INSTITUTO DE FÍSICA
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The physics behind ultrarelativistic hydrodynamics

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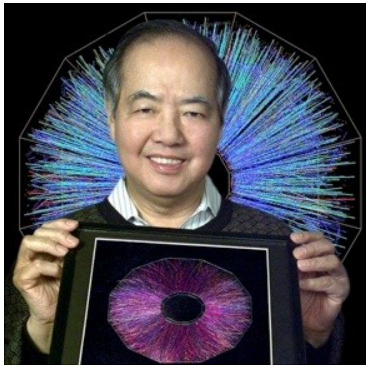
What you will see in this talk

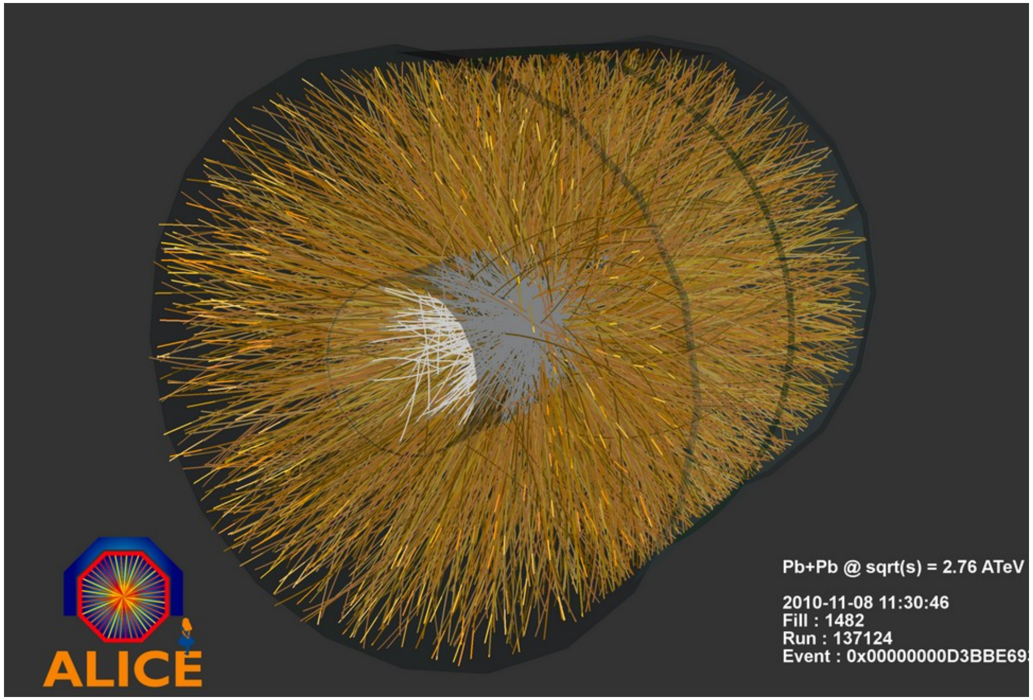
- ✔ Motivation: why study heavy-ion collisions?
- ✔ Fluid-dynamical modeling of heavy-ion collisions
- ✔ Conclusions and perspectives

