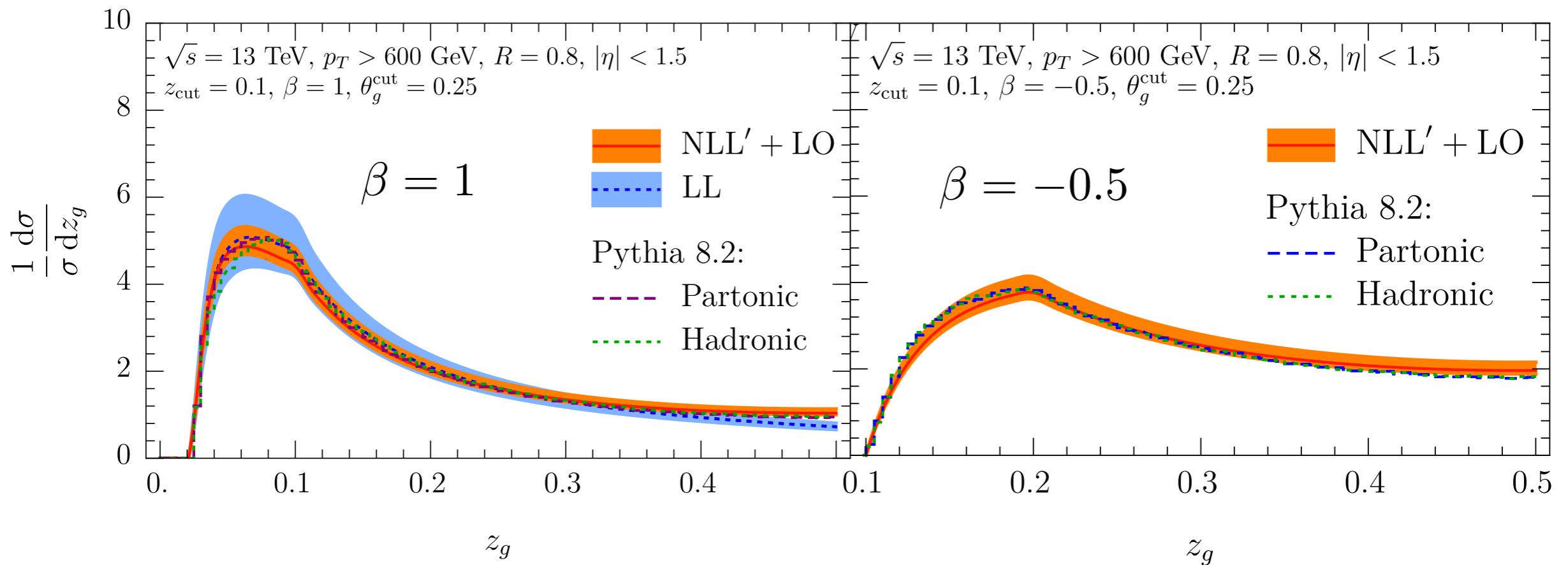
- NLL' uncertainties allow us to see effects of matching to full splitting function



- Around 10% difference between quark/gluon non-singular
- Observable difference between quark/gluon splitting function due to spin

