Update of Bodek-Yang Model 2021
(accounting for difference between Vector and axial structure functions)

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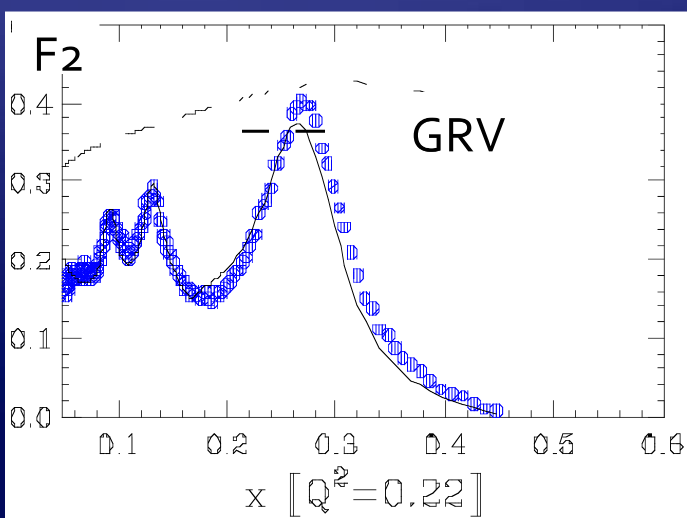
NuCo 2021 Neutrinos en Columbia

https://indico.cern.ch/event/1010475/contributions/4450703/

https://renata.zoom.us/j/88699889284

Bodek-Yang Model

- Bodek-Yang model aims for describing DIS cross section in all Q^2 regions
- Challenges in e/μ/-N DIS (to start with)
  - High x PDFs at low Q^2
  - Resonance region overlapped with a DIS contribution
  - Hard to extrapolate DIS contribution to low Q^2 region from high Q^2 data due to non-perturbative QCD effects.



- A model in terms of quark-parton model (easy to convert charged lepton scattering to neutrino scattering)
  - Understanding of high x PDFs at low Q^2? Wealth of SLAC, JLAB data.
  - Understanding of resonance scattering in terms of quark-parton model? (duality works, many studies by JLAB)

Lessons from previous QCD studies

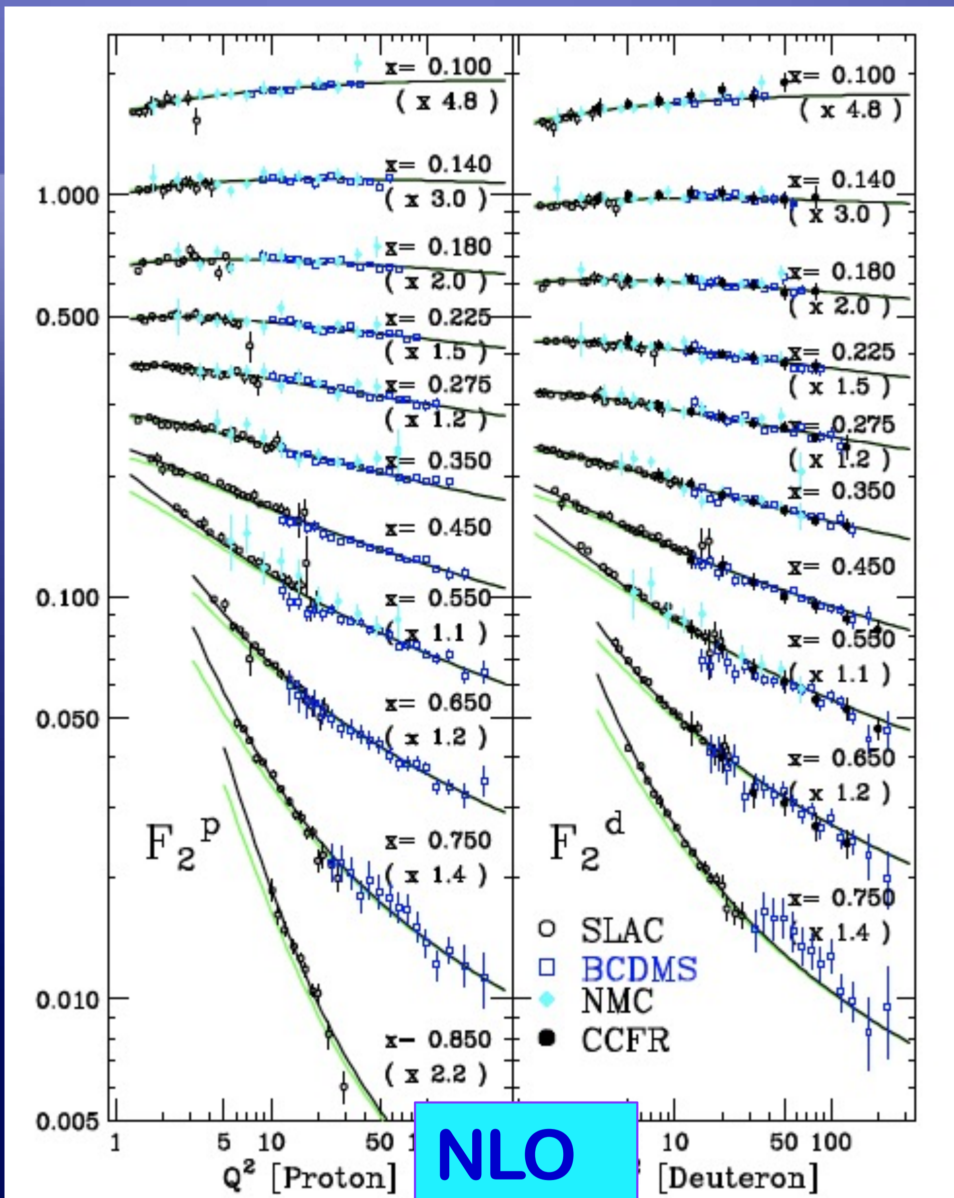
- NLO & NNLO analyses with DIS data: PRL 82, 2467 (1999), Eur. Phys. J. **C13**, 241 (2000) by Bodek and Yang
  - Kinematic higher twist (target mass) effects are large and must be included in the form of Georgi & Politzer x scaling.
  - Resonance region is also well described (duality works).
  - Most of dynamic higher twist corrections (in NLO analysis) are similar to missing NNLO higher order terms.
- NNLO pQCD+TM with NNLO PDFs can describe the non-perturbative QCD effects at low Q^2
- Thus, we reverse the approach to build the model:
  - Use LO PDFs and "effective target mass and final state masses" to account for initial target mass, final target mass, and even missing higher orders

