Emittance Exchange in MICE

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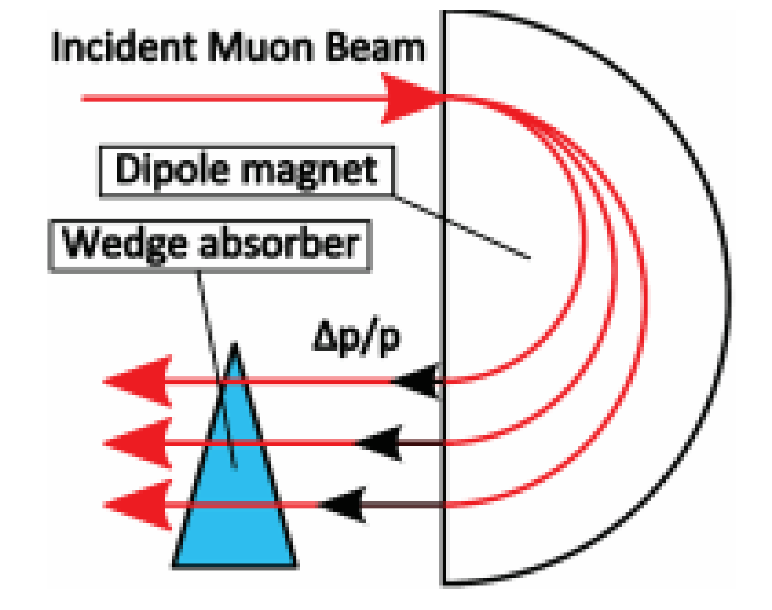
26 March 2021

Aims



- Demonstrate Emittance Exchange and Reverse Emittance Exchange in the Wedge using MICE data
- Emittance Exchange can be demonstrated by looking at the change in phase space density of the particle selection before and after having passed through a Wedge absorber
- Emittance Exchange is shown by a decreased transverse phase space density (x, px, y, py) and increased longitudinal phase space density (z, pz), (and vice versa for Reverse Emittance Exchange)
- Can use a number of techniques to calculate phase space density: KDE, KNN, Voronoi Tessellations, etc.
- MICE beam only has a small natural dispersion → Use beam reweighing techniques to select beams with desired dispersion

MICE – 3 Cooling measurements

- MICE measures cooling using three techniques

- 1. Emittance:

$$\varepsilon_d = \frac{\sqrt[d]{|\Sigma|}}{m_\mu c}$$

- 2. Amplitude:

$$A_d = \varepsilon_d \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x} \qquad = \qquad \frac{\sqrt[d]{|\Sigma|}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}}{m_\mu c}$$

- 3. Phase-Space Density:

  - Kernel Density Estimation (KDE)

$$\hat{\rho}(\vec{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\boldsymbol{H}}(\boldsymbol{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\boldsymbol{x}}{h}\right) = \frac{\sum_{i=1}^{n} exp[-\frac{1}{2}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}]}{n(2\pi)^{d/2}|\Sigma|^{1/2}}$$

  - k-nearest neighbour (KNN)

$$\vec{\rho}(x) = \frac{k}{n\kappa_d R_k^d} = \frac{k\Gamma(\frac{d}{2}+1)}{n\pi^{\frac{d}{2}} R_k^d}$$

MICE – 3 Cooling measurements

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- 1. Emittance:

$$\varepsilon_d = \frac{\sqrt[d]{|\Sigma|}}{m_\mu c}$$

d = dimension
Σ = covariance matrix
m_μ = muon mass
c = speed of light

- 2. Amplitude:

$$A_d = \varepsilon_d \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x} \quad = \quad \frac{\sqrt[d]{|\Sigma|}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}}{m_\mu c}$$

$\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}$ is the Mahalanobis distance

- 3. Phase-Space Density:

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K = Kernel Choice
h = Kernel bandwidth
n = no. pf particles
Last part shows a Gaussian kernel

  - k-nearest neighbour (KNN)

$$\vec{\rho}(x) = \frac{k}{n\kappa_d R_k^d} = \frac{k\Gamma(\frac{d}{2}+1)}{n\pi^{\frac{d}{2}} R_k^d}$$

k = no. of neighbours
R_k^d = Euclidean distance
κ_d = Volume of a unit d-ball
$\Gamma(\frac{d}{2}+1)$ = Euler-Gamma function

MICE – 3 Cooling measurements

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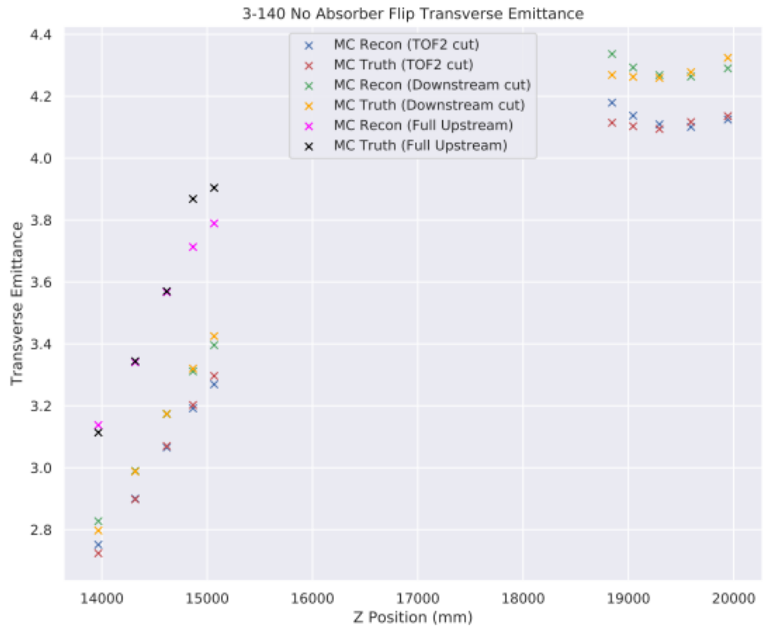
What do all 3 share?
Dependence on the Covariance Matrix

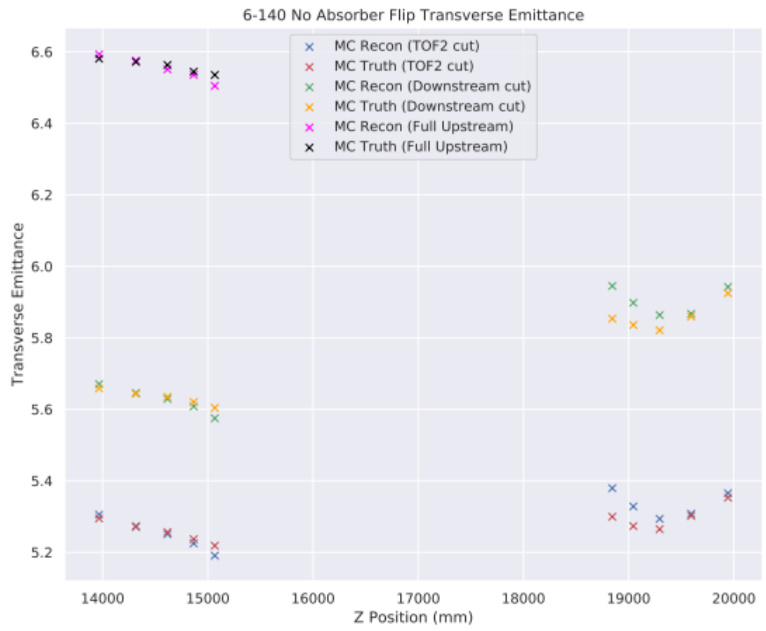
"Independent" Measurement

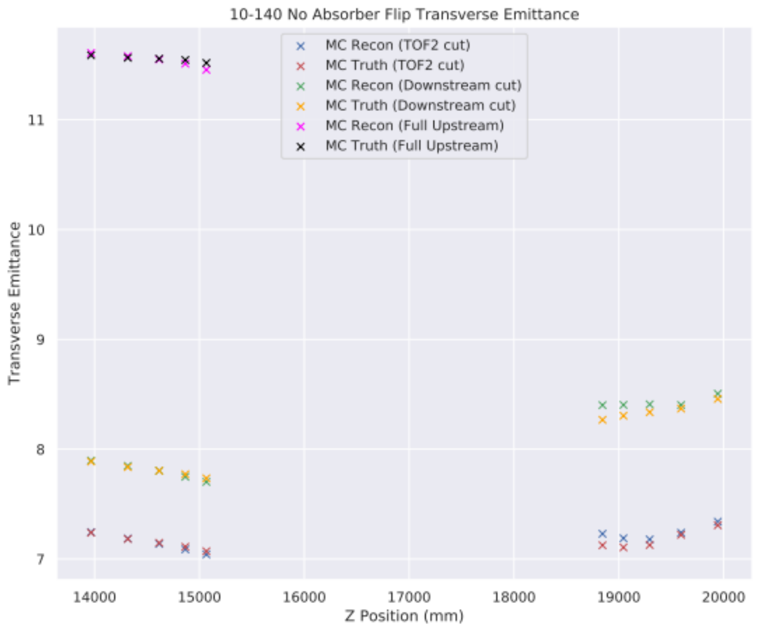
- Emittance, Amplitude and Density depend on Covariance Matrix
- Emittance gives a "measure" of the distribution of the beam
- Amplitude and density are individual particle measurements relative to the rest of the distribution of the beam
- If the distribution changes significantly, e.g. transmission losses, the individual measurement will change
- Caution when comparing beams and particles at different planes
  - Not measuring cooling or heating, but collimation

Emittance

- Upstream Emittance bears no resemblance to Downstream distribution
- Emittance only comparable for a selection
- Emittance growth seen also dependent on particle distribution function. Growth is smaller for a "tighter" selection, i.e. going through less non-linear optics

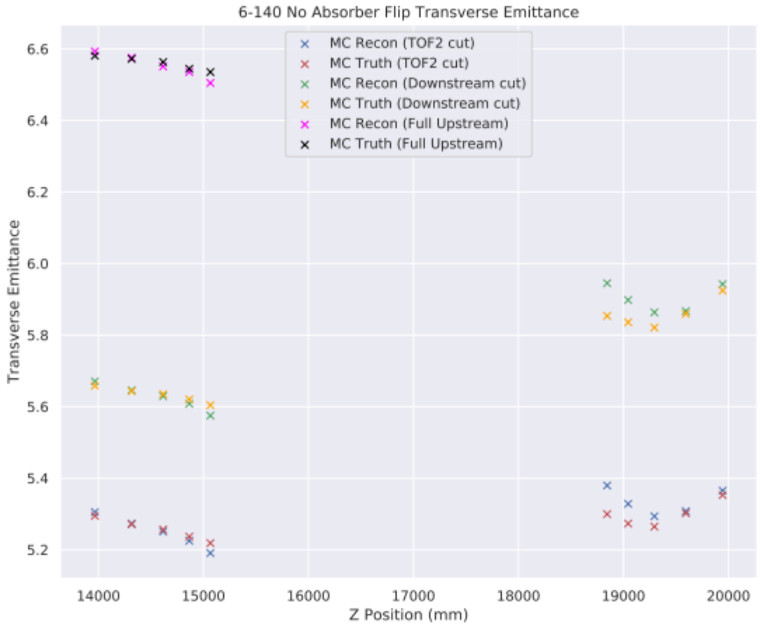


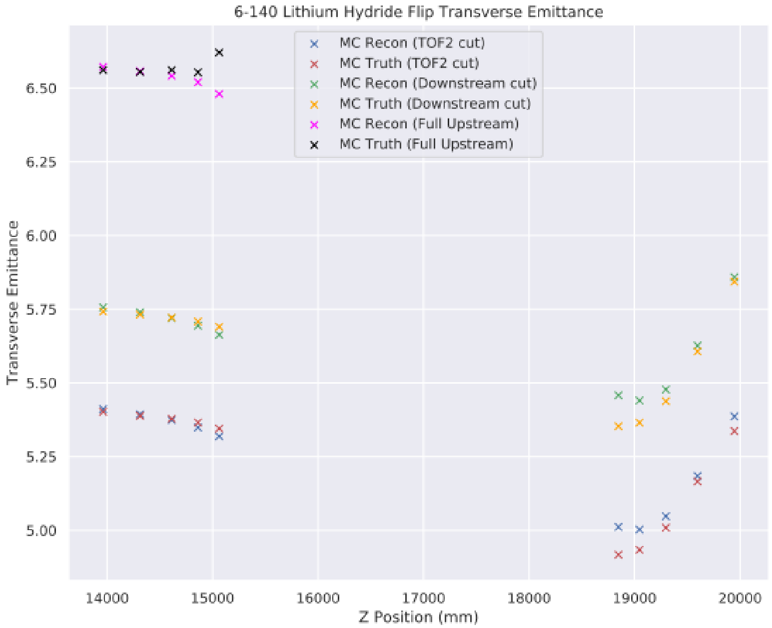


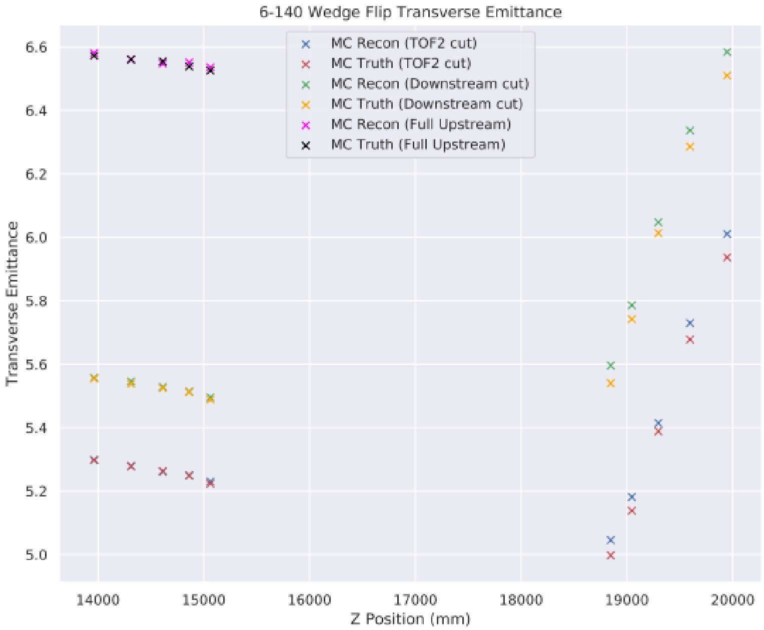


Emittance

- Upstream distributions similar, with similar gradients through Upstream tracker
- In TKD: No Absorber Emittance emittance relatively flat
- For LiH, growth seen within TKD (possibly due to Energy Straggling)
- In Wedge, significant growth in TKD due to dispersion



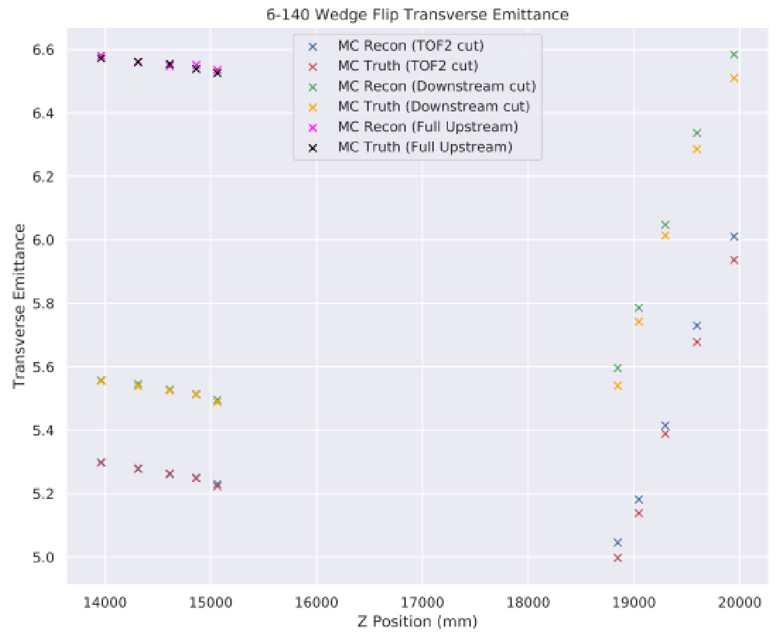


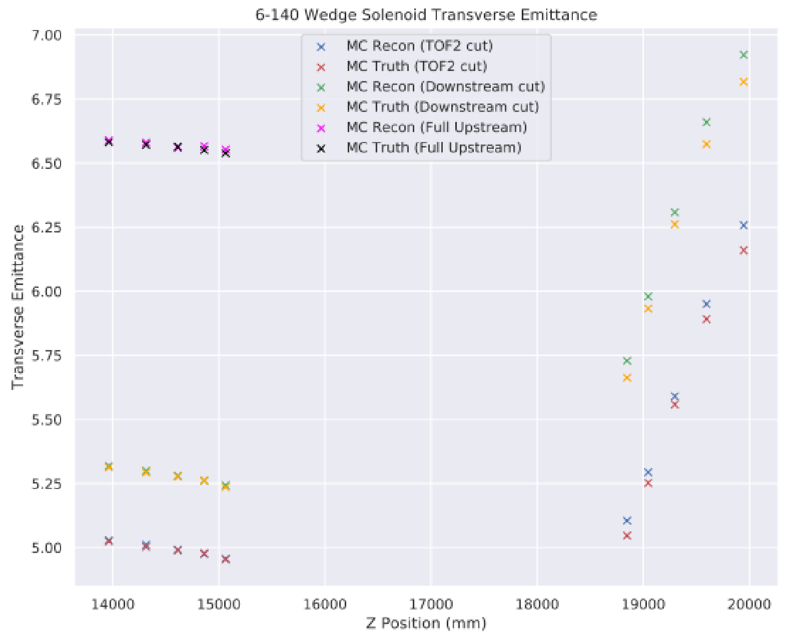


Emittance

- The emittance seen is also dependent on the magnetic field mode i.e. flip or solenoid
- This is because each mode will have different transmission losses and thus affect the downstream survival bias

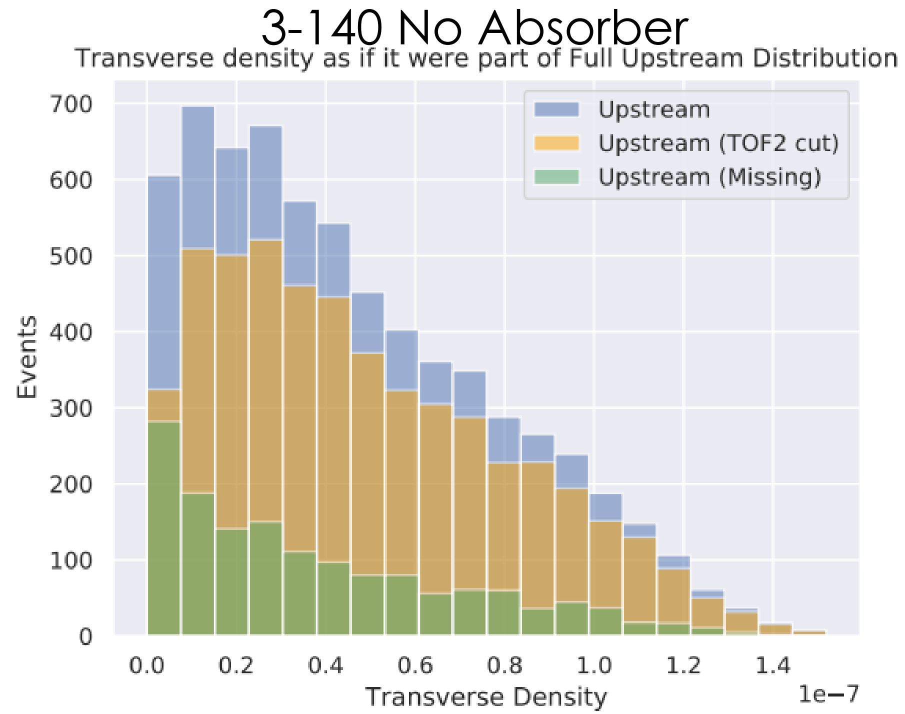
- For Emittance, Amplitude and Phase-Space Density we can only compare the same particles upstream and downstream, however this is biased by transmission losses
- The full Upstream sample is unbiased
- The challenge is to have an unbiased downstream “cooling” signal
- Emittance, Amplitude and Density all have the Covariance Matrix in common.

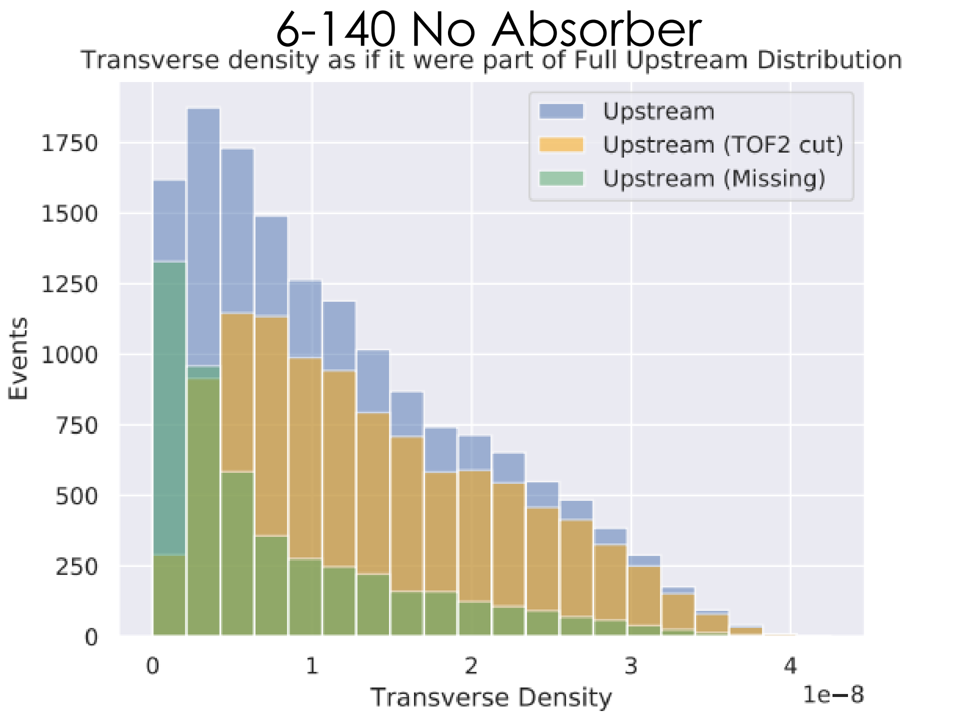


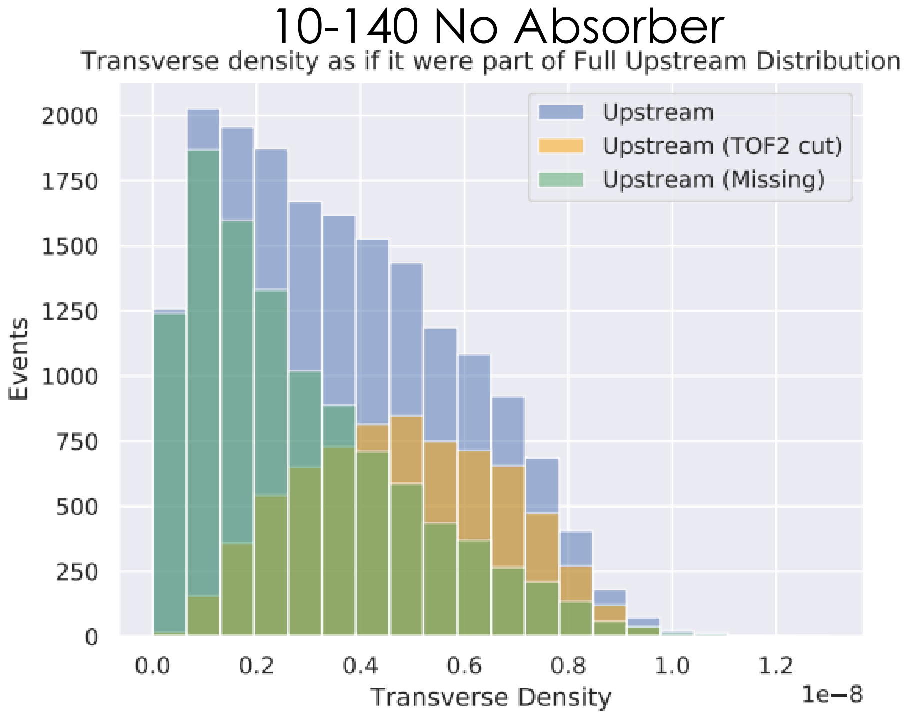


Transverse densities as if they were part of Upstream Distribution making it to TOF2 or not (i.e. missing) – MC Recon

- Wedge Analysis requires TOF2
- TOF2 is even stricter cut than TKD
- More missing particles, even at core



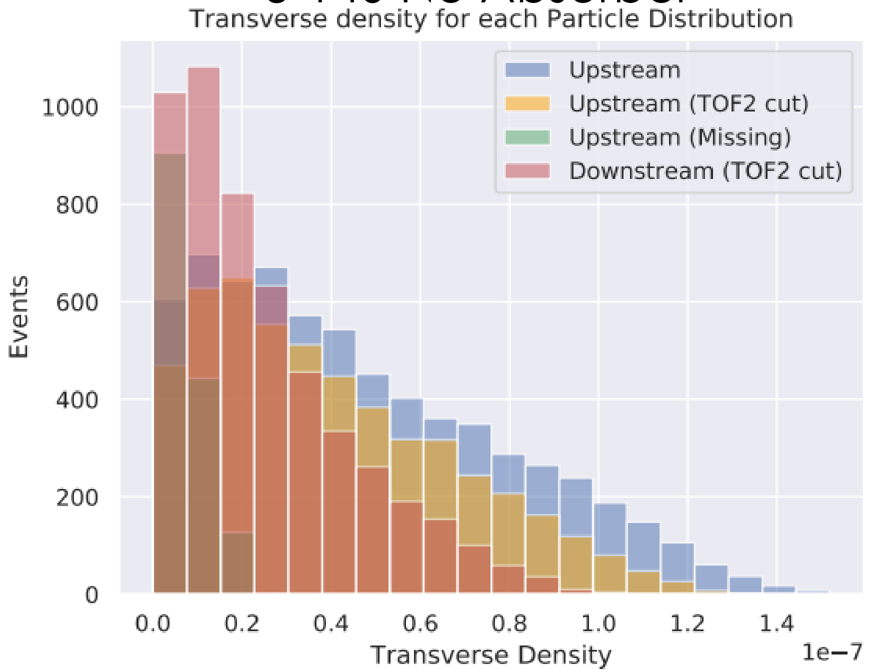




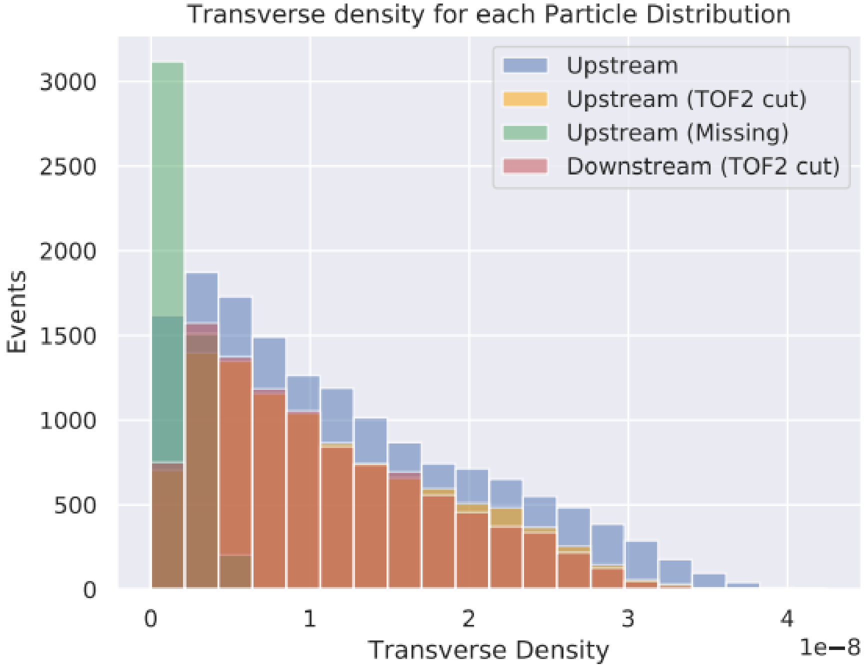
Transverse densities calculated over each measured distribution

- When the Upstream sample (blue) is separated into the sample which makes it downstream (yellow) and the sample which doesn't (green) and the density is calculated for each, their sum doesn't result in the original distribution
- This means there is no simple scaling to compare the full Upstream sample (blue) with the Downstream sample (red)
- We can only compare the yellow and red samples noting it is biased by survivorship, unless we can apply an appropriate correction

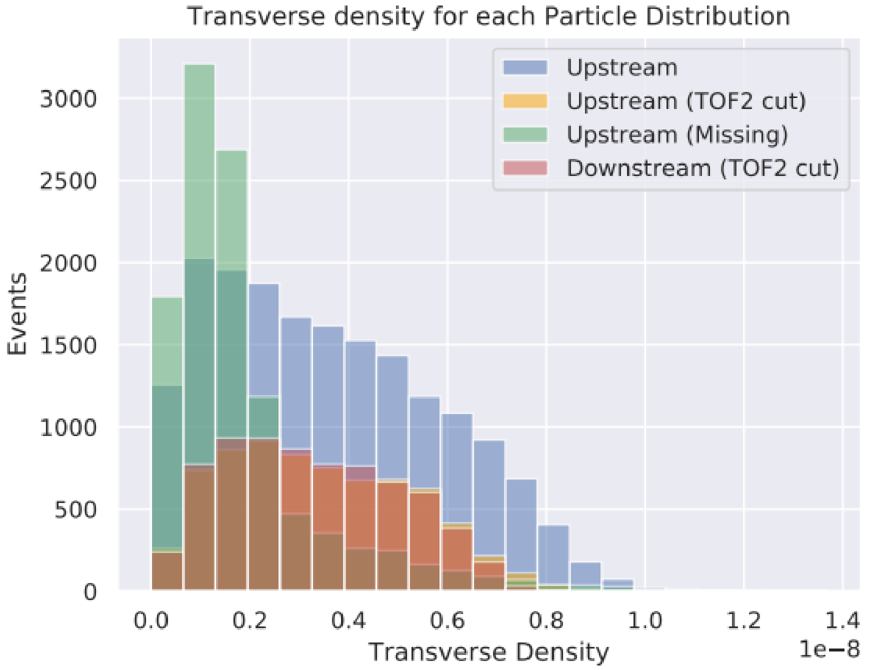
3-140 No Absorber



6-140 No Absorber

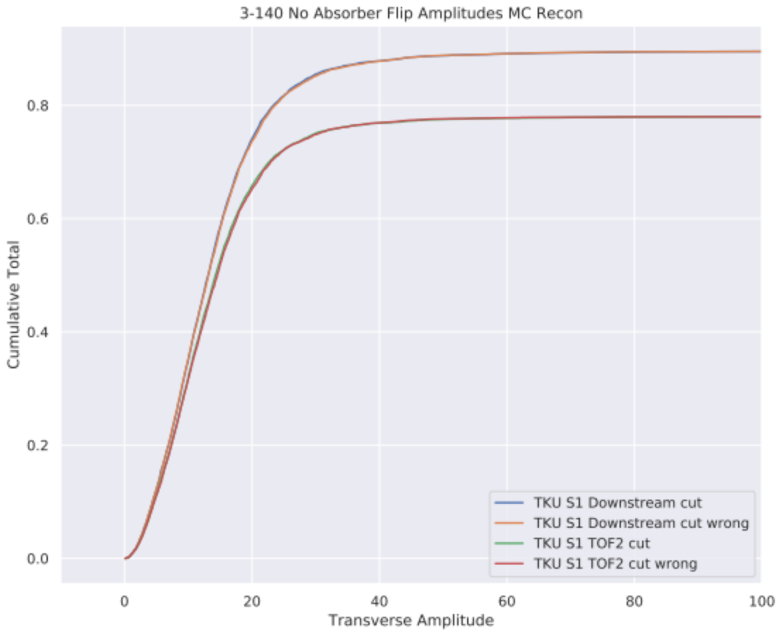


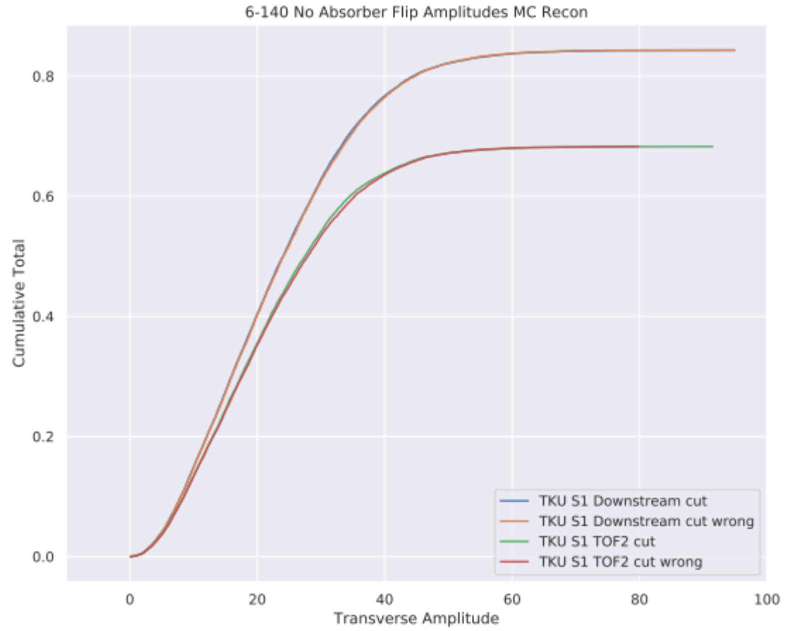
10-140 No Absorber

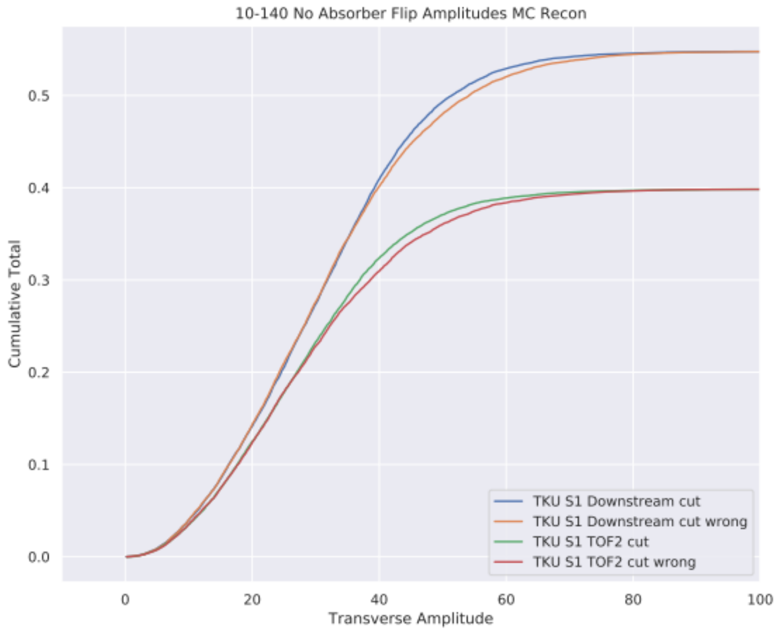


Cumulative Amplitude Plots

- If there are only small variations, does it even matter? Maybe it is just noise
- The cumulative plots also show only some small variation, and mainly at the higher input emittance beams.
- The sampled beam calculation is slightly cooler than that calculated by the full upstream sample
- The cooling performance may be slightly biased (in this case underestimated), although it mainly tells that the input distribution does have some influence

