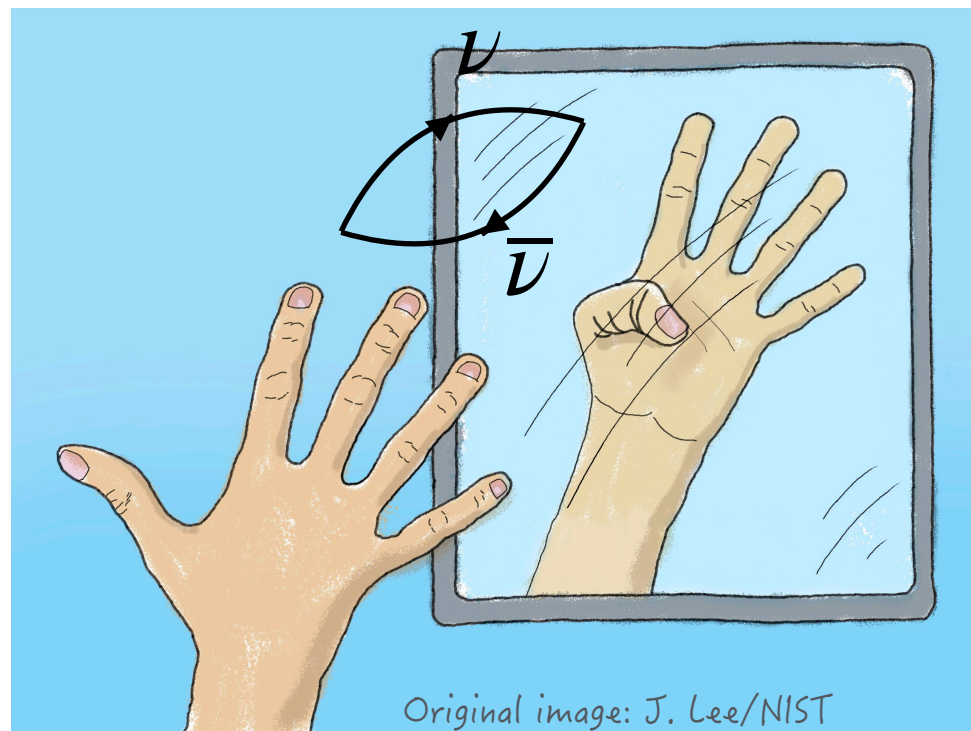


Probing the two-neutrino exchange force using atomic parity violation

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Collaborators:
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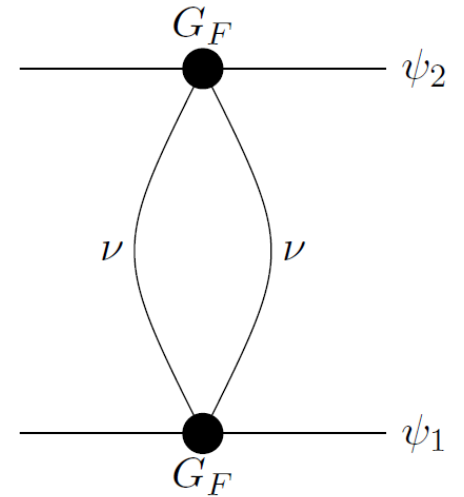
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Motivation

A pair of massless neutrinos mediated mediate a long-range force via one-loop diagrams



At leading order
$$V(r) = \frac{G_F^2}{4\pi^3 r^5}$$

Feinberg & Sucher (1968)
Feinberg, Sucher & Au (1989)
Hsu & Sikivie (1994)

...

At distances larger than 1 nm, this force is weaker than the gravitational force between two protons

Is there any way to probe this force that has not been explored yet?