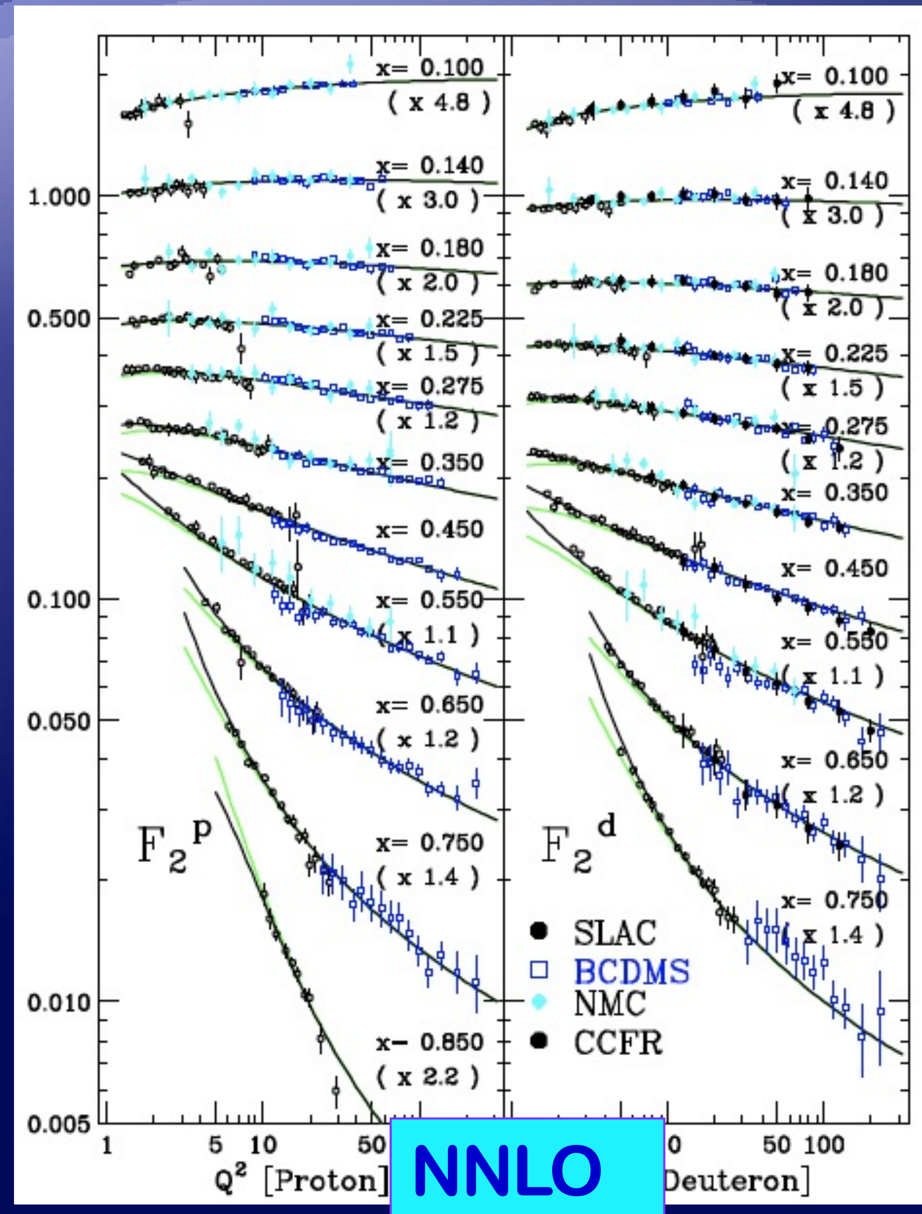
We use LO PDFs and K Factors to be able to go to $Q^2 = 0$ (NLO PDF blow up at low Q^2)