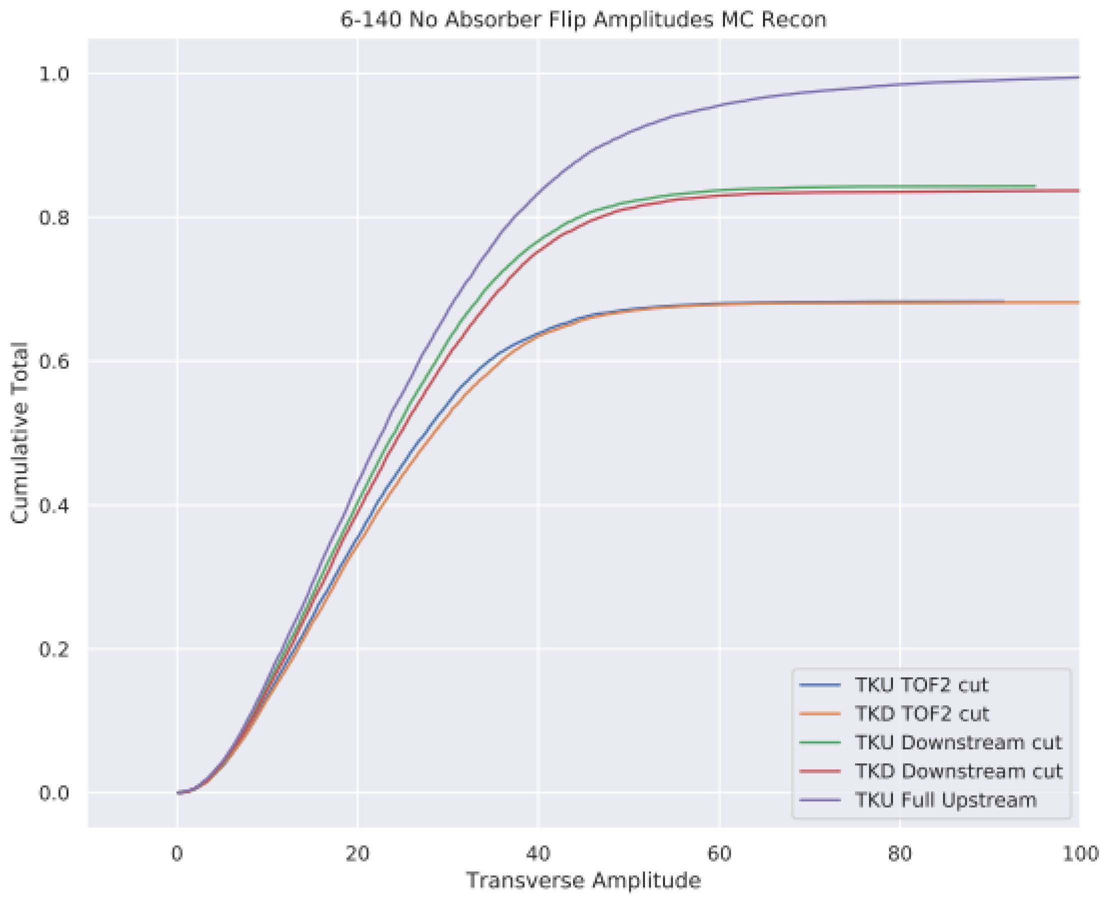


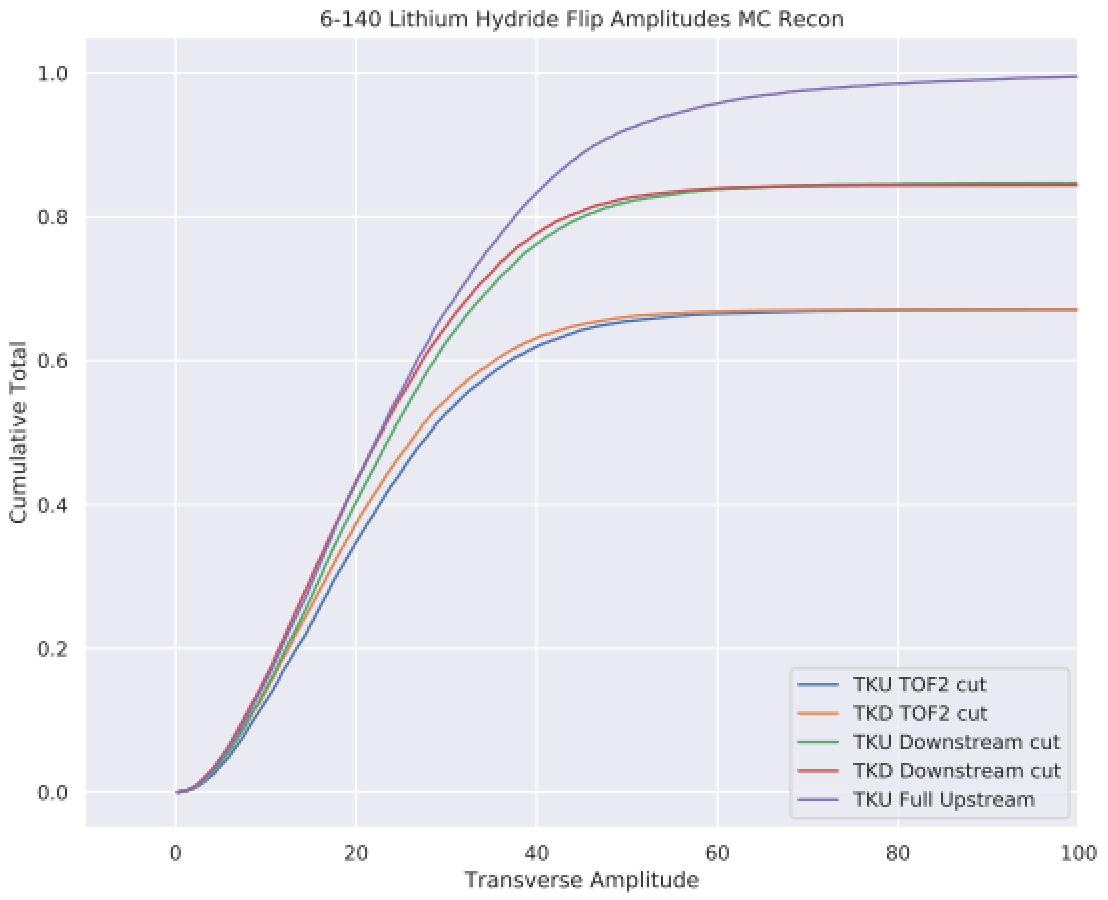




Cumulative Amplitude Plots

- The variation seen for the Upstream sample using the sampled covariance matrix and the full upstream matrix may look small, but then again the expected cooling performance of Lithium Hydride seen in the Cumulative Amplitude plots is also small





Number of points

- Transmission losses bias the downstream distribution. We keep most particles that are cooled, however at higher emittances we lose more heated particles (e.g. scattering beyond experiment aperture)
- At a plane (for the plane symmetric/no absorber case), the distribution of the full sample can be separated into two distributions, the transmitted sample and non transmitted sample using their covariance matrices:

$$N_1\Sigma_1 = N_2\Sigma_2 + N_3\Sigma_3$$

- Where N is the number of particles and Σ is the samples covariance matrix
- Can only normalize by the number of particles if the transmitted and non-transmitted samples have the same covariance matrix as the parent, i.e. $\Sigma_2 = \Sigma_3$
- In MICE we have effectively said $N_1\Sigma_1 = N_2\Sigma_2$ and ignored $N_3\Sigma_3$, creating an error we do not account for.
- There is a question of how to the error analysis when comparing particles at two different planes, if a change in the covariance matrix changes the measured emittance, amplitude and density

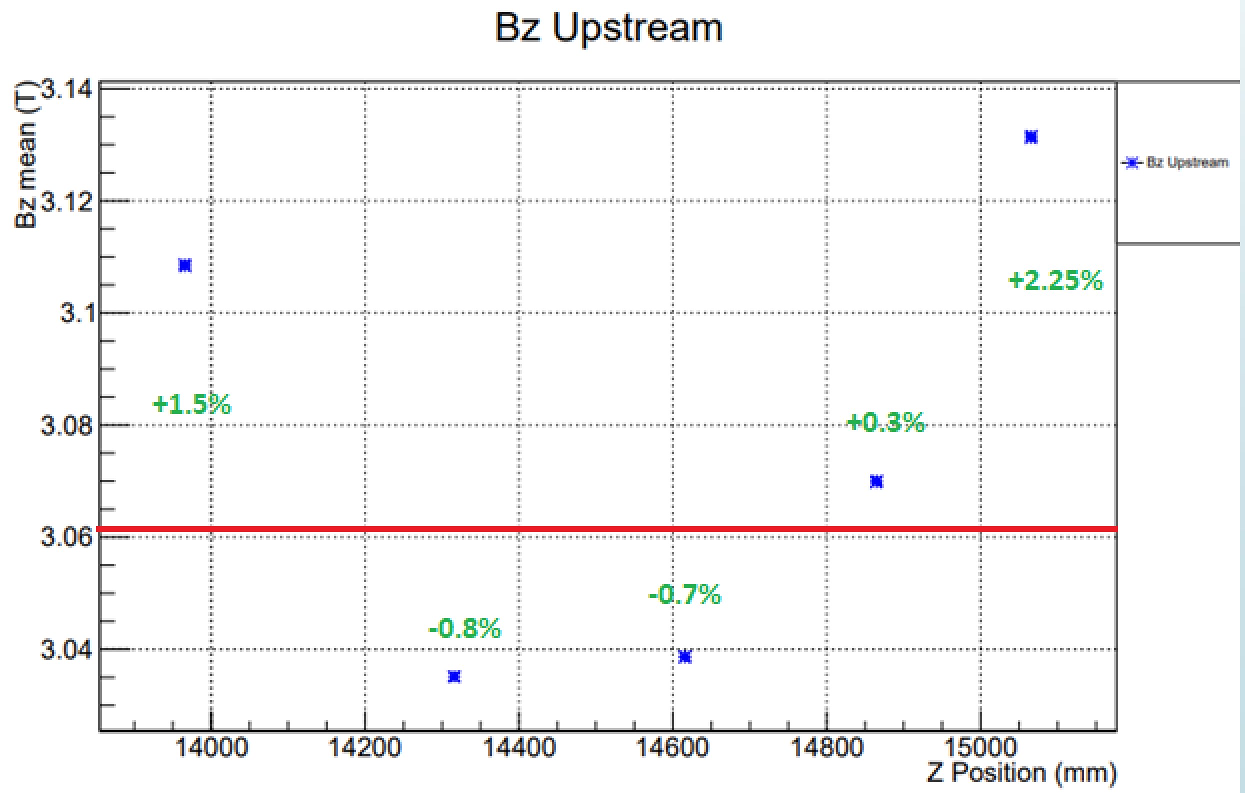
Limitations

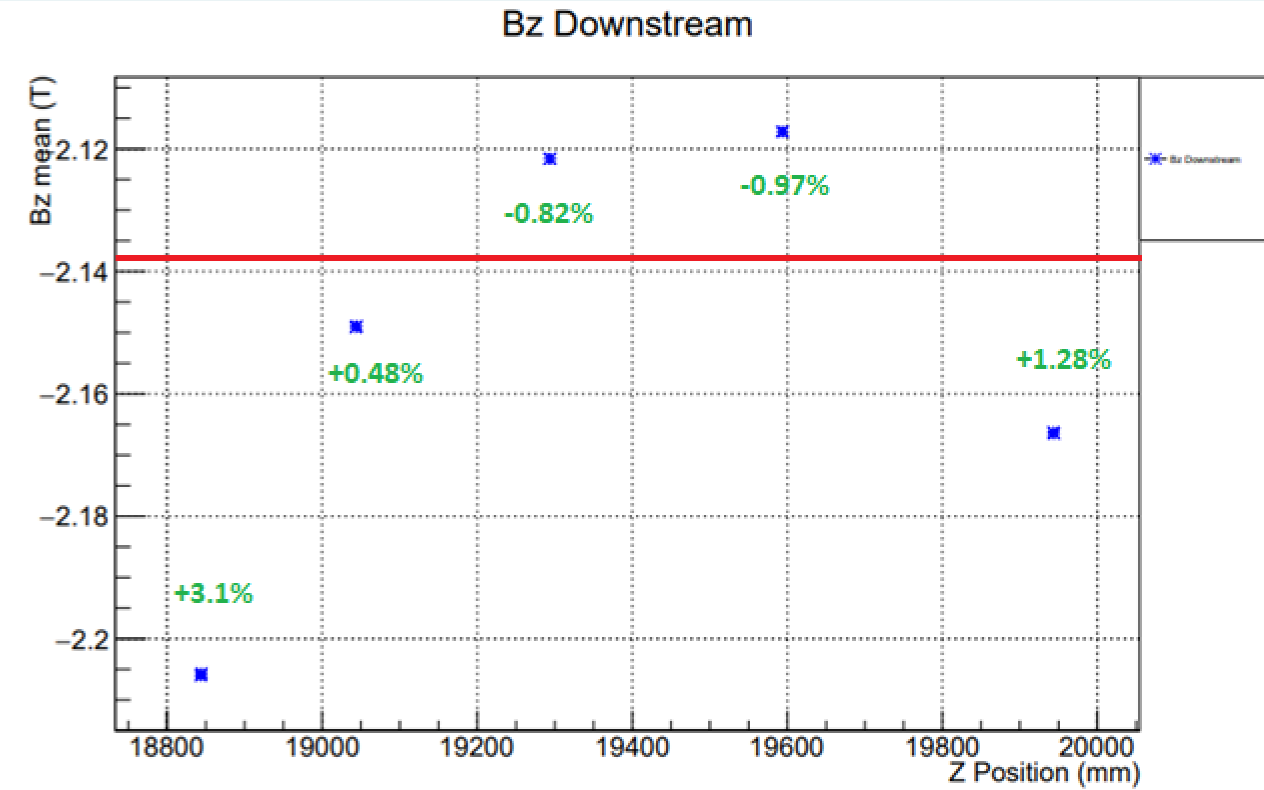
- Saw different distributions within a plane change the measurement
- We want to measure the change in the measurement between two planes
- Emittance and Amplitude conserved to first order and small Pz distribution
- Density conserved when measured in (x,y,z,px,py,pz) space to any order when there are no dissipative forces

- We assume constant solenoid field within tracker. This should allow separation into transverse and longitudinal phase space.

Bz fields in trackers

- Bz fields are not uniform
- Recon assumes mean magnetic field through Tracker
- However field varies significantly and is worst at reference planes (2-3%)
- It is also different between upstream and downstream due to different field strengths





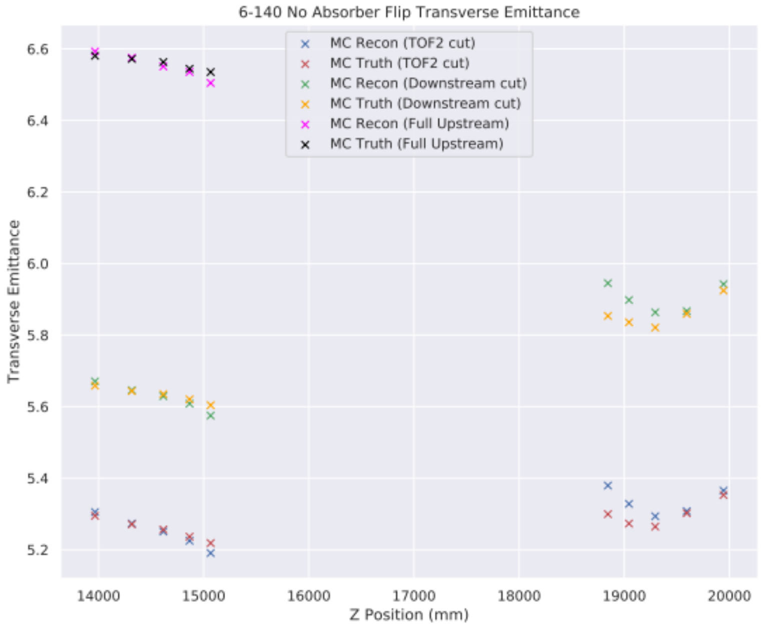
Limitations

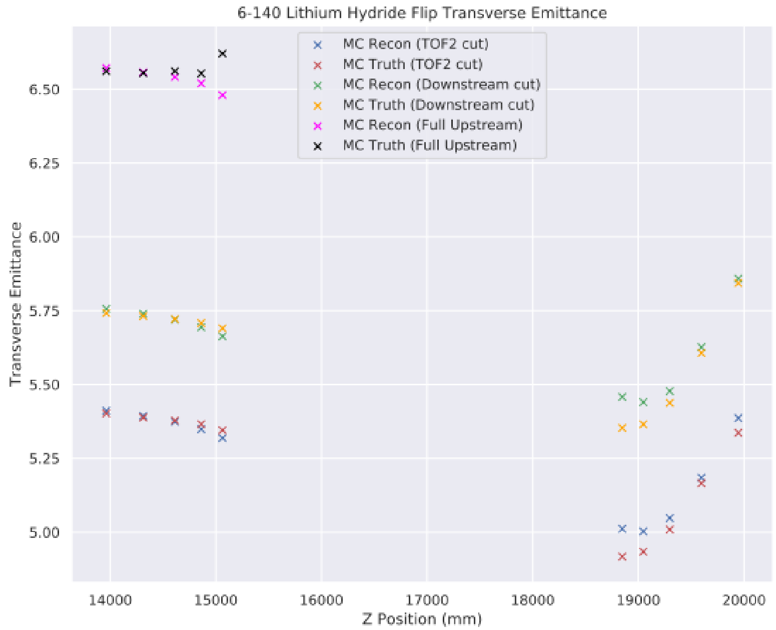
- Saw different distributions within a plane change the measurement
- We want to measure the change in the measurement between two planes
- Emittance and Amplitude conserved to first order and small Pz distribution
- Density conserved when measured in (x,y,z,px,py,pz) space to any order when there are no dissipative forces (Liouville)

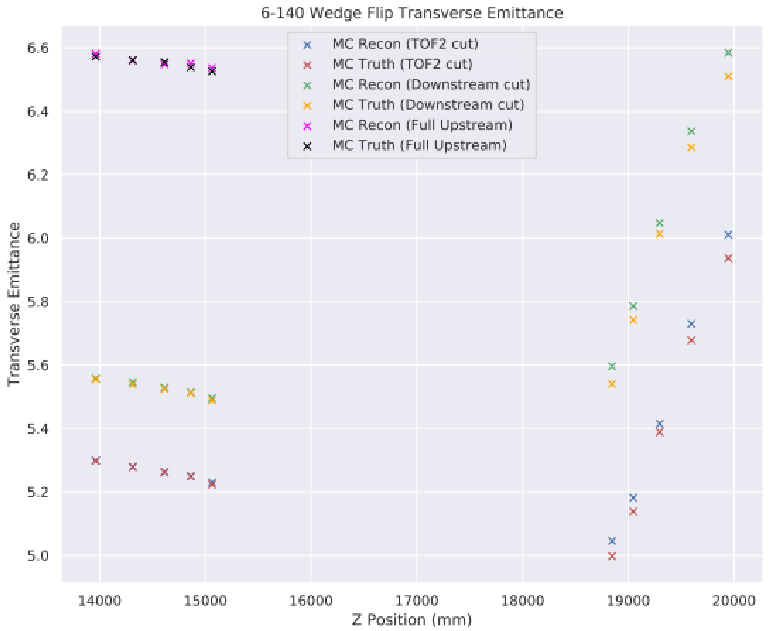
- We assume constant solenoid field within tracker. This should allow separation into transverse and longitudinal phase space.
- We assume small Pz distribution (as well as Pt to be negligible) to make a measurement at a longitudinal z plane.
- Problem for the wedge case as we are creating the Pz dispersion.

Emittance

- Upstream distributions similar, with similar gradients through Upstream tracker
- In TKD: No Absorber Emittance emittance relatively flat
- For LiH, growth seen within TKD (possibly due to Energy Straggling)
- In Wedge, significant growth in TKD due to dispersion

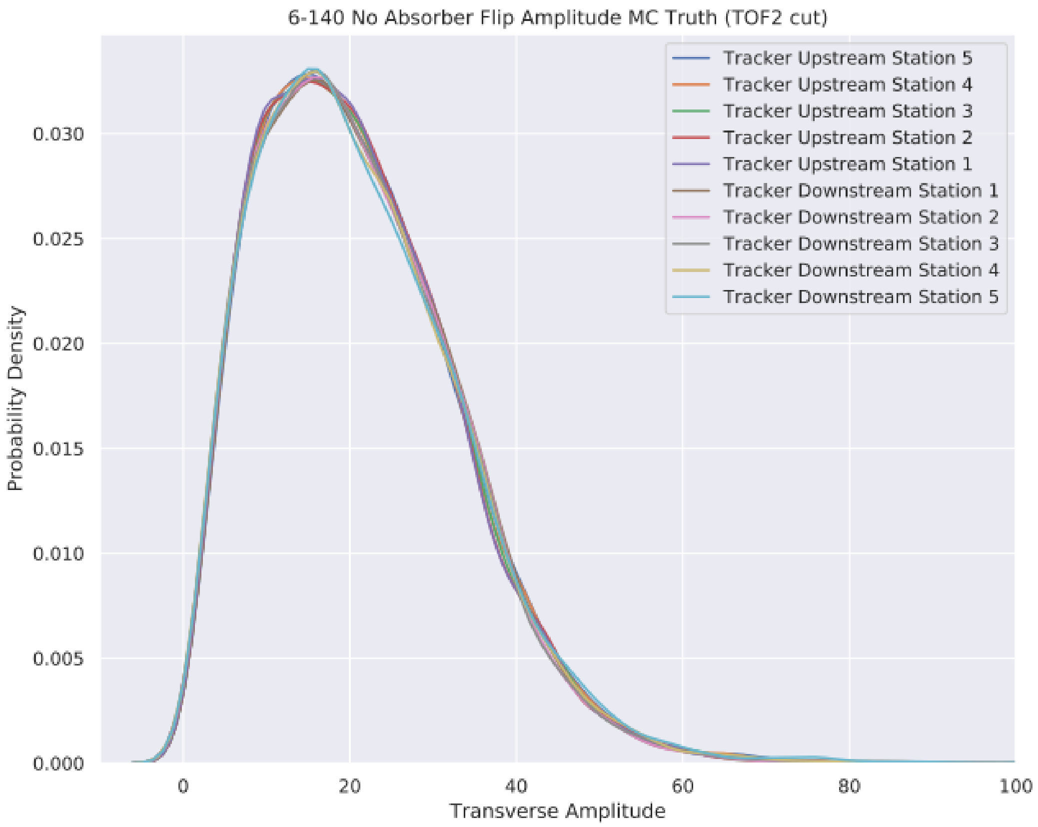


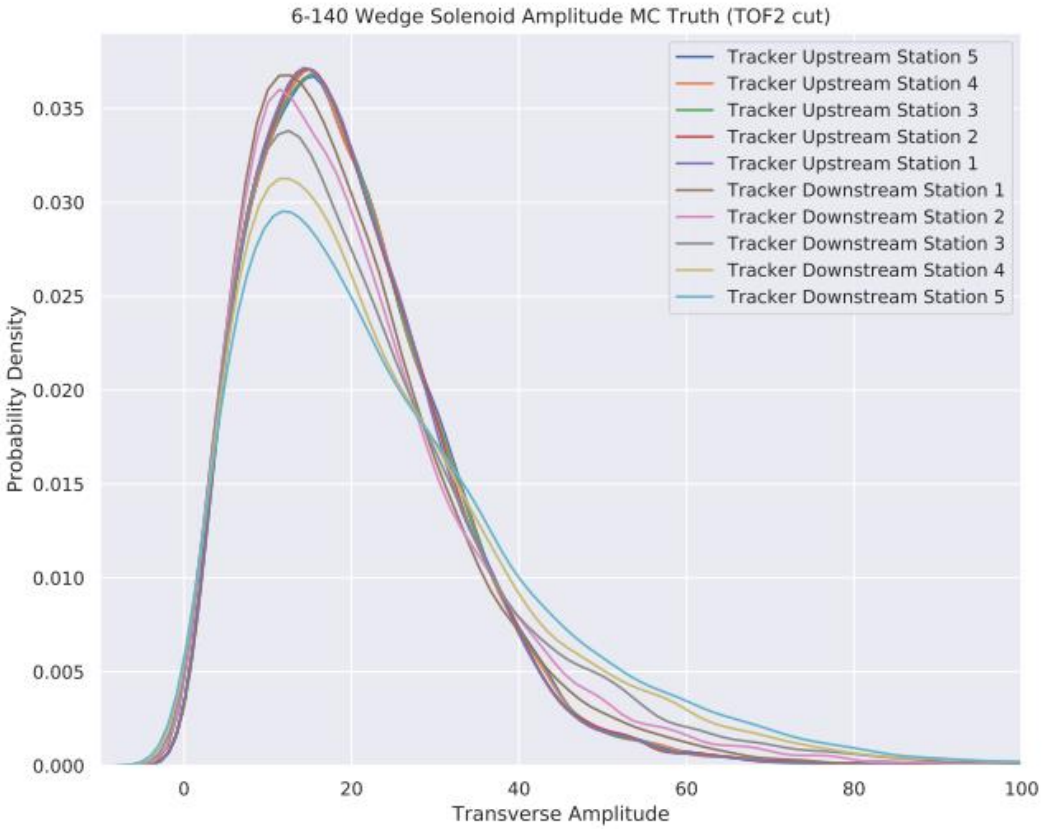




Amplitude

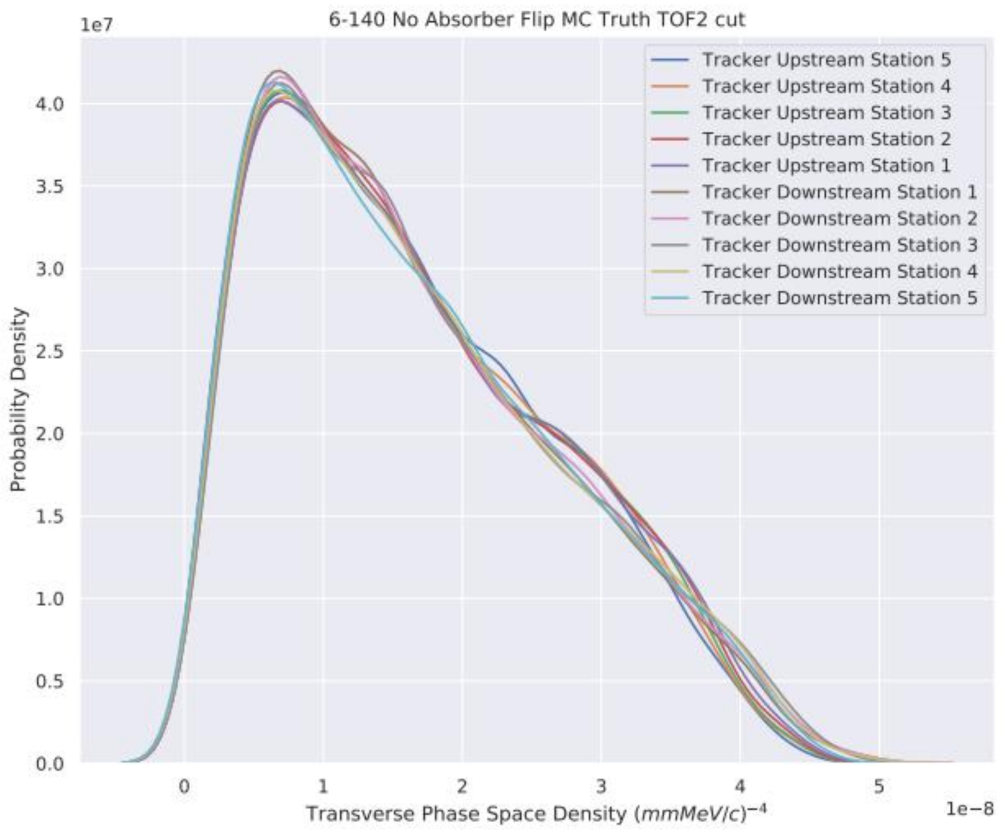
- No Absorber shows slight variation between TKU and TKD
- More pronounced for Wedge case
- Wedge case shows change within TKD
- TKU to TKD comparison becomes arbitrary if it is plane dependent

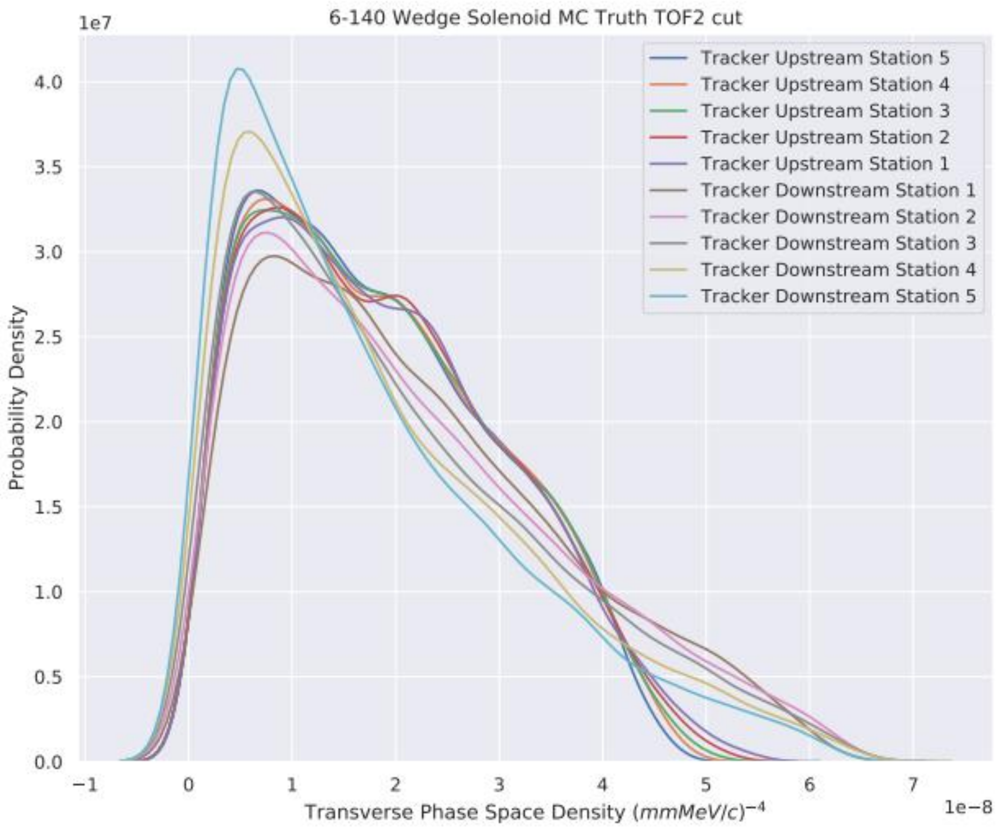




Density

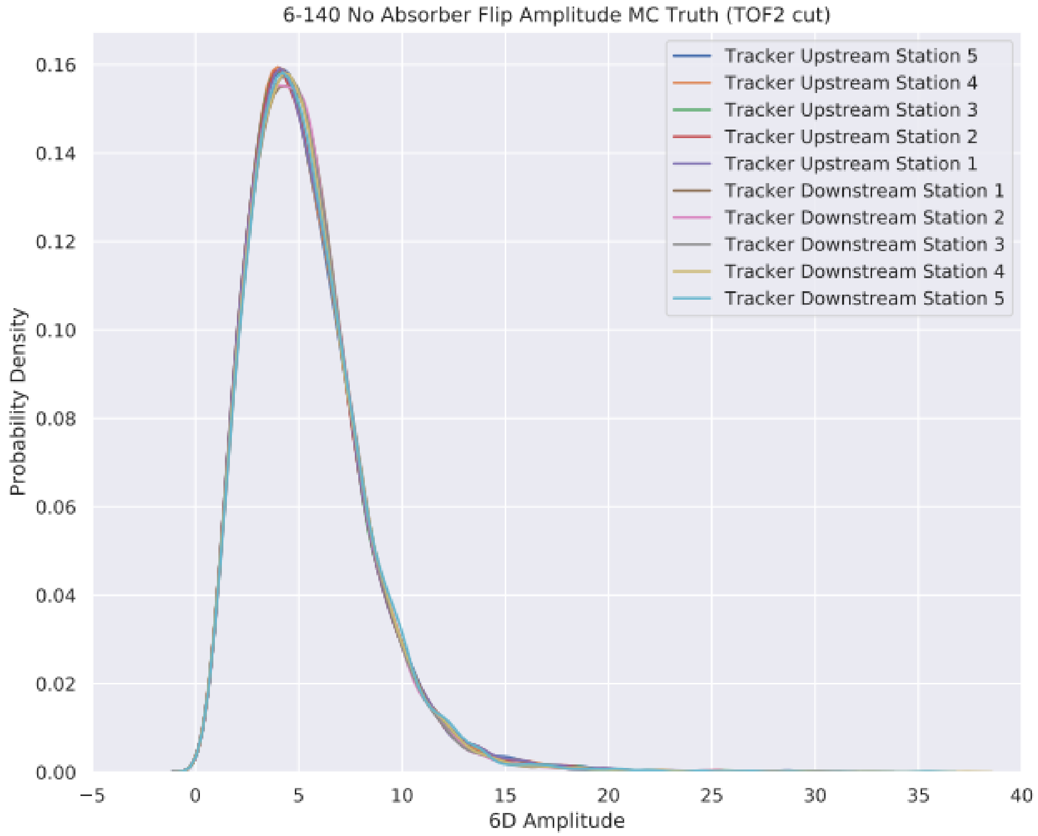
- Similar issues as seen for Amplitude

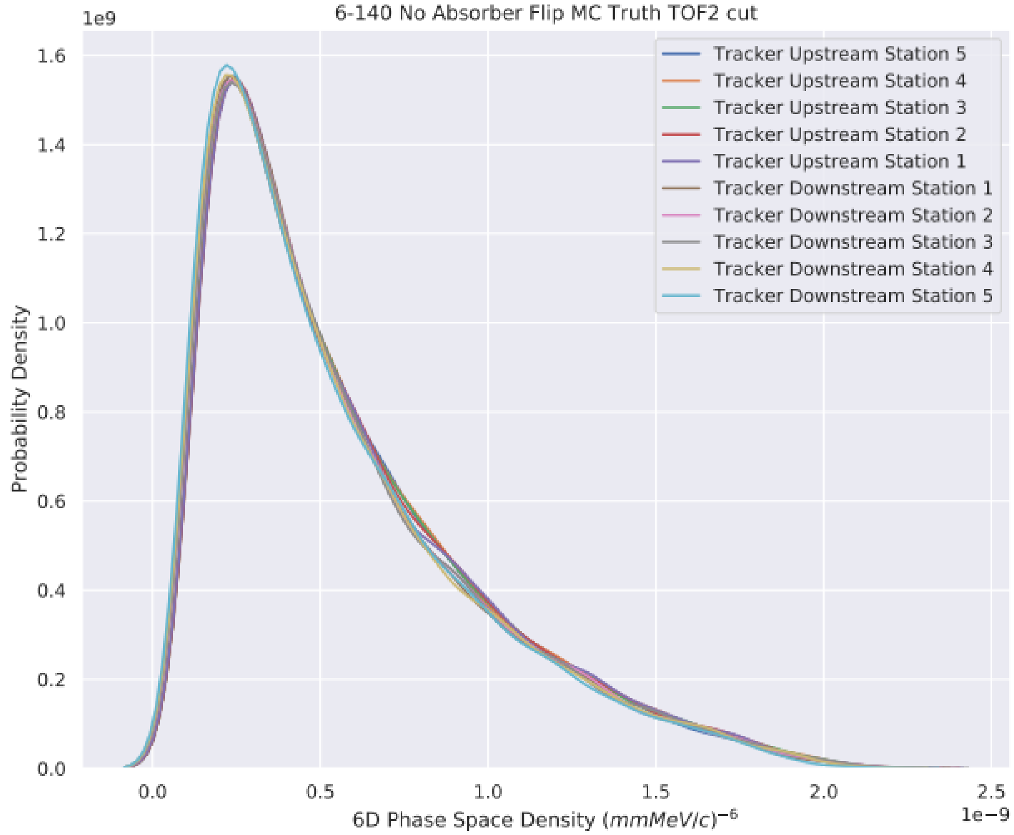




What works well

- No Absorber 6D Amplitude and Density show near conservation in MC Truth
- Amplitude peak changes slightly between trackers and then grows in TKD
- Density remains near constant bar the last station (Liouville = 6D conservation = good)
- Both also have slight wobbles in mid range (there is small Energy loss, scattering = dissipative force)



