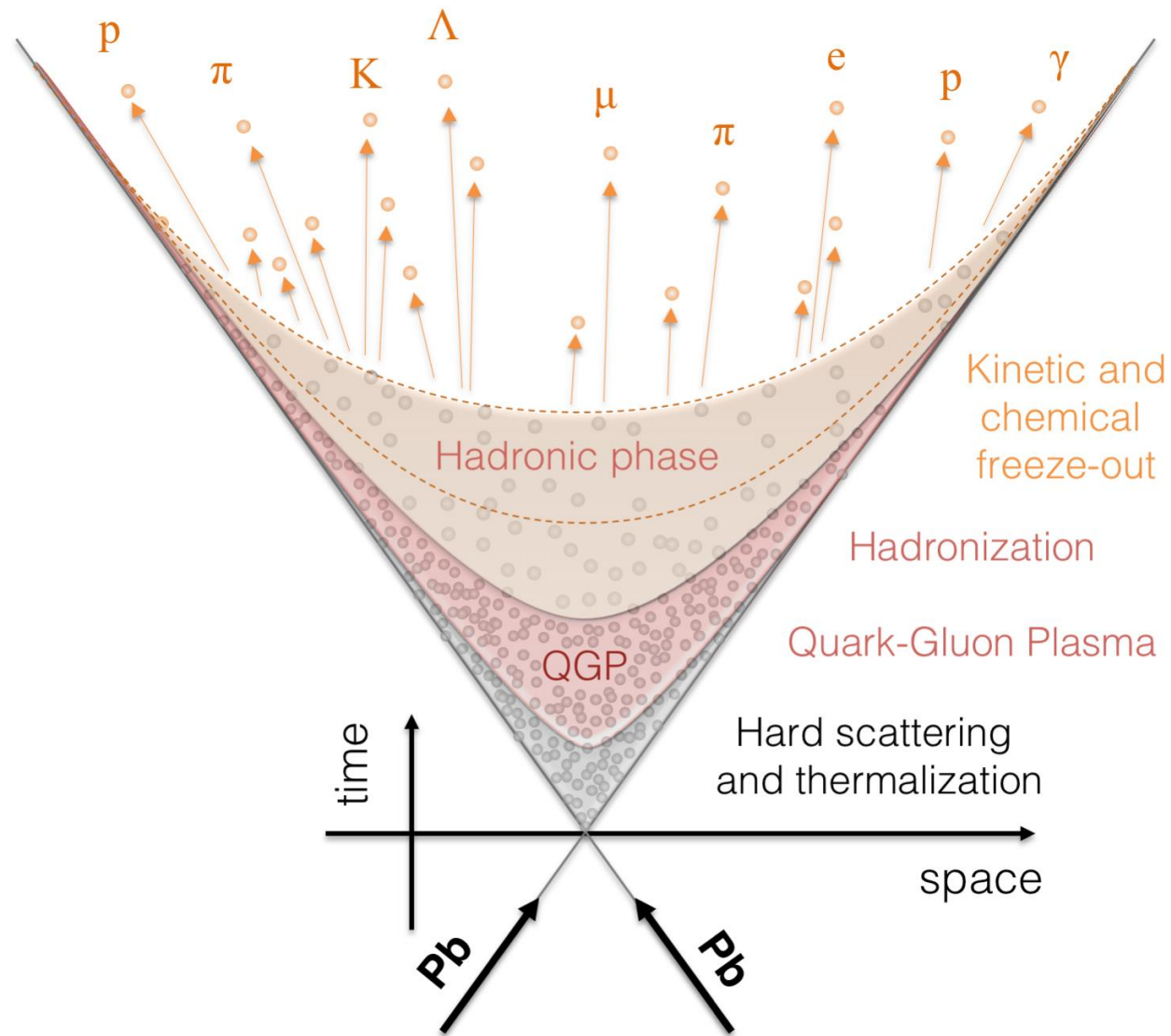


# Late Stages

Particlization and Hadronic Phase

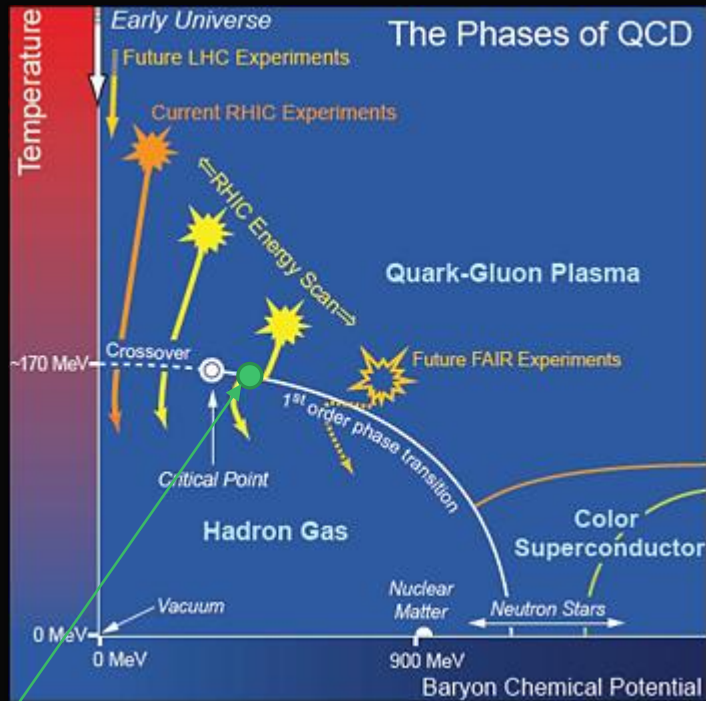
# Overview

- ◇ We have seen
  - ◇ Initial condition generation
  - ◇ Pre-equilibrium dynamics
  - ◇ Hydrodynamic evolution
- ◇ How does the fluid prescription connect to particles?



