

# *Neutrino oscillations in extended theories of gravity*

Luciano Petruzzello

Università degli Studi di Salerno, Dipartimento di Fisica “E. R. Caianiello”  
INFN Sezione di Napoli, Gruppo collegato di Salerno



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## Outline

- Quadratic theories of gravity
- Neutrino oscillations in curved spacetime
- Strong equivalence principle violation
- Future perspectives

## Quadratic theories of gravity

The most general gravitational Lagrangian quadratic in the curvature invariants\*

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa^2} \left\{ \mathcal{R} + \frac{1}{2} \left[ \mathcal{R}\mathcal{F}_1(\square)\mathcal{R} + \mathcal{R}_{\mu\nu}\mathcal{F}_2(\square)\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{F}_3(\square)\mathcal{R}^{\mu\nu\rho\sigma} \right] \right\}$$

For a suitable choice of the form factors we have:

- Non-locality
- UV completion of General Relativity valid at all energy scales
- Freedom from ghost fields (i.e. avoids unitarity problem)

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\*T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Phys. Rev. Lett. (2012)

## Linearized regime

In the limit

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

The Lagrangian then becomes

$$\begin{aligned}\mathcal{L} = & \frac{1}{4} \left\{ \frac{1}{2} h_{\mu\nu} f(\square) \square h^{\mu\nu} - h_\mu^\sigma f(\square) \partial_\sigma \partial_\nu h^{\mu\nu} + h g(\square) \partial_\mu \partial_\nu h^{\mu\nu} \right. \\ & \left. - \frac{1}{2} h g(\square) \square h + \frac{1}{2} h^{\lambda\sigma} \frac{f(\square) - g(\square)}{\square} \partial_\lambda \partial_\sigma \partial_\mu \partial_\nu h^{\mu\nu} \right\}\end{aligned}$$

$$h = \eta_{\mu\nu} h^{\mu\nu} \quad f(\square) = 1 + \frac{1}{2} \mathcal{F}_2(\square) \square \quad g(\square) = 1 - 2\mathcal{F}_1(\square) \square - \frac{1}{2} \mathcal{F}_2(\square) \square$$

## Solutions

For a static point-like source of gravity

$$T_{\mu\nu} = m \delta_\mu^0 \delta_\nu^0 \delta^{(3)}(\mathbf{r})$$

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\psi)(dr^2 + r^2 d\Omega^2)$$

$$\phi(r) = -\frac{4Gm}{\pi r} \int_0^\infty \frac{f - 2g}{f(f - 3g)} \frac{\sin(kr)}{k} dk$$

$$\psi(r) = \frac{4Gm}{\pi r} \int_0^\infty \frac{g}{f(f - 3g)} \frac{\sin(kr)}{k} dk$$

$$f = g = 1 \Rightarrow \phi = \psi = -\frac{Gm}{r} \Rightarrow \text{GR}$$

## Neutrino oscillations

According to the standard picture\*

$$|\nu_\alpha\rangle = \sum_{k=1,2} U_{\alpha k}(\theta) |\nu_k\rangle$$

In a semiclassical fashion

$$|\nu_k(x)\rangle = \exp[-i\varphi_k(x)]|\nu_k\rangle, \quad \varphi_k = \int_{\lambda_P}^{\lambda_D} P_{\mu,k} \frac{dx_{\text{null}}^\mu}{d\lambda} d\lambda$$