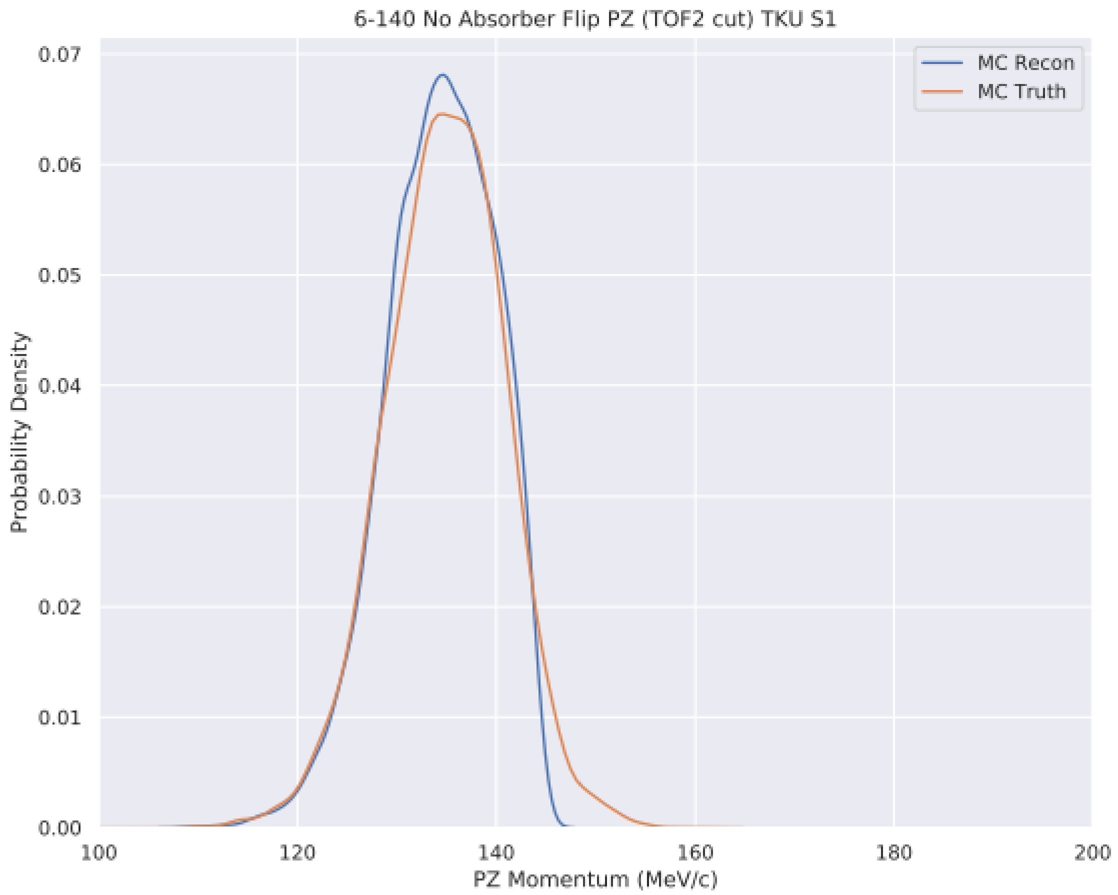


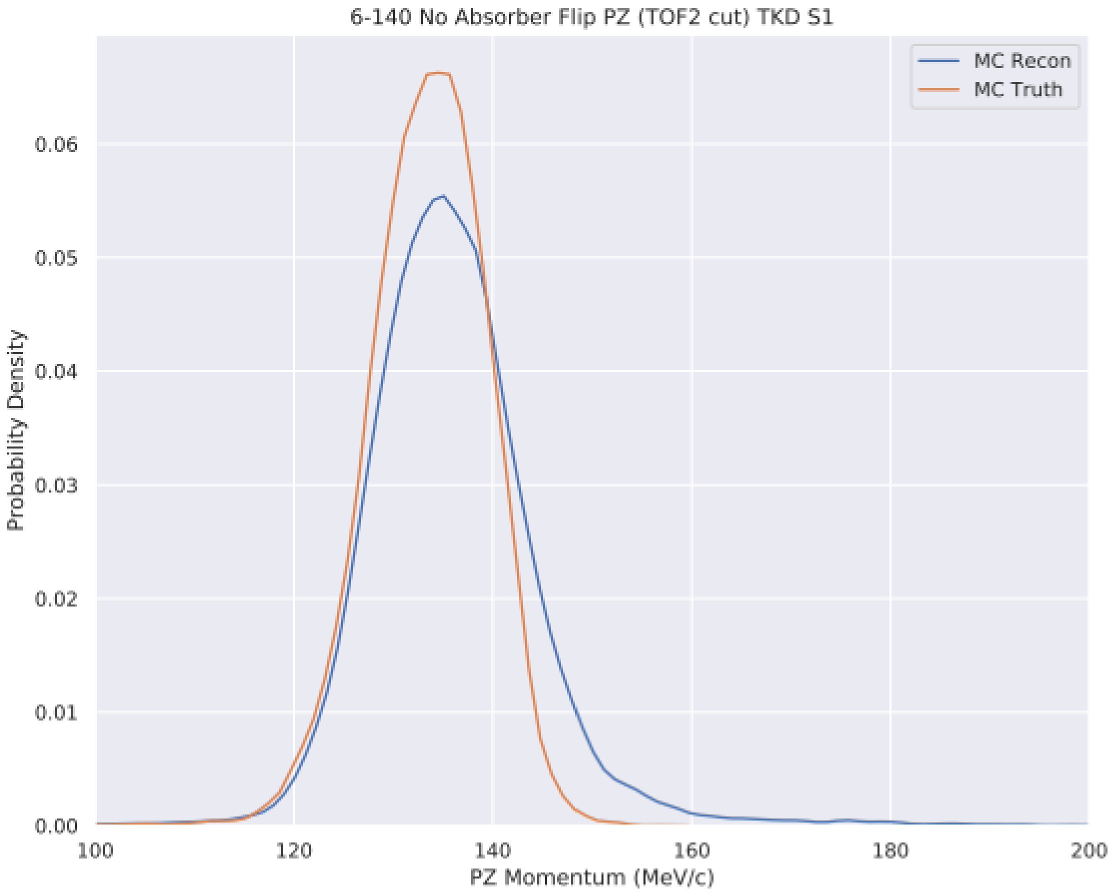
Where the problems arise

- Transmission cut – Initial beam is biased (will see more cooling)
- Use full initial beam – Emittance, Amplitude and Density measurements change depending on input distribution. If it changes significantly it changes the result (i.e. bad for high emittance beam with low transmission)
- Pz Dispersion – Assumptions may no longer apply
- Reconstruction – Pt Recon Error is small enough, but Pz isn't. Changes the calculated Emittance, Amplitude and Density
- Recon depends on many parameters. Better/worse in different conditions

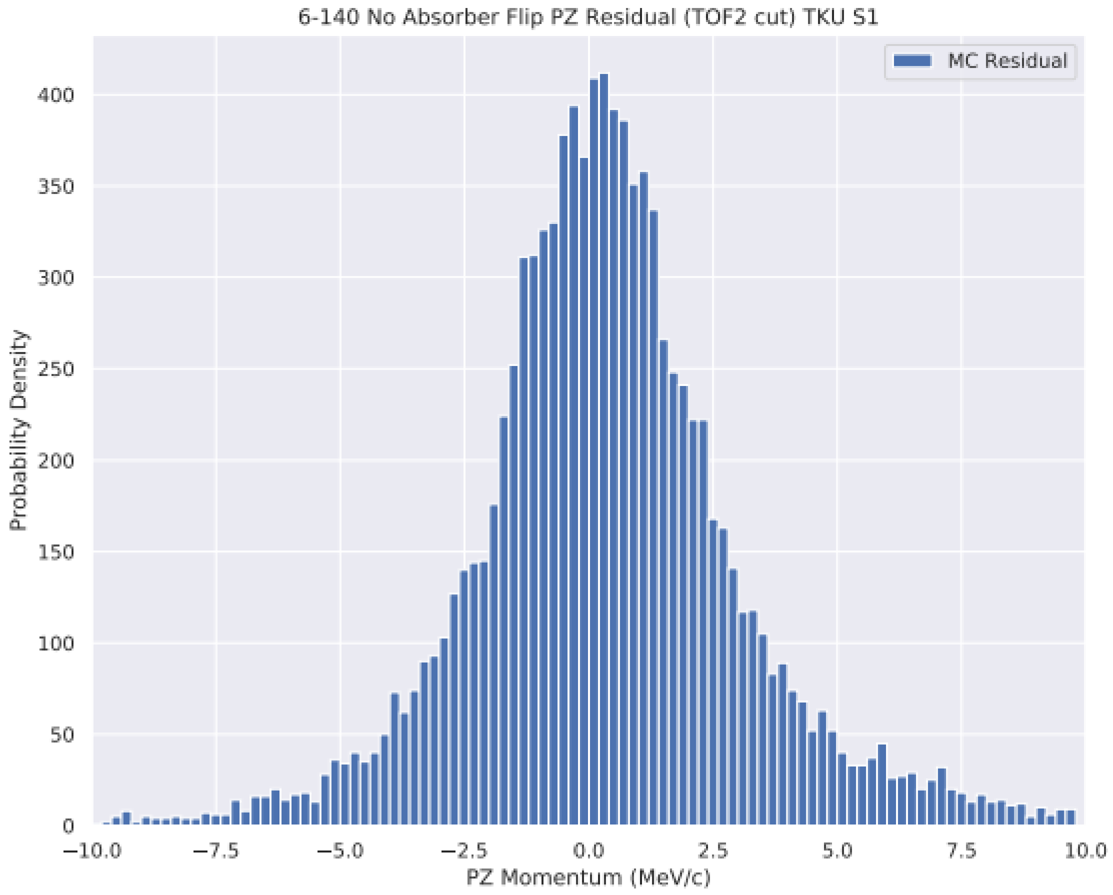
Pz Distribution at Reference Planes

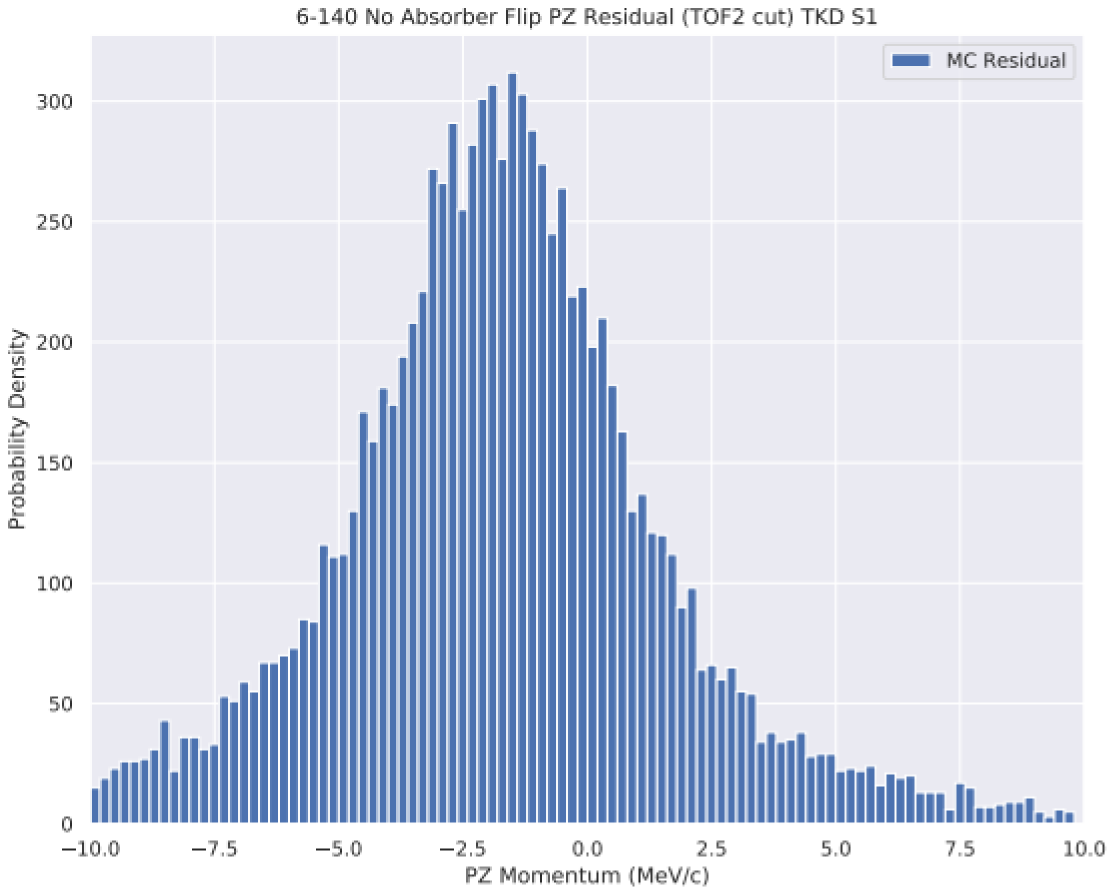
- TKU: Recon Probability Distribution is narrower and taller
- TKD: Recon Probability Distribution is broader and smaller
- Trackers are not identical in their reconstruction -> systematic bias





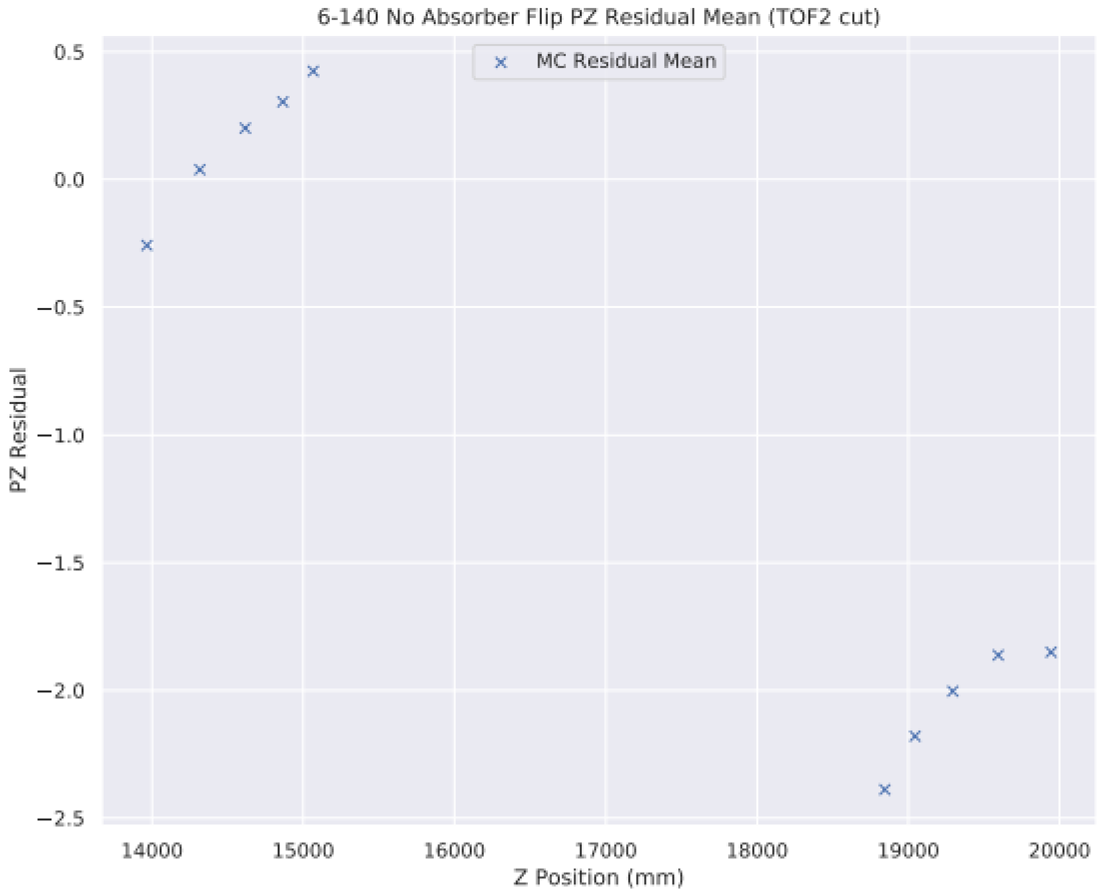
Pz Residuals at Reference Planes

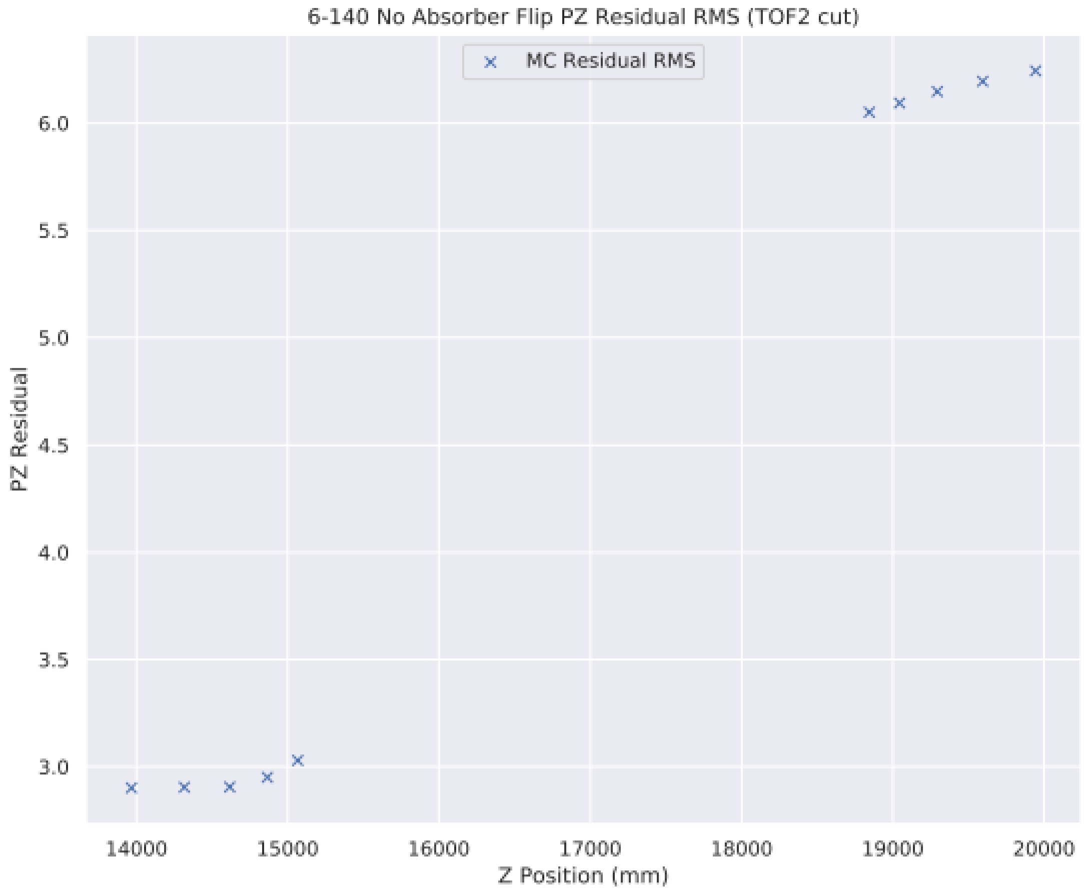




Pz Mean Residual and RMS

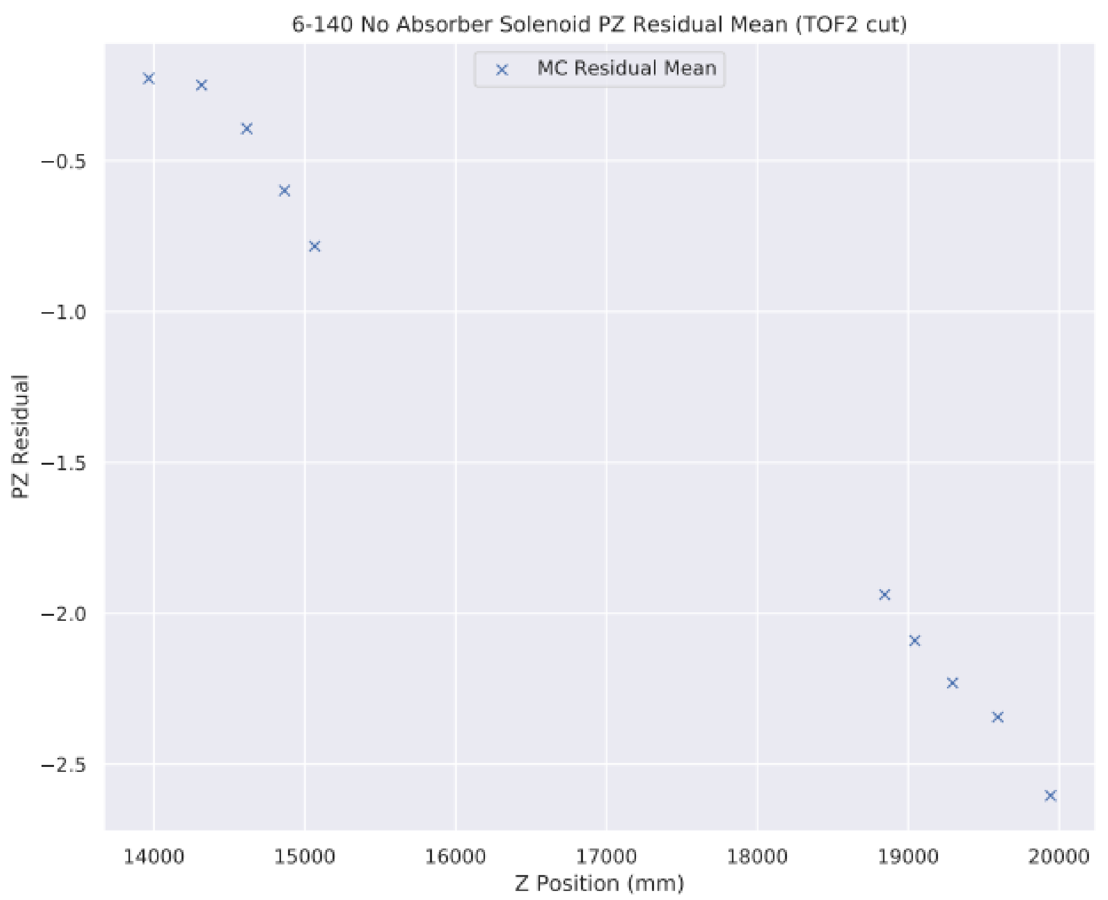
- Recon adds more Pz in TKD

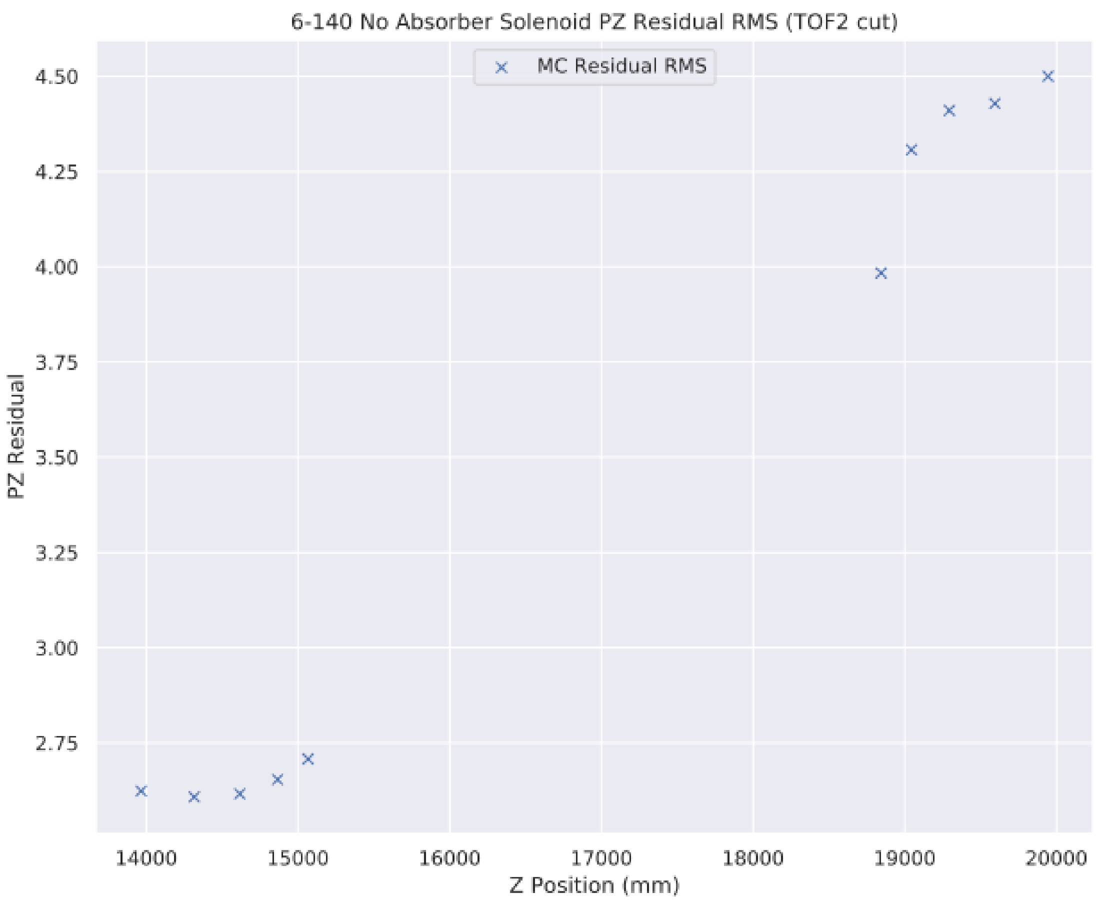


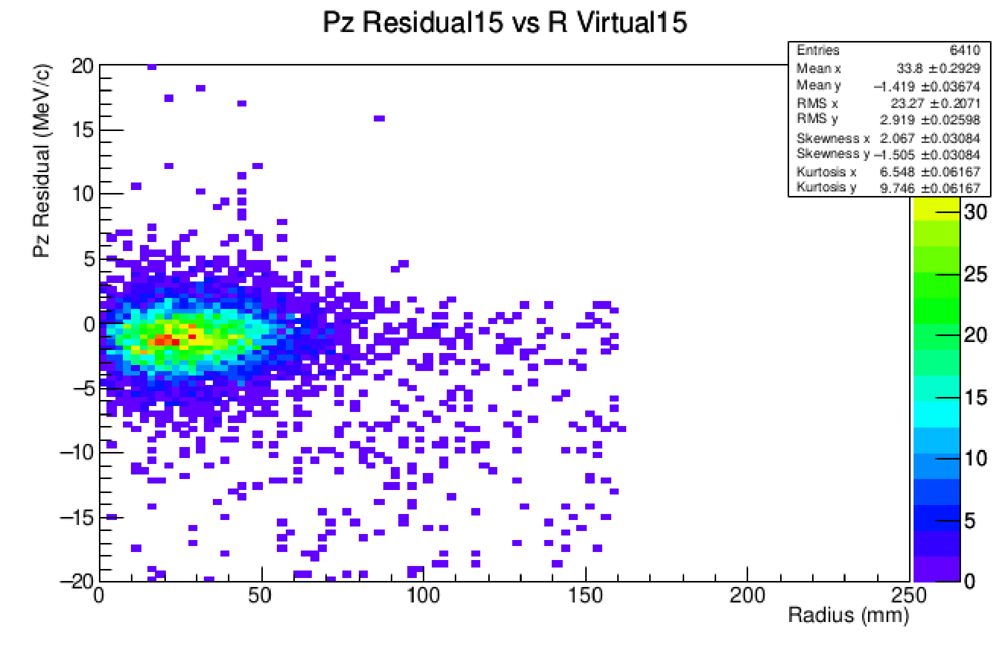


Pz Mean Residual and RMS

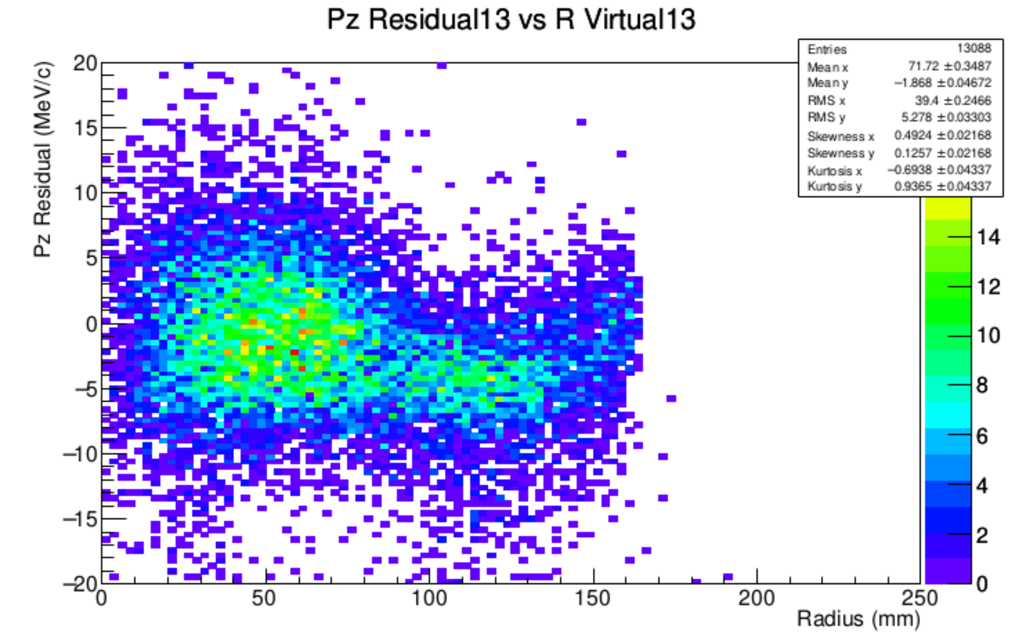
Going from Flip to solenoid changes the Mean Residual



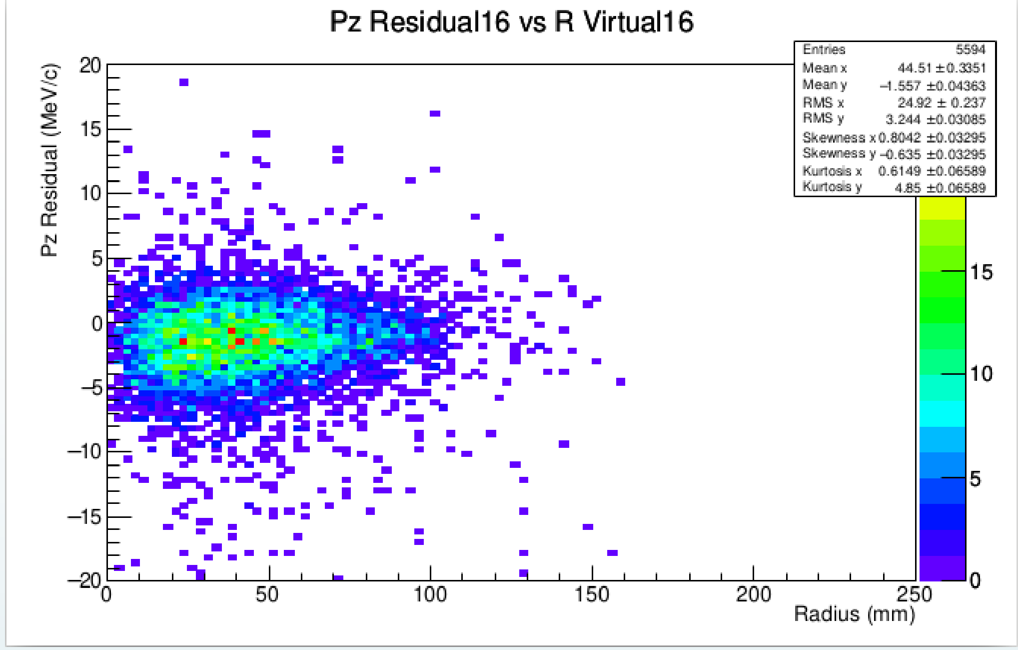




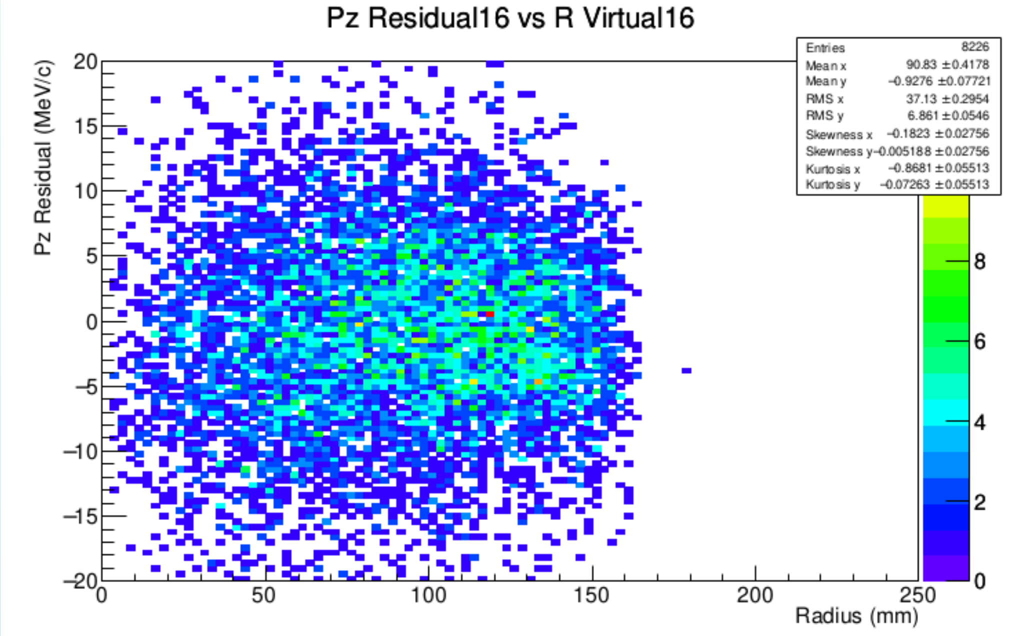
TKU Reference Plane: 3-140 above
TKU Reference Plane: 3-240 Below

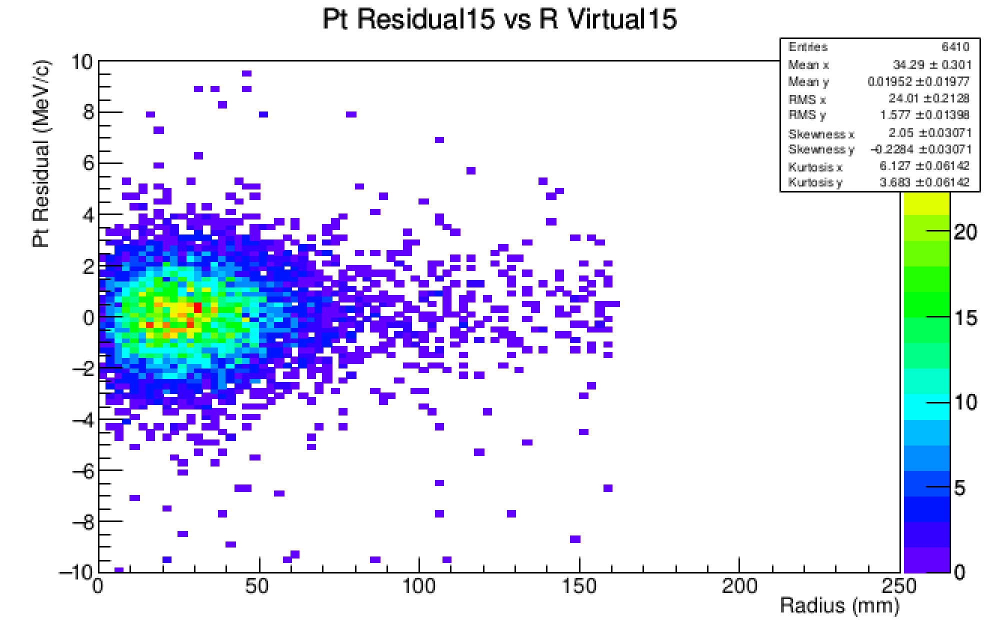


- Pz Residual vs Radial Position at Station
- The Radial Position can affect the Residual, as well as the parent momentum
- It can also be different between trackers



TKD Reference Plane: 3-140 above
TKD Reference Plane: 3-240 Below





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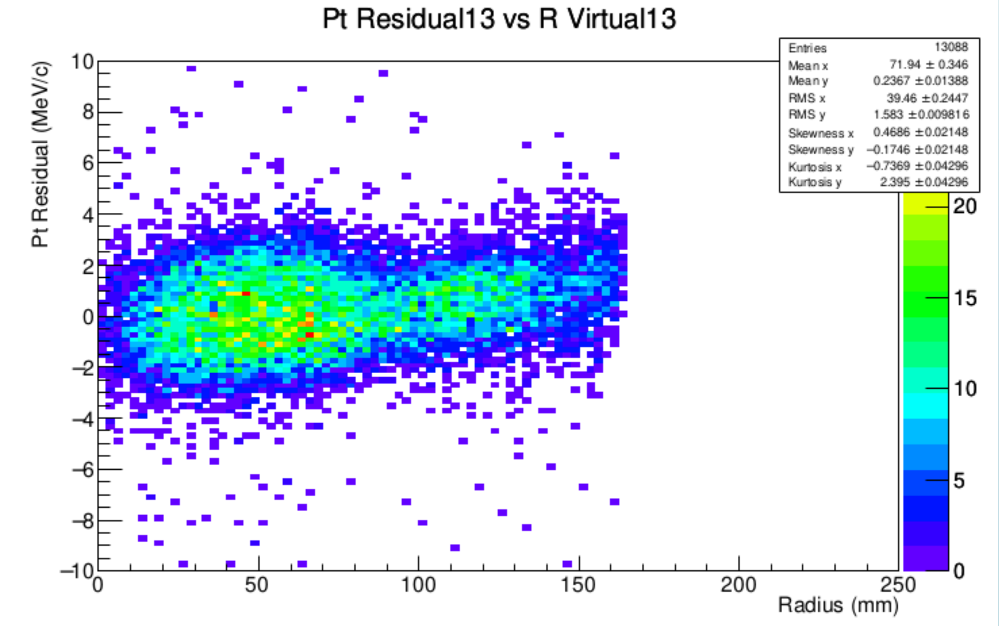