# Hadronization vs Particlization

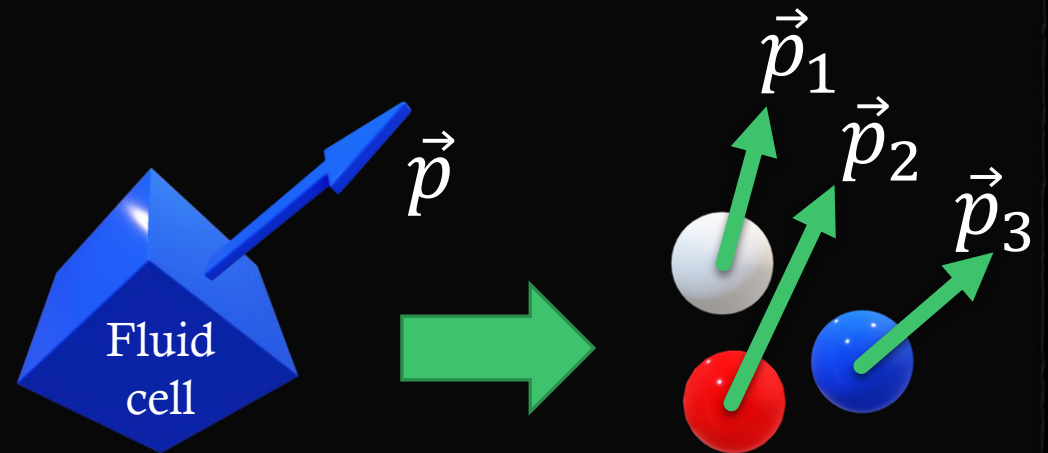
## Hadronization



Partons in QGP becomes hadrons

## Particlization

- ◆ The procedure of transforming fluid cells into hadrons

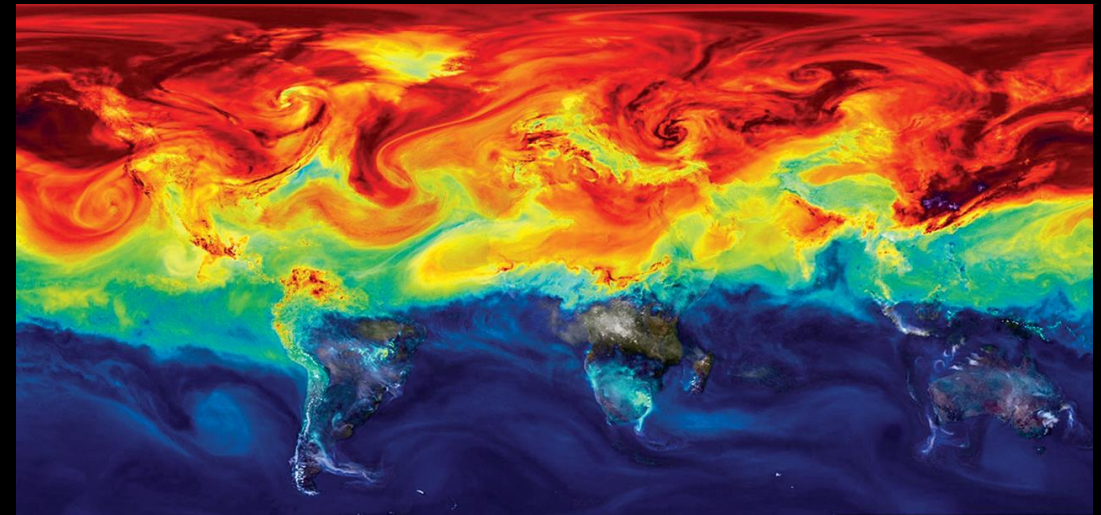


# Particlization procedure

- ◇ QGP and Hydrodynamics are not the same thing
- ◇ Hadron gas model and hydrodynamics have an overlap of applicability

# Particlization procedure

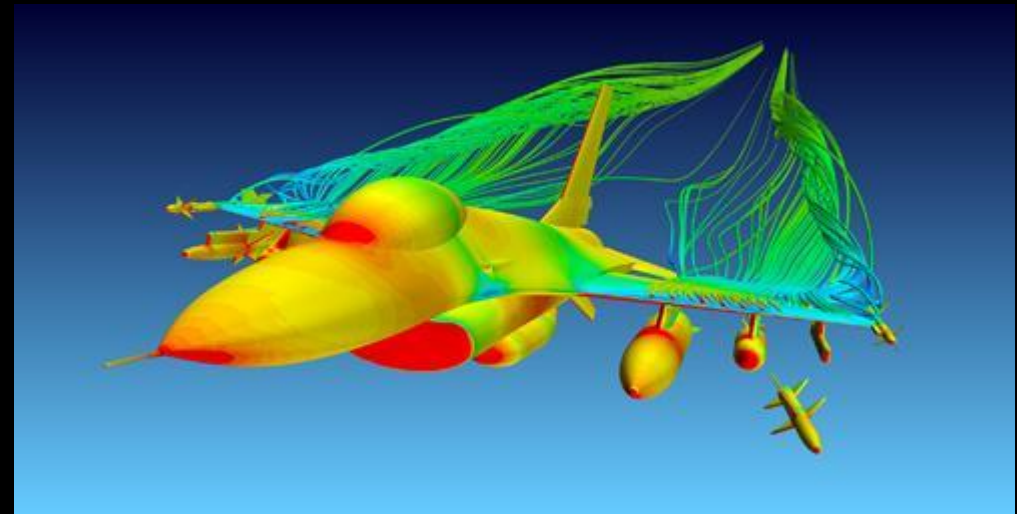
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  - ◇ Climate modelling



Soden, Collins, Feldman. *Science*, Vol. 361, Issue 6400, pp. 326-327

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<https://www.designnews.com/design-hardware-software/cfd-tool-adds-moving-object-simulation>

Accessed on March 11<sup>th</sup>, 2021

# Particlization procedure

- ◇ QGP and Hydrodynamics are not the same thing
- ◇ Hadron gas model and hydrodynamics have an overlap of applicability
- ◇ Macroscopic examples:
  - ◇ Climate modelling
  - ◇ Modelling of airplanes
- ◇ Particlization is done by exploiting the overlap between Hydrodynamic and **Kinetic Theory** (Hadron gas)