# Results

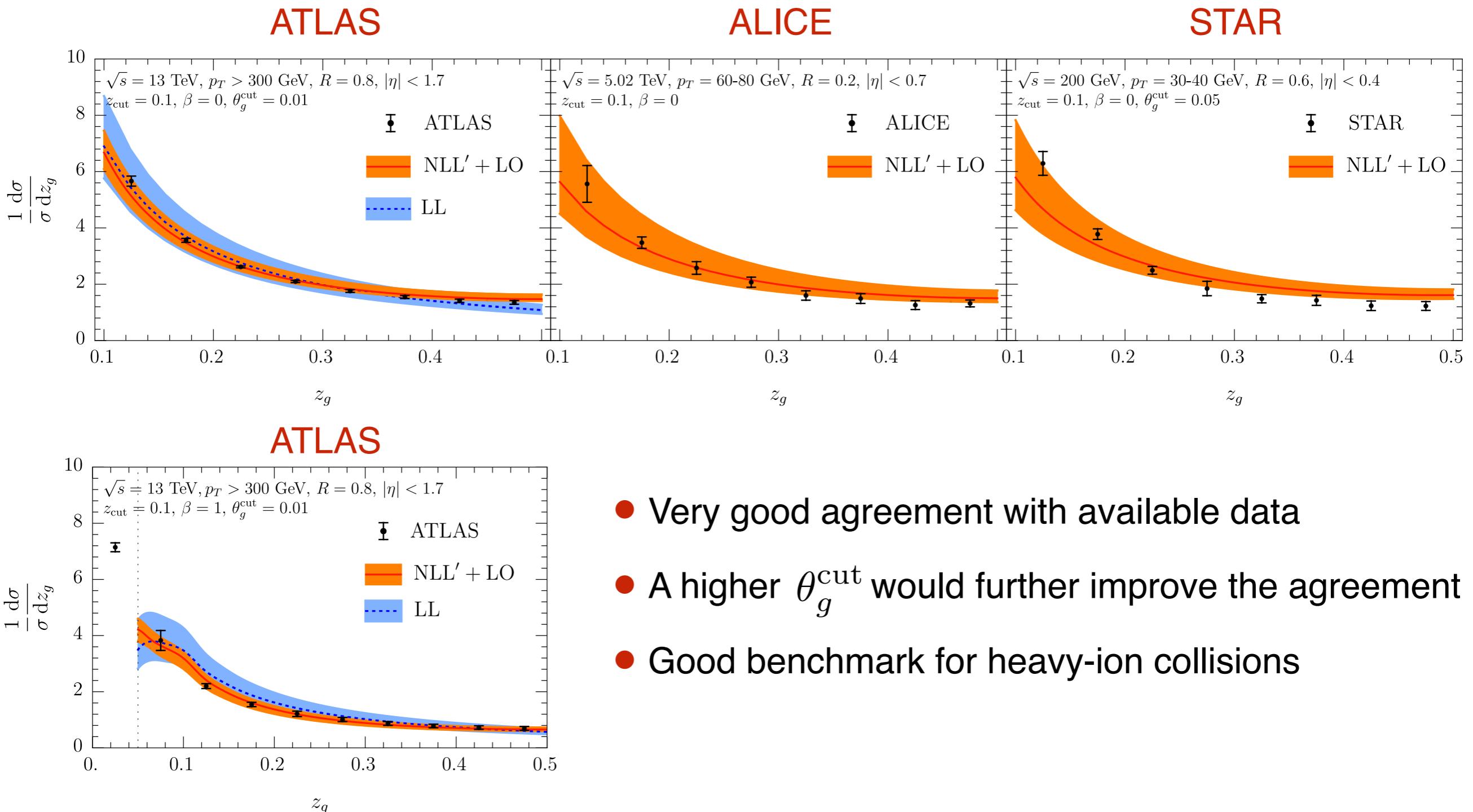
- Comparison to Pythia



- Effects of matching to full splitting function similar to  $\beta = 0$
- (SCET) LL compared to NLL': Larger bands, no matching