Possible answer

To observe a small effect, look for symmetries that this force violates:

The two-neutrino force is the largest long-range parity-violating interaction in the Standard Model

Spoiler:

We find that the effect of the parity non-conserving force on atomic systems is tiny

Observing atomic parity violation in atoms

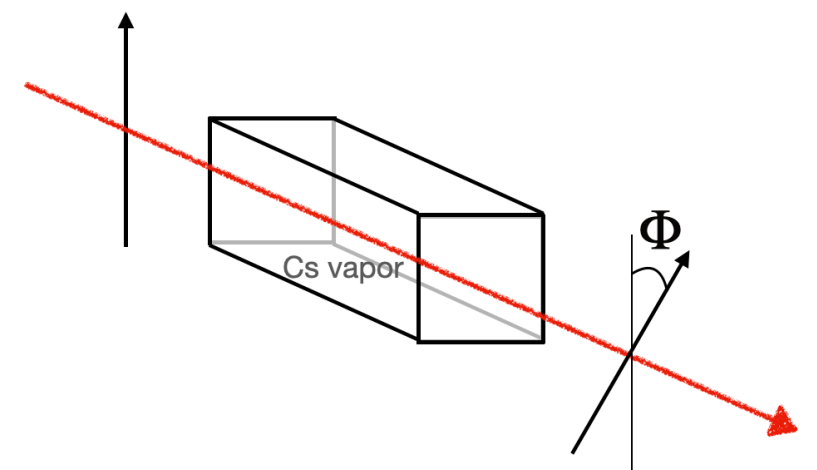
Consider stimulated emission in an atom:

- Electric dipole transitions $E1$: between states of opposite parity
- Magnetic dipole transitions $M1$: between states of same parity

If the Hamiltonian contains a perturbation that violates parity, its eigenstates will contain a small mixture of opposite-parity corrections



Optical rotation: Left-polarized and right-polarized light will refract with different index of refraction in a sample of atomic vapors



$$\Phi = \frac{\pi L}{\lambda} \text{Re} (n_R(\lambda) + n_L(\lambda)) \approx \frac{2\pi L}{\lambda} \text{Re} (n_R(\lambda) + n_L(\lambda) - 2) R$$

near resonance

$$R \equiv \text{Im} \left(\frac{E1_{PV}}{M1} \right)$$

Reviews: Khriplovich (1991), Bouchiat & Bouchiat (1997),...

Parity violating forces in the hydrogen atom

Assuming a) a static nucleus and b) that the electron velocity is a small parameter, the most general PV-potential is

$$V_{PNC}(r) = H_1 F(r) \vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r) \vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N) \cdot \vec{\nabla} [F(r)]$$

Tree-level

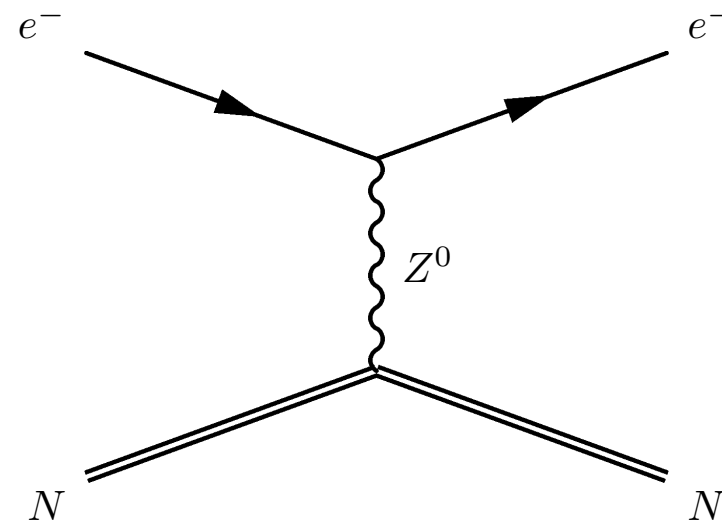
$$\mathcal{L}_{Z\bar{\psi}\psi} = \frac{1}{2} \frac{g}{\cos \theta_W} \bar{\psi} \left[(g_V^\psi - g_A^\psi \gamma^5) \not{Z} \psi \right]$$