# Particlization procedure

◇ Cooper-Frye

$$\diamond E \frac{d^3N}{dp^3} = \int d\sigma_\mu p^\mu f(x, p)$$



# Particlization procedure

- ◇ Cooper-Frye

- ◇  $E \frac{d^3N}{d^3p} = \int d\sigma_\mu p^\mu f(x, p)$

- ◇ Momentum distribution of particles

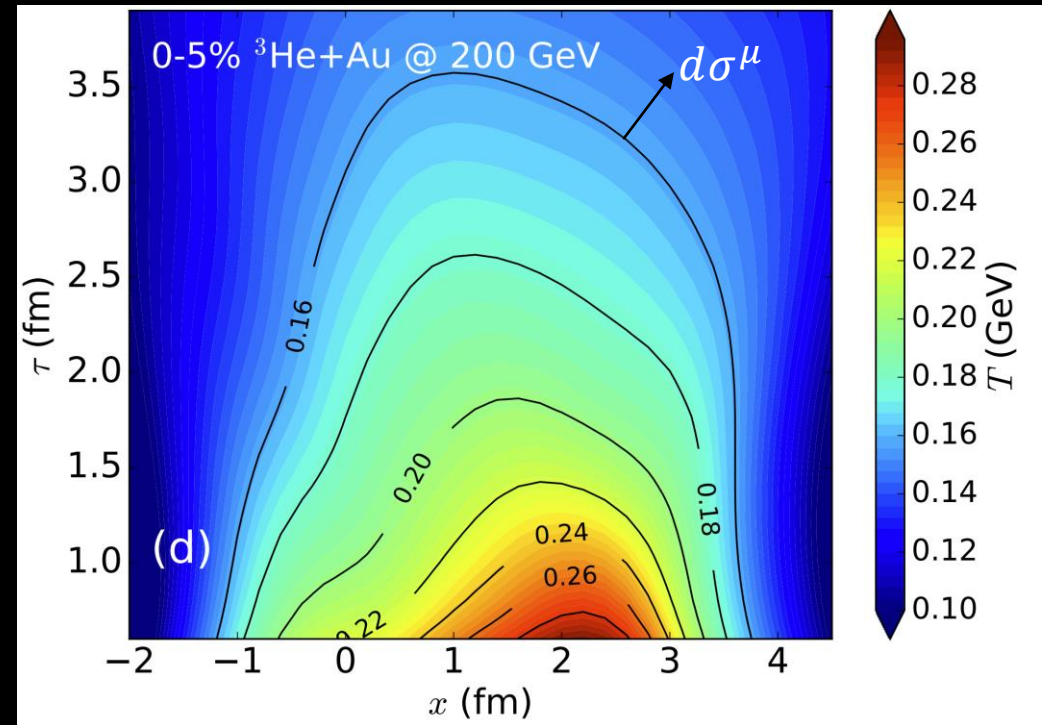
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C. Shen, et al. Phys.Rev. C95 (2017) no.1, 014906

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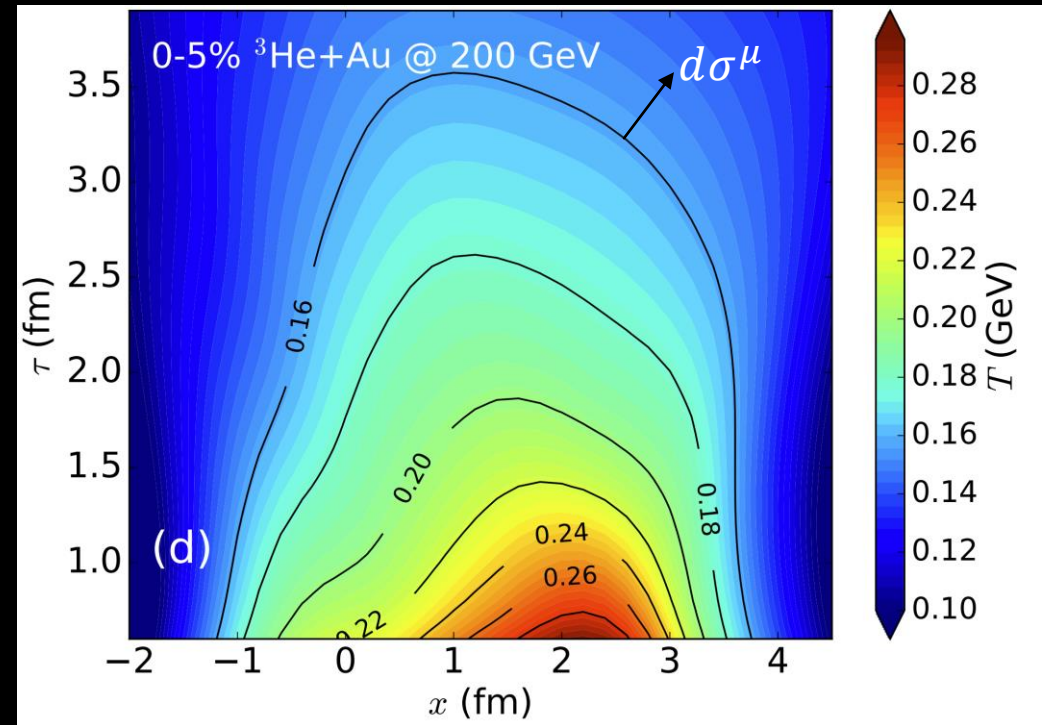
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# Particlization procedure