- Pt Residual vs Radial Position at Station
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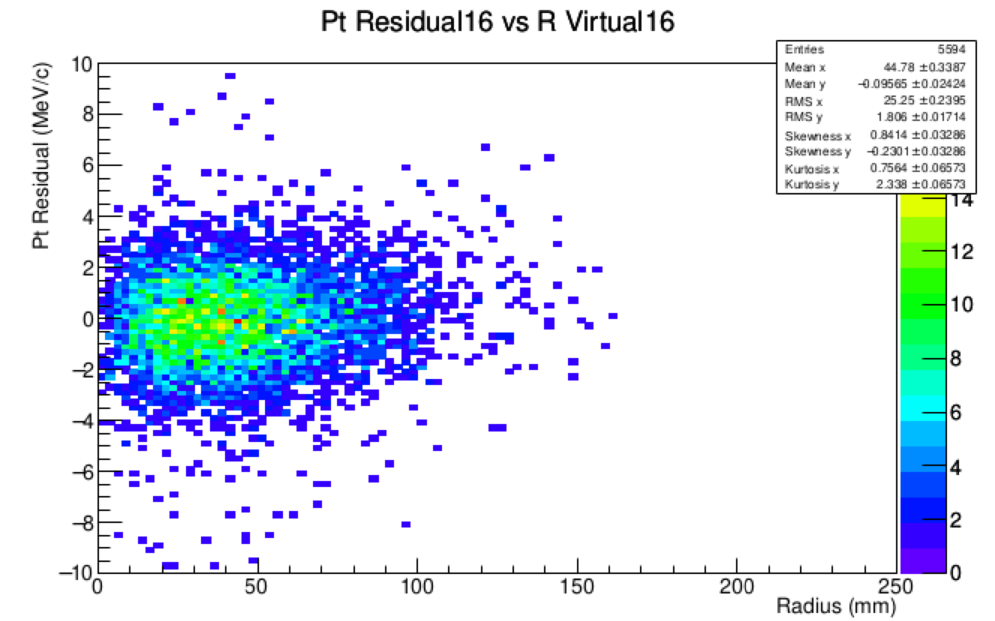
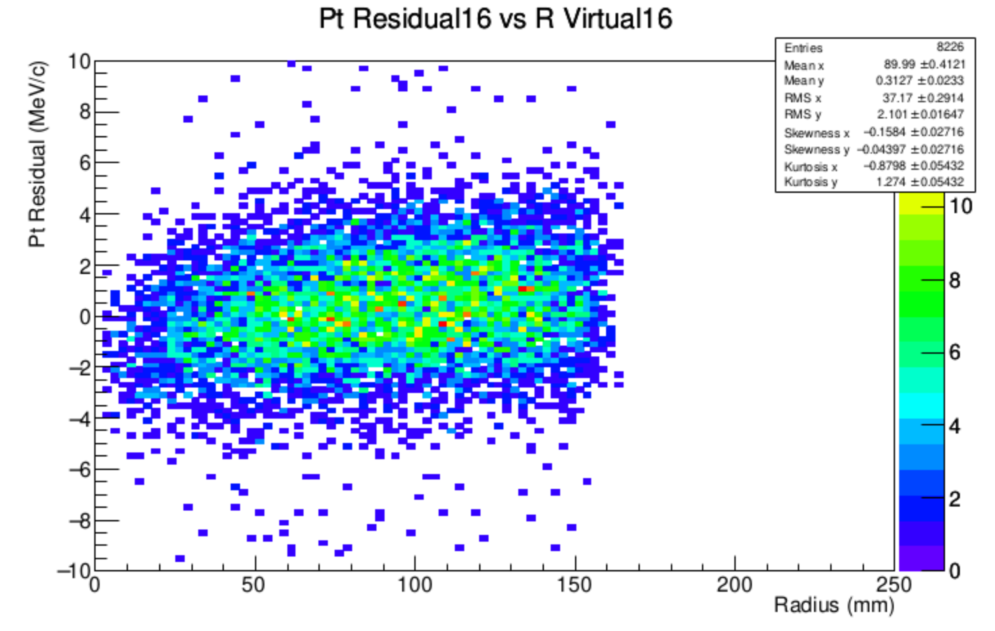
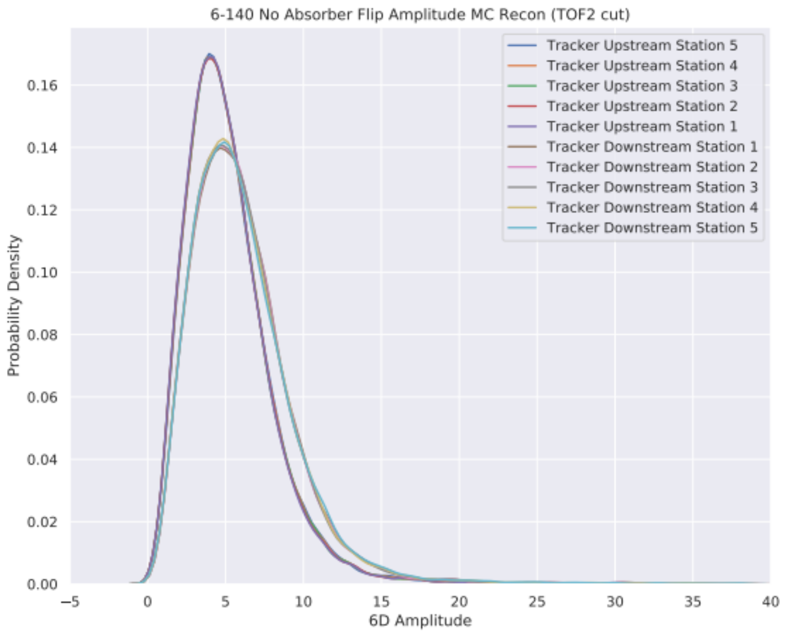


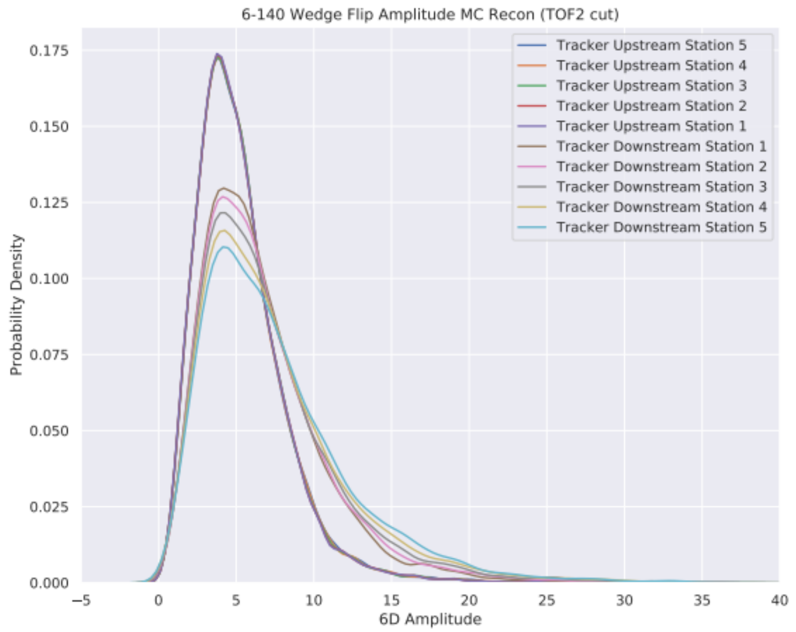
TKD Reference Plane: 3-140 above
TKD Reference Plane: 3-240 Below

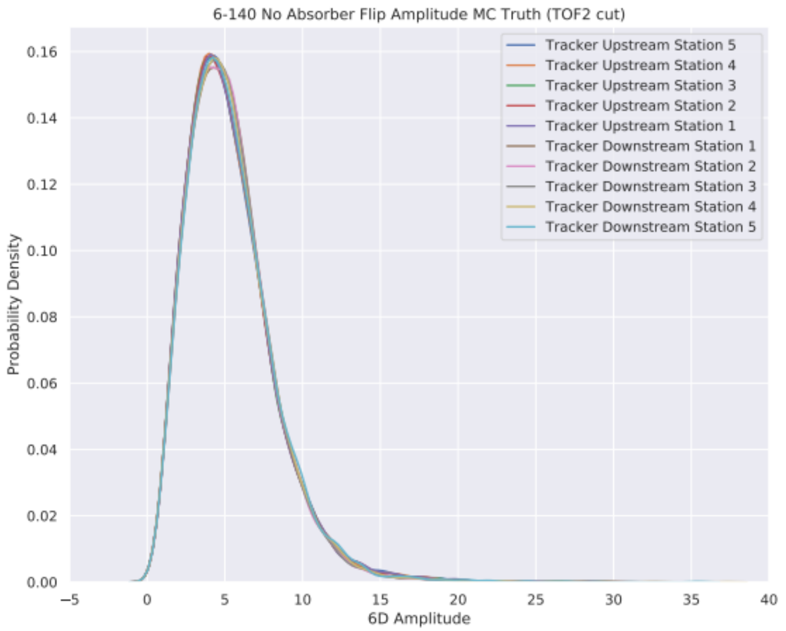


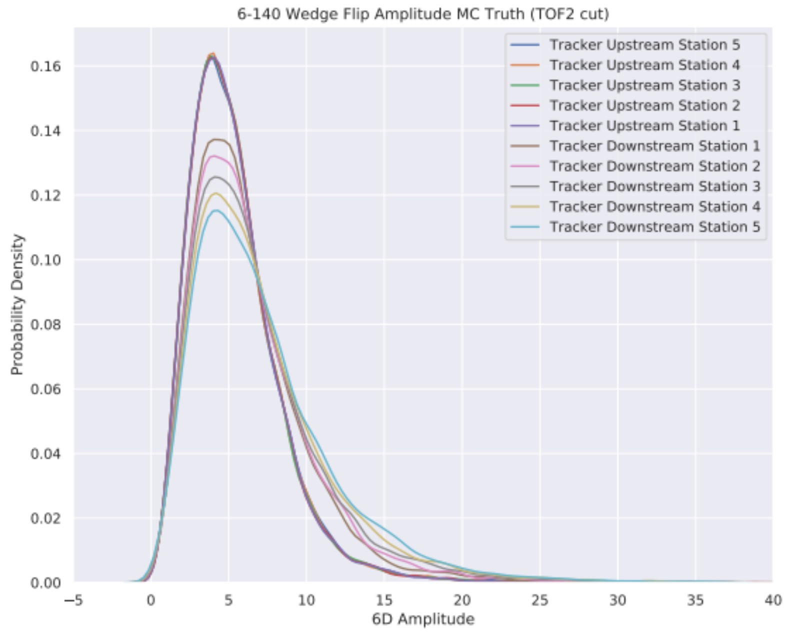
6D Amplitude

- Truth shows 6D conservation for No Absorber case
- Wedge shows change between TKU and TKD and within TKD due to dispersion
- Likely need to correct Transverse components for extra rotation
- Makes separation of 6D into Transverse and Longitudinal components tricky



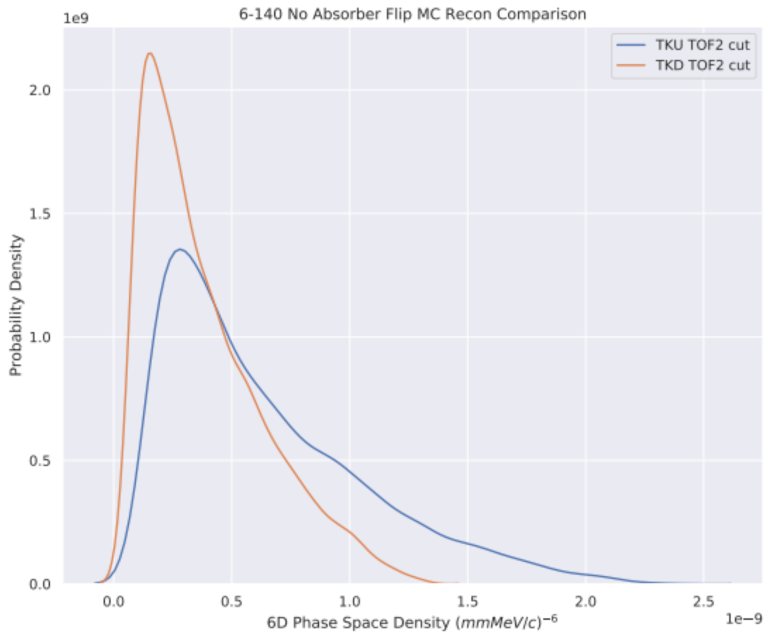


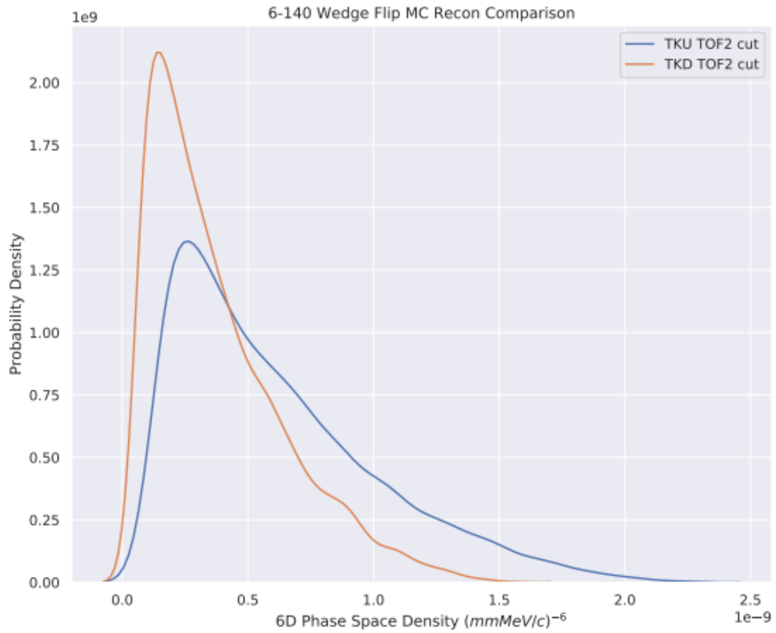


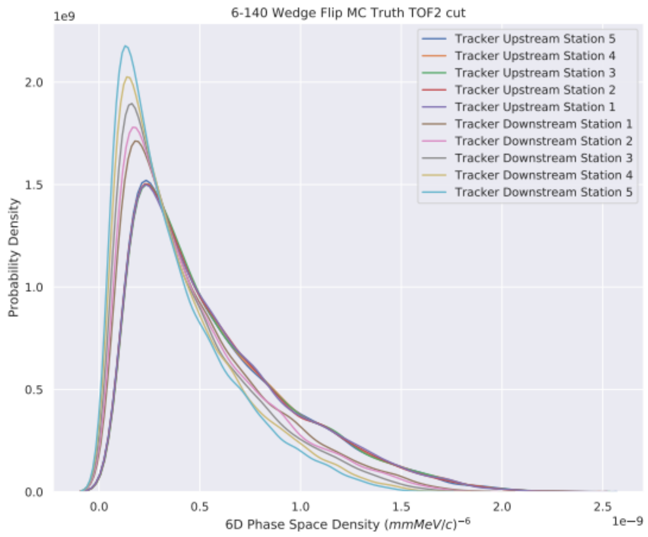


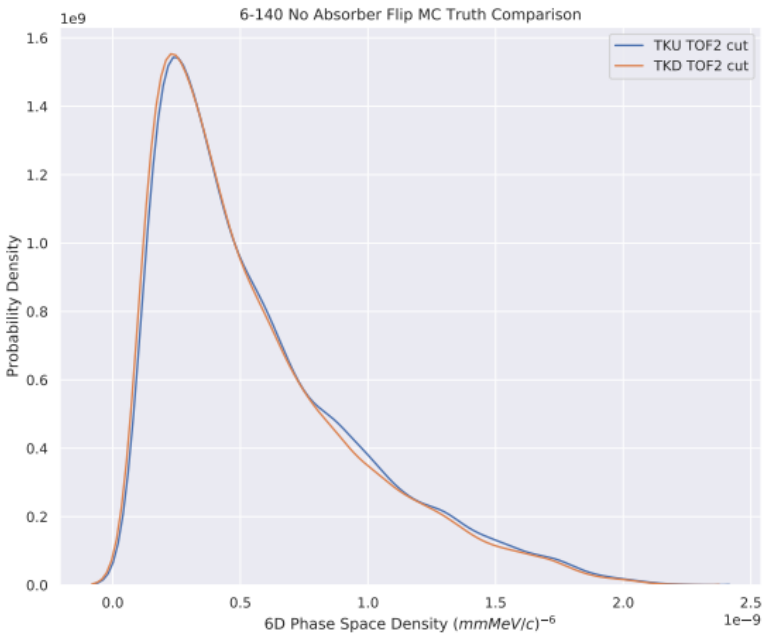
6D Density

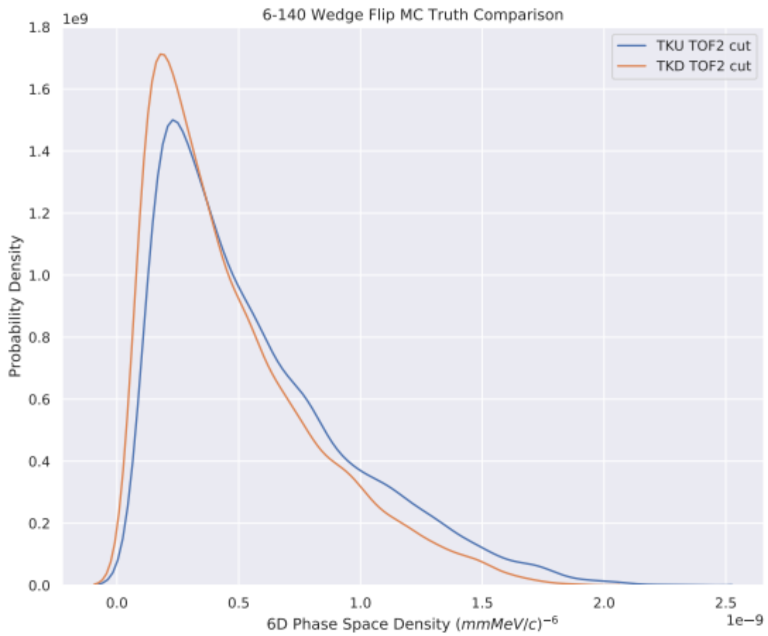
- 6D density is also similar to 6D Amplitude
- It also has same effect through TKD -> Transverse Components need correction





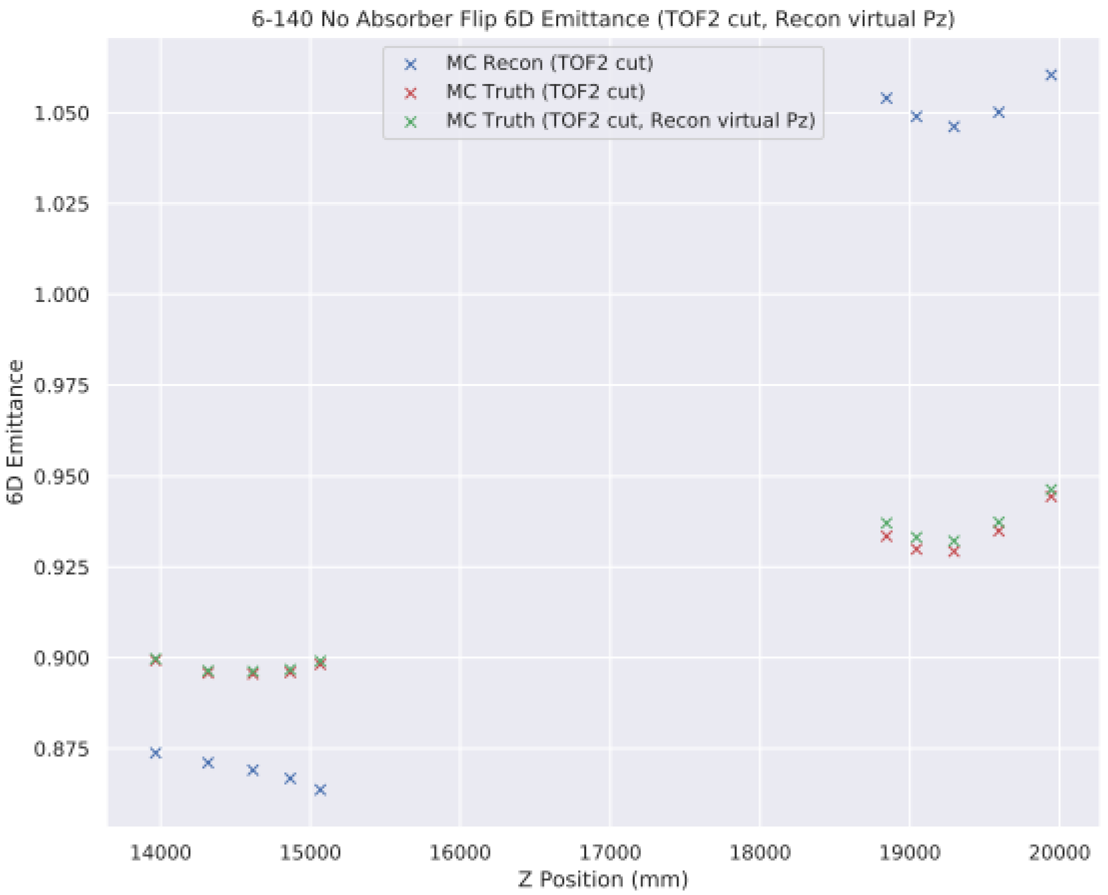


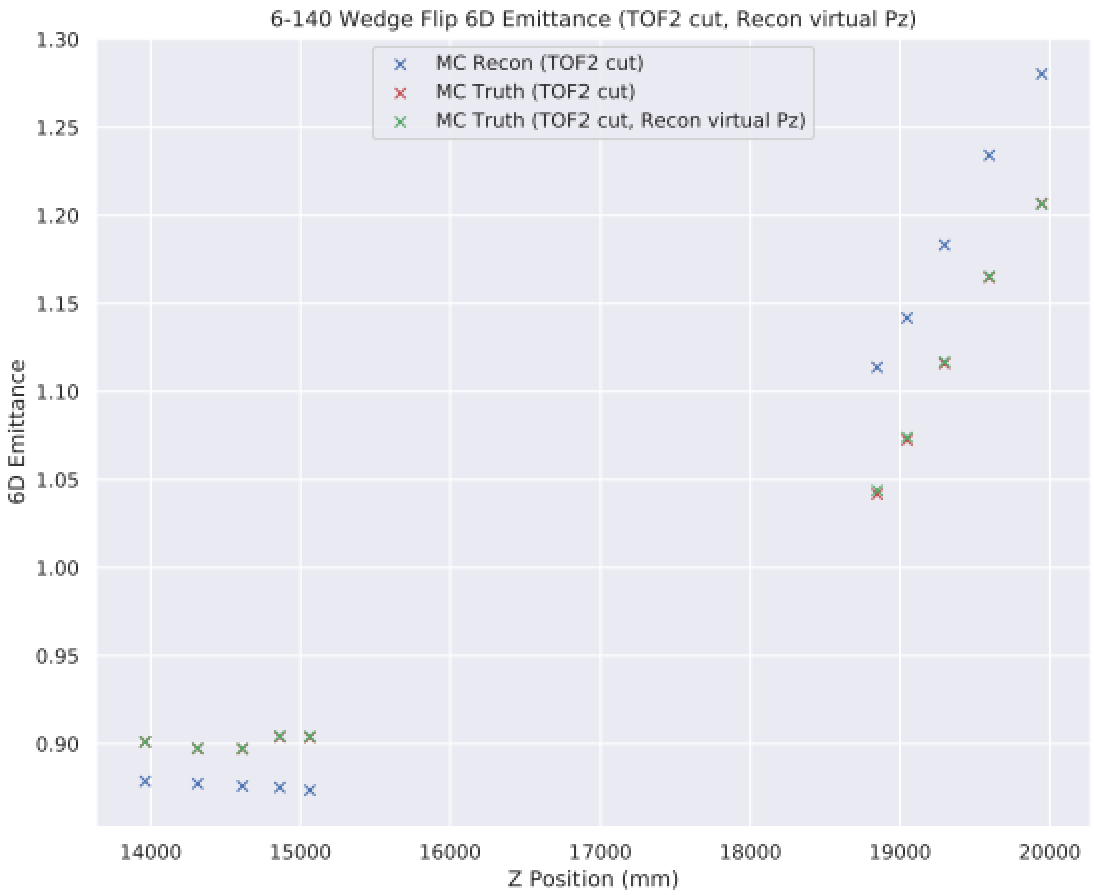




6D emittance

- Green is same as Truth except Time has been reconstructed using Truth Pz
- Recon shows larger discrepancies (Resolution effect)
- Could take larger momentum bite, but would then need to correct transverse components. Probably need to in Wedge case due to dispersion downstream



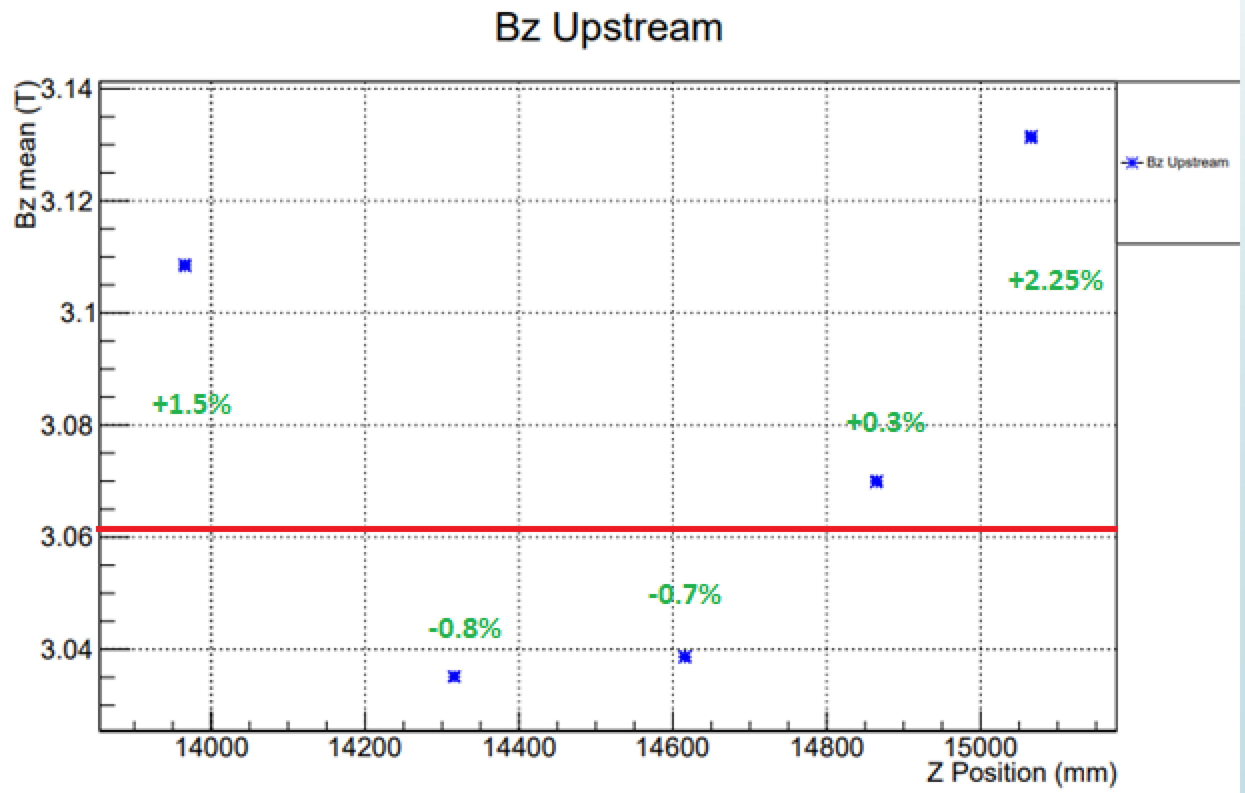


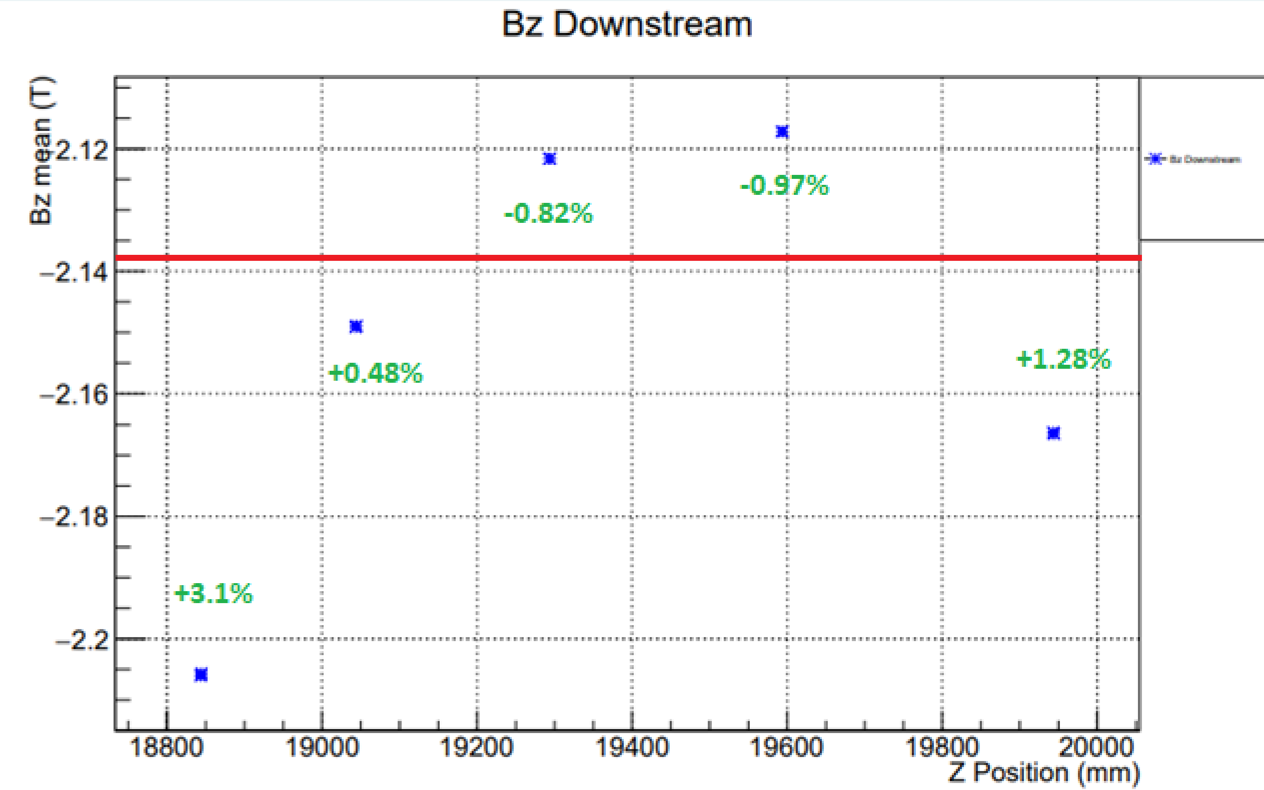
Why does Recon cause bias (plots follow)

- Use 3T and 2T fields – different Resolutions, also Flip vs Solenoid
- Recon assumes Constant Uniform field
  - MICE field deviates by up to 3% from this value
  - Within tracker, the separated Transverse and Longitudinal Emittance, Amplitude and Density will show slight deviations between stations
- Misalignments
  - Tracker, Solenoid and Magnetic field have inherent misalignments
  - If between Tracker and Magnetic field, then transverse and longitudinal planes aren't fully separated
- Energy loss
  - Recon uses circle fit for helix. Doesn't account for deviation of helix radius due to Energy loss. Energy loss correction done in Kalman stage
- Scattering (potentially)
  - Can cause transfer between Pz and Pt and Vice Versa. Assume scattering is symmetric, however the acceptance of the scattered particle is not symmetric. More likely to keep particles whose helix radius decreases rather than increases