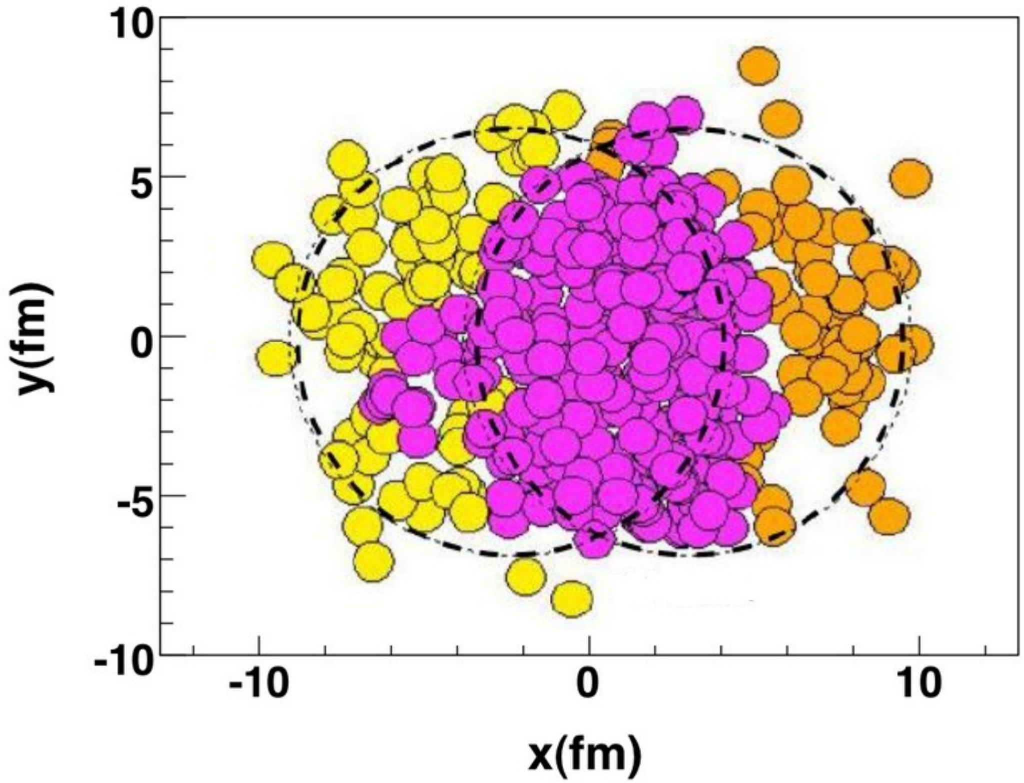
Why collide heavy ions?