## ◇ Cooper-Frye

$$\diamond E \frac{d^3N}{dp^3} = \int d\sigma_\mu p^\mu f(x, p) \approx \sum_\sigma \Delta\sigma_\mu p^\mu f(x, p)$$

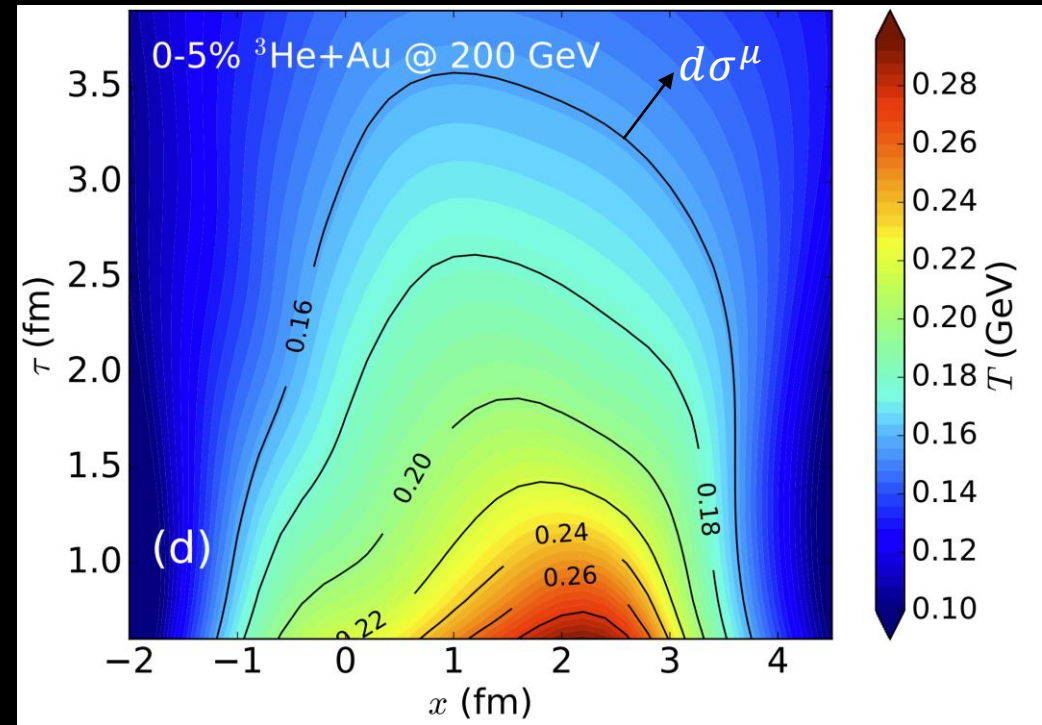
◇ Momentum distribution of particles

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## ◇ Particle sampling

◇ Particles placed on the place of hypersurface following its momentum distribution



C. Shen, et al. Phys.Rev. C95 (2017) no.1, 014906

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## ◇ References to learn more:

◇ [\[1206.3371\] Particlization in hybrid models](#) – Huovinen, Petersen

◇ [Phys. Rev. D 10, 186 \(1974\) - Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production](#) – Cooper, Frye

◇ [The iEBE-VISHNU code package for relativistic heavy-ion collisions](#) – Shen et al. – iSS article

# Hadron gas - UrQMD

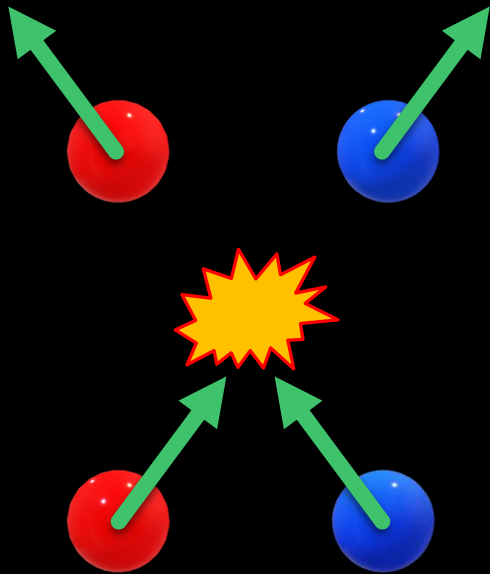
- ◇ Up to thousand of particles produced in each collision
- ◇ If system is dense enough, hadrons may scatter on each other