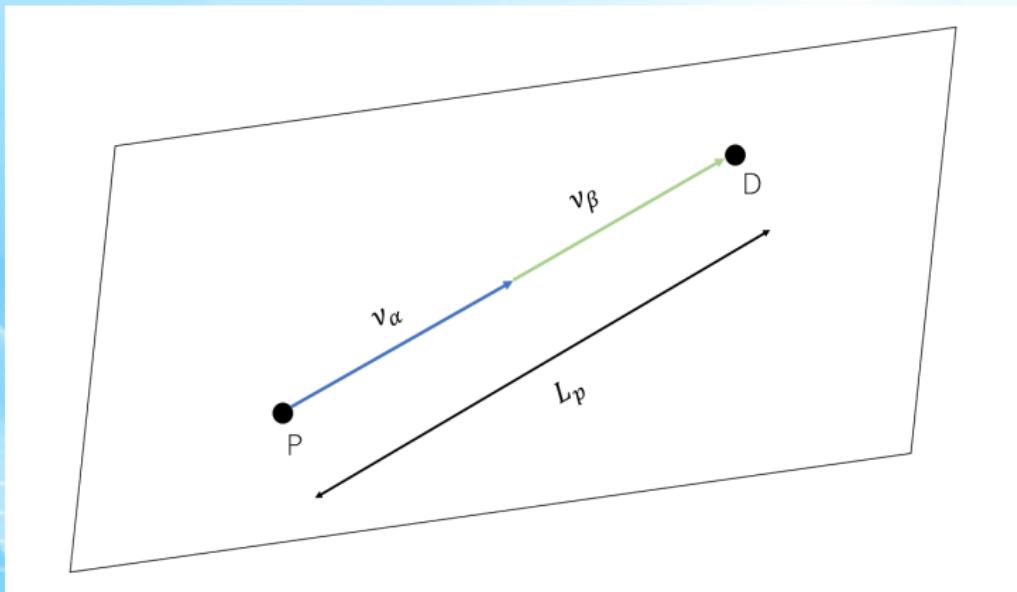
The flavor transition probability

$$\mathcal{P}_{\alpha \rightarrow \beta} = \left| \langle \nu_\beta(t_D, \mathbf{x}_D) | \nu_\alpha(t_P, \mathbf{x}_P) \rangle \right|^2 = \sin^2(2\theta) \sin^2\left(\frac{\varphi_{12}}{2}\right)$$

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\*S. M. Bilenky and B. Pontecorvo, Phys. Rep. (1978)

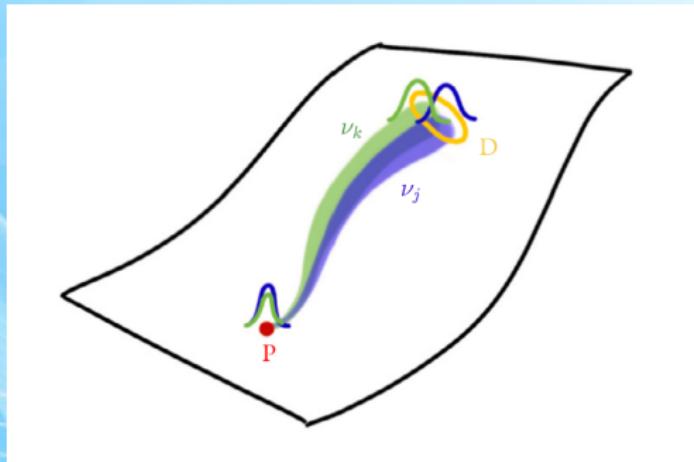
## Setup (flat case)



$$U(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad \varphi_{12} = \frac{\Delta m^2}{2E} L_p \quad \Delta m^2 = m_2^2 - m_1^2$$

## Setup (Schwarzschild case)

In curved spacetime the situation is different\*



$$\varphi_{12} = \varphi_0 + \varphi_{GR}$$

$$\varphi_0 = \frac{\Delta m^2}{2E_\ell} L_p$$

$$\varphi_{GR} = \frac{\Delta m^2 L_p}{2E_\ell} \left[ \frac{Gm}{r_D} - \frac{Gm}{L_p} \ln \left( \frac{r_D}{r_P} \right) \right]$$

\*C. Y. Cardall and G. M. Fuller, Phys. Rev. D (1997)

## Extended model scenario

If the spacetime is not simply described by GR\*

$$\varphi_{12} = \varphi_0 + \varphi_{GR} + \varphi_Q$$

Assuming  $\phi = \phi_{GR} + \phi_Q$  and  $\psi = \psi_{GR} + \psi_Q = \phi_{GR} + \psi_Q$

$$\varphi_Q = \frac{\Delta m^2 L_p}{2E_\ell} \left[ \frac{1}{L_p} \int_{r_P}^{r_D} \phi_Q(r) dr - \phi_Q(r_D) \right]$$