$$H_1 = H_1^{\text{tree}} = \frac{g^2}{2 \cos^2 \theta_W} g_A^e g_V^p,$$

$$H_2 = H_2^{\text{tree}} = \frac{g^2}{2 \cos^2 \theta_W} g_V^e g_A^p,$$

$$C = C^{\text{tree}} = \frac{g^2}{2 \cos^2 \theta_W} \frac{g_V^e g_A^p}{2m_e},$$

$$F(r) = F^{\text{tree}}(r) = \frac{e^{-m_Z r}}{4\pi r}.$$



$$V_{PNC}^{\text{tree}} \sim \frac{g^2}{m_e} \left[\frac{e^{-m_Z r}}{r} \vec{\sigma}_e \cdot \vec{p} + \frac{e^{-m_Z r}}{r} \vec{\sigma}_p \cdot \vec{p} + (\vec{\sigma}_e \times \vec{\sigma}_p) \cdot \vec{\nabla} \left(\frac{e^{-m_Z r}}{r} \right) \right]$$

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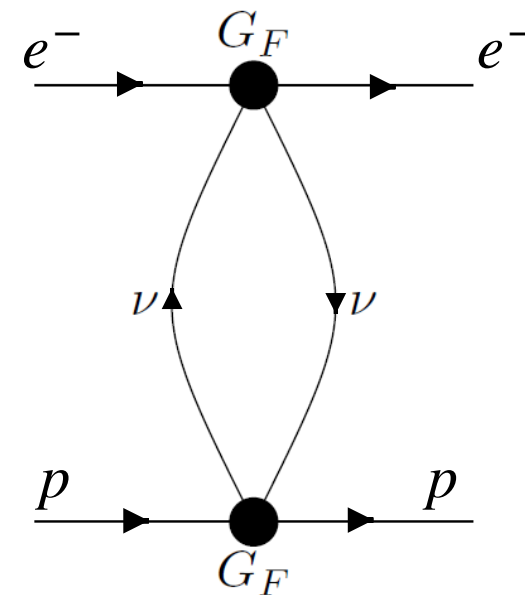
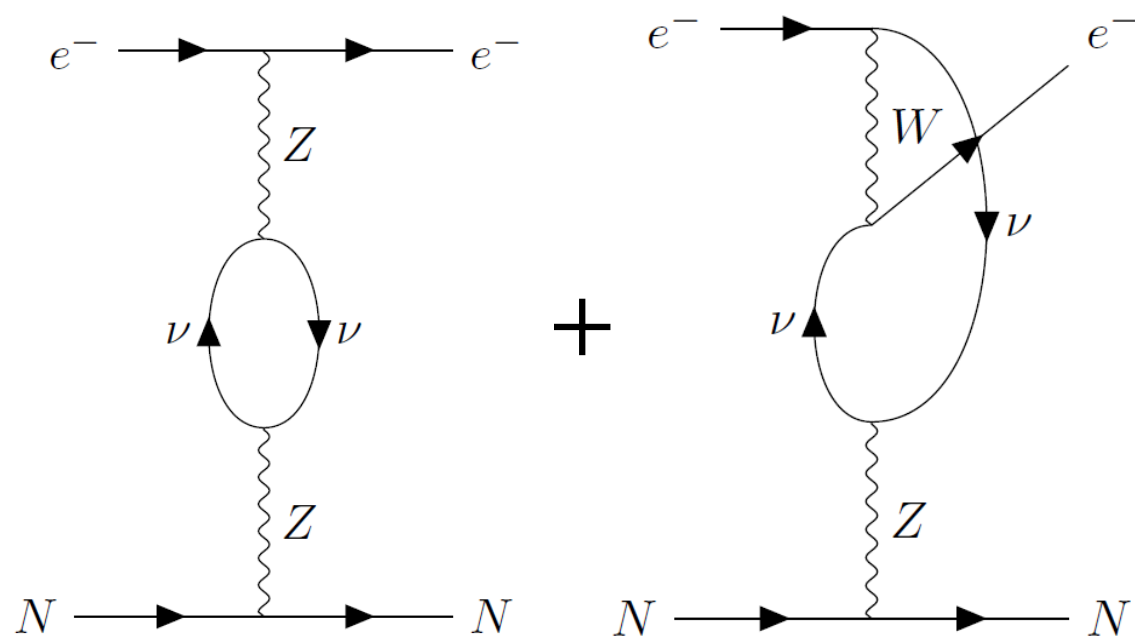
$$V_{PNC}(r) = H_1 F(r) \vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r) \vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N) \cdot \vec{\nabla} [F(r)]$$