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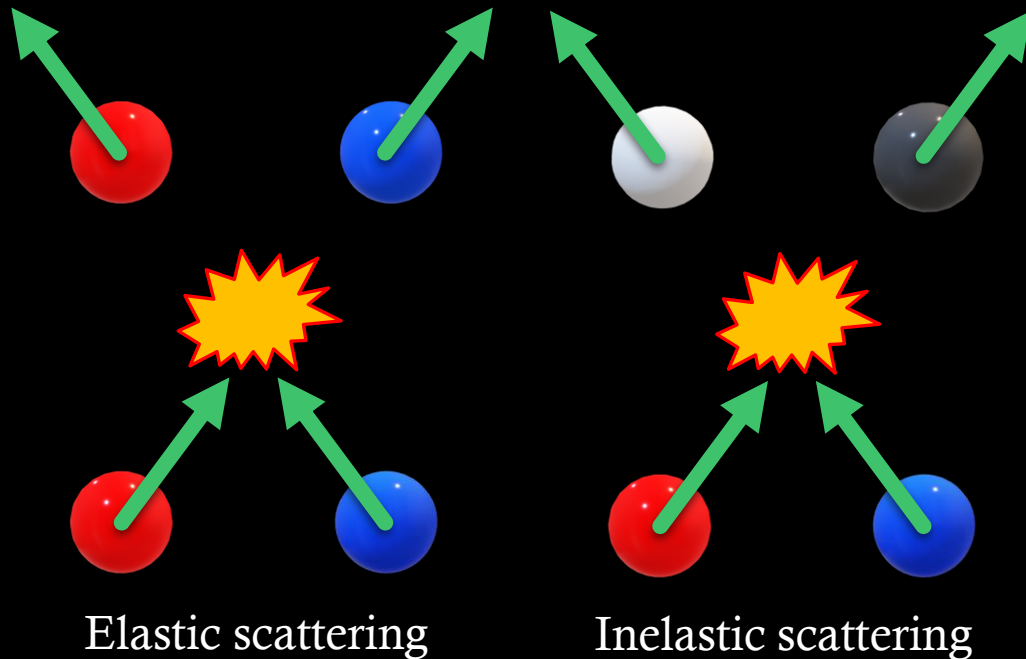