Equivalently, for an observer at infinity  $E_\ell = e^0 \hat{e}_0 E$

$$\varphi_{12} = \frac{\Delta m^2}{2E} \int_{r_P}^{r_D} [1 + \phi(r) - \psi(r)] dr$$

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\*L. Buoninfante, G. G. Luciano, L. Petrucciello and L. Smaldone, Phys. Rev. D (2020)

## Some examples

- $\mathcal{R}^2$  gravity ( $\mathcal{F}_1 = \alpha$ ,  $\mathcal{F}_2 = 0$ )

$$\varphi_Q = \frac{\Delta m^2 L_p}{2E_\ell} \left\{ \frac{Gm e^{-m_0 r_D}}{3r_D} - \frac{Gm}{3L_p} \left[ \text{Ei}(-m_0 r) \right]_{r_P}^{r_D} \right\}$$

- Fourth-order gravity ( $\mathcal{F}_1 = \alpha$ ,  $\mathcal{F}_2 = \beta$ )

$$\begin{aligned} \varphi_Q = & \frac{\Delta m^2 L_p}{2E_\ell} \left\{ \frac{Gm e^{-m_0 r_D}}{3r_D} - \frac{4Gm e^{-m_2 r_D}}{3r_D} \right. \\ & \left. - \frac{Gm}{3L_p} \left[ \text{Ei}(-m_0 r) \right]_{r_P}^{r_D} + \frac{4Gm}{3L_p} \left[ \text{Ei}(-m_2 r) \right]_{r_P}^{r_D} \right\} \end{aligned}$$

## Some examples (2)

- Infinite derivative gravity ( $\mathcal{F}_1 = -\frac{1}{2}\mathcal{F}_2 = \frac{1-e^{\Box/M_s^2}}{2\Box}$ )

$$\begin{aligned}\varphi_Q &= \frac{\Delta m^2 L_p}{2E_\ell} \left\{ -\frac{Gm}{r_B} \operatorname{Erfc} \left[ \frac{M_s r_D}{2} \right] + \frac{Gm}{L_p} \ln \left( \frac{r_D}{r_P} \right) \right. \\ &\quad \left. - \frac{Gm}{L_p} \left[ \frac{M_s r}{\sqrt{\pi}} {}_2F_2 \left( \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{M_s^2 r^2}{4} \right) \right]_{r_P}^{r_D} \right\}\end{aligned}$$

- IR modification of GR ( $\mathcal{F}_1 = \frac{\alpha}{\Box}$ ,  $\mathcal{F}_2 = 0$ )