# Results

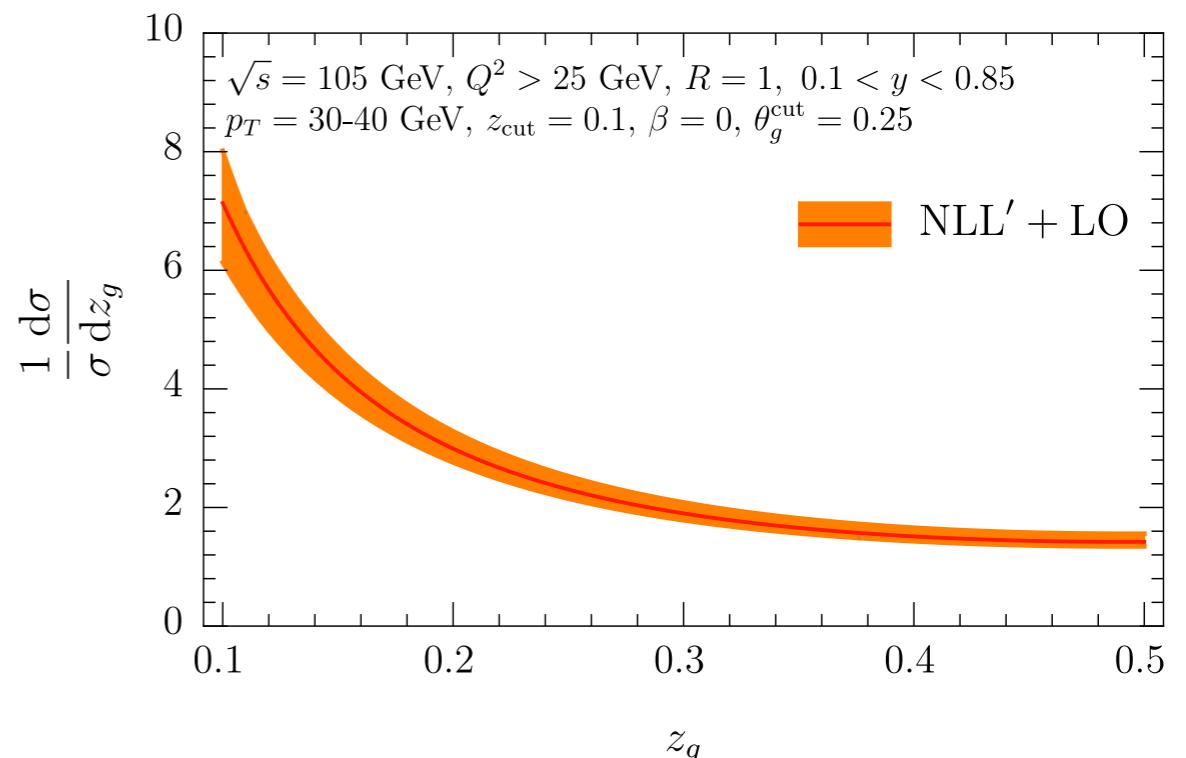
## Comparison to Data



# Conclusions

- Joint resummation in  $\theta_g$  and  $z_g$  at NLL' accuracy achieved through factorization in SCET
- NLL' + LO accuracy probes the full splitting splitting function
- Excellent agreement with Pythia for any  $\beta$
- Very good agreement with data from multiple collider experiments
- Important to have  $\theta_g^{\text{cut}}$  in order to control nonperturbative effects

Prediction for future EIC!



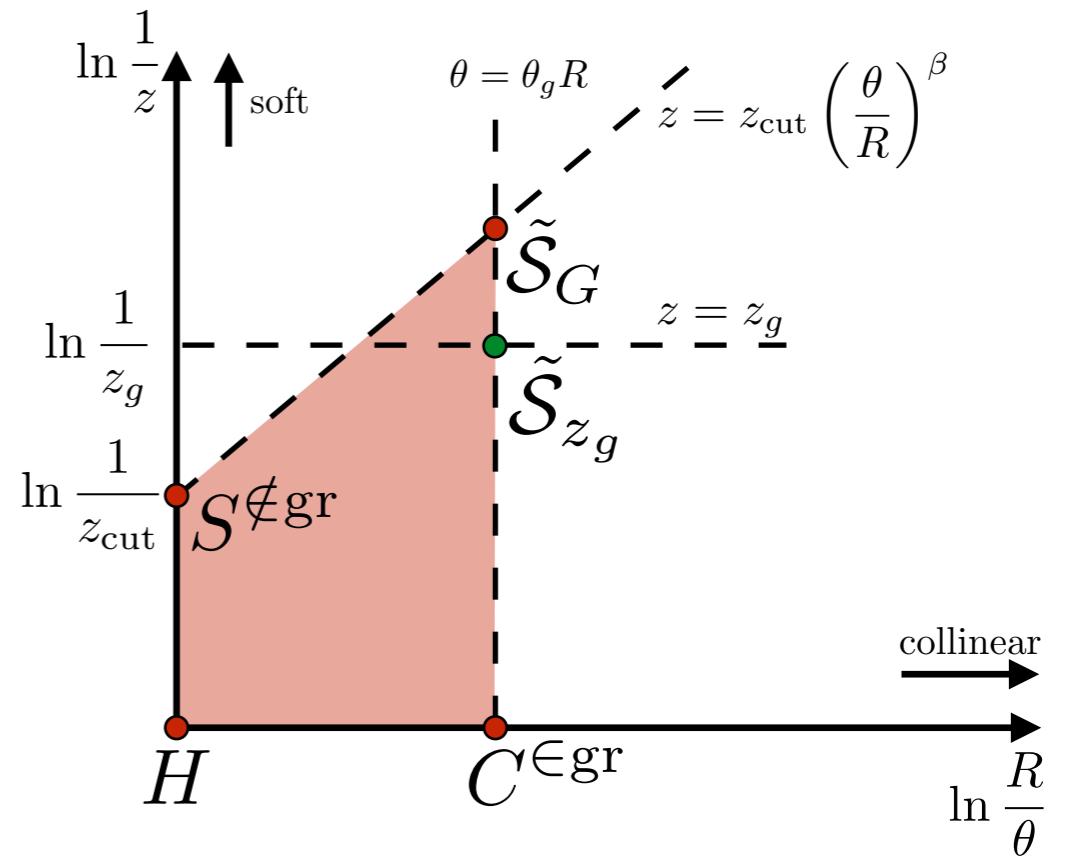
# **Backup slides**

# S\_zg

$$\tilde{\mathcal{S}}'_{z_g} \equiv \frac{d}{d\theta_g} \tilde{\mathcal{S}}_{z_g}$$

$$\tilde{\mathcal{S}}'_{z_g} = -\frac{2\alpha_s C_i}{\pi} \frac{1}{\theta_g} \ln \left( \frac{\mu}{z_g \theta_g p_T R} \right)$$