Elastic scattering



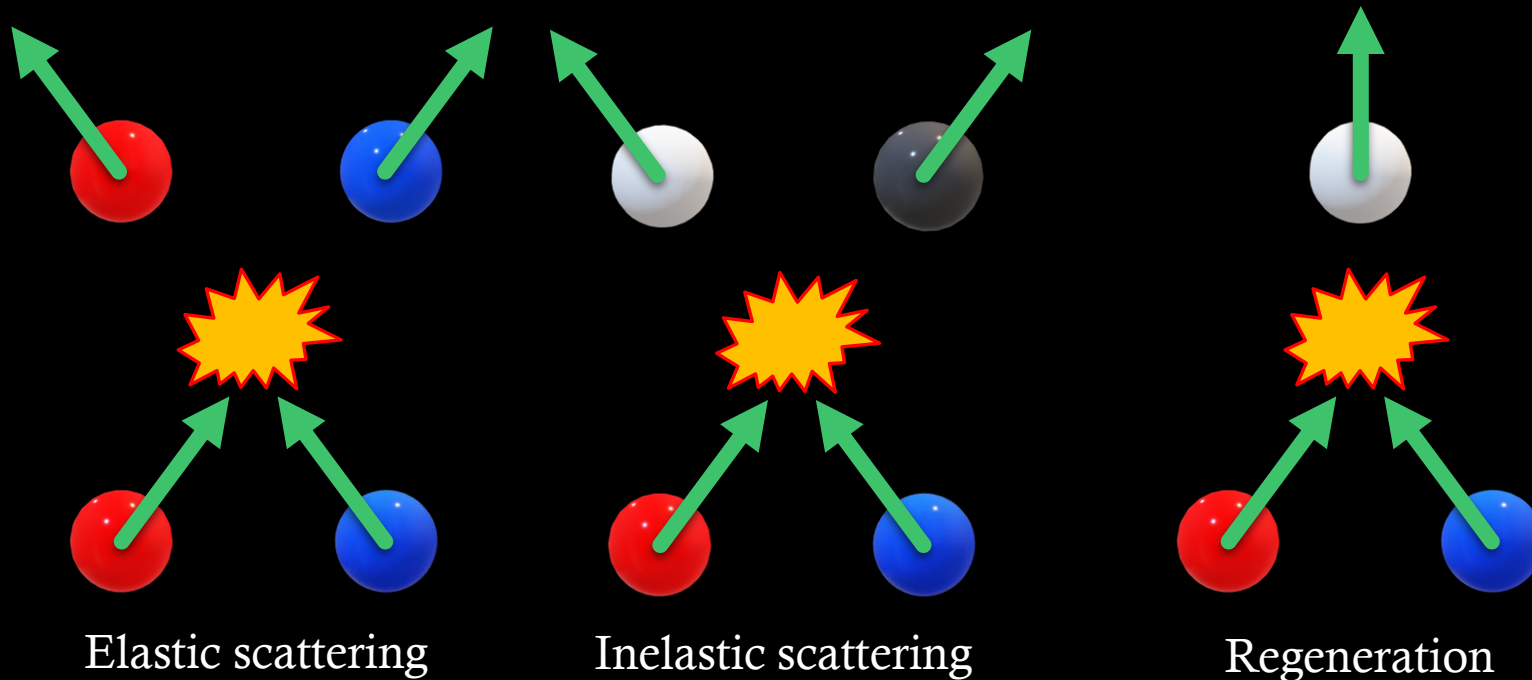
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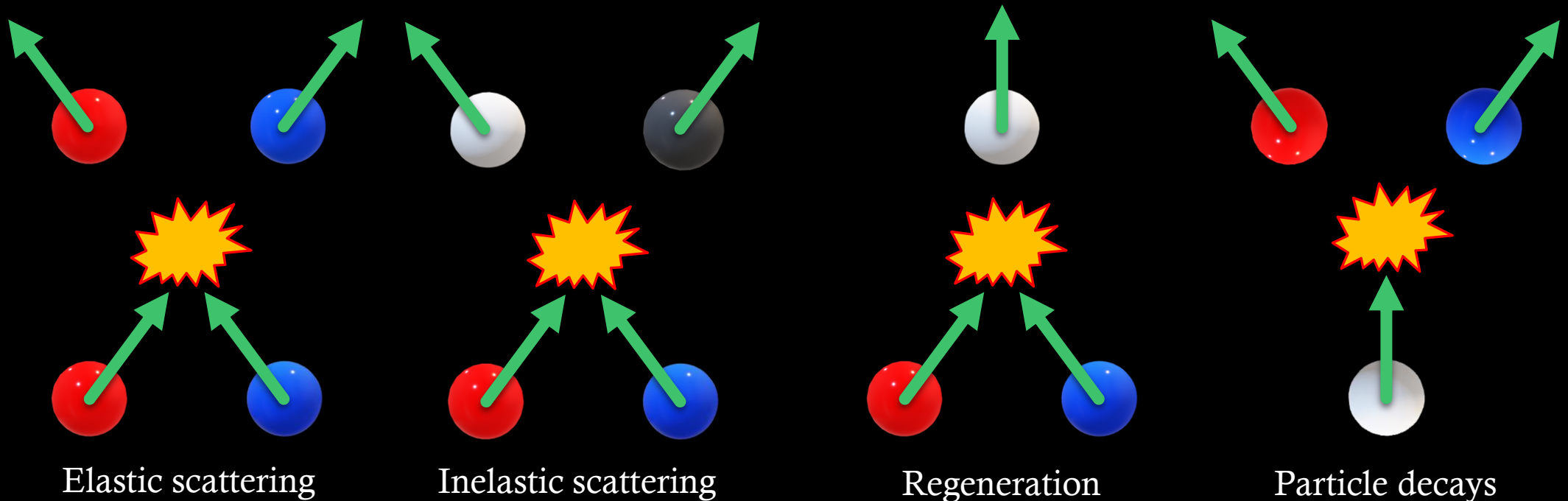