Bz fields in trackers

- Bz fields are not uniform
- Recon assumes mean magnetic field through Tracker

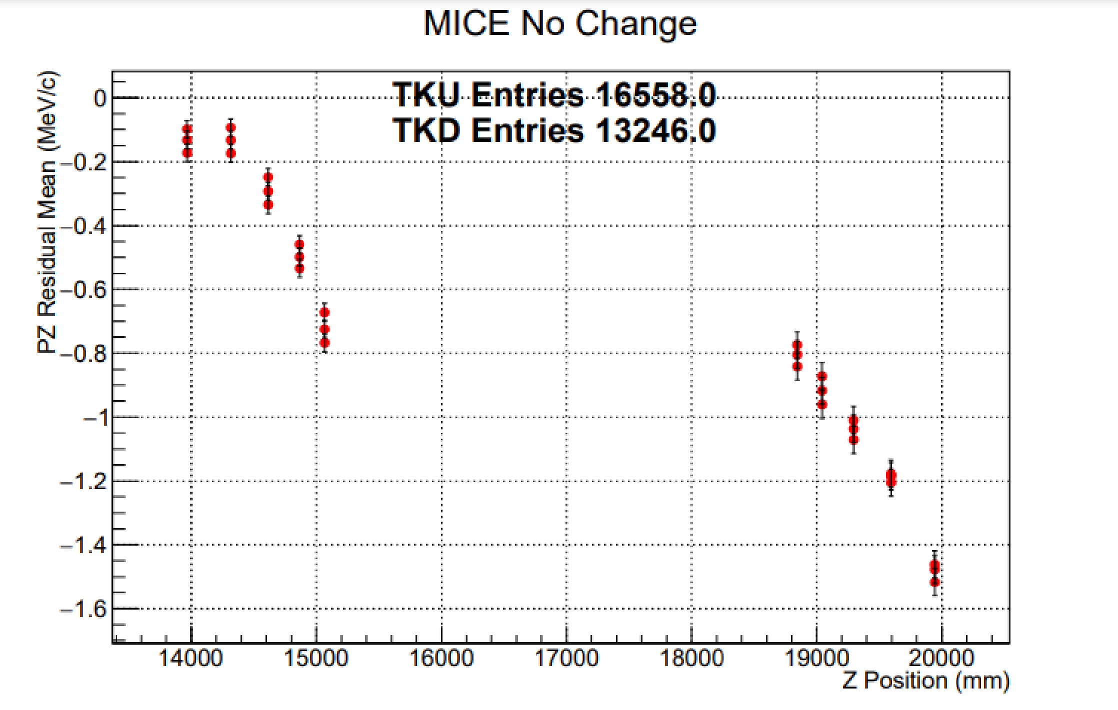


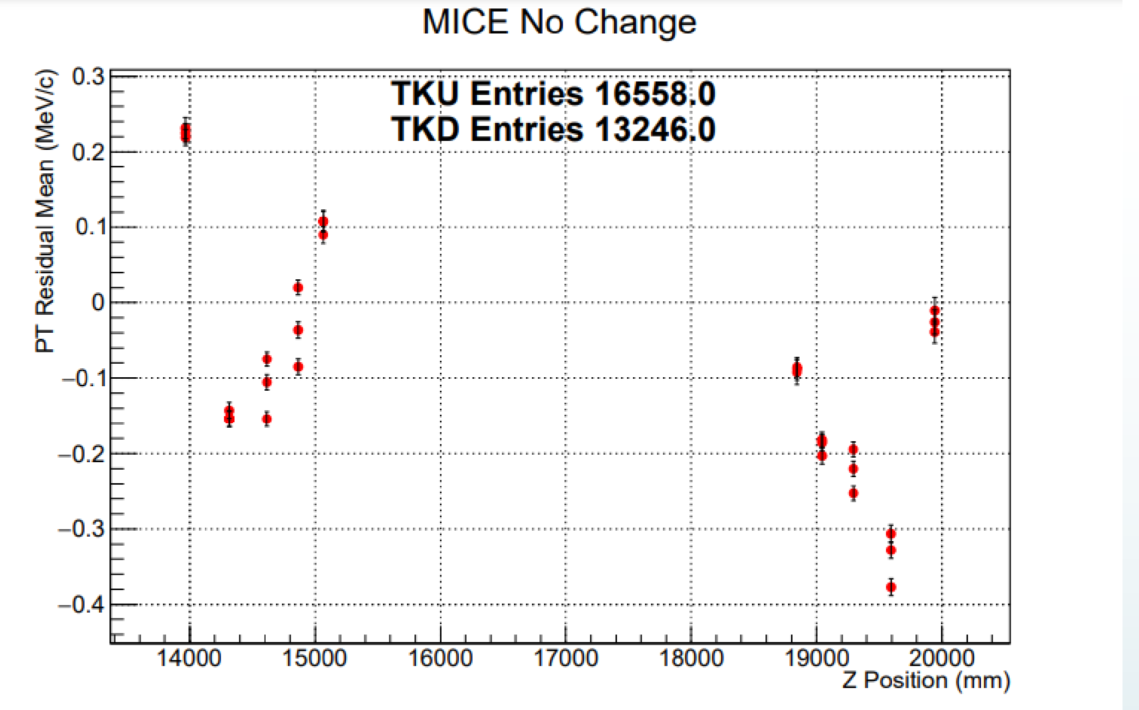


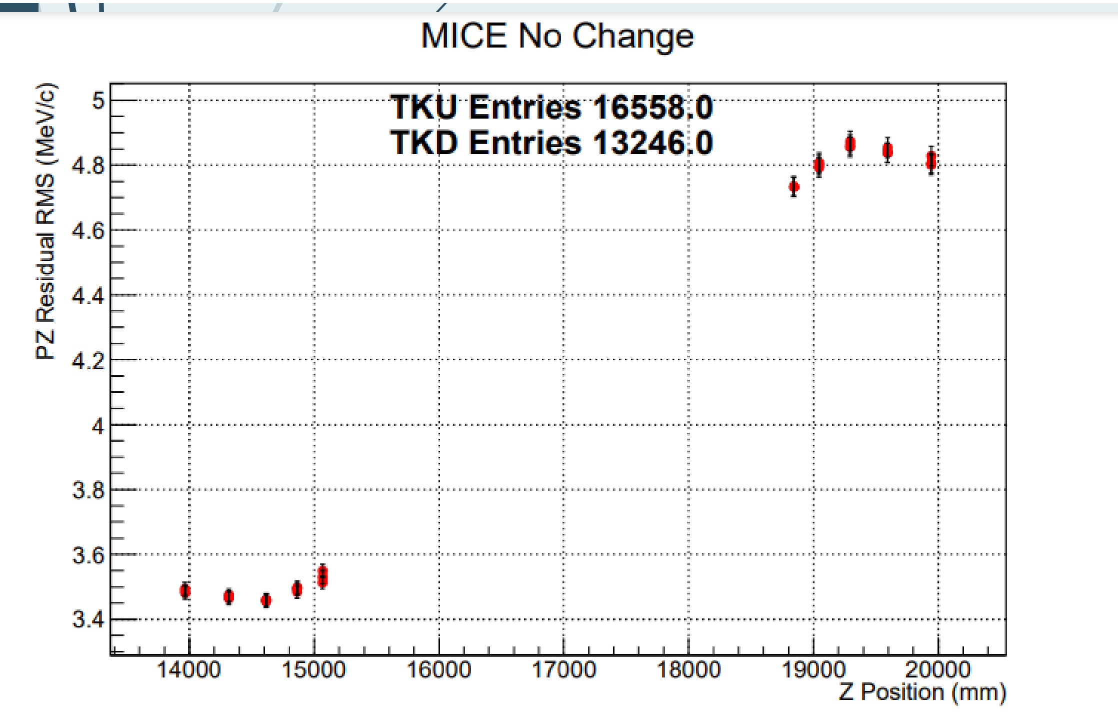
Pz and Pt Mean and RMS Residuals

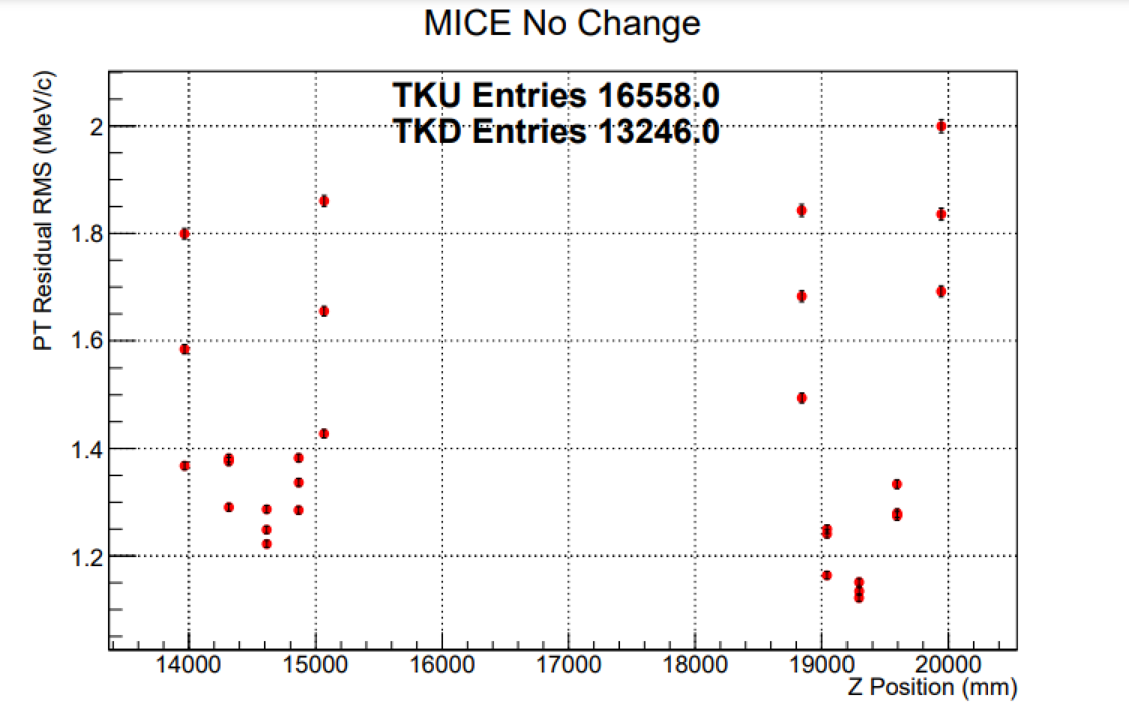
Truth minus Recon

Uses normal MICE fields and Alignments



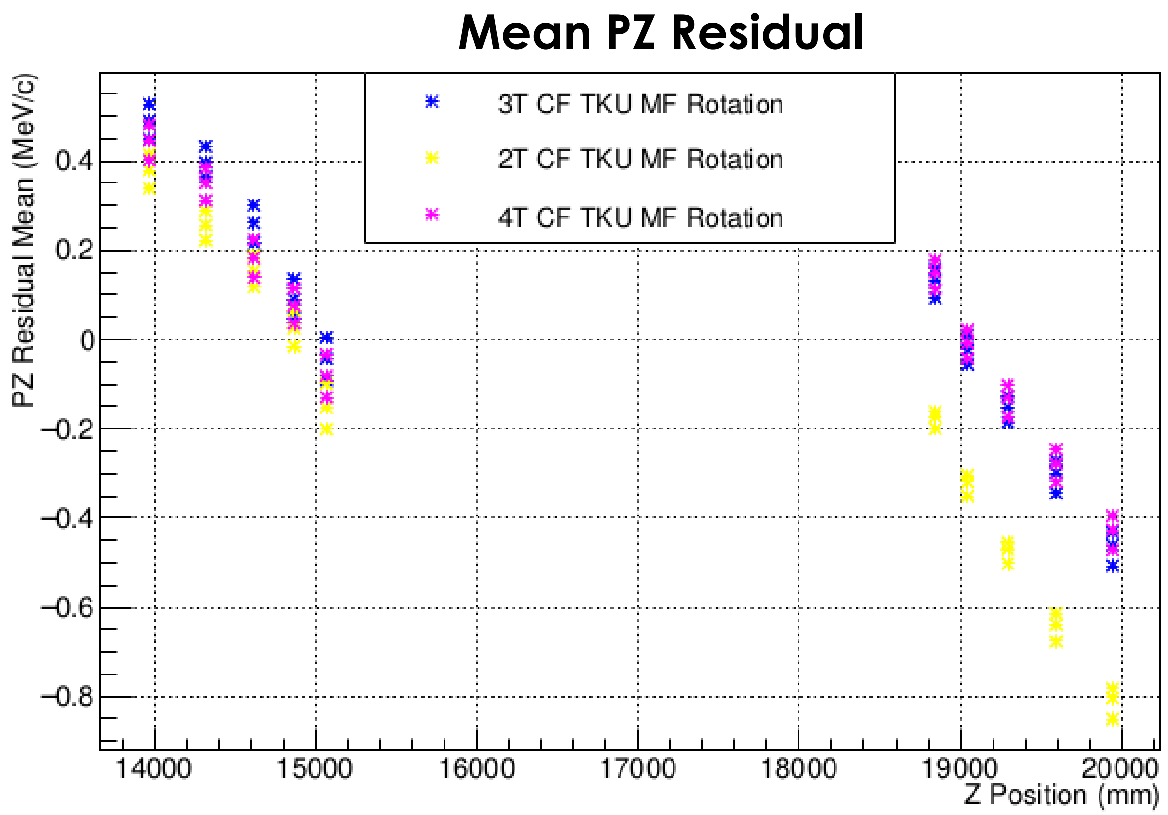


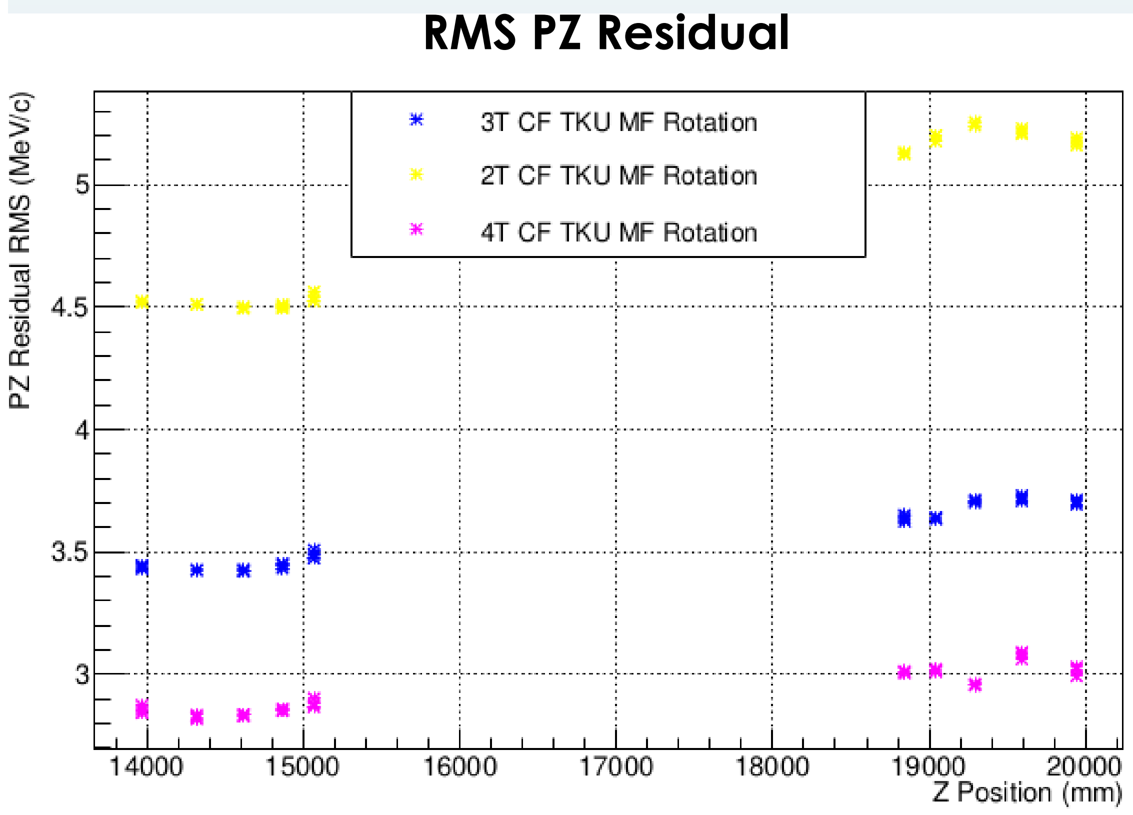




Constant Solenoid Field

- Higher fields - > Better RMS
- Residual in tracker decreases linearly

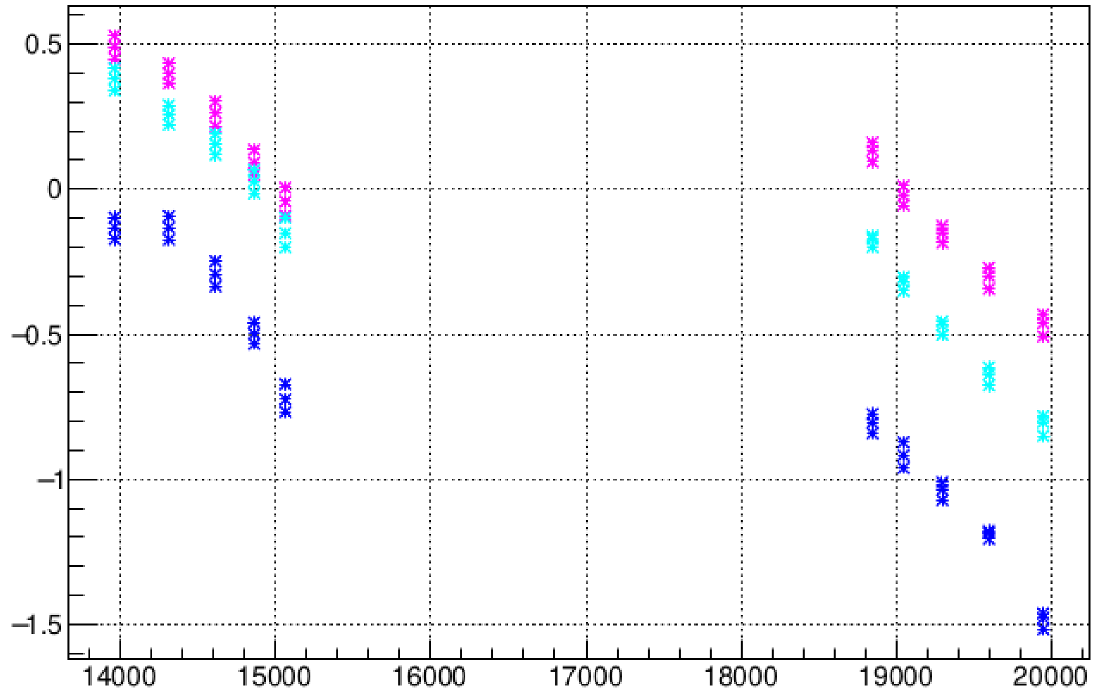




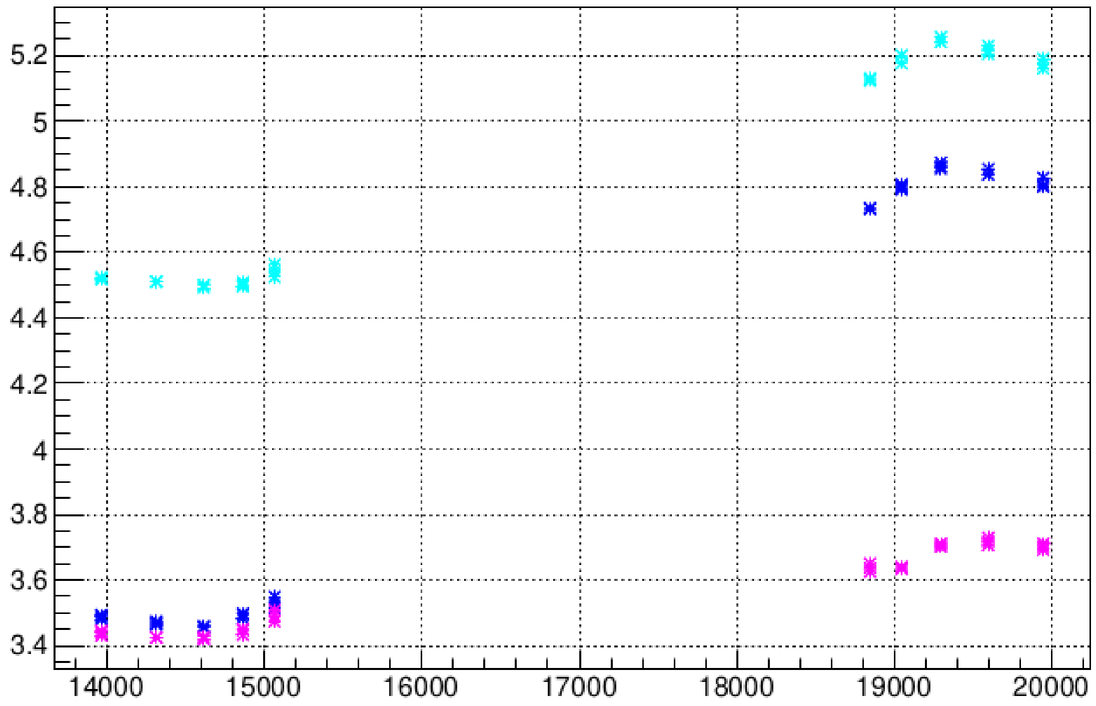
Blue = MICE field No Change
Cyan = Constant Field 2T
Magenta = Constant Field 3T

PZ Bias depends on magnetic field

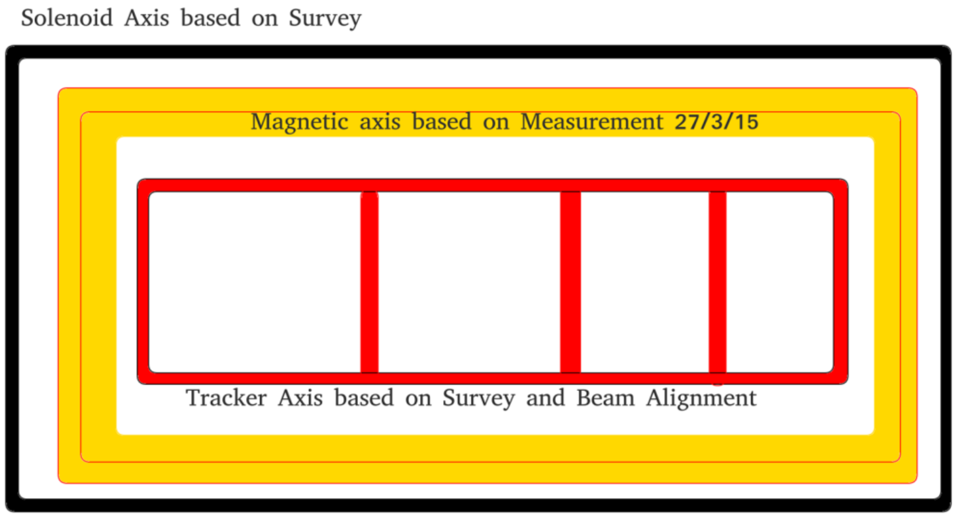
Mean PZ Residual



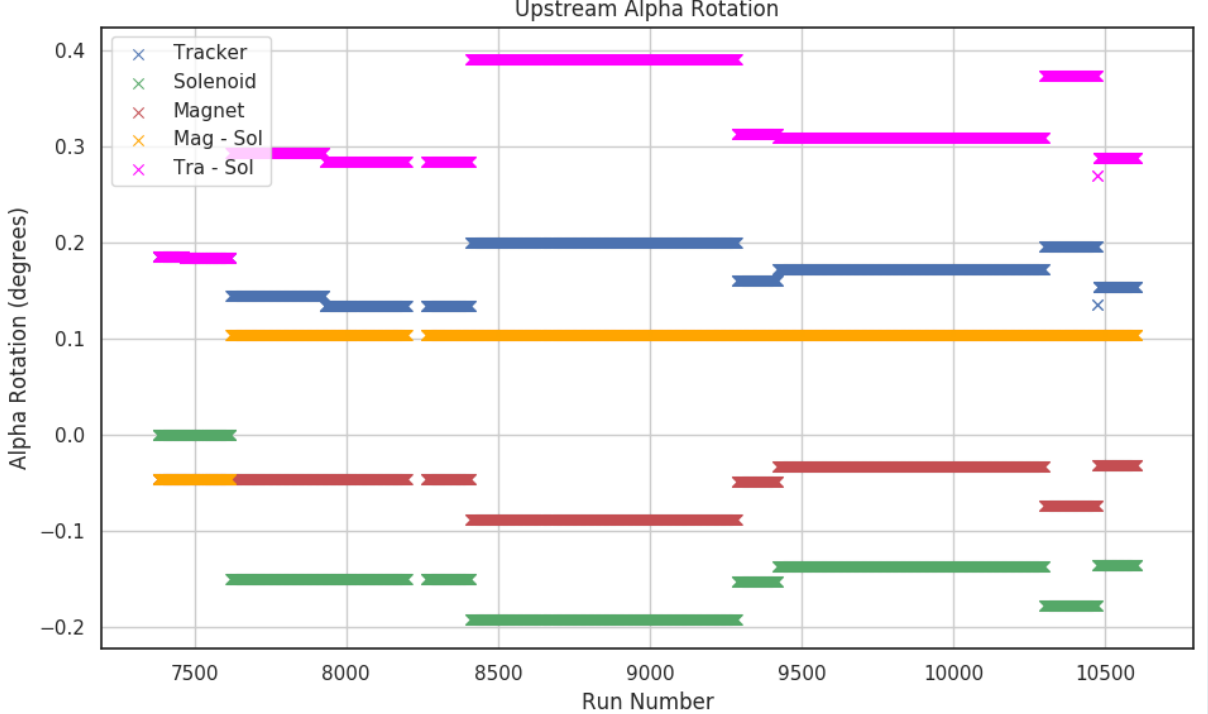
RMS PZ Residual

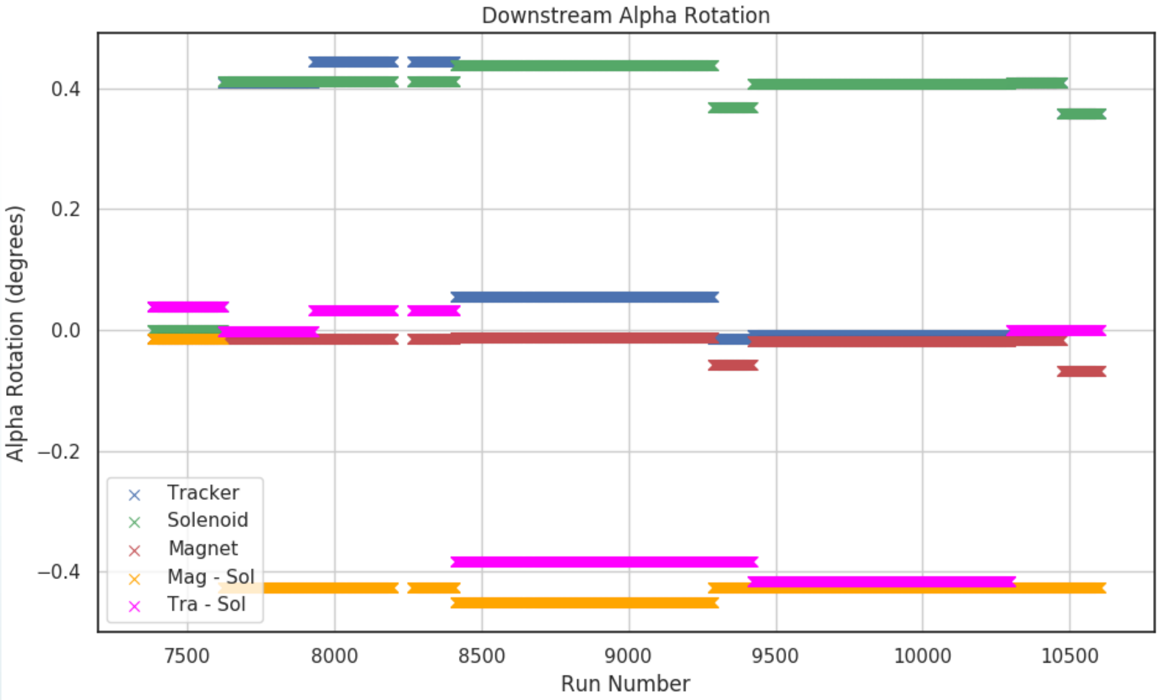


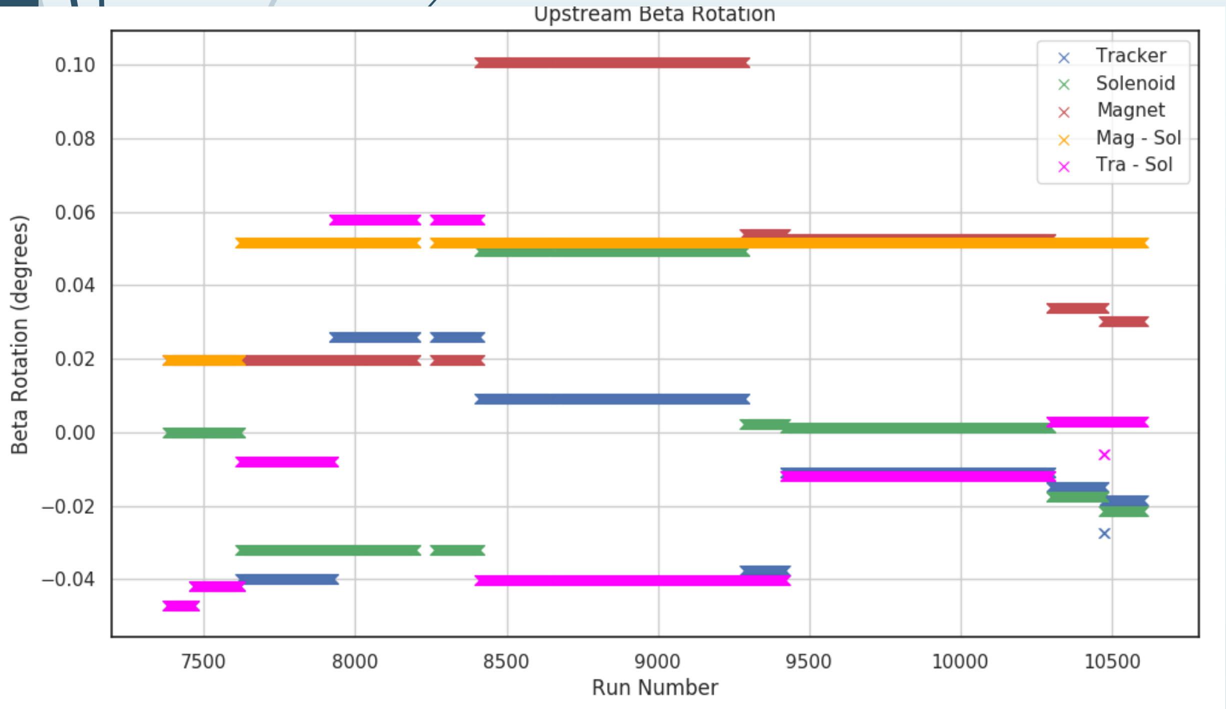
Misalignments

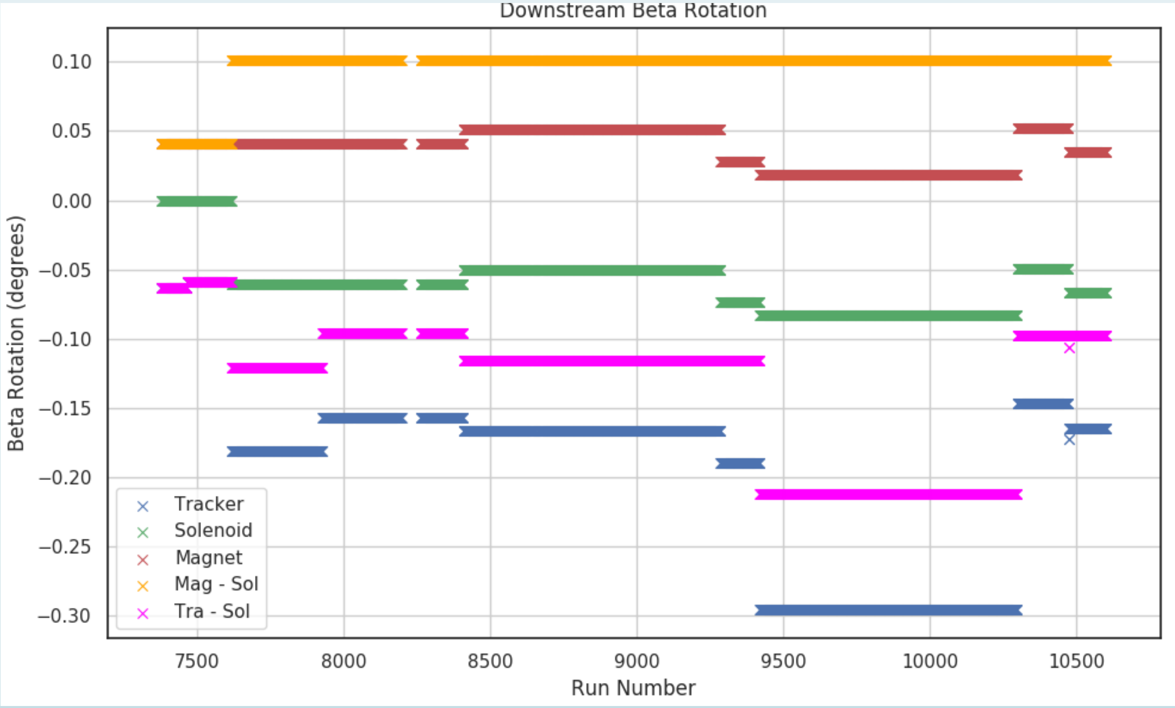


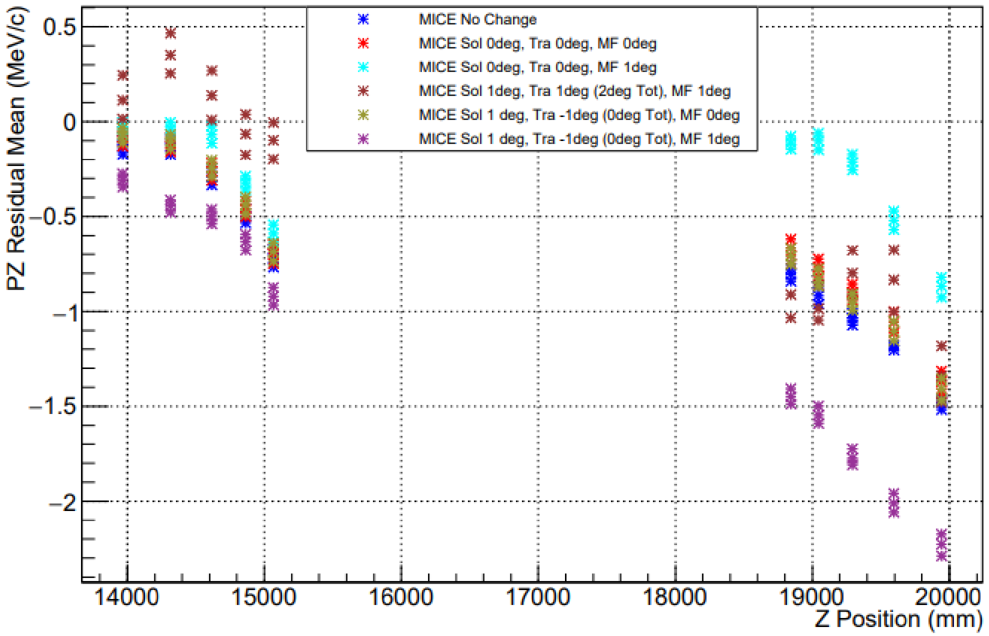
- Misalignments of the solenoid, tracker and magnetic field are all less than ~0.5 degrees (< 9 mrad)
- Magnets and Solenoids are in global co-ordinates, trackers are relative to Solenoids
- Magnet Alignment based on one measurement (27/3/15), and then changes with Solenoid Alignment
- Beam Alignment gives new tracker Alignment after every absorber change
- Curiously tracker to Solenoid Alignment changes by up to 9 mrad
- Important comparison is between the red and pink lines
  - Determines how well transverse and longitudinal planes are separated
  - Helix path becomes skewed if too large


Upstream Alpha Rotation
Alpha Rotation (degrees)
Run Number
Tracker
Solenoid
Magnet
Mag - Sol
Tra - Sol
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
7500
8000
8500
9000
9500
10000
10500

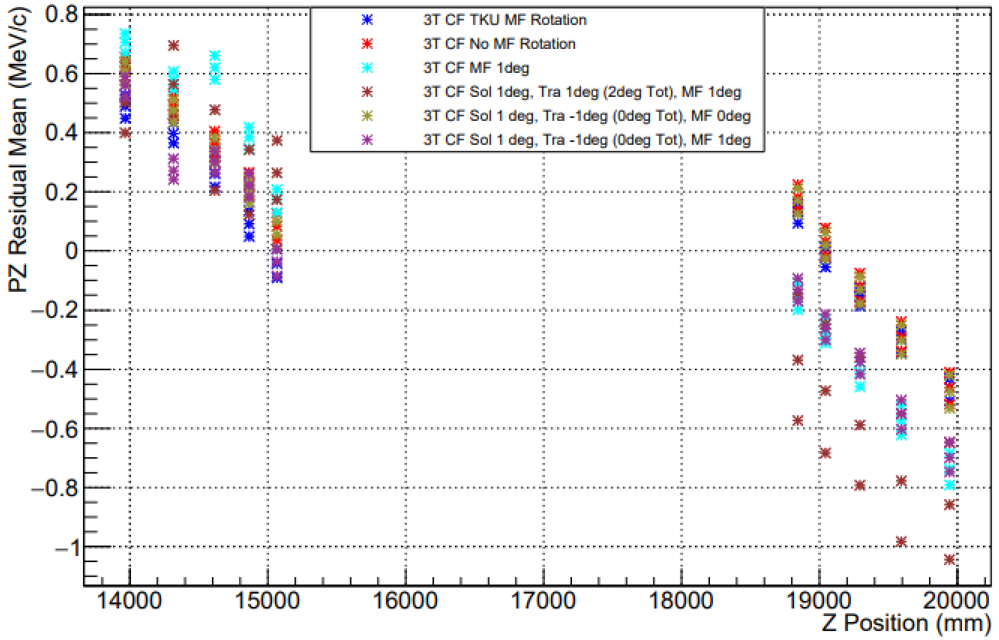

Downstream Alpha Rotation
Alpha Rotation (degrees)
Run Number
Tracker
Solenoid
Magnet
Mag - Sol
Tra - Sol
0.4
0.2
0.0
−0.2
−0.4
7500
8000
8500
9000
9500
10000
10500

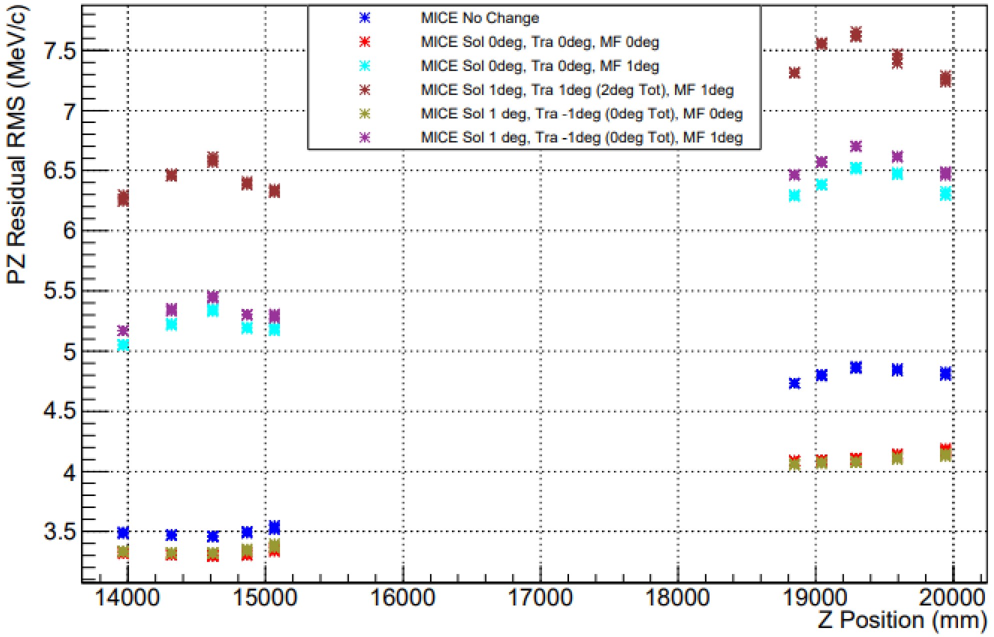

Upstream Beta Rotation
Beta Rotation (degrees)
Run Number
Tracker
Solenoid
Magnet
Mag - Sol
Tra - Sol
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
7500
8000
8500
9000
9500
10000
10500


Downstream Beta Rotation
Beta Rotation (degrees)
Run Number
Tracker
Solenoid
Magnet
Mag - Sol
Tra - Sol
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
7500
8000
8500
9000
9500
10000
10500



Comparing the MICE and Constant Field Pz Mean Residual and Pz RMS Residual for different Alignments

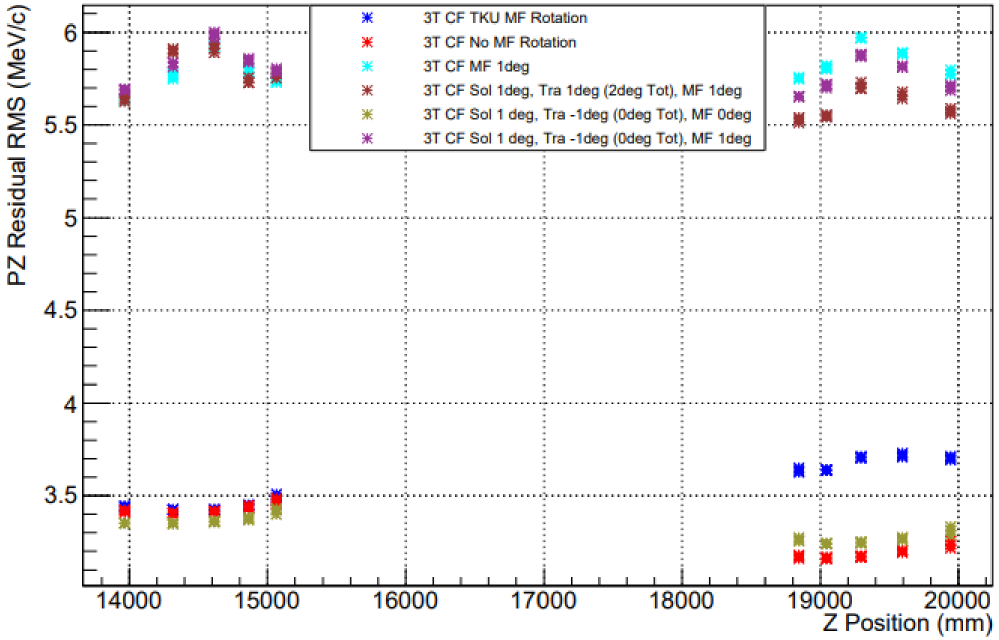




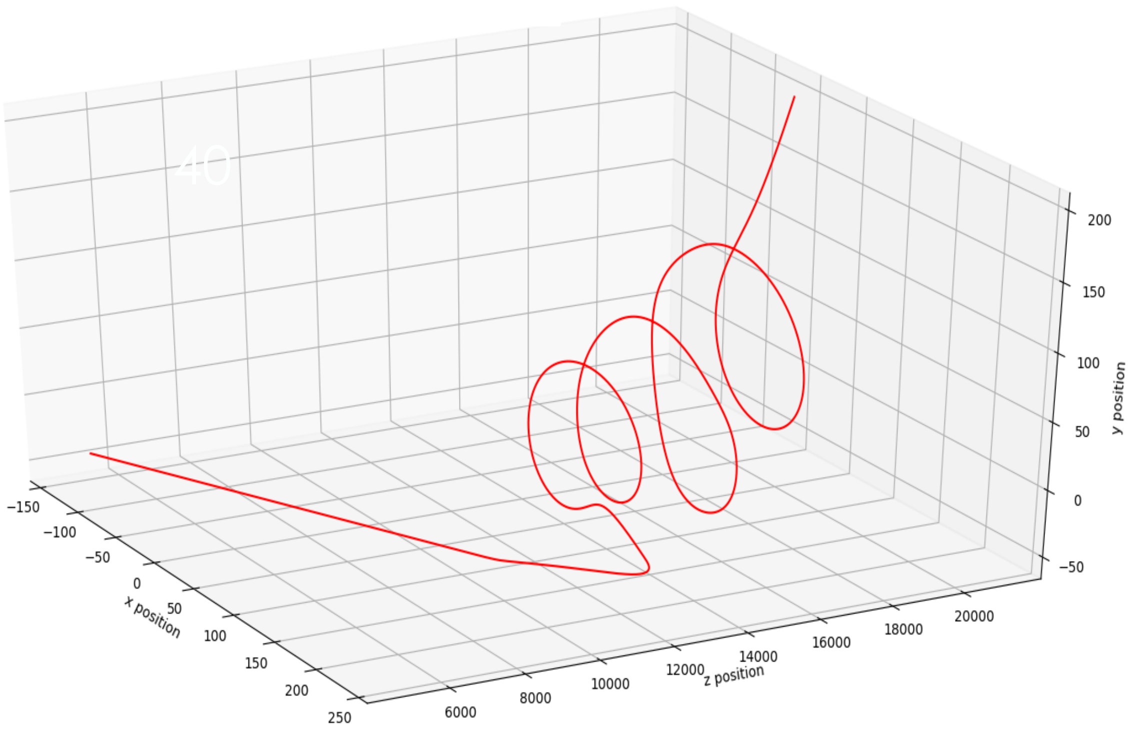
Main Takeaway:

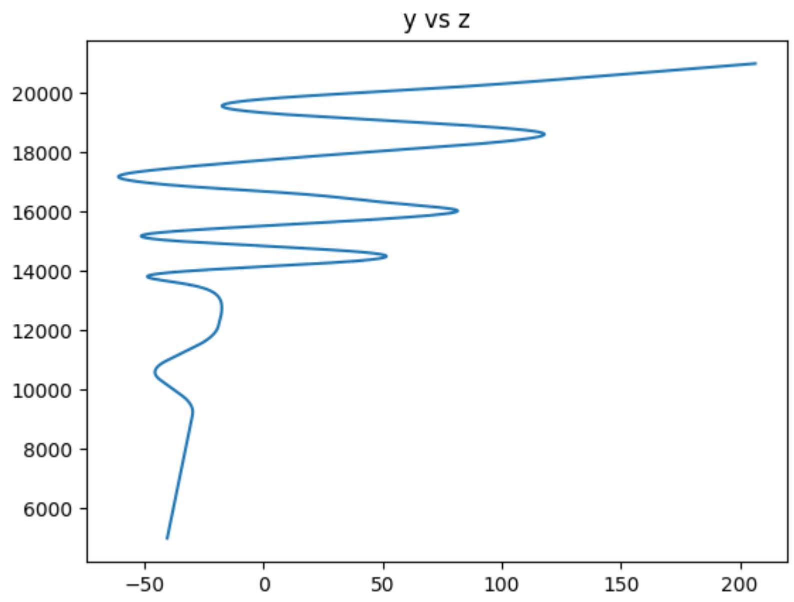
Misalignments change the Recon

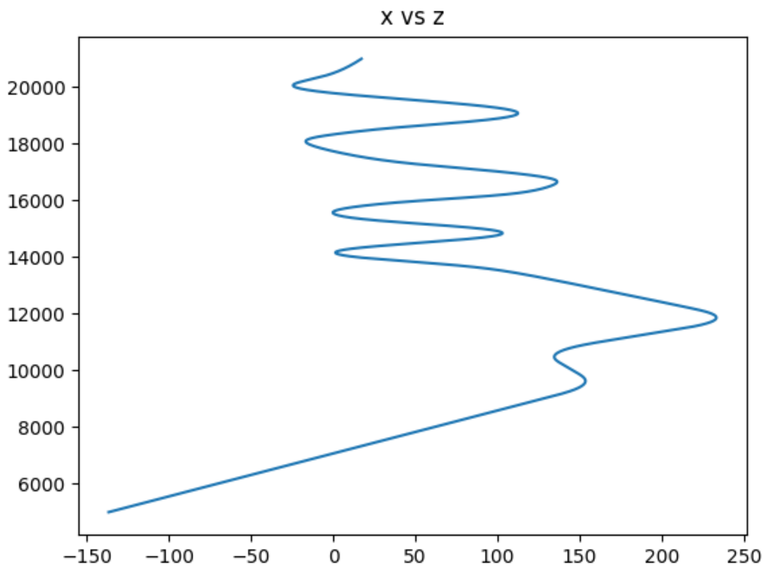
Note, there is a distribution dependence

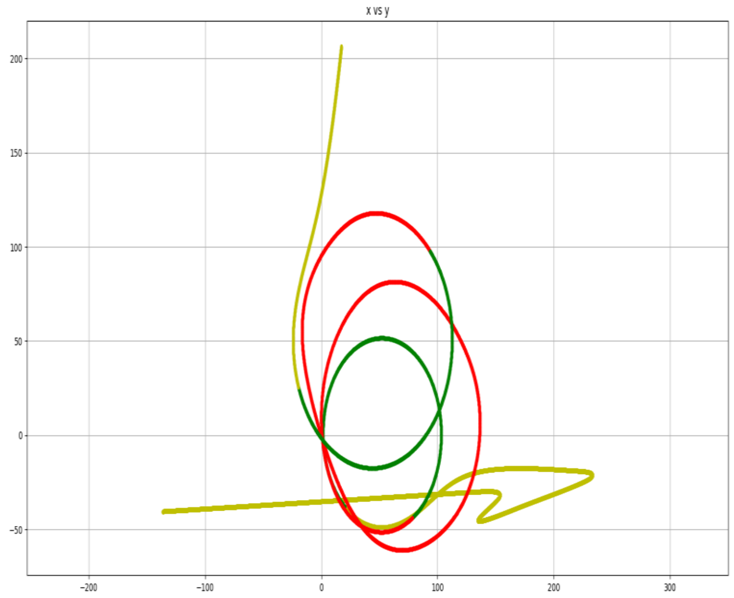


Example of a particle's trajectory in MICE (no stochastic processes)









Green paths show trajectory in each tracker. Circle fit is applied to five points from which Pt and Pz is derived

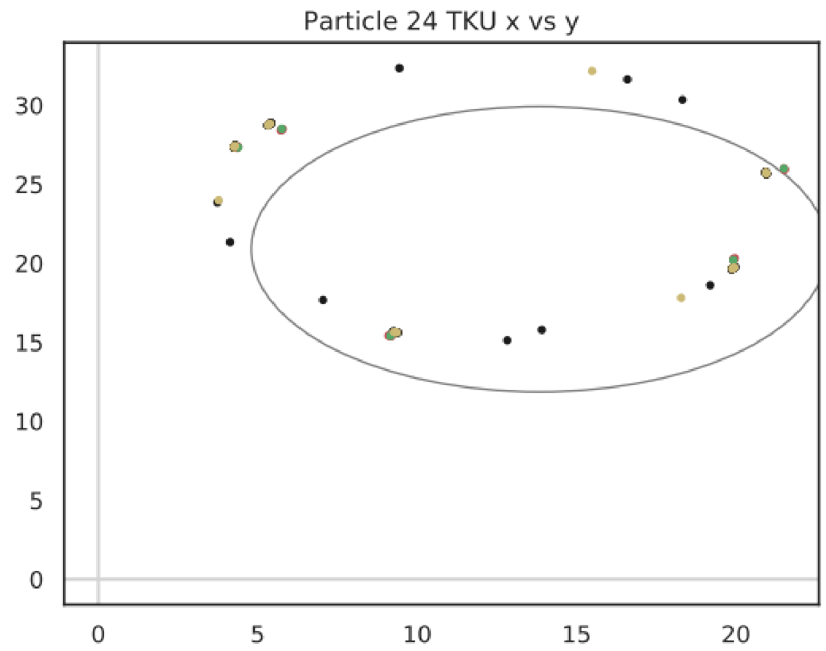
Using MAUS defaults to track

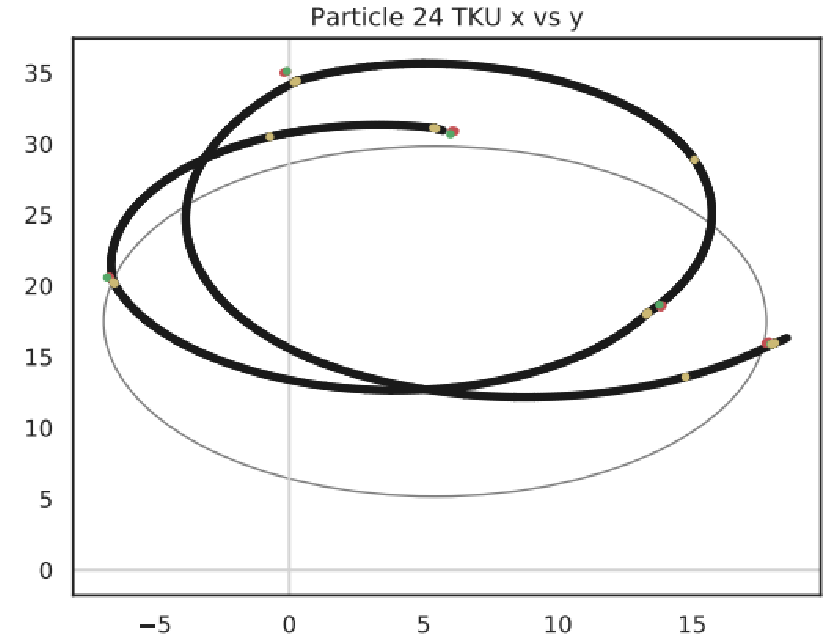
- Apologies, may include mistakes

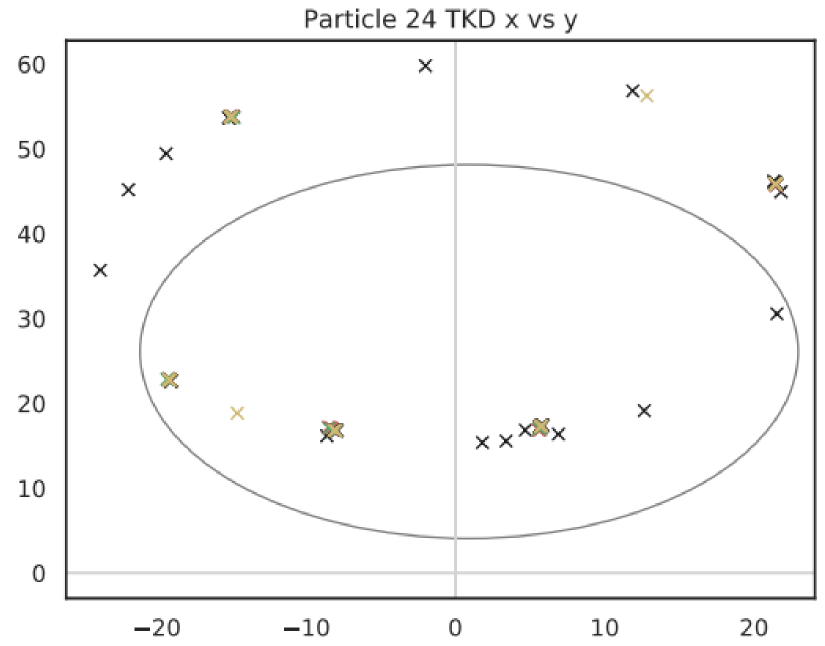
- Includes scattering, misalignments, energy loss
- Default tracking distance is 100mm or until encounters material
- Very few points, so decreased distance to 1mm
- Scattering may be larger as it may scatter between station more

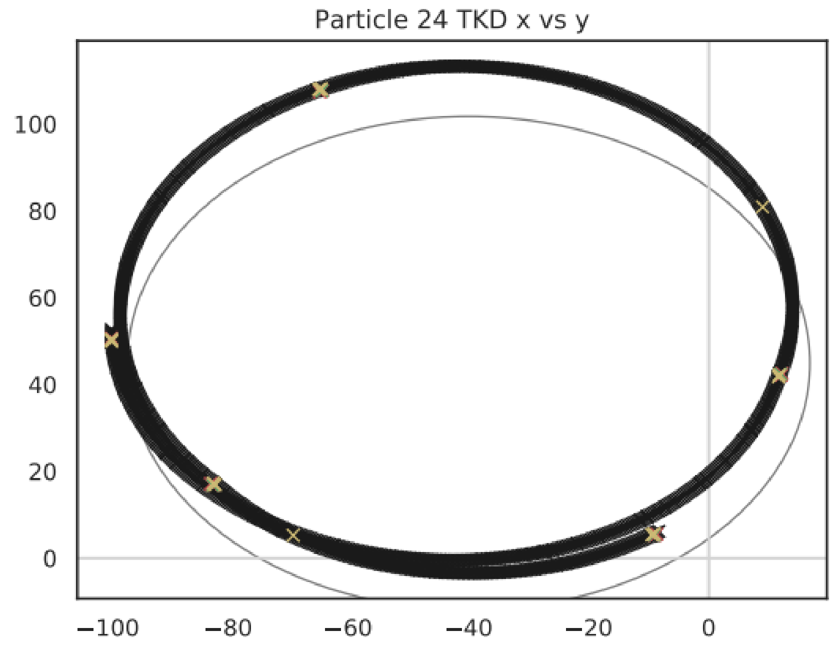
- Draw Circle fit based on X0, Y0 and R0
- Position ot representative as they are in local not global co-ordinates

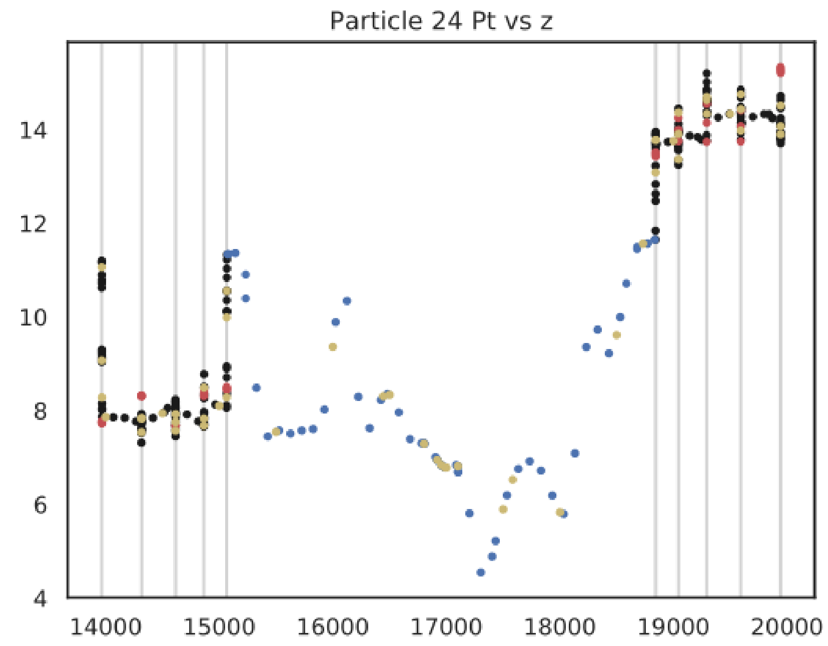
- Black is tracking, yellow are virtual hits
- Green are Spacepoints, Red Recon points

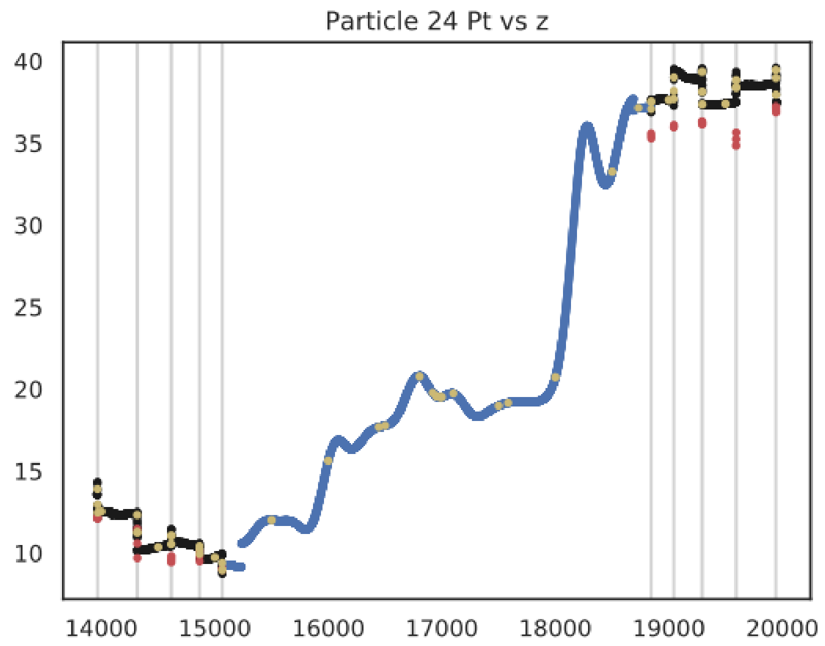

Particle 24 TKU x vs y
30
25
20
15
10
5
0
0
5
10
15
20

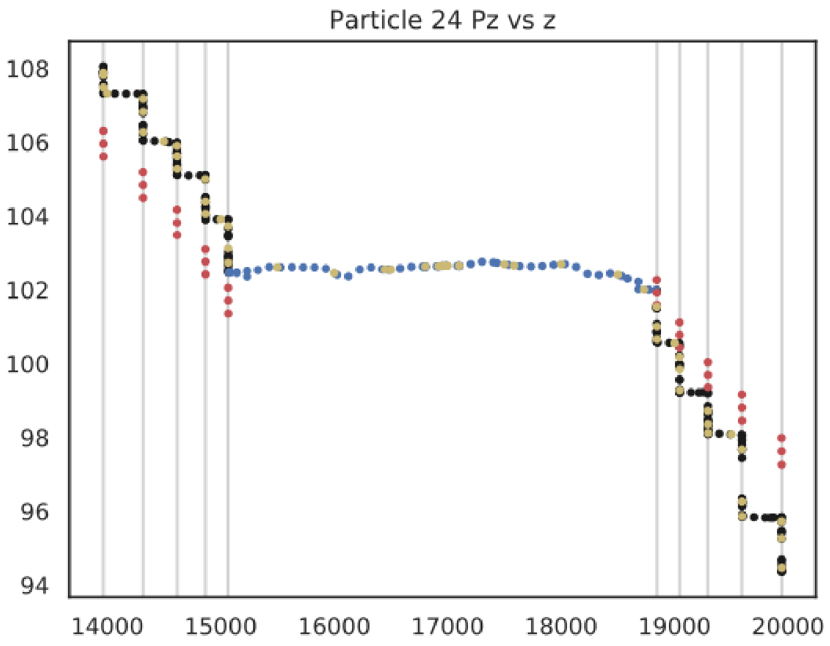

Particle 24 TKU x vs y
35
30
25
20
15
10
5
0
−5
0
5
10
15

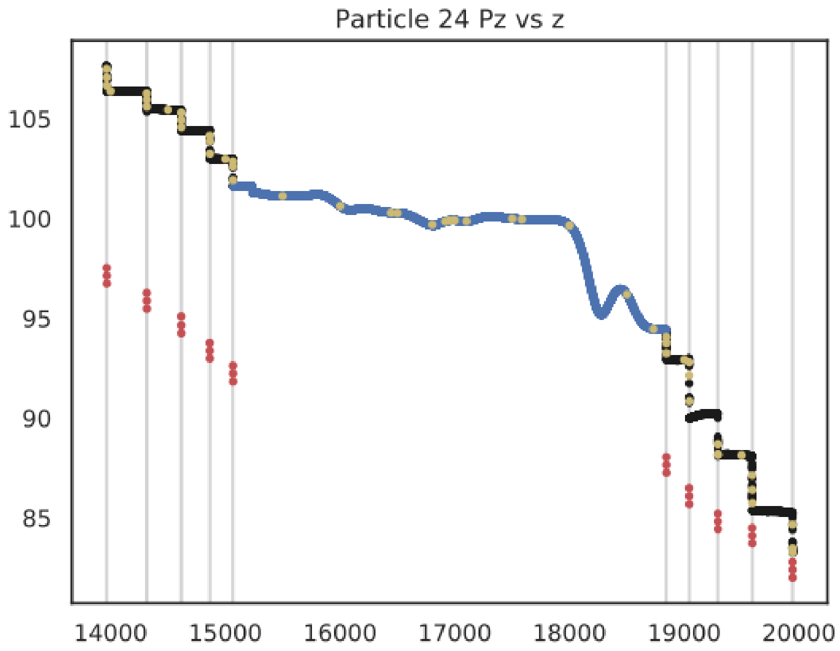

Particle 24 TKD x vs y
60
50
40
30
20
10
0
−20
−10
0
10
20


Particle 24 TKD x vs y
100
80
60
40
20
0
−100
−80
−60
−40
−20
0


Particle 24 Pt vs z
4
6
8
10
12
14
14000
15000
16000
17000
18000
19000
20000

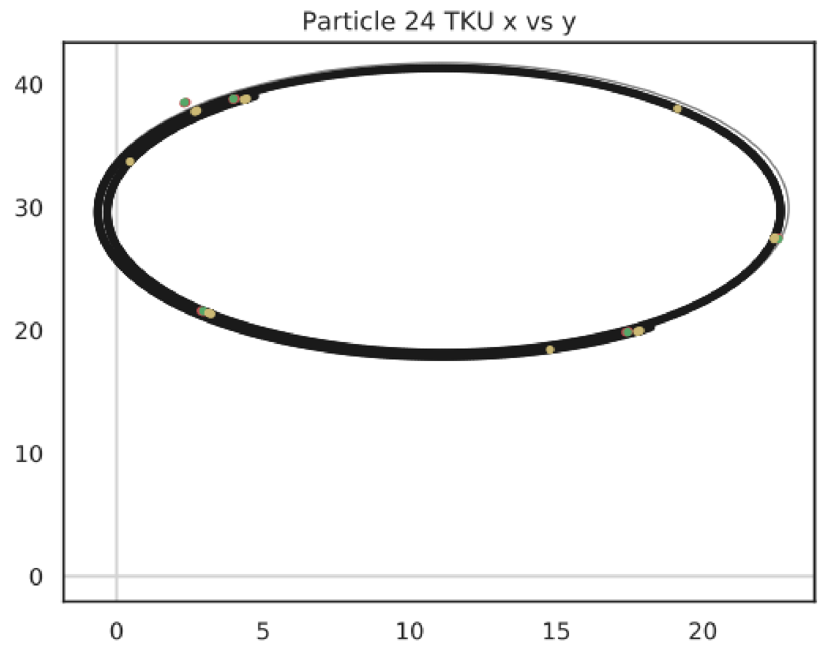

Particle 24 Pt vs z
10
15
20
25
30
35
40
14000
15000
16000
17000
18000
19000
20000


Particle 24 Pz vs z
94
96
98
100
102
104
106
108
14000
15000
16000
17000
18000
19000
20000


Particle 24 Pz vs z
85
90
95
100
105
14000
15000
16000
17000
18000
19000
20000

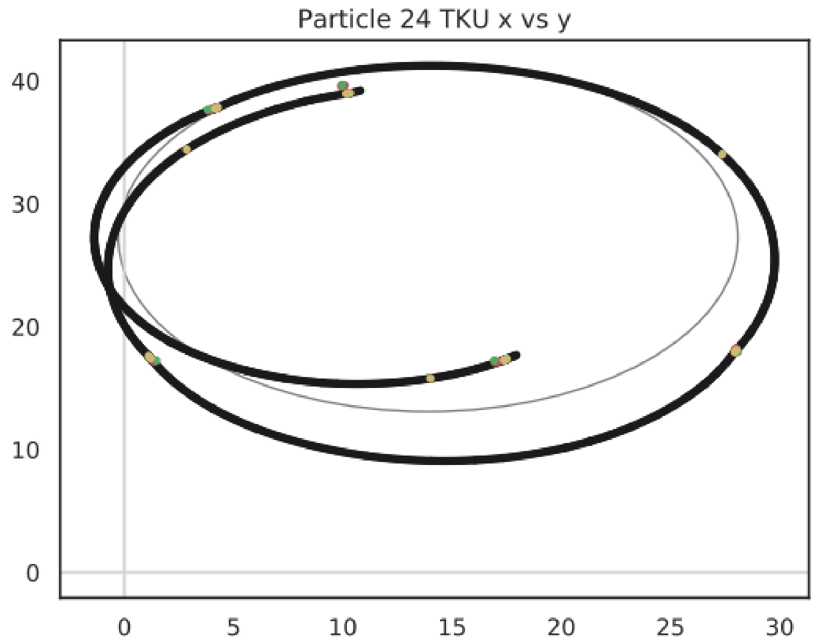
Breaking it down

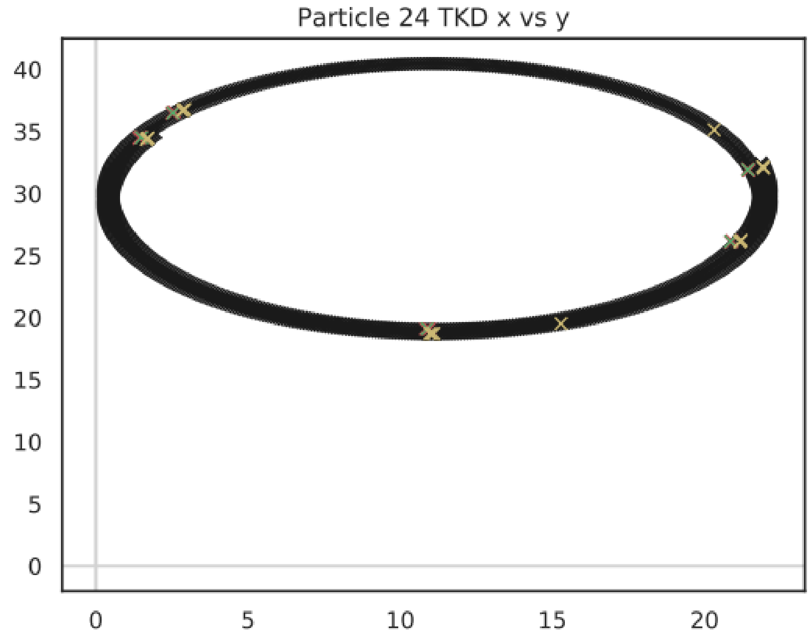
- Too many interfering actions, will break it down
- Use Constant Field
- No Misalignments
- Compare no Scattering to Scattering case



<- No Scattering

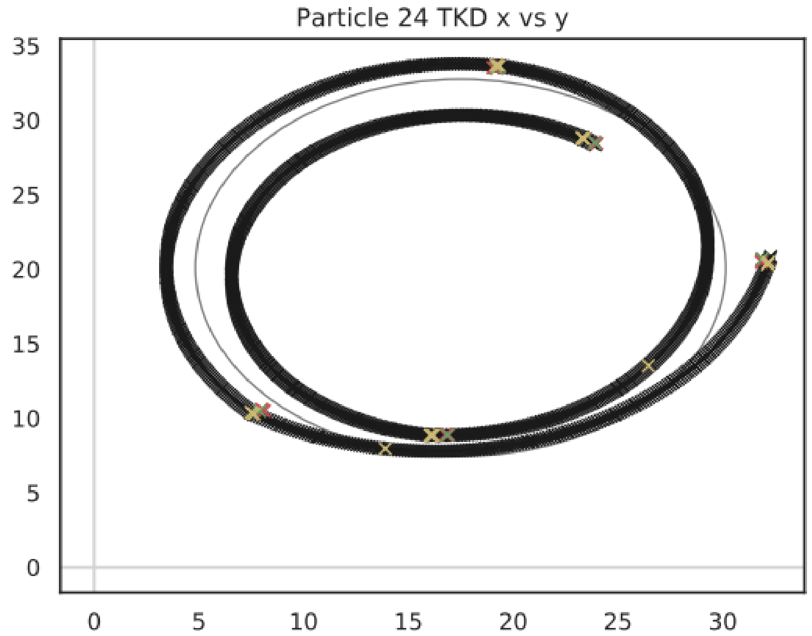
Scattering ->

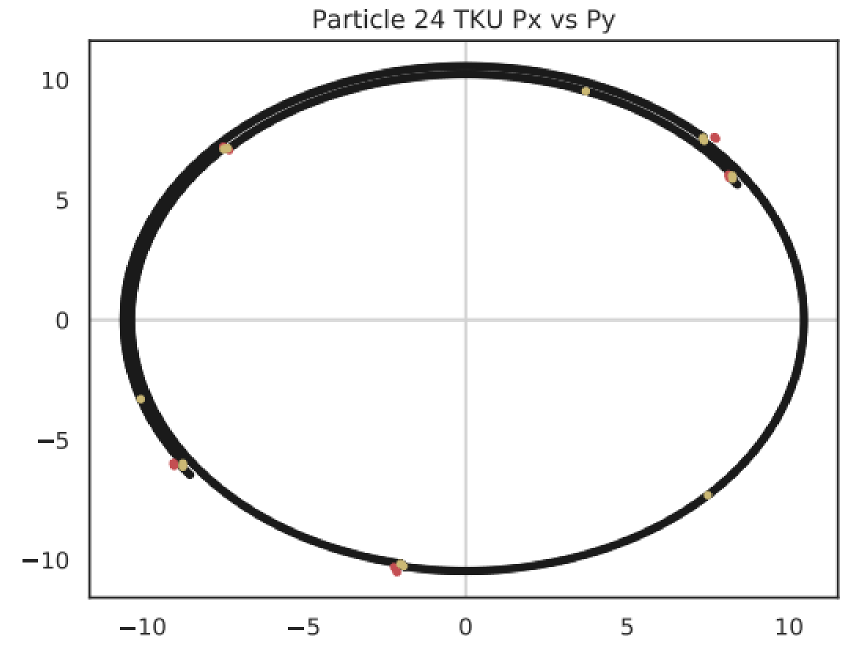


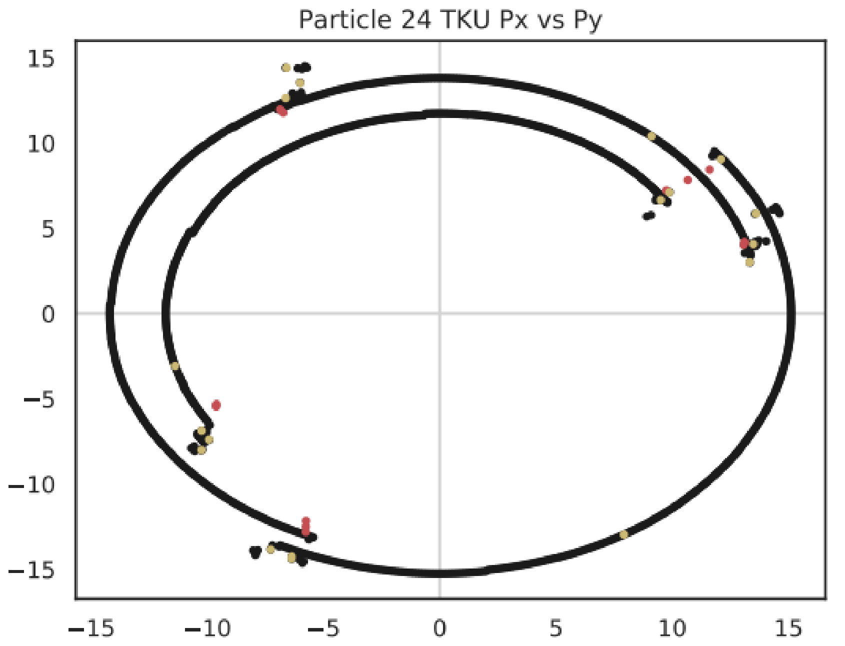


Constant Field

No Misalignments





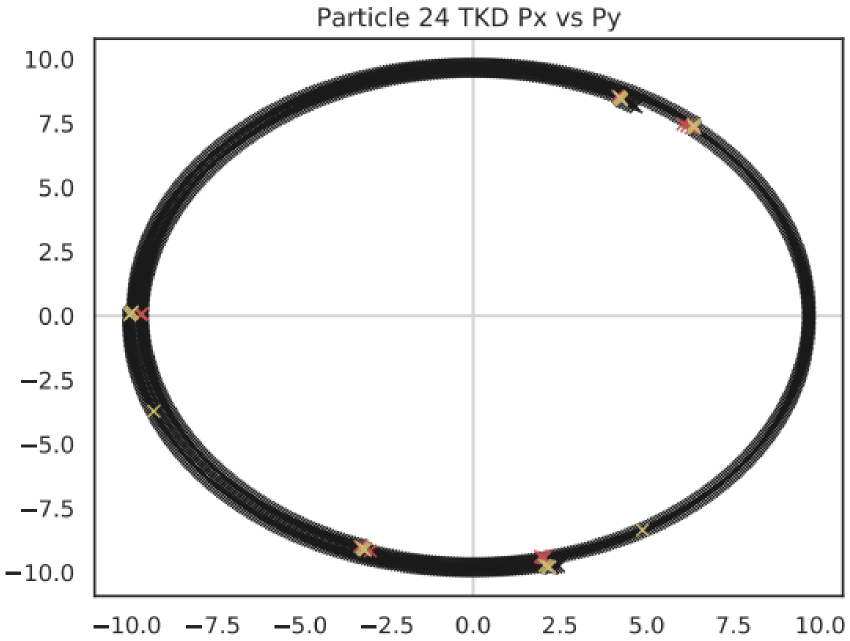


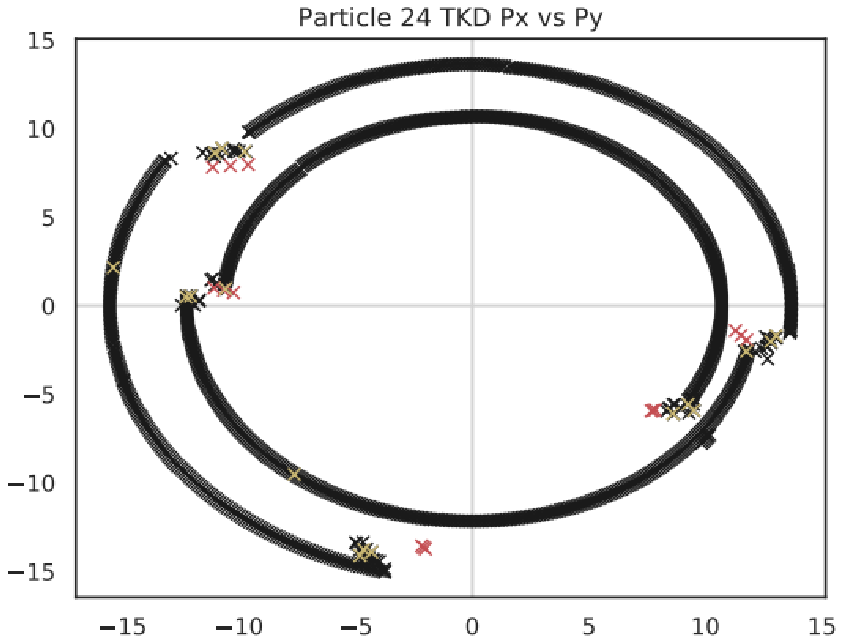
<- No Scattering

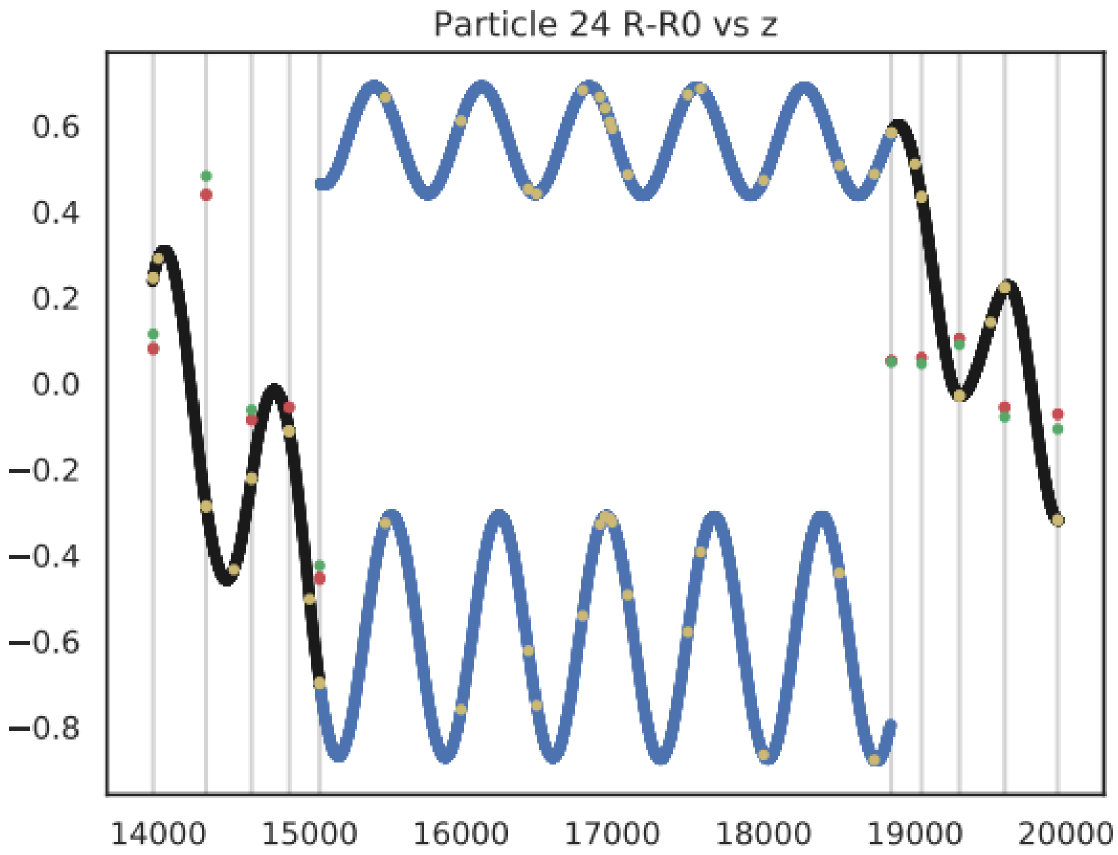
Scattering ->

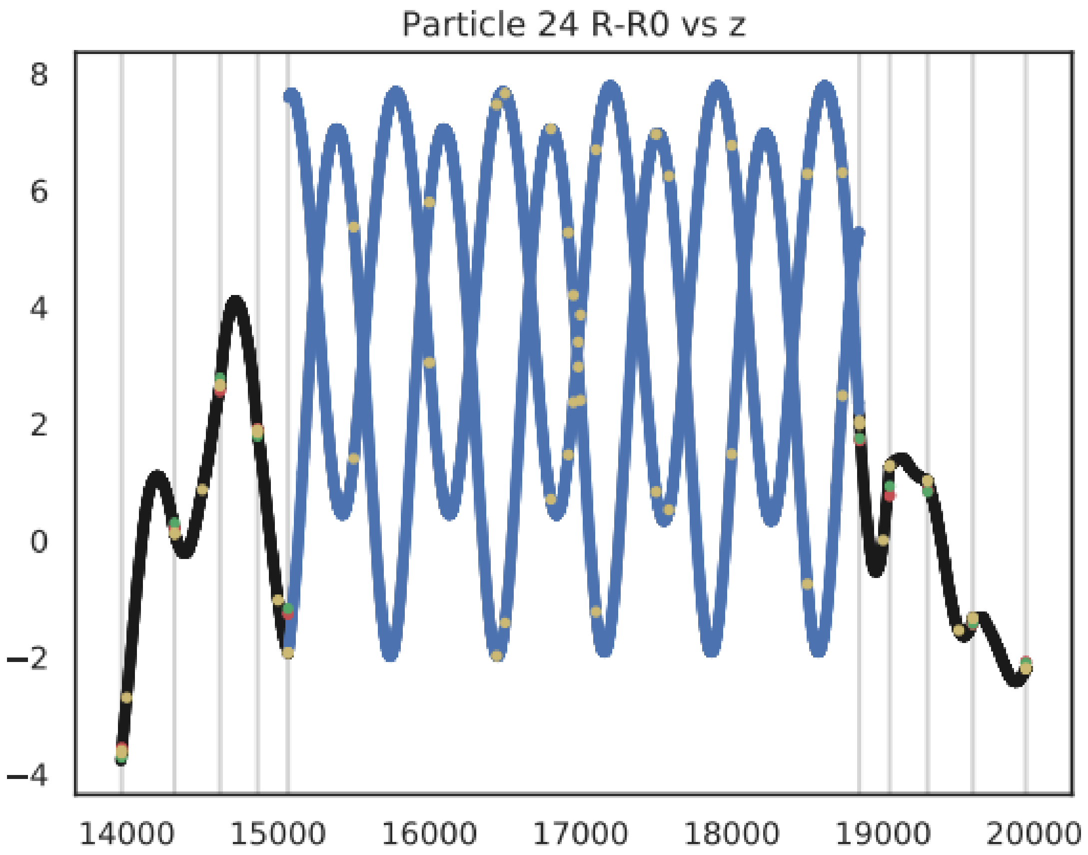
Constant Field

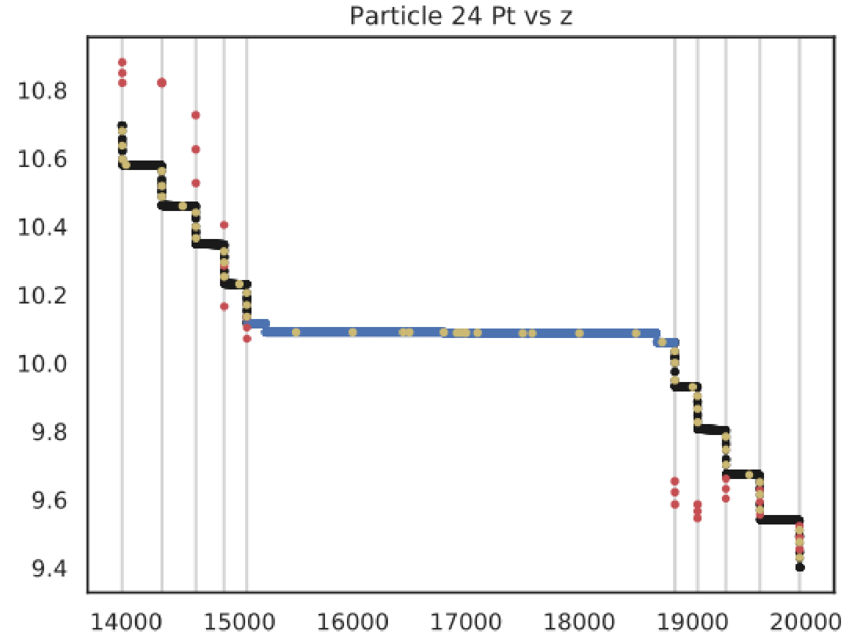
No Misalignments

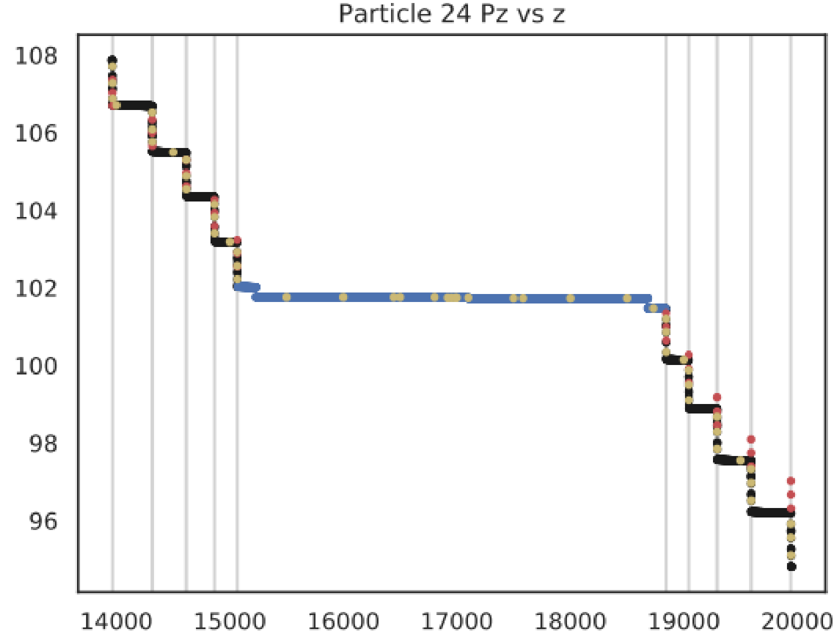





Particle 24 R-R0 vs z
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
14000
15000
16000
17000
18000
19000
20000