$$\varphi_Q = \frac{\alpha}{3\alpha - 1} \varphi_{GR}.$$

## How to constrain extended models

Due to the linearized approximation

$$|\varphi_0| > |\varphi_{GR}|$$

Due to gravitational phenomenology

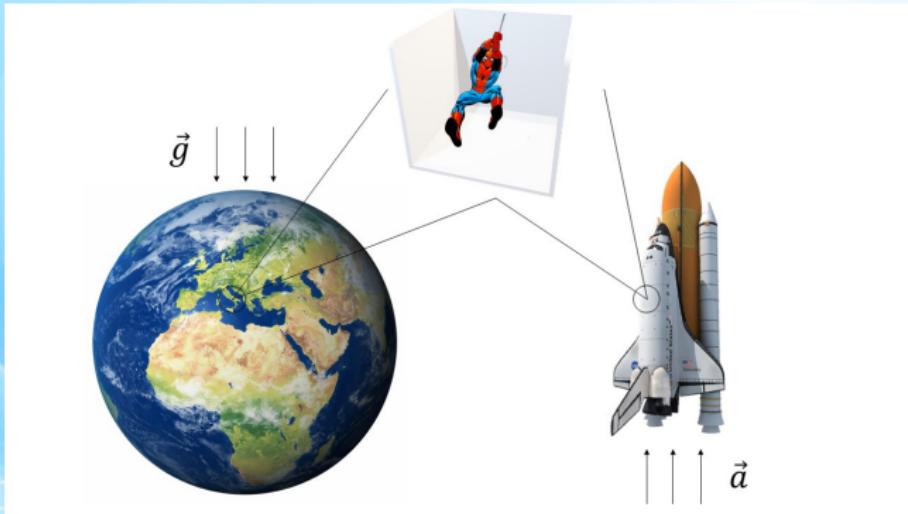
$$|\varphi_{GR}| \gtrsim |\varphi_Q|$$

Therefore

$$\left| \frac{\varphi_Q}{\varphi_0} \right| < 1$$

Physics entails a restriction on the free parameters of the extended theories

## Weak Equivalence Principle (WEP)



$$|\mathbf{a}| = |\mathbf{g}| \implies m_i = m_g$$

$$\lambda \equiv \frac{Gm}{rc^2} \ll 1$$

# Strong Equivalence Principle (SEP)

Simultaneous requirements\*

$$\text{SEP} = \text{GWEP} + \text{LLI} + \text{LPI}$$

In order to quantify SEP, it is possible to resort to the PPN formalism†

Parameter	Meaning	Value in GR
$\gamma$	Space curvature produced by unit rest mass	1
$\beta$	Nonlinearity effects for gravity	1
$\xi$	Preferred-location effects	0
$\alpha_1$		0
$\alpha_2$	Preferred-frame effects	0
$\alpha_3$		0
$\alpha_3$		0
$\zeta_1$		0
$\zeta_2$	Violation of total momentum conservation	0
$\zeta_3$		0
$\zeta_4$		0

Nordtvedt parameter

$$\eta = 4(\beta - 1) - (\gamma - 1)$$

$$\eta \neq 0 \iff \text{No SEP}$$

\*C. M. Will, Living Rev. Rel. (2006)

†S. Weinberg, *Gravitation and Cosmology* (1972)

## Neutrino oscillations as a witness of SEP

In the linearized regime

$$\beta = 1 \quad \implies \quad \eta = \frac{\phi_Q - \psi_Q}{\phi}$$

$$\varphi_{12} = \frac{\Delta m^2}{2E} \int_{r_P}^{r_D} [1 + \phi(r) \eta(r)] dr$$

The flavor transition rate *does* depend on the hypothetical SEP violation

$$\eta \rightarrow 0 \implies \mathcal{P}_{\alpha \rightarrow \beta}^{\text{curved}} \rightarrow \mathcal{P}_{\alpha \rightarrow \beta}^{\text{flat}}$$

## Future perspective

- To go beyond the linearized approximation
- To go beyond the semiclassical approximation
- To further explore the interplay between mixed particles and gravity\*
- To further explore the interplay between mixed particles and the equivalence principle†

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\* M. Blasone, G. Lambiase, G. G. Luciano and L. Petrucciello, Phys. Rev. D (2018); Phys. Lett. B (2020); Eur. Phys. J. C (2020); G. G. Luciano and L. Petrucciello, Int. J. Mod. Phys. D (2020); M. Blasone, G. Lambiase, G. G. Luciano, L. Petrucciello and L. Smaldone, Class. Quant. Grav. (2020)

† M. Blasone, P. Jizba, G. Lambiase and L. Petrucciello, Phys. Lett. B (2020)

**GRACIAS**

DANKSCHEEN  
SPASSIBO  
NUHUN  
SNACHALHUYA  
TASHAKKUR ATU  
YAQHANYELAY  
CHALTU  
DARHY ABRAAD  
WABEEJA MAITEKA YUSPAGABATAM  
ANNA SUKSAMA EKHMET  
ATTÖ HIRSH SPASIBO DENKAUJA UNAHLACHEESH  
MAAKE MAKEE GE  
KOMAPSUMNIDA LAH MEHRBANI PÄLDIES  
GAEJTHO  
GOZAIMASHITA  
EFCHARISTO AGUY-JE  
FAKAUE  
BAINKA  
TAVTAPUCH MEDAWACSE  
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