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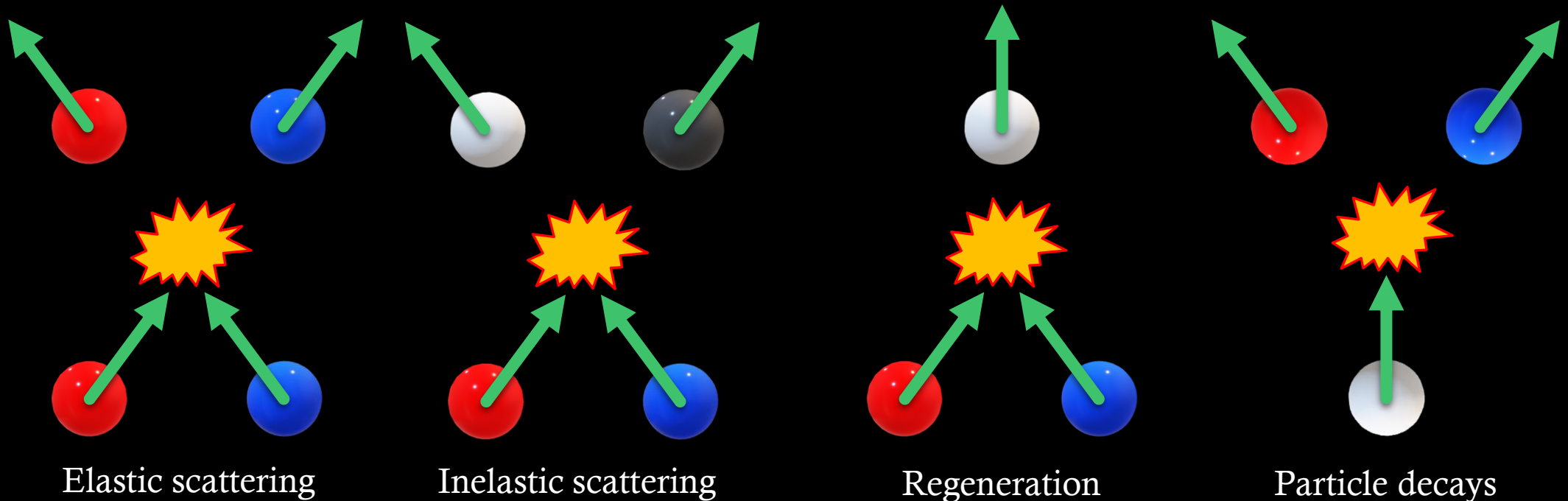
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# Hadron gas - UrQMD