Particle 24 R-R0 vs z
8
6
4
2
0
−2
−4
14000
15000
16000
17000
18000
19000
20000



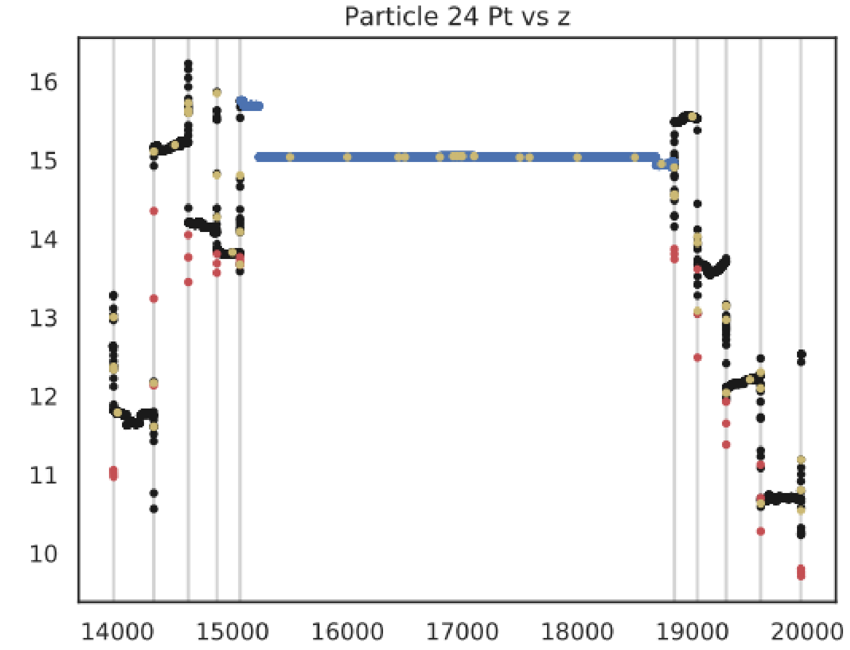


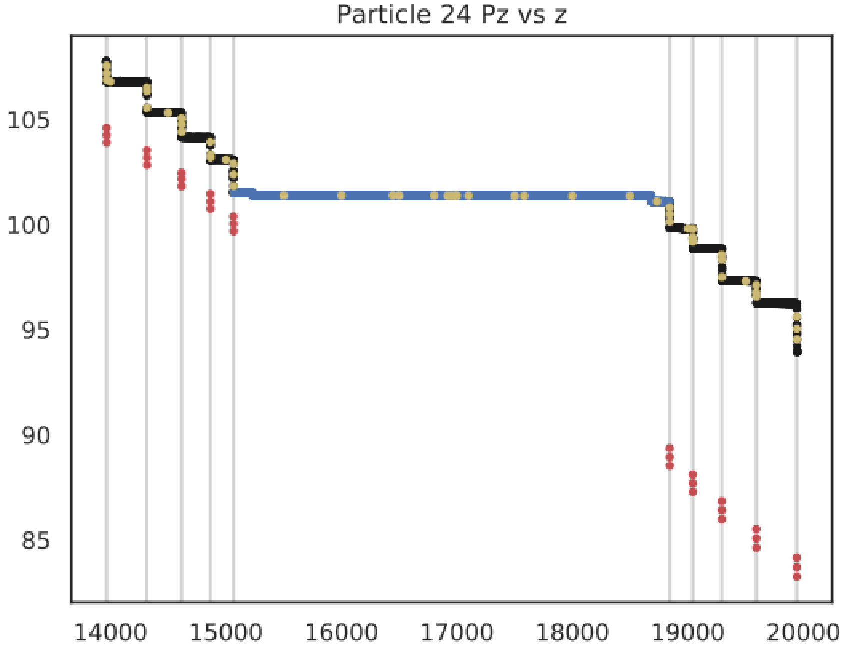
<- No Scattering

Scattering ->

Constant Field

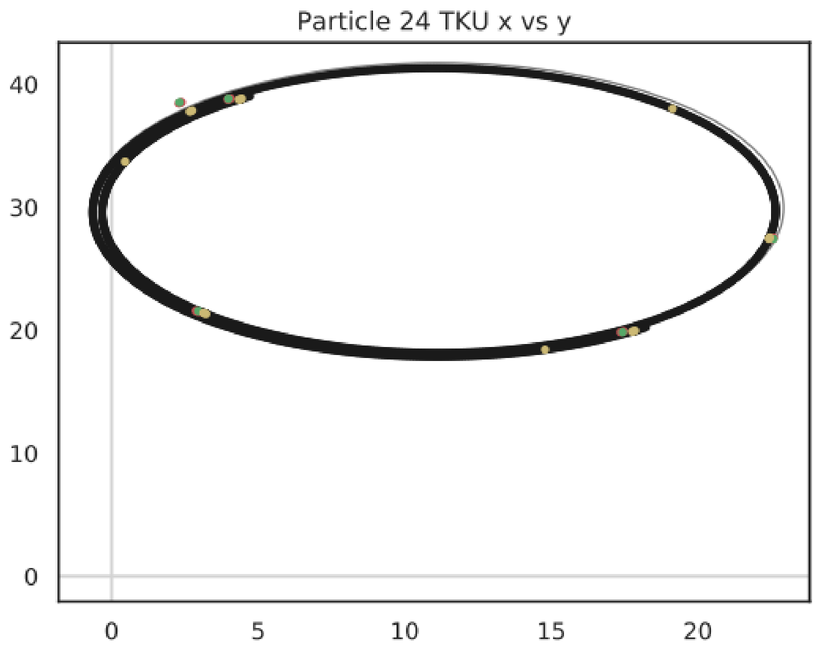
No Misalignments

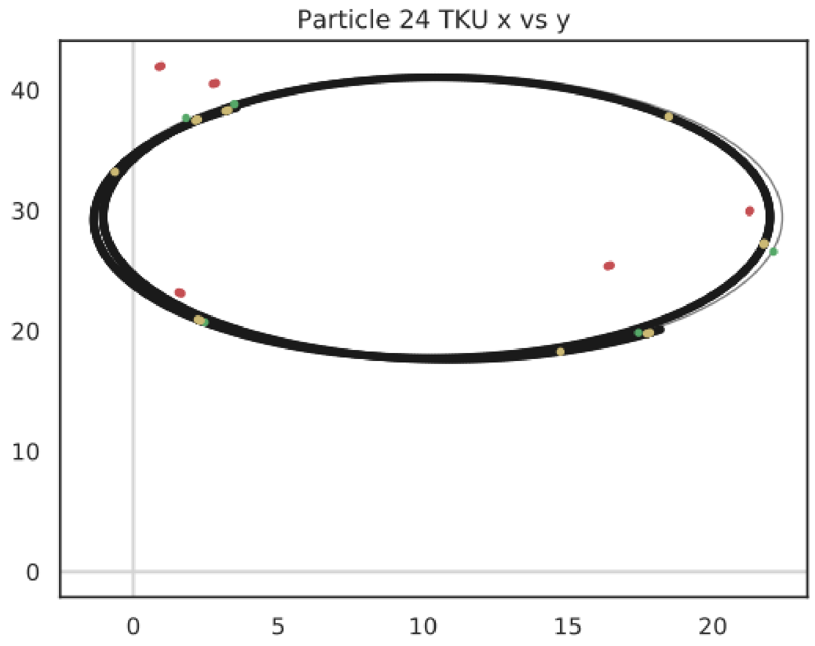




Breaking it down

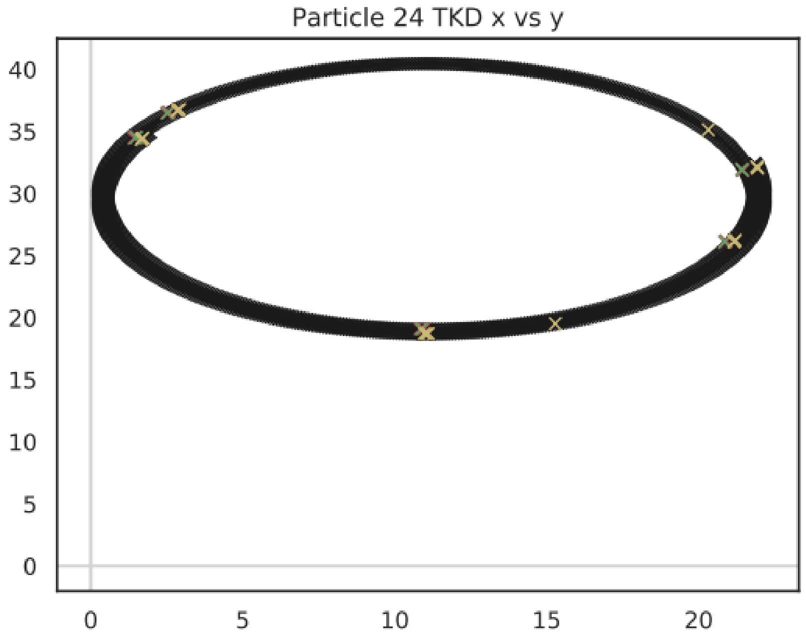
- Too many interfering actions, will break it down
- Use Constant Field
- No Scattering
- Compare no misalignment case to TKD Misalignment





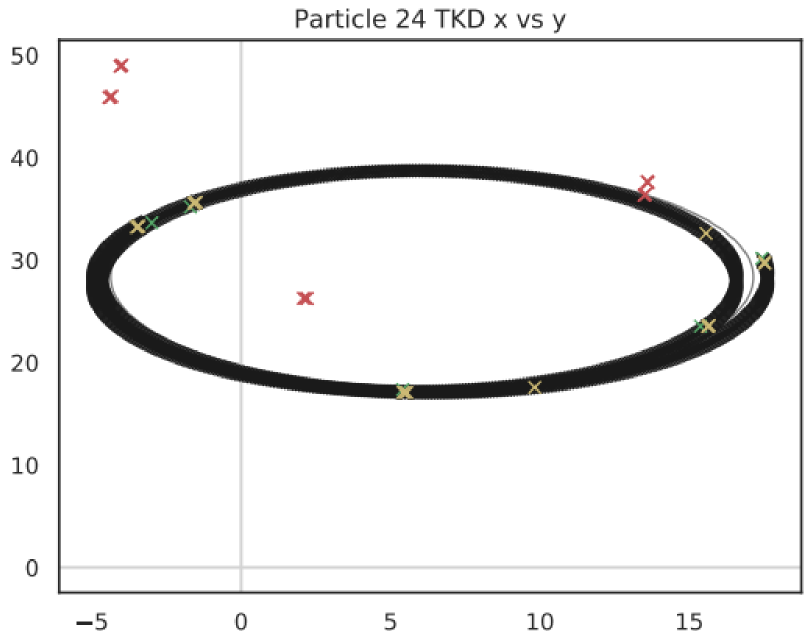
<- No Misalignment

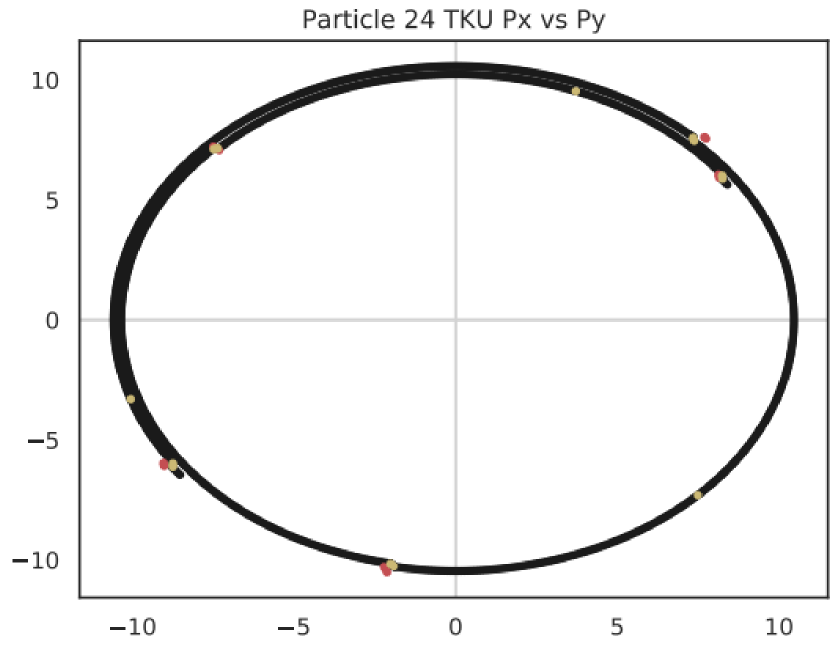
TKD Misalignmnet ->

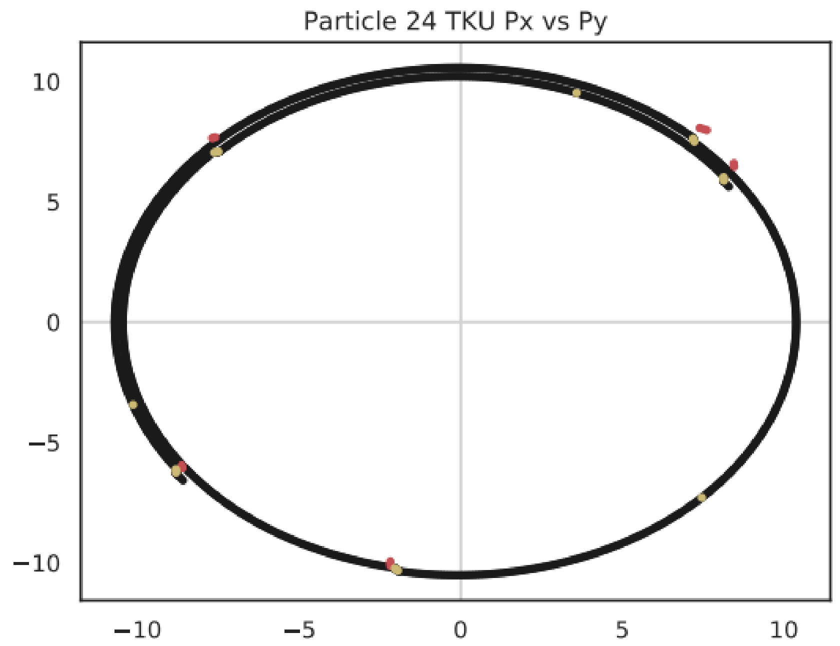


Constant Field

No Scattering





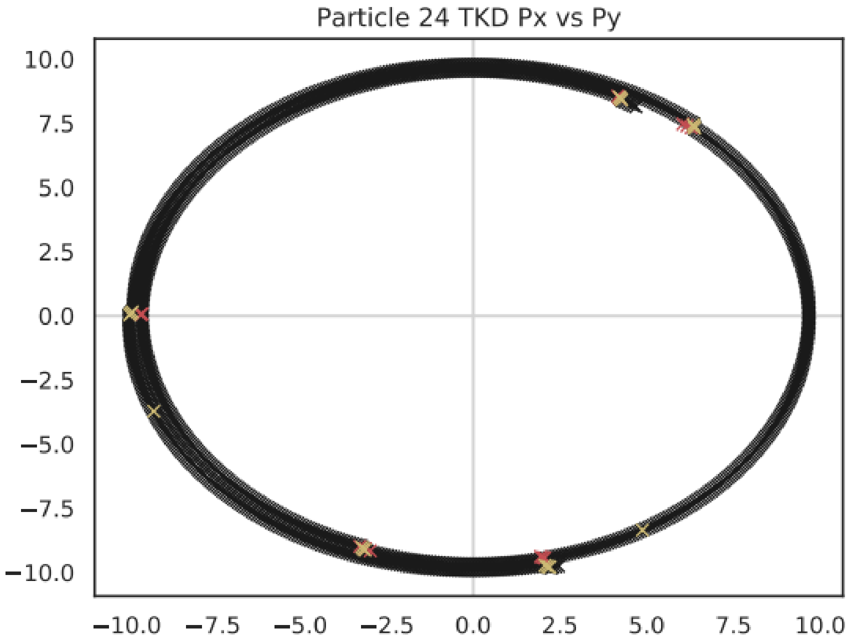


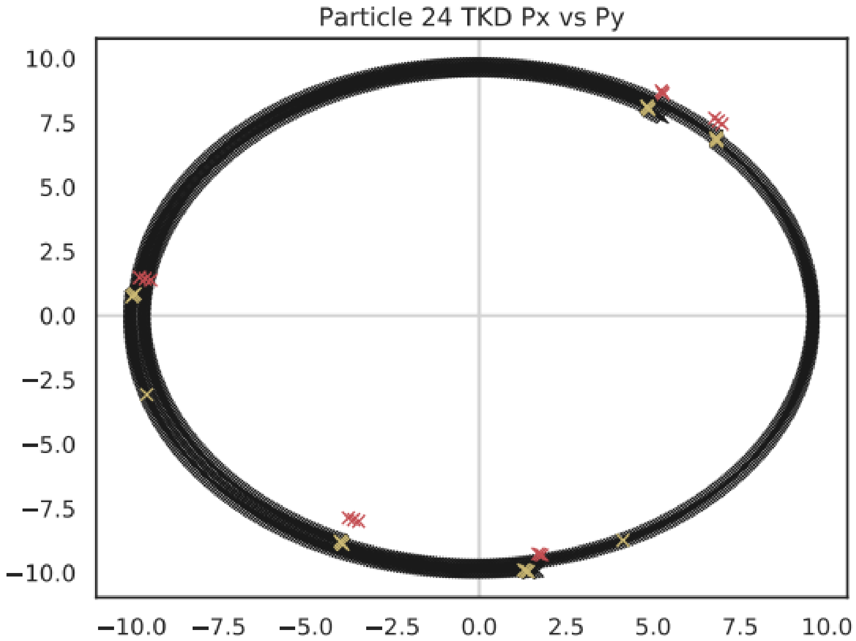
<- No Misalignment

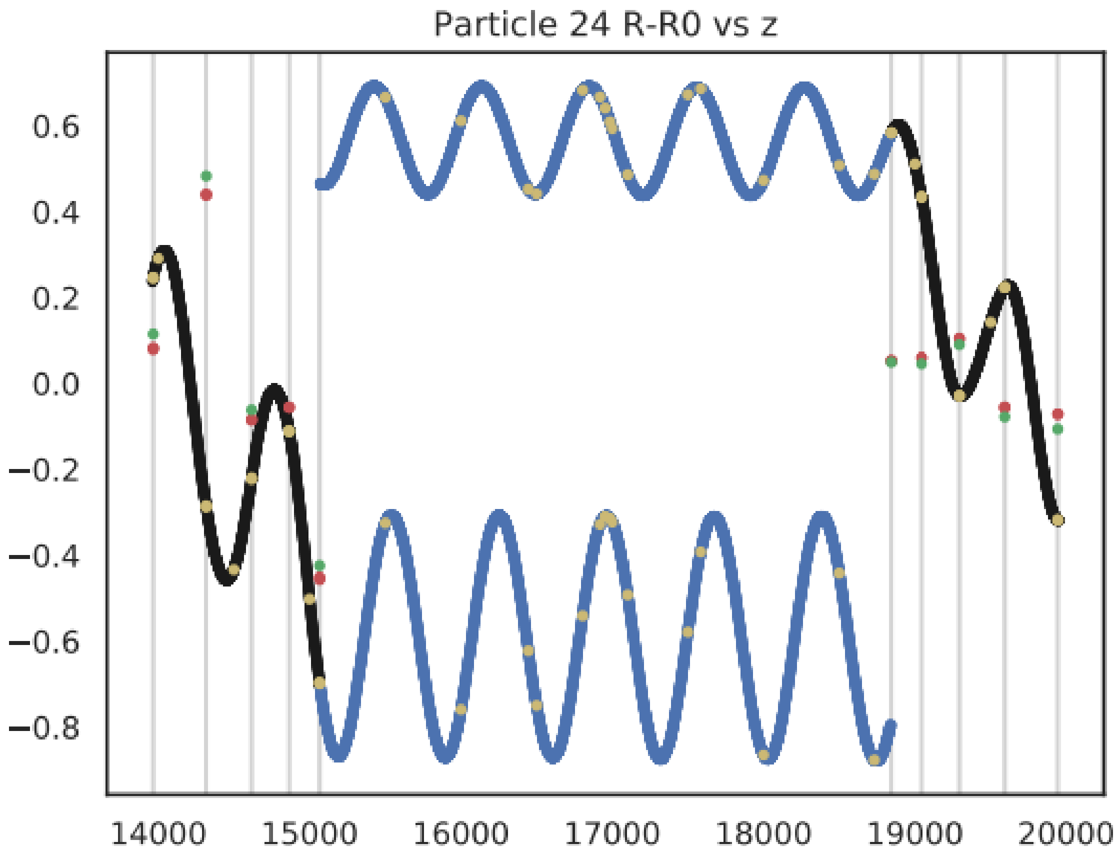
TKD Misalignmnet ->

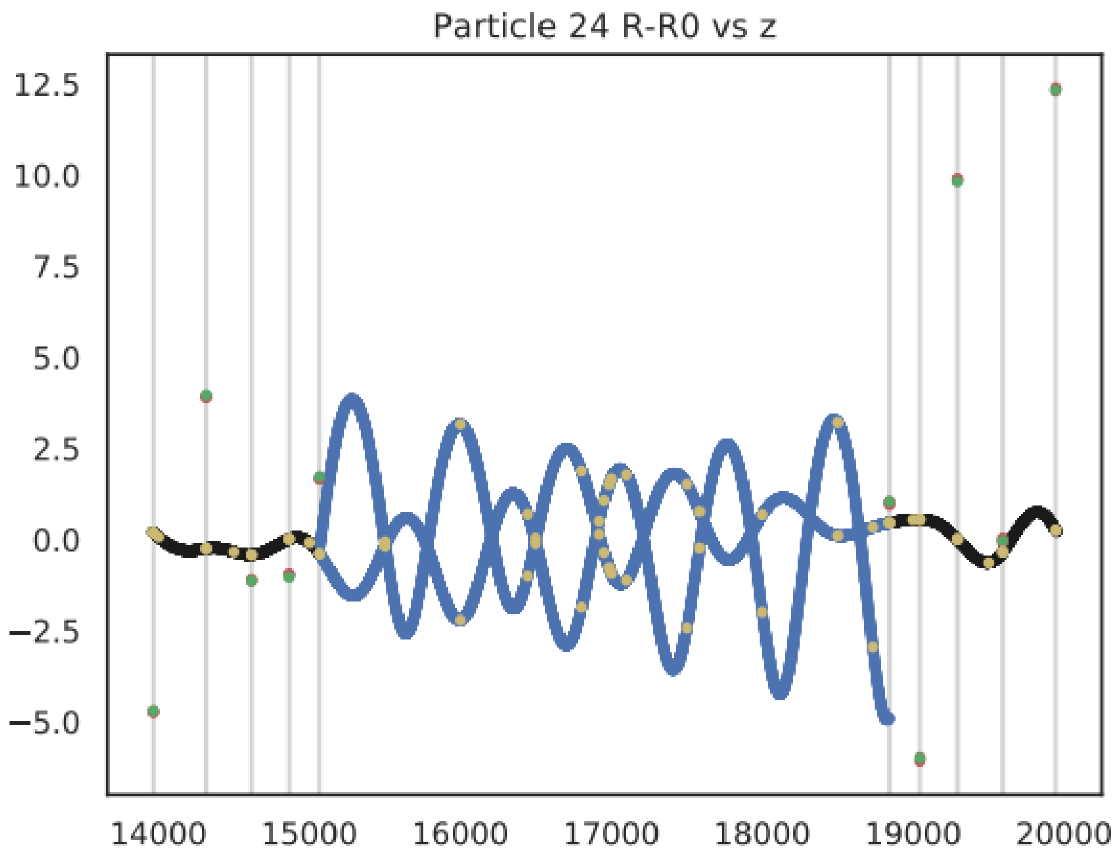
Constant Field

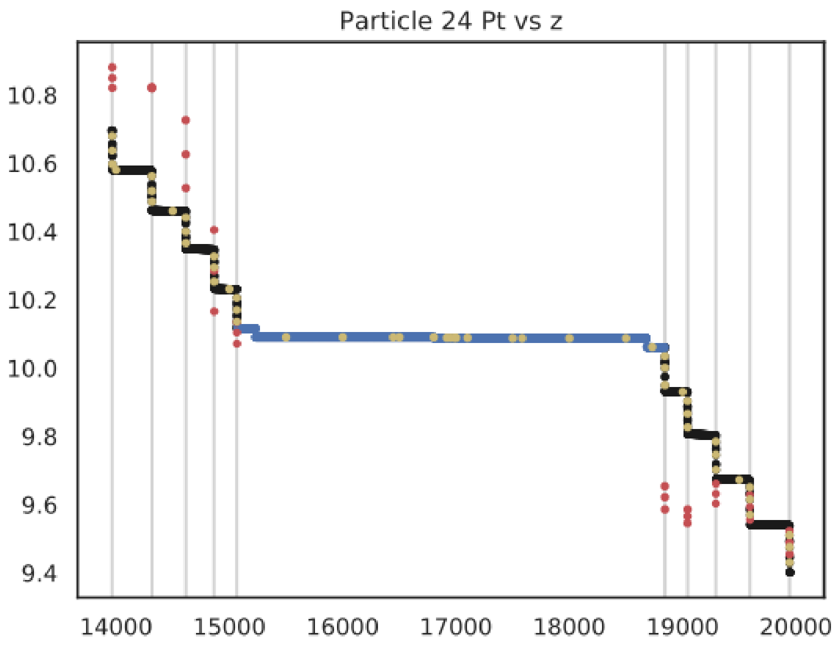
No Scattering





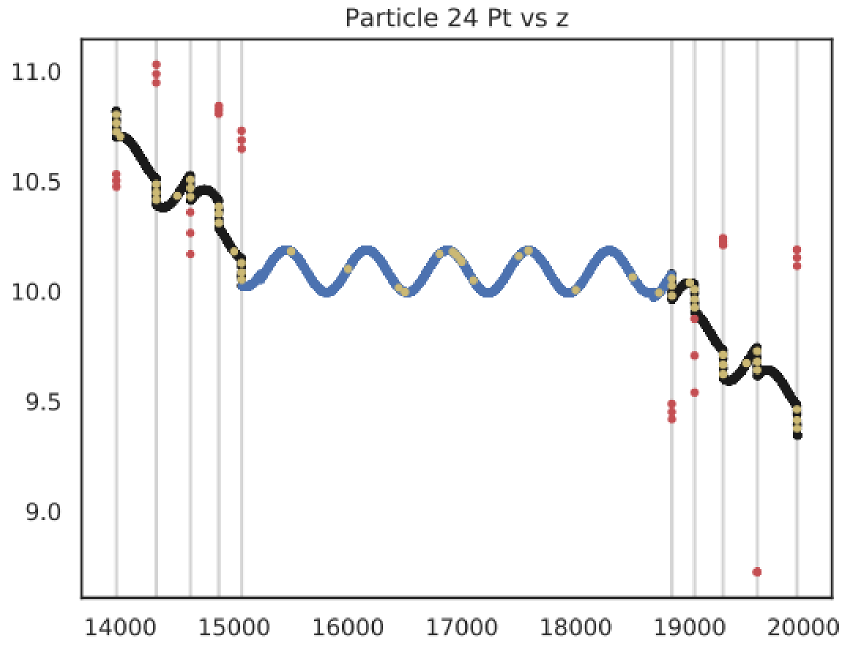

Particle 24 R-R0 vs z
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
14000
15000
16000
17000
18000
19000
20000

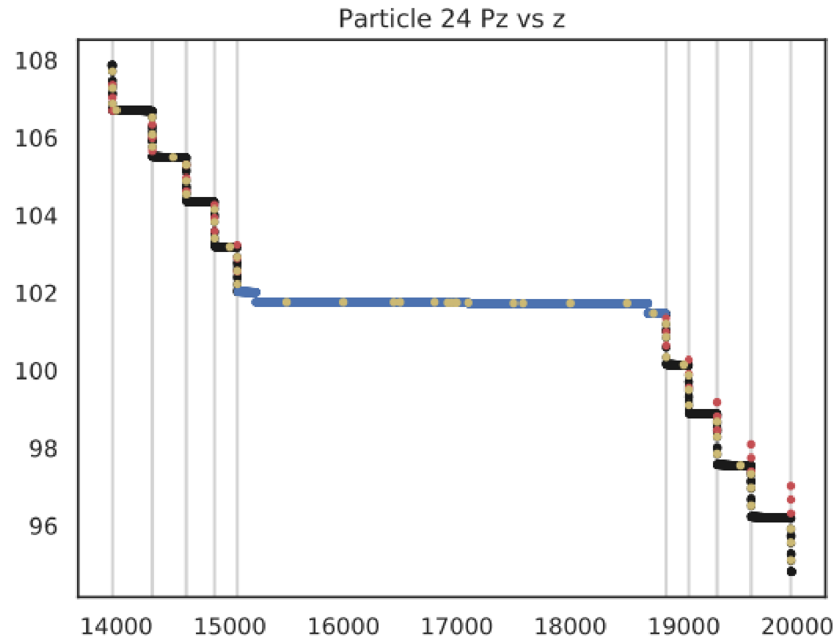

Particle 24 R-R0 vs z
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
14000
15000
16000
17000
18000
19000
20000



