Towards

Neutrino oscillations

without mass diagonalization nor Lagrangians

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QFT - F

(on-shell methods)

- QFT = a very successful framework
- Still, there are some drawbacks:
  - Lorentz invariance forces us to work with redundant fields (gauge redundancy)
  - Fields redefinitions are sometimes needed to make the physics manifest (e.g. $\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 + \frac{2}{\Lambda}\phi(\partial\phi)^2 - \frac{m^2}{\Lambda}\phi^3 + \frac{2}{\Lambda^2}\phi^2(\partial\phi)^2 - \frac{m^2}{2\Lambda^2}\phi^4$ is a free theory!)
  - In EFTs some work is needed to find a basis of independent operators

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Can we bypass these drawbacks? YES!

- In the 1960's QFT and EFT techniques put a stop to the S-matrix program

compute directly amplitudes from first principles

- In the 2010's we saw a resurgence of the S-matrix program

Idea

- Back to the basics: quantum particle = irrep of the Poincaré group
- The irreps of Poincaré are induced by the LITTLE GROUP (LG) irreps
    - For massive particles (in $d = 4$) $\longrightarrow$ SO(3)
    - For massless particles (in $d = 4$) $\longrightarrow$ U(1) (ignoring continuous-spin)
- Amplitude $\mathscr{A} = \langle f \,|\, T \,|\, i \rangle$ inherits the LG transformations:

$$\mathscr{A} \to \prod_{a=i,f} D_a \mathscr{A}$$

- **Whole point**: determine $\mathscr{A}$ solely from its transformation under the LG
- There is no notion of quantum field nor Lagrangian!

[Technical parenthesis:

- LG transformations are complicated functions of momenta (not easy to work with)
- HOWEVER: they become simple using the $SO(1,3) \leftrightarrow SL(2,\mathbb{C})$ correspondence $\Rightarrow$ use **spinor variables**!]

Benincasa-Cachazo 0705.4305

Arkani Hamed-Huang-Huang 1709.04891

Massless particles

- 3-points amplitudes COMPLETELY FIXED by helicity
- Important results:
    - GR and YM as the unique consistent theories of massless self-interacting spin-2 and spin-1 particles
    - Anomaly condition recovered without worrying about the path integral measure
    - higher points amplitudes can be determined recursively from lower-point amplitudes (using polology)
    - (High energy) QCD phenomenology simplified!

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Massive particles

- LG a bit more complicated, amplitudes a bit more complicated
- Important results: Yang theorem, massive particles with $s > 2$ cannot be elementary
- The Higgs mechanism can be recovered completely bottom-up
- The result is automatically valid at ALL ORDERS in a $1/\Lambda$ expansion!
  (because we obtain "kinematic structures" with the right transformations under the LG)

What has been done for the SM

- The SM on-shell amplitude has been written a couple of years ago, but the couplings were put “by hand”

- BUT…we know that not all couplings are equivalent in the SM: some involve unitary matrices, and these constitute a prediction of the theory