T.D. Lee

Need to approach thermodynamic limit

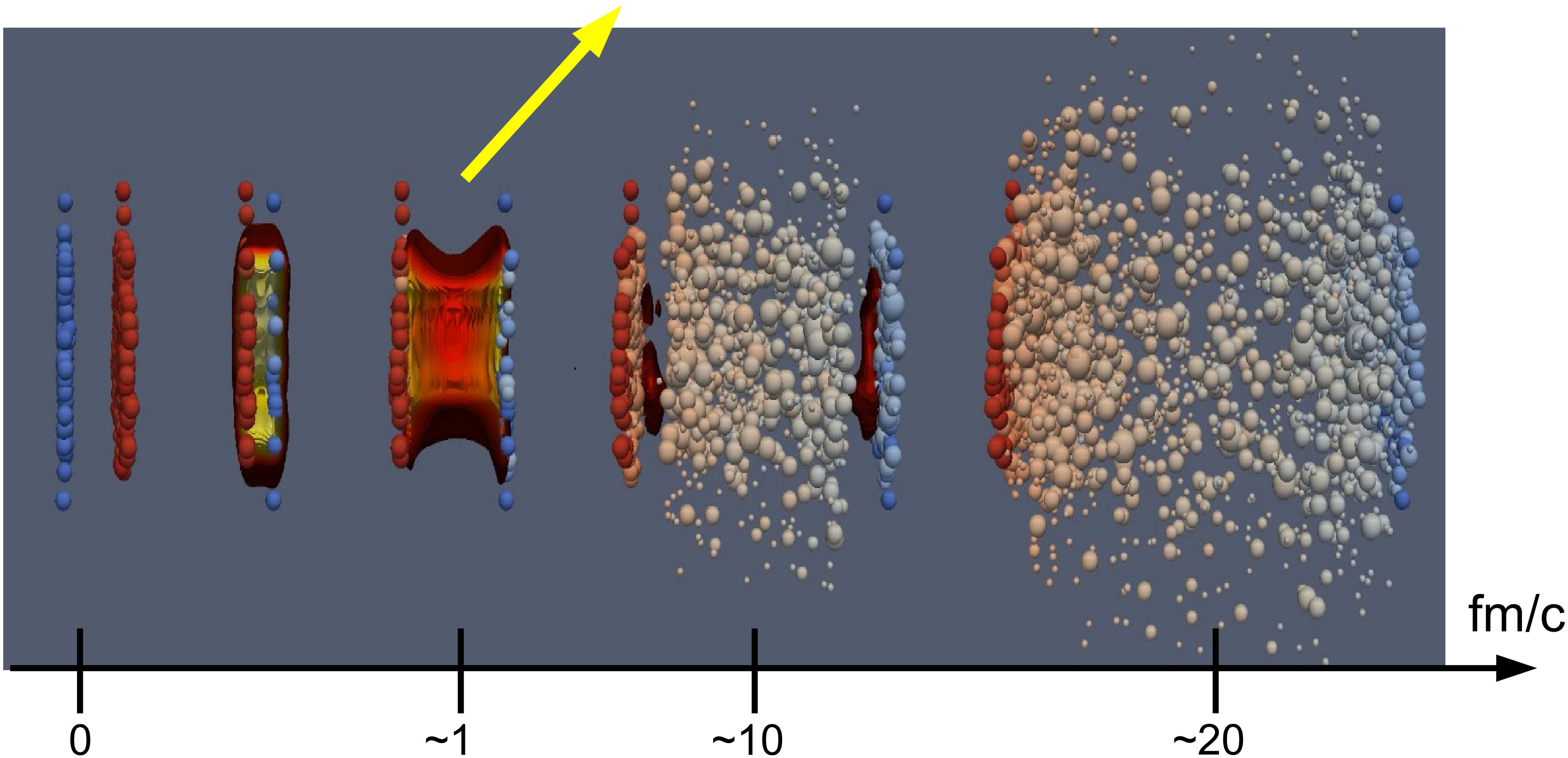




- **~thousands of particles**
- **large volumes ~1000 times larger than of a proton**

Heavy Ion Collisions

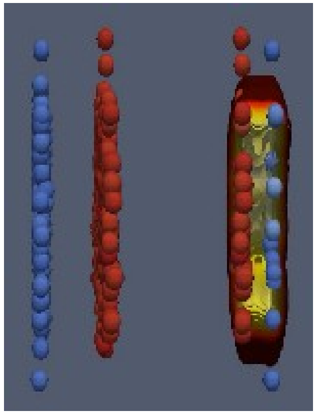
QCD matter is only created transiently ~10 fm/c



Hydrodynamic expansion expected

In this sense, use of hydrodynamics is not a surprise

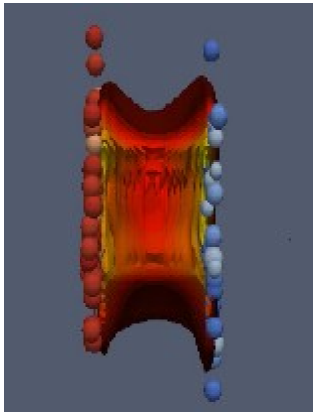
Hydrodynamic modeling of heavy ion collisions