<- No Misalignment

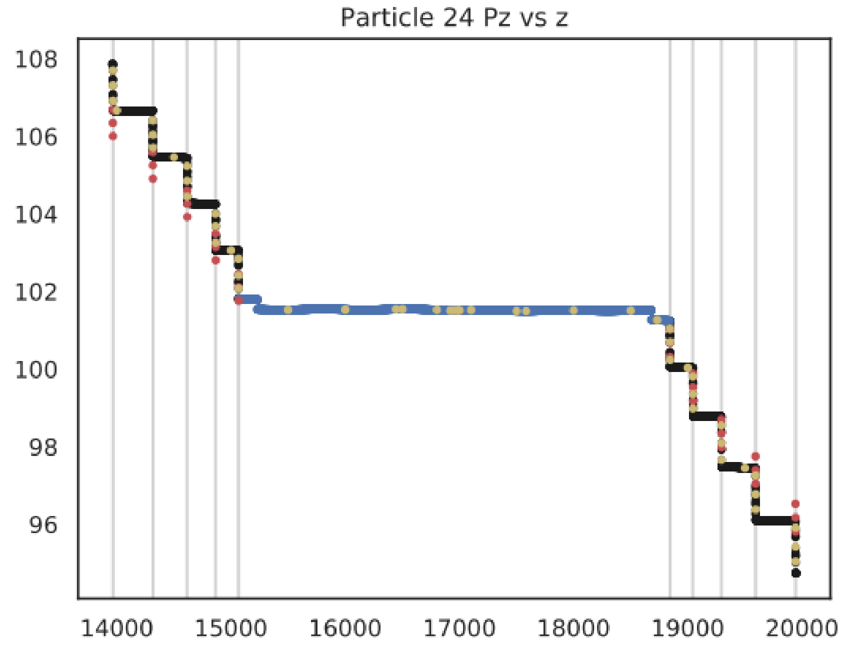
TKD Misalignmnet ->

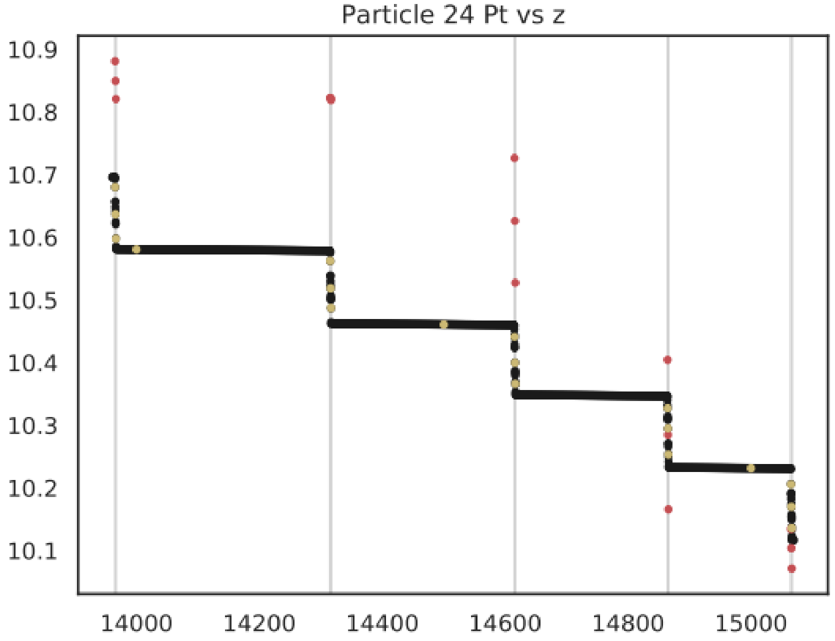




Constant Field

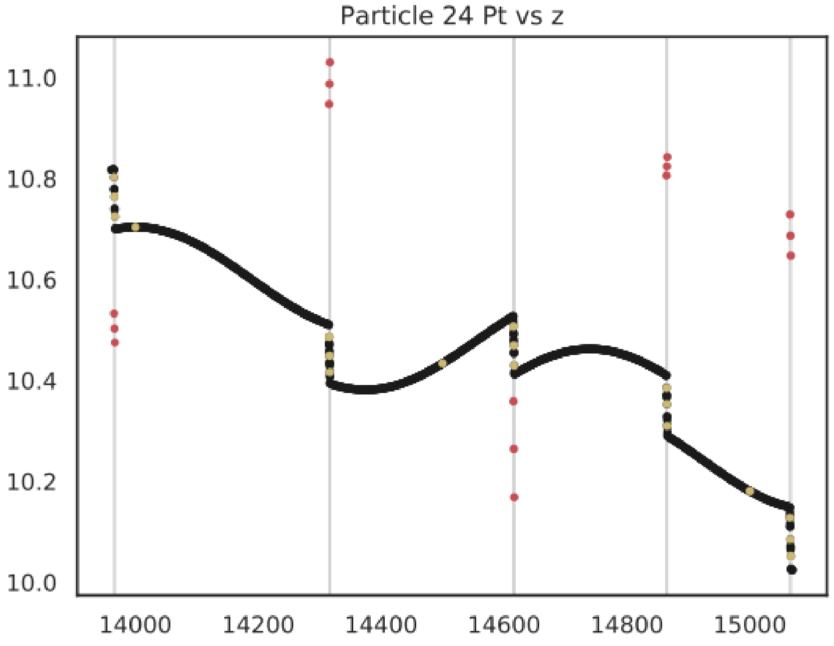
No Scattering

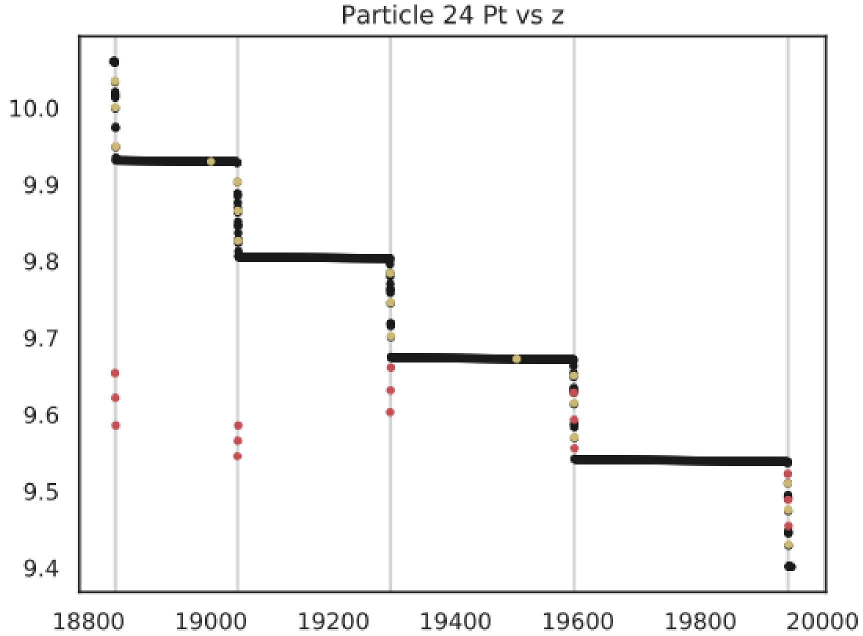




<- No Misalignment

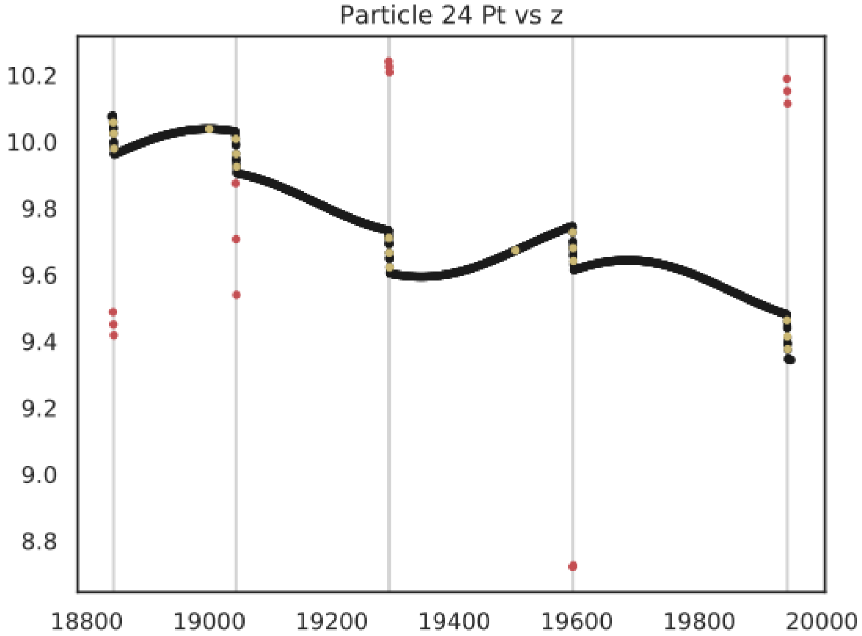
TKD Misalignmnet ->





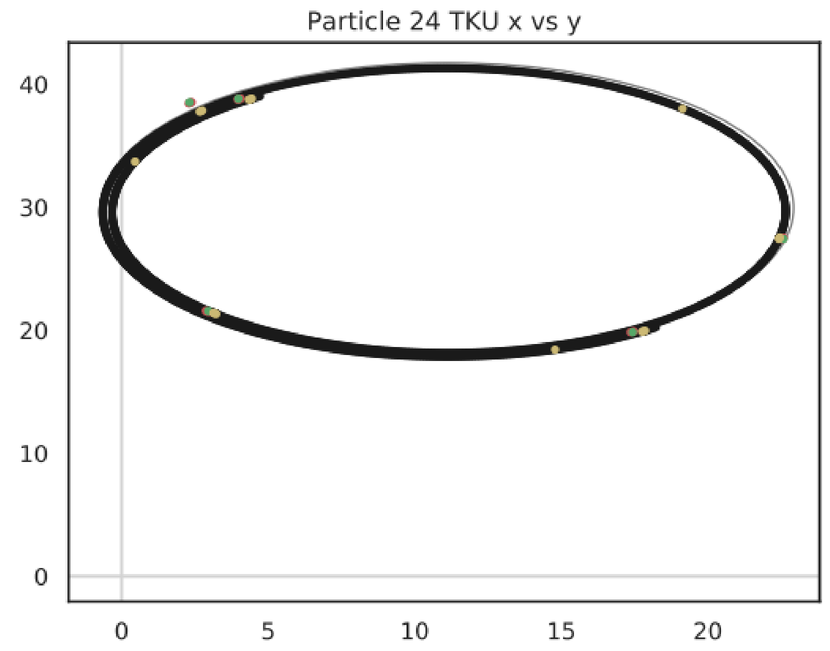
Constant Field

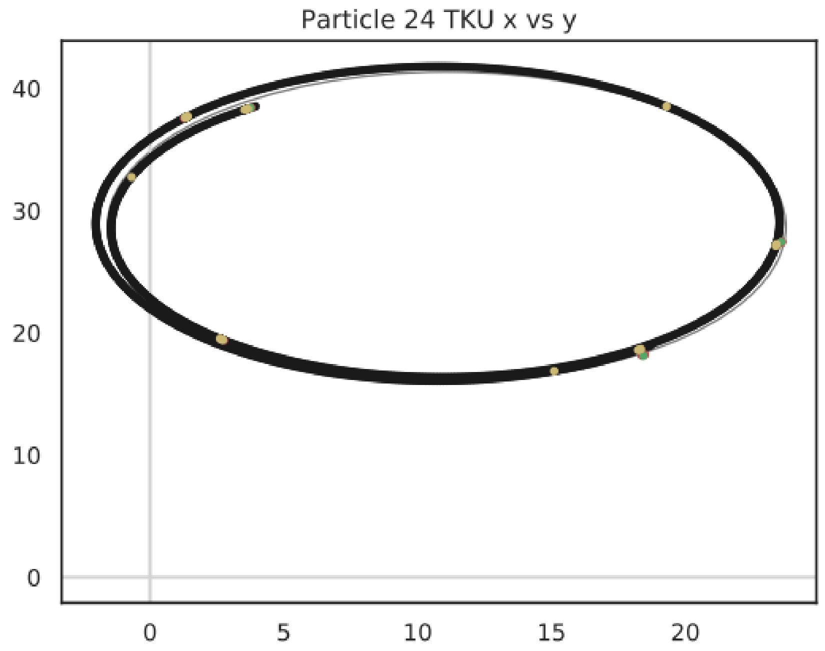
No Scattering



Breaking it down

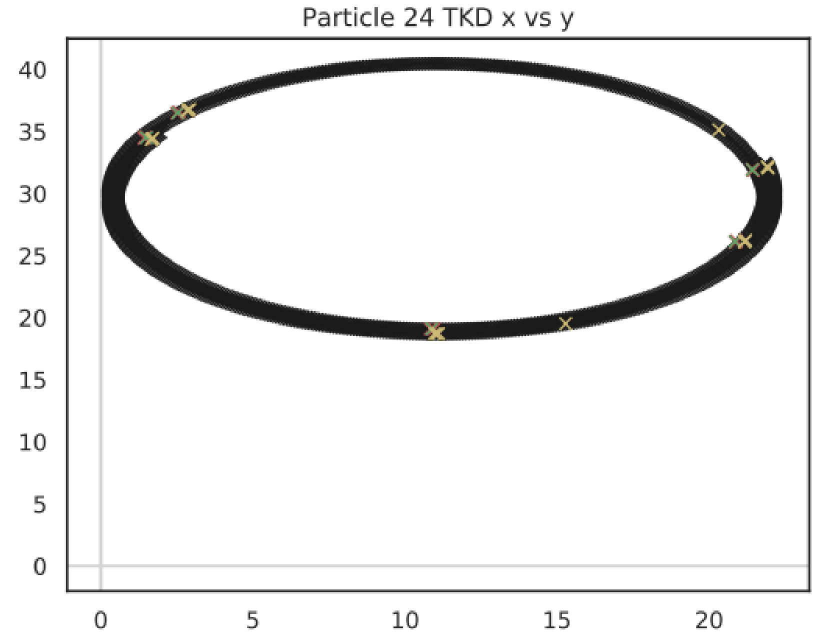
- Too many interfering actions, will break it down
- No Scattering
- No Misalignments
- Compare Constant Field to MICE field case





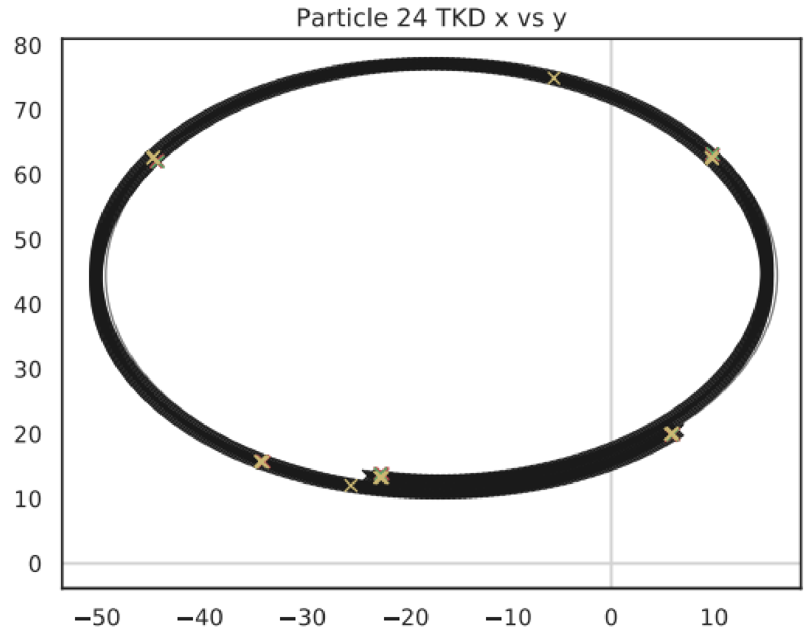
<- MICE field

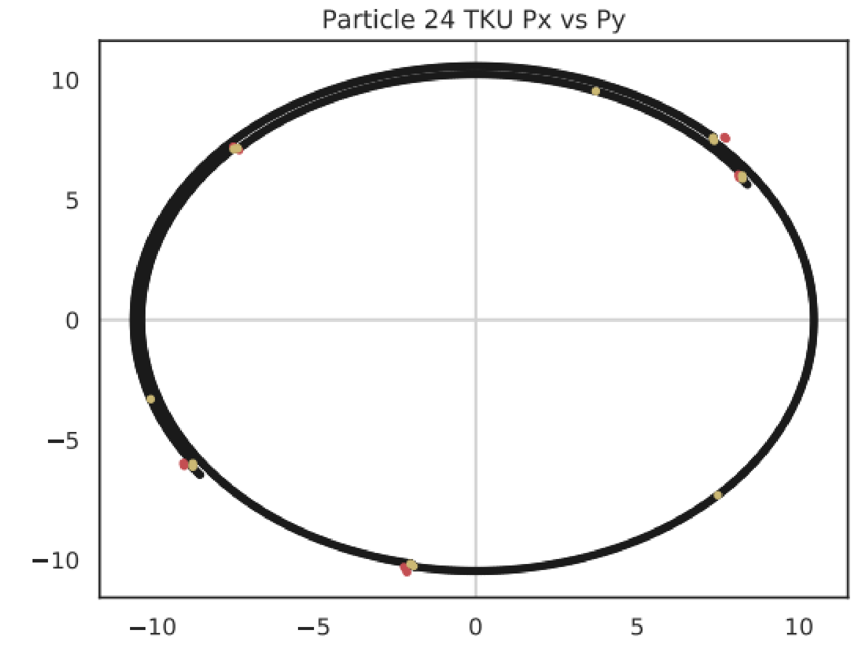
MICE field ->



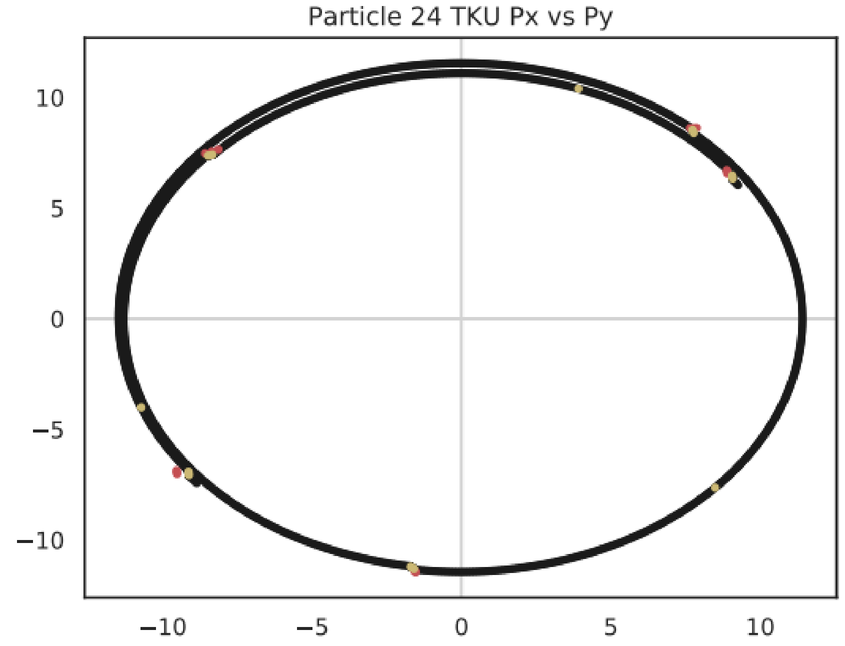
No Misalignment

No Scattering

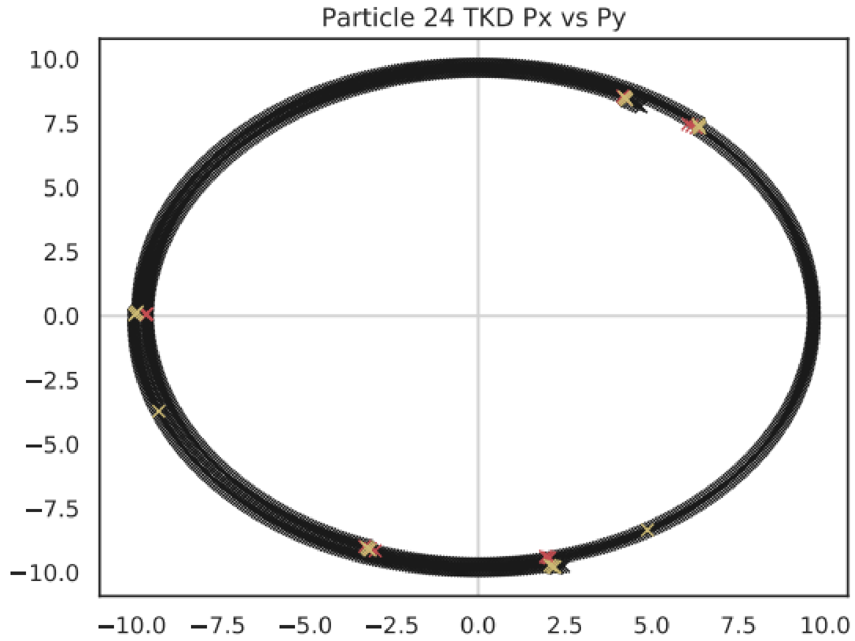




<- MICE field

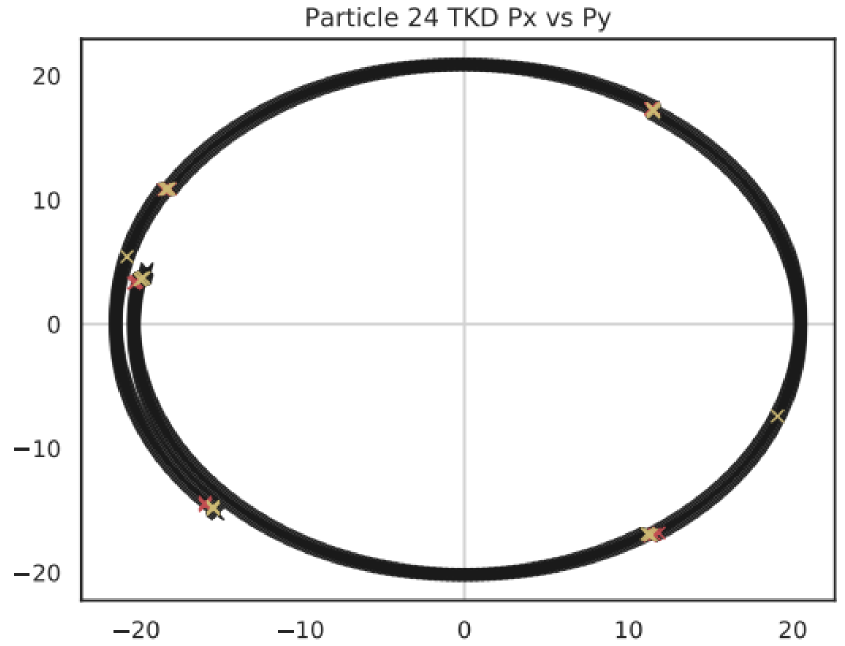


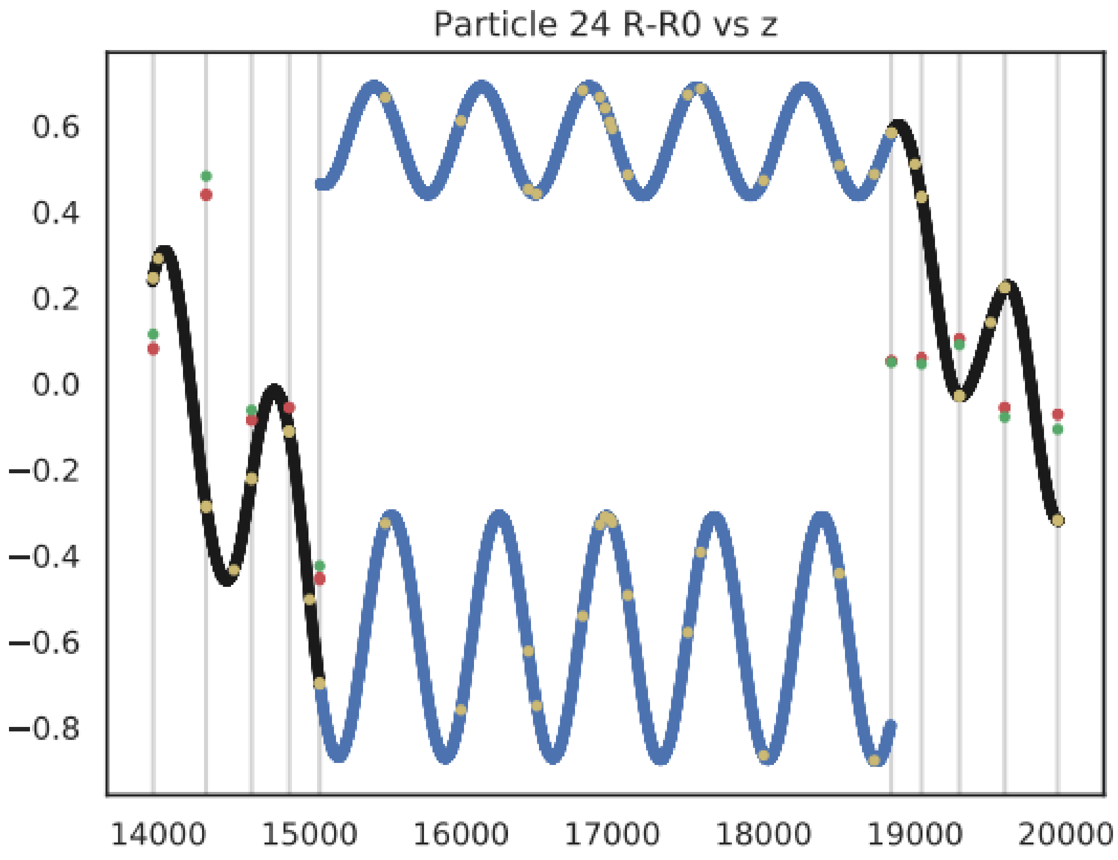
MICE field ->

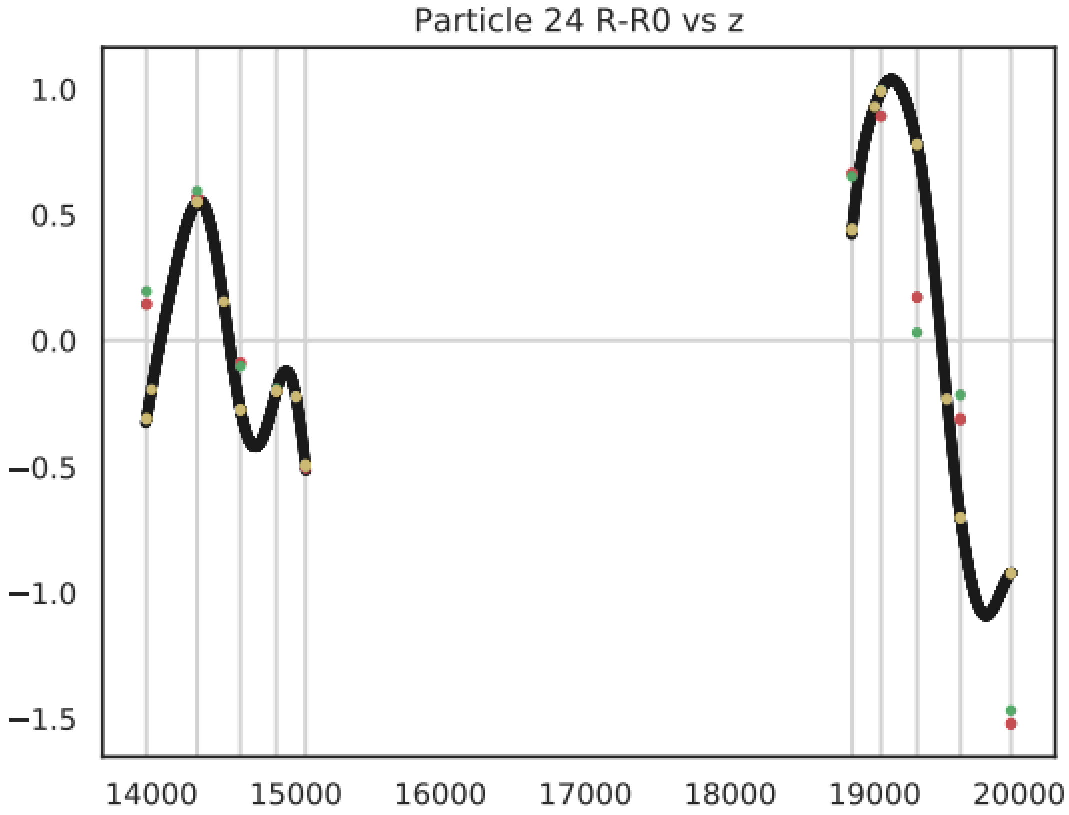


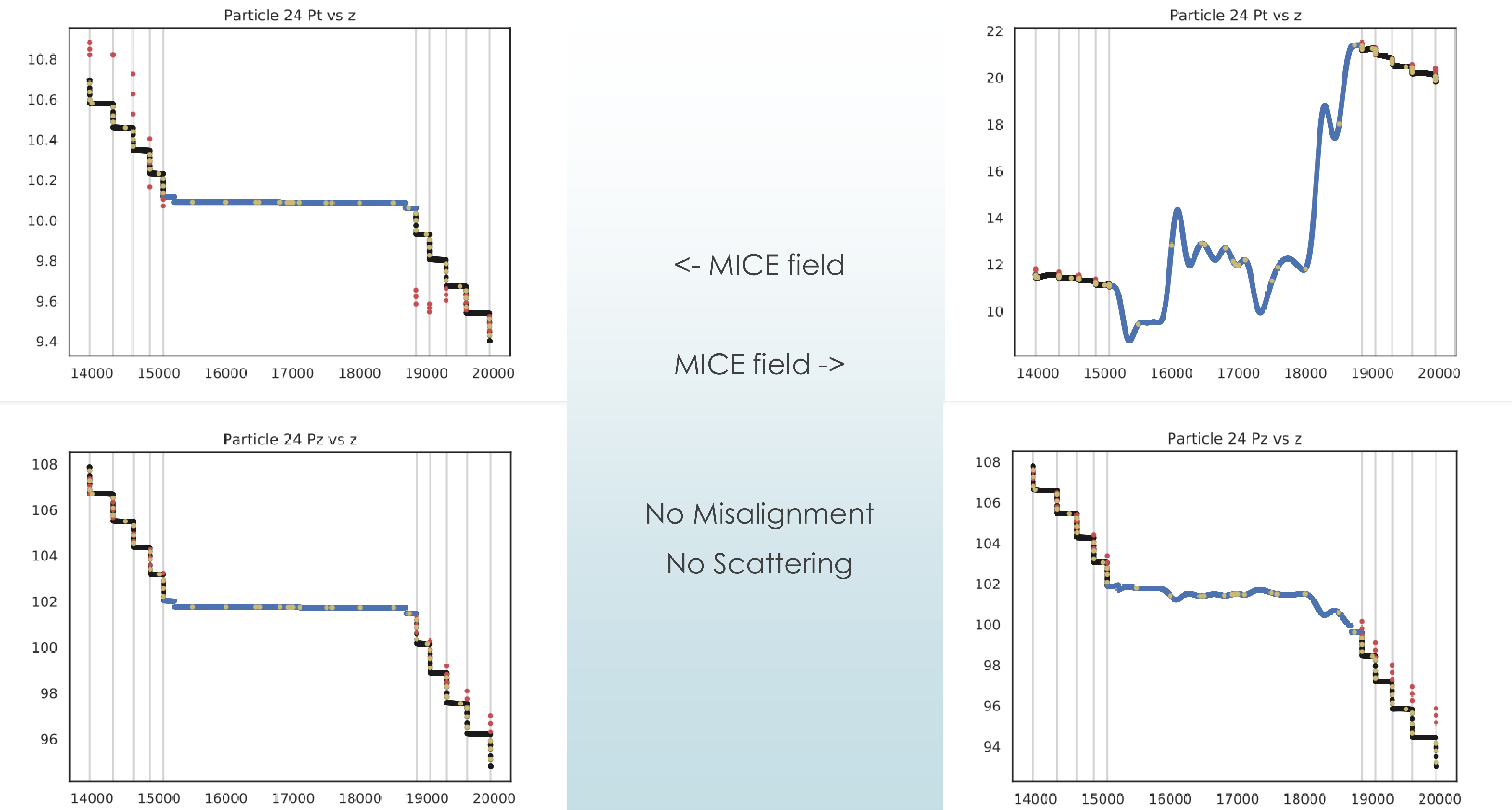
No Misalignment

No Scattering




Particle 24 R-R0 vs z
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
14000
15000
16000
17000
18000
19000
20000


Particle 24 R-R0 vs z
1.0
0.5
0.0
−0.5
−1.0
−1.5
14000
15000
16000
17000
18000
19000
20000

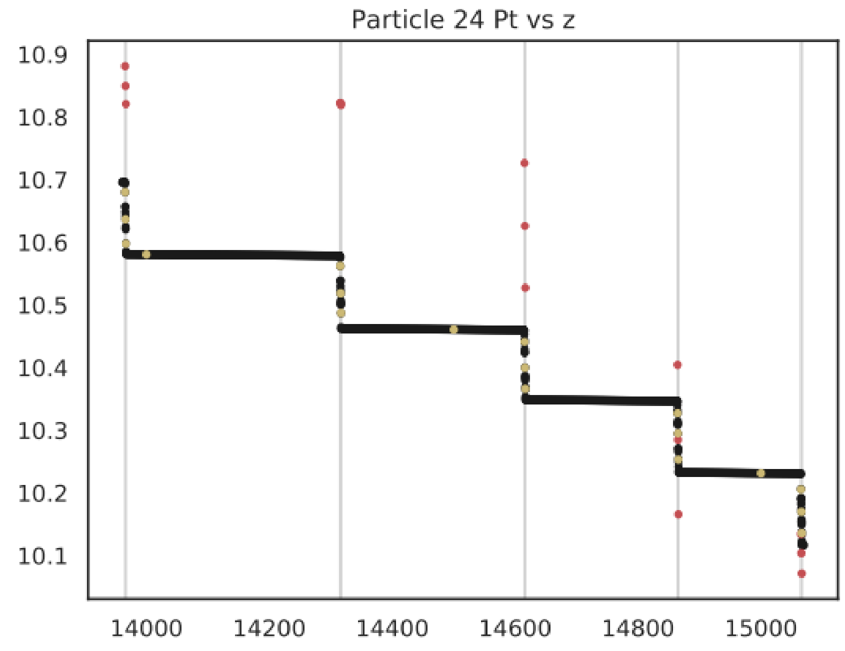


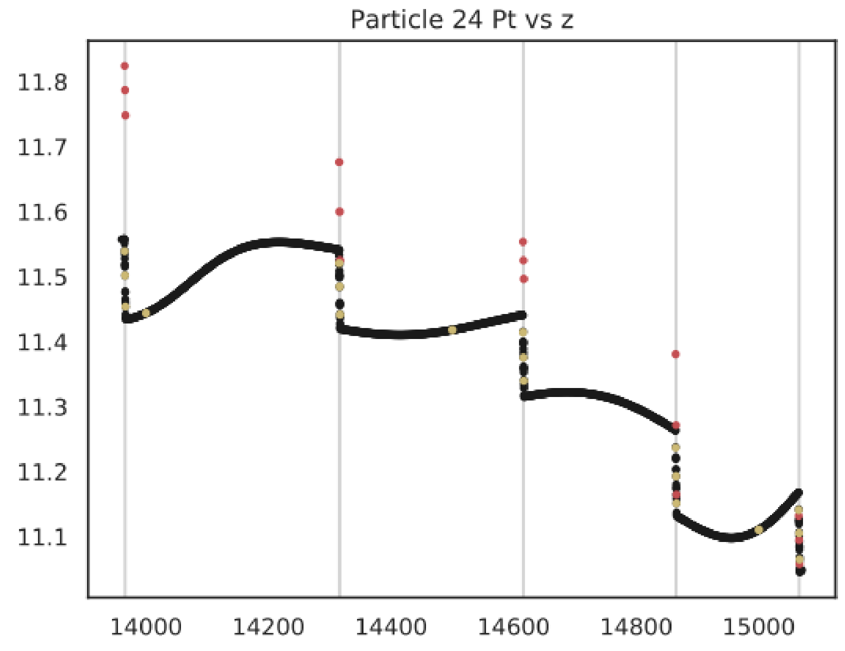
<- MICE field

MICE field ->

No Misalignment

No Scattering



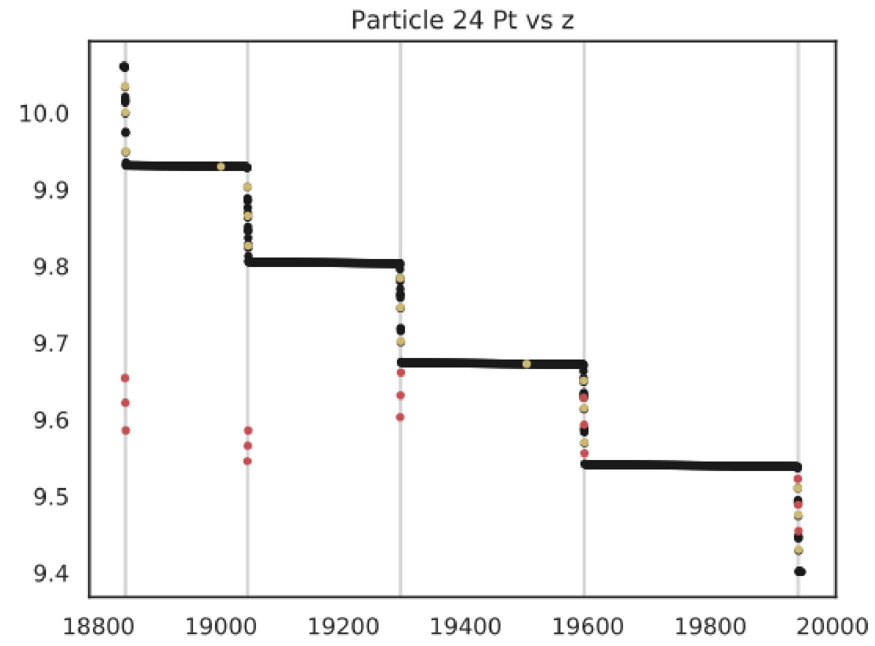


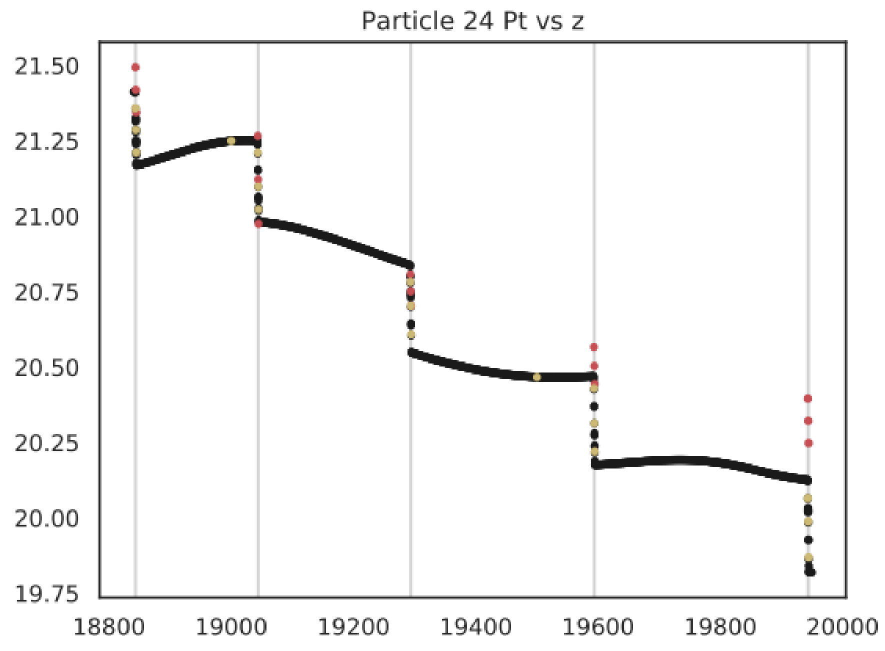
<- MICE field

MICE field ->

No Misalignment

No Scattering





Conclusions

- Transmission losses may spoil effects we want to say
- Distributions are changed and thus the calculated Emittance, Amplitude and Density changes
- Difficult to characterise the transmission loss error and its effect on Particle selection

- Recon introduces biases, most pronounced for Pz
- Significantly biases comparisons between TKU and TD
- MICE field not uniform. Emittance, Amplitude and density when separated into transverse and longitudinal components do not remain uniform at stations. What is the error between Reference planes due to this?