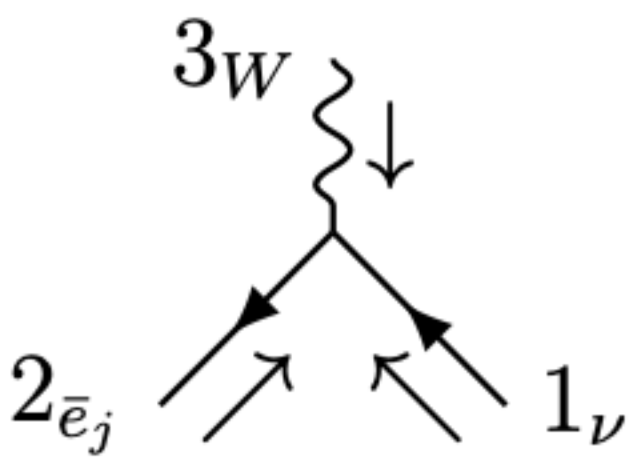
OUR QUESTION 1

The SM predicts UNITARY CKM and PMNS

<u>can we obtain this result without mass diagonalization?</u>

G. Alves, EB, G. Salla 2103.16362

Start with the 3-point amplitude



$$\mathscr{A} = \underbrace{\frac{y_L^{ij}}{M}\langle \mathbf{1}_i\mathbf{3}\rangle\langle \mathbf{2}_j\mathbf{3}\rangle}_{\boxed{\bar{e}_R\sigma^{\mu\nu}\nu_L W^-_{\mu\nu}}} + \underbrace{\frac{g_L^{ij}}{m_W}\langle \mathbf{1}_i\mathbf{3}\rangle[\mathbf{2}_j\mathbf{3}]}_{\boxed{\text{SM}}} + \underbrace{\frac{g_R^{ij}}{m_W}[\mathbf{1}_i\mathbf{3}]\langle \mathbf{2}_j\mathbf{3}\rangle}_{\boxed{\bar{e}_R\gamma^{\mu}\nu_R W^-_{\mu}}} + \underbrace{\frac{y_R^{ij}}{M}[\mathbf{1}_i\mathbf{3}][\mathbf{2}_\mathbf{j}\mathbf{3}]}_{\boxed{\bar{e}_L\sigma^{\mu\nu}\nu_R W^-_{\mu\nu}}}$$

UV origin depend on the Majorana/Dirac ν nature
(at $d = 6$ for Dirac , at $d = 7$ for Majorana)

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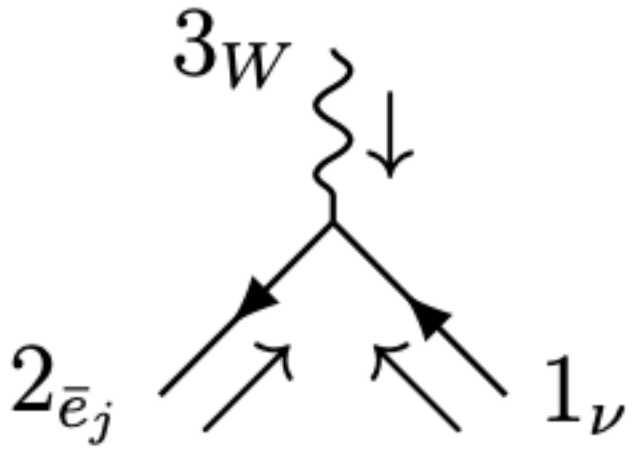

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taking the massless limits we can match to the massless amplitude

+

in the massless limit we are free to redefine the states using flavor transformations

$$|\nu_i(\boldsymbol{p},h)\rangle \to (U_\nu^*)_{ji}\,|\nu_j(\boldsymbol{p},h)\rangle\,, \qquad |\bar{e}_i(\boldsymbol{p},h)\rangle \to (U_e)_{ji}\,|\bar{e}_j(\boldsymbol{p},h)\rangle$$

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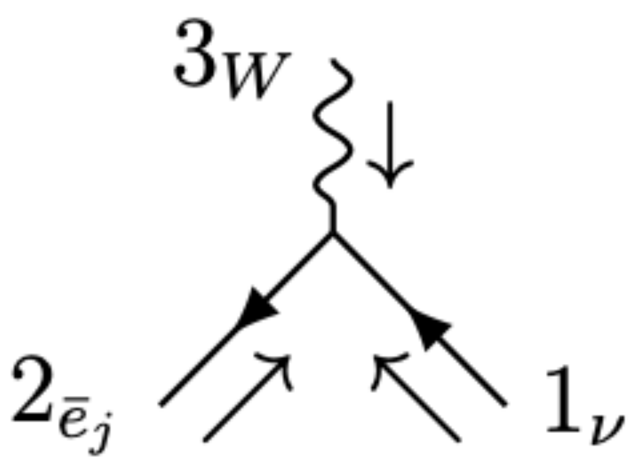