Initial state and "pre-equilibrium" dynamics

description of early time-dynamics and thermalization

"hydrodynamization" by hand

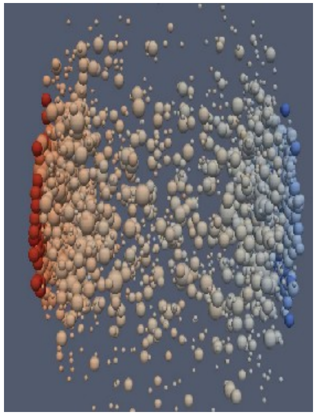


Fluid-dynamical expansion of QGP

Description of QGP as a relativistic dissipative fluid

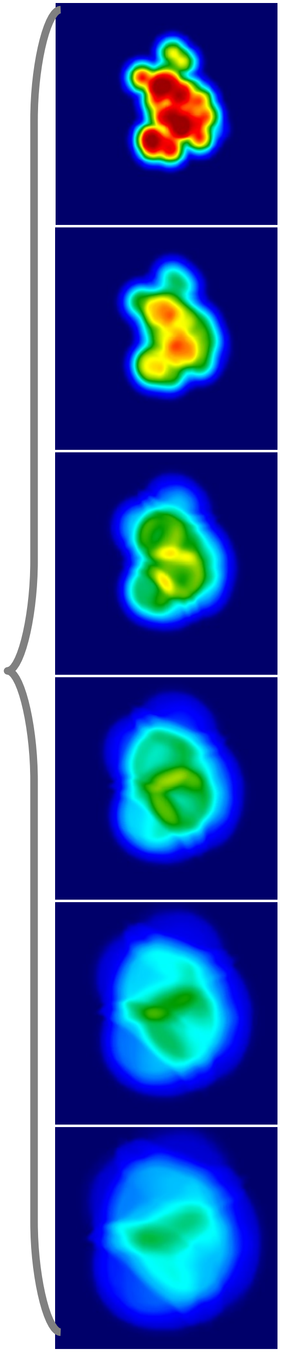
EoS, viscosities, ...