Loop-level: Enter the neutrino force

$$(\mathcal{O}_Z)_{ij} = -\frac{g^2}{8m_Z^2 c_W^2} [\bar{\psi} \gamma^\mu (g_V^\psi - g_A^\psi \gamma^5) \psi] \delta_{ij} [\bar{\nu}_j \gamma_\mu (1 - \gamma^5) \nu_i] \quad + \quad (\mathcal{O}_W)_{ij} = -\frac{g^2}{8m_W^2} U_{\alpha j} U_{\alpha i}^* [\bar{\psi} \gamma^\mu (1 - \gamma^5) \psi] [\bar{\nu}_j \gamma_\mu (1 - \gamma^5) \nu_i]$$



$$\mathcal{O}_4 = -\frac{G_F}{\sqrt{2}} [\bar{\psi} \gamma^\mu (a^\psi - b^\psi \gamma^5) \psi] [\bar{\nu} \gamma_\mu (1 - \gamma^5) \nu]$$



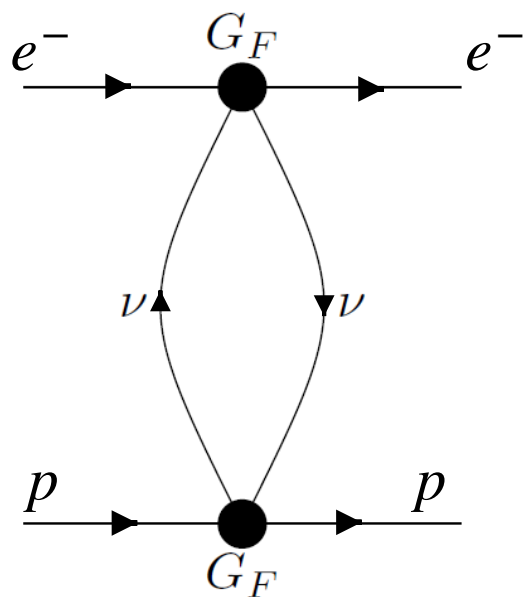
Parity violating forces in the hydrogen atom

$$V_{PNC}(r) = H_1 F(r) \vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r) \vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N) \cdot \vec{\nabla} [F(r)]$$

Loop-level

$$\mathcal{O}_4 = -\frac{G_F}{\sqrt{2}} [\bar{\psi} \gamma^\mu (a^\psi - b^\psi \gamma^5) \psi] [\bar{\nu} \gamma_\mu (1 - \gamma^5) \nu]$$

$$i\mathcal{M} = -\frac{(-iG_F)^2}{2} \bar{e} \bar{N} [\Gamma_\mu^e \Gamma_\nu^N] \int \frac{d^4 k d^4 k'}{(2\pi)^4} \delta^4(q - k - k') \text{Tr} \left[i\Gamma^\mu \frac{i(-\not{k}' + m)}{k'^2 - m^2} i\Gamma^\nu \frac{i(\not{k} + m)}{k^2 - m^2} \right] e N.$$



$$V_{PNC}^{\text{loop}} \approx \sum_i \frac{G_A}{m_e} \left(-\frac{1}{4} + s_W^2 + \frac{1}{2} |U_{ei}|^2 \right) \left[(2\vec{\sigma}_p \cdot \vec{p}_e) V_{\nu_i \nu_i}(r) + (\vec{\sigma}_e \times \vec{\sigma}_p) \cdot \vec{\nabla} V_{\nu_i \nu_i}(r) \right]$$