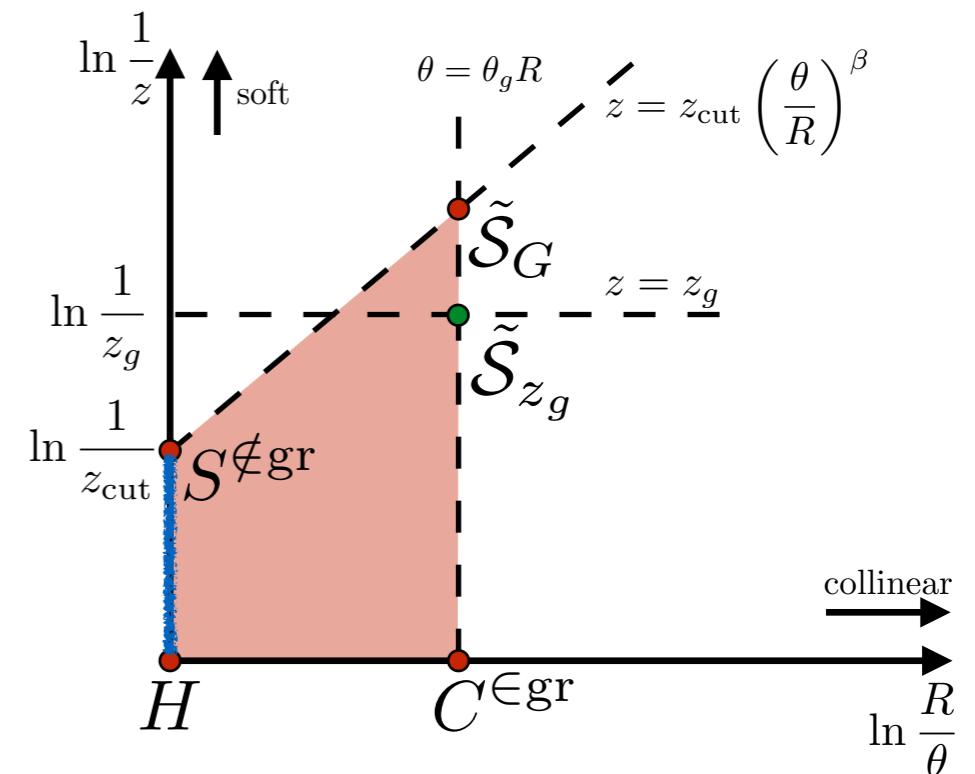
$$\frac{d}{d \ln \mu} \tilde{\mathcal{S}}'_{z_g} = -\frac{2\alpha_s C_i}{\pi} \frac{1}{\theta_g}$$



# NGLs

Outer boundary:

- Standard hemisphere NGLs in  $z_{\text{cut}}$
- Anti-kt, hard boundary: no clustering effects



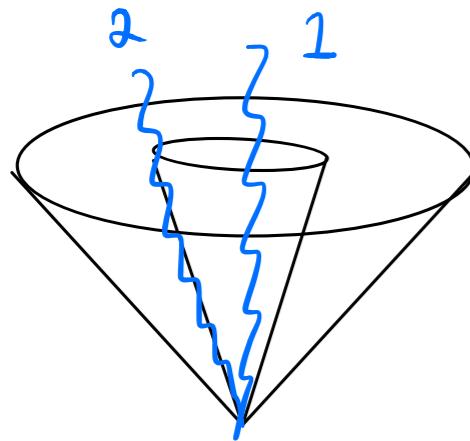
$$S_q^{\text{NG}}(\hat{L}) = 1 - \frac{\pi^2}{24}\hat{L}^2 + \frac{\zeta_3}{12}\hat{L}^3 + \frac{\pi^4}{34560}\hat{L}^4 + \left(-\frac{\pi^2\zeta_3}{360} + \frac{17\zeta_5}{480}\right)\hat{L}^5 + \mathcal{O}(L^6)$$