fluid elements converted to particles

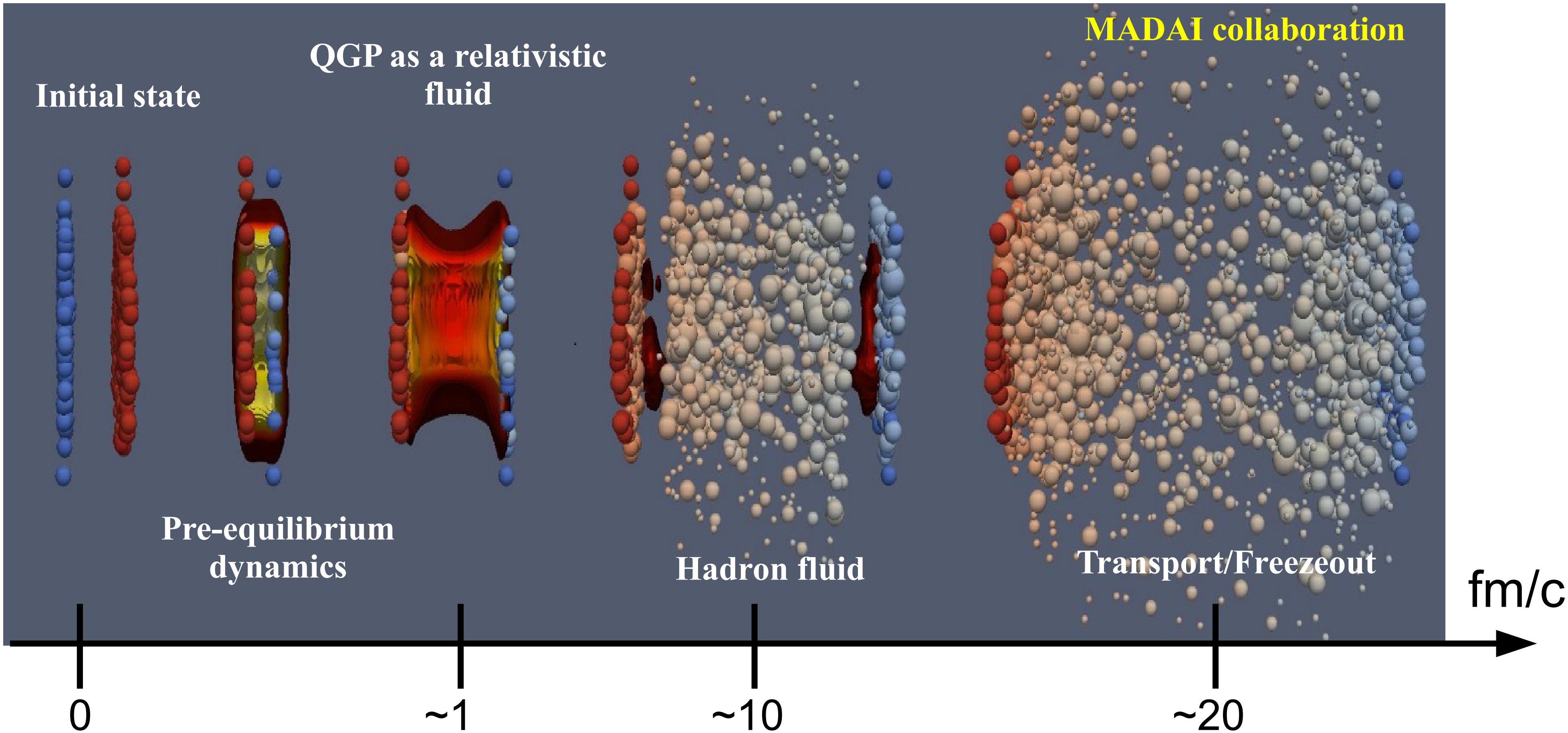


Transport description of Hadron Gas

Matter described by cross sections and decay probabilities



Empirical: "fluid-dynamical" modeling of heavy ion collisions works well at RHIC and LHC energies



Main assumption: system approaches "local equilibrium" on very small time scales ~1 fm

Does this make sense?

Basics of fluid dynamics

Effective theory describing the dynamics of a system over long-times and long-distances

Separation of scales → <u>**macroscopic:**</u> L <u>**microscopic:**</u> ℓ

Knudsen number: $K_N \sim \dfrac{\ell}{L} \ll 1$

Conservation laws

+

simple constitutive relations

Basics of fluid dynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net charge conservation

$$\partial_\mu N_s^\mu = 0 \quad \text{strangeness}$$

$$\partial_\mu N_e^\mu = 0 \quad \text{electric charge}$$

$$\partial_\mu N_b^\mu = 0 \quad \text{Baryon number}$$

Tensor decomposition

$$N_q^\mu = n_q u^\mu + n_q^\mu$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

net-charge diffusion 4-current (n_q^μ)

Bulk viscous pressure (Π)

Shear stress tensor ($\pi^{\mu\nu}$)

Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Basics of fluid dynamics

Conservation laws

[redacted]

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Tensor decomposition

[redacted]

$$T^{\mu\nu} \quad = \quad \varepsilon u^\mu u^\nu - (P_0 + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

net-charge diffusion 4-current

Bulk viscous pressure

Shear stress tensor

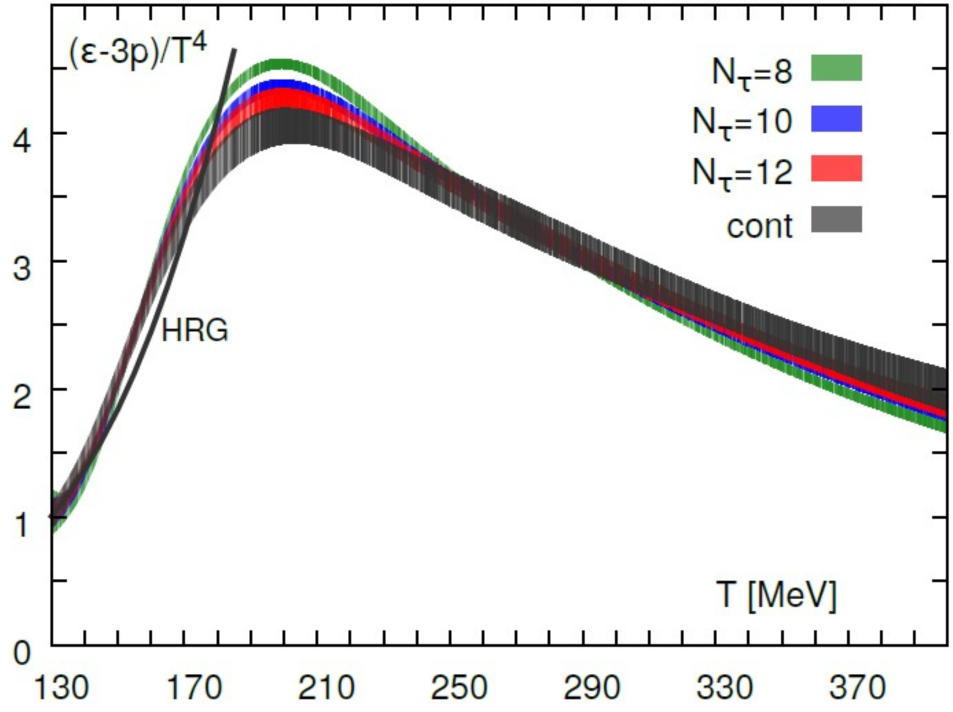
Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