$V_{\nu\nu}$ is computed by taking the Fourier transform of the parity-conserving part of the amplitude (using the Cutkosky cutting rules)

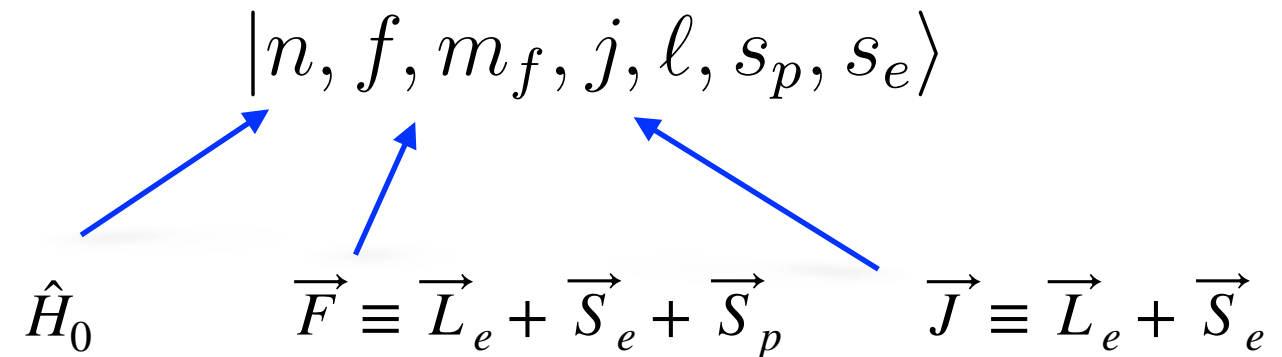
$$V_{\nu\nu}^{\text{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3} \frac{K_3(2m_\nu r)}{r^2}$$

$$V_{\nu\nu}^{\text{Majorana}}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3} \frac{K_2(2m_\nu r)}{r^3}$$

Effect of the neutrino force on the hydrogen atom

Ghosh, Grossman & Tangarife PRD (2020)

Unperturbed eigenstates



The energy of a state with $f, j, \ell, s_e = s_p = \frac{1}{2}$

$$E_{nfj\ell} = (E_0)_n + (E_{\text{fine}})_{nj} + (E_{\text{hyperfine}})_{nfj\ell}$$

$$(E_0)_n = -\frac{\alpha^2 m_e}{2n^2} \quad (E_{\text{fine}})_{nj} = -\frac{\alpha^4 m_e}{2n^4} \left(\frac{n}{j + \frac{1}{2}} - \frac{3}{4} \right)$$

$$(E_{\text{hyperfine}})_{nfj\ell} = \frac{\alpha^4 g_p}{m_p} a_0^3 \frac{\ell(\ell + 1) m_e^2 (f(f + 1) - j(j + 1) - \frac{3}{4})}{4j(j + 1)} \left\langle \frac{1}{r^3} \right\rangle_{n\ell}$$

The only degeneracy remains in m_f

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We treat $V_{\nu\nu}$ as a perturbation

$$|\psi_q^1\rangle = |\psi_q^0\rangle + \sum_{p \neq q} \frac{\langle \psi_p^0 | V | \psi_q^0 \rangle}{E_q^0 - E_p^0} |\psi_p^0\rangle$$

$$\langle n\ell m | V_{PNC}^{\text{tree}} | n'\ell' m' \rangle \sim \int_0^\infty d\eta \, \eta^2 \, \eta^{\ell'} V_{PNC}^{\text{tree}}(\eta) \eta^\ell \sim \frac{\alpha^{2\ell+5} m_e^{2\ell+3}}{m_Z^{2\ell+2}} = m_e \alpha^{2\ell+5} \left(\frac{m_e}{m_Z} \right)^{2\ell+2}$$