- ◇ Up to thousand of particles produced in each collision
- ◇ If system is dense enough, hadrons may scatter on each other

- ◇ Different kinds of interactions
- ◇ Hadrons travel following classical trajectories
- ◇ Interact stochastically following experimental hadron-hadron cross-section



# Additional References

- ◇ [\[nucl-th/9803035\] Microscopic Models for Ultrarelativistic Heavy Ion Collisions](#) – Bass et al.
- ◇ [\[hep-ph/9909407v1\] Relativistic Hadron-Hadron Collisions in the Ultra-Relativistic Quantum Molecular Dynamics Model \(UrQMD\)](#) – Bleicher et al.
- ◇ <https://urqmd.org/> – UrQMD webpage
- ◇ Please, do not redistribute UrQMD
- ◇ If you plan to start a research project using UrQMD, ask permission from the authors first (see UrQMD page on the left)

# Backups

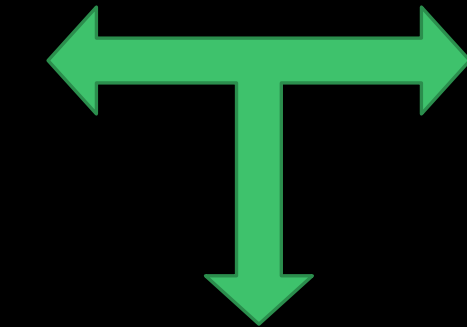
# Wigner functions

Kinetic Theory to Hydrodynamics

$$\diamond T^{\mu\nu} = \int \frac{d^3p}{\varepsilon} p^\mu p^\nu f(x, p)$$

Quantum Field Theory to Hydrodynamics

$$\diamond T^{\mu\nu} = Tr(\hat{\rho} : \hat{T}^{\mu\nu}(x) :)$$



Wigner function (from QFT)

$$W(x, k) = \frac{1}{(2\pi)^4} \int d^4y \frac{1}{2} e^{-ik \cdot y} \langle : \hat{\psi}^\dagger(x + y/2) \hat{\psi}(x - y/2) : \rangle$$

$$\langle \cdot \rangle = Tr(\hat{\rho} \cdot)$$



$f(x, p)$  defined as

$$W^+(x, k) = \theta(k^0) W(x, k) = \int \frac{d^3p}{\varepsilon} \delta^4(k - p) f(x, p)$$