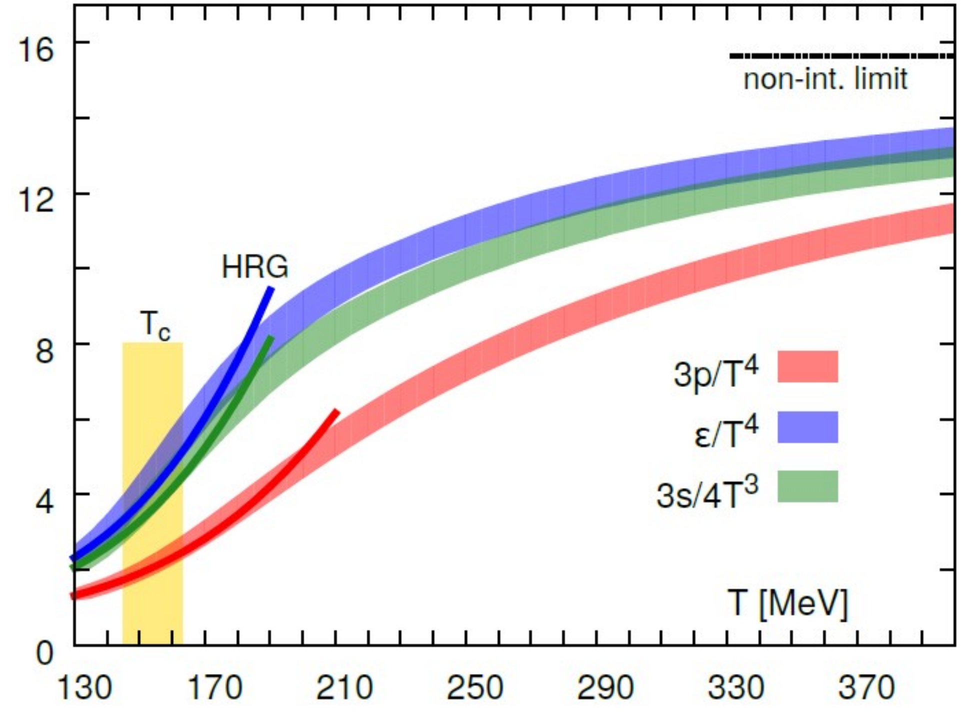
Equation of state

<u>*Thermodynamic pressure:*</u> $P_0 \quad = \quad P_0\left(T, \mu_b, \mu_e, \mu_s\right)$

Lattice QCD: entropy density increases near T = 180 MeV

phase transition





PRD 90, 094503 (2014)

Equation of state

Thermodynamic pressure: $P_0 = P_0(T, \mu_b, \mu_e, \mu_s)$

Taylor expansion up to 4th order: $$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \underbrace{\frac{\chi_{l,m,n}^{B,Q,S}}{l!m!n!}}_{\text{lQCD}} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

- matched to *hadron resonance gas* model at small T
- matched to Stefan-Boltzmann limit at large T

- Prescription employed by:

Monnai, Schenke, Shen, PRC 100, 024907 (2019)
Noronha-Hostler, Parotto, Ratti, Stafford, PRC 100, 064910 (2019)

Basics of fluid dynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net charge conservation

$$\begin{aligned} \partial_\mu N_s^\mu &= 0 \\ \partial_\mu N_e^\mu &= 0 \\ \partial_\mu N_b^\mu &= 0 \end{aligned}$$

strangeness

electric charge

Baryon number

Tensor decomposition

$$\begin{aligned} N_q^\mu &= n_q u^\mu + n_q^\mu \\ T^{\mu\nu} &= \varepsilon u^\mu u^\nu - (P_0 + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \end{aligned}$$