$$\eta \equiv r/a_0$$

$$\langle n\ell m | V_{PNC}^{\text{loop}} | n'\ell' m' \rangle \sim \frac{G_F^2}{m_e a_0^6} \int d\eta \, \eta^2 \eta^{\ell'} \left(\frac{1}{\eta^6} \right) \eta^\ell \exp \left[-\eta \left(\frac{1}{n} + \frac{1}{n'} \right) \right]$$

for $\ell = 0$ and $\ell = 1$, the radial integral does not converge,
indicating the failure of four-Fermi theory

for $\ell \geq 2$

$$\frac{\alpha^2}{m_e m_Z^4 a_0^6} \int_0^\infty d\eta \, \eta^{2\ell-3} \exp(-n_{\text{sup}} \eta) \sim m_e \alpha^8 \left(\frac{m_e}{m_Z} \right)^4$$

Effect of the neutrino force on the hydrogen atom

Ghosh, Grossman & Tangarife PRD (2020)

Let's look now at electric and magnetic transitions: Use states with $\ell = 3$ since they can mix with states with $\ell = 2$ and $\ell = 4$

Our goal: To compute $R \equiv \text{Im} \left(\frac{E1_{PV}}{M1} \right)$

$$|A\rangle = |4, 3, 3, 5/2, 3\rangle \equiv 4F_{5/2, F=3}$$

$$|B\rangle = |4, 3, 3, 7/2, 3\rangle \equiv 4F_{7/2, F=3}$$

Before adding $V_{\nu\nu}$, these states have the same ℓ and there can be an $M1$ transition but not an $E1$ transition

But they are corrected

$$|A'\rangle = |A\rangle + \frac{\langle \Delta | V_{PNC} | A \rangle}{E_A - E_\Delta} |\Delta\rangle + \dots \quad |B'\rangle = |B\rangle + \frac{\langle \Delta | V_{PNC} | B \rangle}{E_B - E_\Delta} |\Delta\rangle + \dots$$

$$|\Delta\rangle = |4, 3, 3, 5/2, 2\rangle$$

Effect of the neutrino force on the hydrogen atom

$$|A\rangle = |4, 3, 3, 5/2, 3\rangle \equiv 4F_{5/2, F=3}$$

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$$|\Delta\rangle = |4, 3, 3, 5/2, 2\rangle$$

$$|B'\rangle = |B\rangle + \frac{\langle \Delta | V_{PNC} | B \rangle}{E_B - E_\Delta} |\Delta\rangle + \dots$$

So we can finally compute

$$R = \text{Im} \left(\frac{E1_{PV}}{M1} \right) = \text{Im} \left(\frac{\langle A' | \hat{P} | B' \rangle}{\langle A' | \hat{M} | B' \rangle} \right) \approx \left(-\frac{1}{4} + s_W^2 + \frac{1}{2} |U_{ei}|^2 \right) \left(-7.7 \times 10^{-33} + 3.7 \times 10^{-32} \left(\frac{m_{\nu_i}}{\alpha m_e} \right)^2 \right)$$

The rotation due to the neutrino force would be $\Phi \sim 10^{-32}$ rads

This is about 23 orders of magnitude smaller than what can be measured in the lab (with Cs) Lintz, Guéna & Bouchiat (2006)

Can we probe the neutrino force using atomic parity violation?

Not yet! The measurement of optical rotation due to the neutrino loop is extremely challenging given the resolutions we can achieve today.

Nonetheless, this calculation, performed for other systems, could lead to somewhat larger quantities and the next step would most likely be an application of this idea to many-electron atoms, beyond the simple hydrogen case. The matrix elements in these atoms are amplified by an additional Z^3 factor.

Although the effects of the neutrino force on the hydrogen atom are extremely small to measure in an experiment, the neutrino force is the largest long-range parity-violating force there is.

Thank you!