$$S_g^{\text{NG}}(\hat{L}) = [S_q^{\text{NG}}(\hat{L})]^2.$$

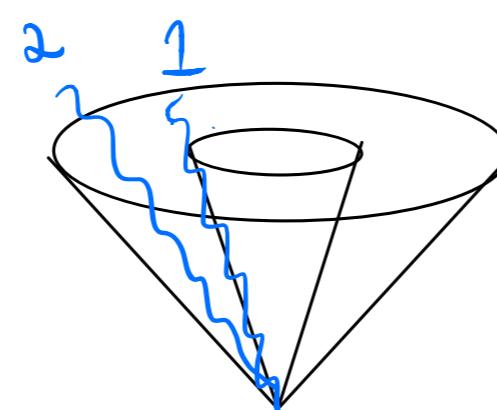
$$\hat{L} = -\frac{\alpha_s N_c}{\pi} \ln z_{\text{cut}}$$

# NGLs

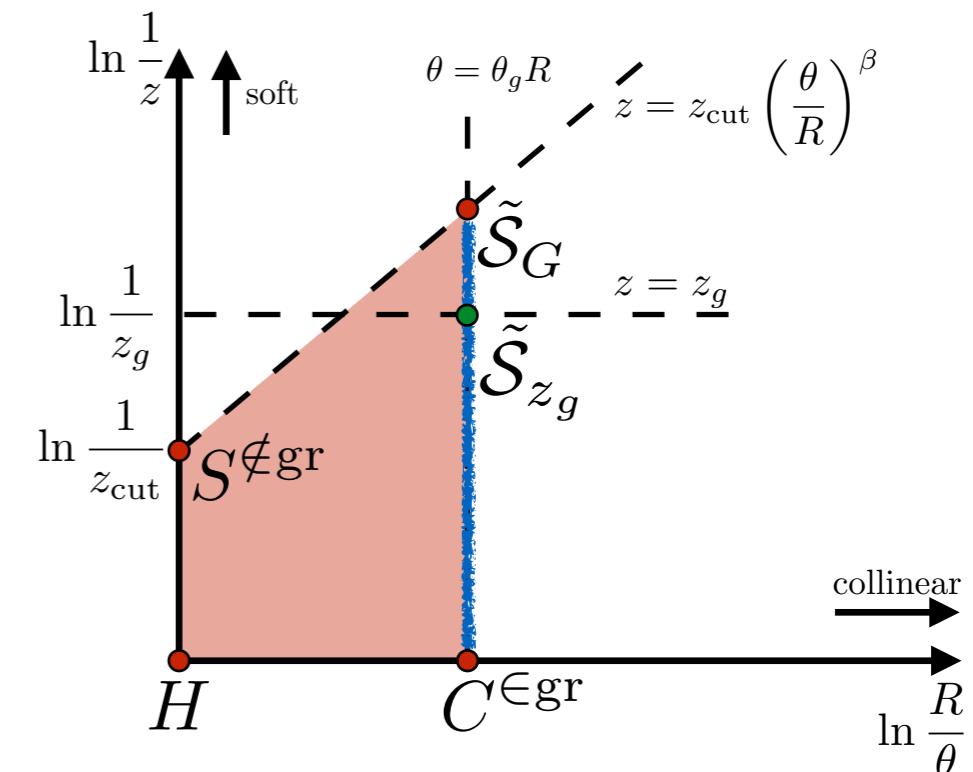
- Inner boundary, two NGL contributions
- Need to work differentially in  $z_g$  and  $t_g$



$$\tilde{\mathcal{S}}_{z_g} \leftrightarrow C^{\text{egr}}$$



$$\tilde{\mathcal{S}}_{z_g} \leftrightarrow \tilde{\mathcal{S}}_G$$



$$\tilde{\mathcal{S}}_{i,1}^{\prime \text{NG},(2)}(z_g) = 1.29 C_i C_A \left( \frac{\alpha_s}{2\pi} \right)^2 \frac{1}{z_g \theta_g} \ln z_g , \quad (7)$$

$$\tilde{\mathcal{S}}_{i,2}^{\prime \text{NG},(2)} \left( \frac{z_{\text{cut}} \theta_g^\beta}{z_g} \right) = -1.29 C_i C_A \left( \frac{\alpha_s}{2\pi} \right)^2 \frac{1}{z_g \theta_g} \ln \left( \frac{z_{\text{cut}} \theta_g^\beta}{z_g} \right) .$$