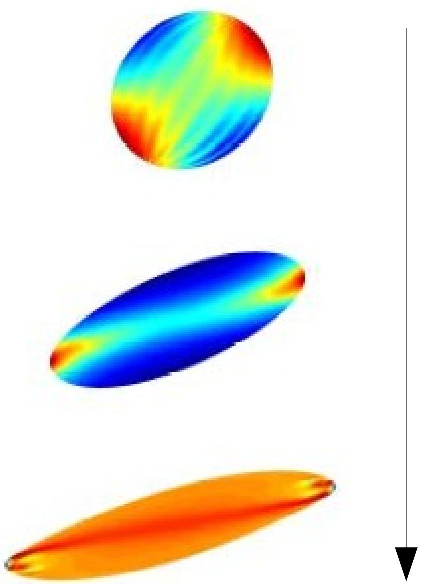
Challenge: What are the EoM's for the dissipative currents?

Relativistic Navier-Stokes theory

Shear Viscosity

(Resistance to deformation)

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$



$$\eta\left(T,\mu_q\right)$$

Bulk Viscosity

(Resistance to expansion)

$$\Pi = -\zeta\nabla_\mu u^\mu$$

$$\zeta\left(T,\mu_q\right)$$

Net-Charge Diffusion

$$n_q^\mu = \kappa_q\nabla^\mu\frac{\mu_q}{T}$$

$$\kappa_q\left(T,\mu_q\right)$$

Navier-Stokes Theory

Dissipative currents $\pi_{\mu\nu}$ are proportional to gradients

$$\pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + \dots$$

The equations are **acausal** and **unstable** !!!!

Israel-Stewart Theory (transient theory)

Dissipative currents $\pi_{\mu\nu}$ become dynamical variables …

$$\tau_\pi\,\dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + \dots$$

The equations can be **causal** and **stable** !!!!

General theory

$$F^\mu = \nabla^\mu P_0$$
$$I^\mu = \nabla^\mu \alpha_0$$

More accurate than Israel-Stewart theory

$$\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta\theta - \ell_{\Pi n}\nabla \cdot n - \tau_{\Pi n} n \cdot F - \delta_{\Pi\Pi}\Pi\theta \\
&- \lambda_{\Pi n} n \cdot I + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \ , \\
\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu &= \kappa_n I^\mu \ - n_\nu \omega^{\nu\mu} - \delta_{nn} n^\mu \theta - \ell_{n\Pi}\nabla^\mu \Pi \\
&+ \ell_{n\pi}\Delta^{\mu\nu}\nabla_\lambda \pi^\lambda_\nu + \tau_{n\Pi}\Pi F^\mu - \tau_{n\pi}\pi^{\mu\nu}F_\nu \\
&- \lambda_{nn} n_\nu \sigma^{\mu\nu} + \lambda_{n\Pi}\Pi I^\mu - \lambda_{n\pi}\pi^{\mu\nu} I_\nu, \\
\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle} \\
&+ \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi n} n^{\langle\mu}F^{\nu\rangle} + \ell_{\pi n}\nabla^{\langle\mu} n^{\nu\rangle} \\
&+ \lambda_{\pi n} n^{\langle\mu} I^{\nu\rangle}
\end{aligned}$$

All transport coefficients must be provided!

Theory we solve

$$F^\mu = \nabla^\mu P_0$$
$$I^\mu = \nabla^\mu \alpha_0$$

More accurate than Israel-Stewart theory

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \text{[redacted]} - \text{[redacted]} - \delta_{\Pi\Pi}\Pi\theta$$
$$- \text{[redacted]} + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \ ,$$

[redacted]

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle}$$
$$+ \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \text{[redacted]} + \text{[redacted]}$$
$$+ \text{[redacted]}$$

All transport coefficients must be provided!

Conclusions and outlook

Fluid-dynamical models that describe several energy heavy ion collisions are under construction – but appear to be able to fit the data

- Equation of state

- dynamical equations for dissipative currents

- Many transport coefficients must be specified as functions of T, μ_B