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$$\frac{g_L}{m_W} \longrightarrow \frac{U_\nu^\dagger g_L}{m_W} \longrightarrow \frac{U_\nu^\dagger g_L U_e}{m_W} \longrightarrow U_\nu^\dagger g_L U_e = g\mathbf{1}$$

$$\Downarrow$$

$$\boxed{g_L = g\, U_{PMNS}}$$

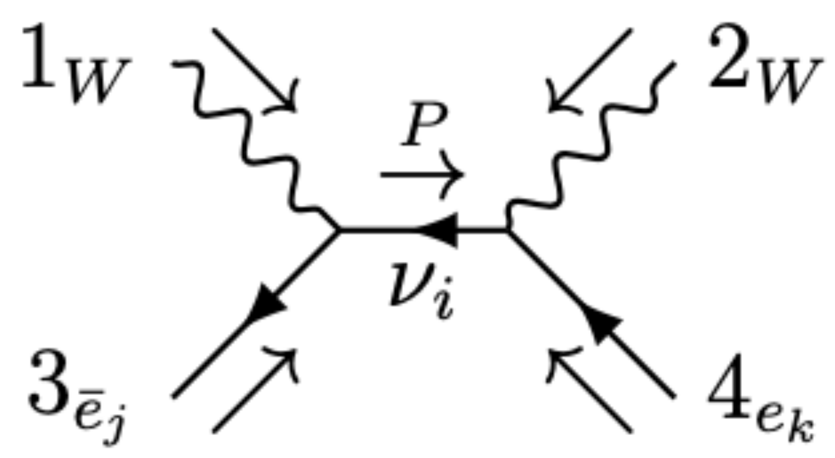
exactly the same reasoning can be applied to quarks to obtain the CKM matrix

OUR QUESTION 2

What happens with neutrino oscillations?

G. Alves, EB, G. Salla 2103.16362

A (very) simplified picture of neutrino oscillations



$$\mathscr{A} = \mathscr{A}_L \frac{1}{s - m_\nu^2} \mathscr{A}_R + \mathscr{A}_{contact}$$

dominated by almost on-shell neutrinos! *

$$\mathscr{A} = \mathscr{A}_L \frac{1}{s - m_\nu^2} \mathscr{A}_R = \mathscr{A}_{SM} + \mathscr{A}_{int} + \mathscr{A}_{NP}$$

complicated kinematic structures, but an exact result at all orders in the EFT

$$\mathscr{A}_{SM} = \frac{g^2}{m_W^2} U^T \frac{1}{s - m_\nu^2} U^* \times \left(\text{kinematic structure}\right)$$

* Provided the W is off-shell and the amplitude is part of a larger amplitude (not considered for simplicity)

Possible extensions

On the neutrino side

- Extend the deduction to $\mathscr{A}(qq\ell\ell)$ (alternative all-order formulation of non-standard interactions)
- Extend the deduction at $\mathscr{A}(\pi\ell\nu)$: how do we match to quarks?
- Neutrino oscillations in matter: how to treat them in this formalism?

More in general

- Phenomenological analysis of the all-order amplitudes with the SM particle content (on-shell generalization of the SMEFT fit…at all orders!)
- More conceptual/technical: how to take the high energy limit of an amplitude that in QFT is generated by the Higgs vev? Soft limit?
- …

Backup

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Massless particles

- 3-points amplitudes COMPLETELY FIXED by helicity:

$$\mathscr{A}(h_1,h_2,h_3) = g\,\langle 12\rangle^{h_3-h_1-h_2}\langle 13\rangle^{h_2-h_1-h_3}\langle 23\rangle^{h_1-h_2-h_3} \qquad \sum_i h_i < 0$$

$$\mathscr{A}(h_1,h_2,h_3) = g\,[12]^{h_1+h_2-h_3}[13]^{h_1+h_3-h_2}[23]^{h_2+h_3-h_1} \qquad \sum_i h_i > 0$$

The Higgs mechanism bottom-up

Example: $\mathscr{A}(\psi\psi^c Z) \sim \left(g_L - g_R\right) \dfrac{m_\psi}{m_Z}\langle 12\rangle$ well behaved as $m_\psi \to 0$ only if

- $g_L = g_R$ (vector-like fermion)
- $m_\psi \to 0$ as $m_Z \to 0$ (chiral fermion mass has the same origin as the vector mass)

Durieux, Kitahara, Shadmi and Weiss 1909.10551