$$\xi = \frac{2xQ'^2}{Q^2(1+\sqrt{1+4M^2x^2/Q^2})},$$

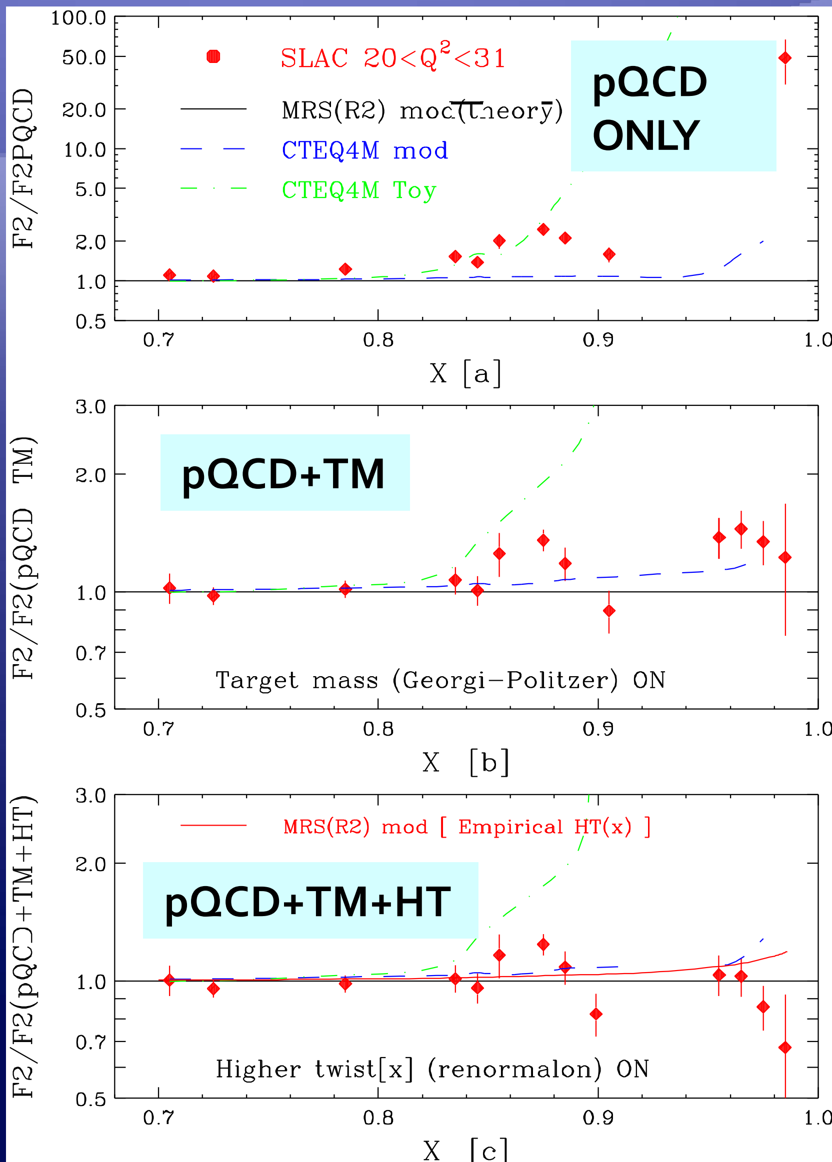
$$2Q'^2 = [Q^2 + M_f{}^2 - M_i{}^2] + \sqrt{(Q^2 + M_f{}^2 - M_i{}^2)^2 + 4Q^2(M_i{}^2 + P_T^2)}.$$

NLO vs NNLO Analyses





Very high x and low Q² data

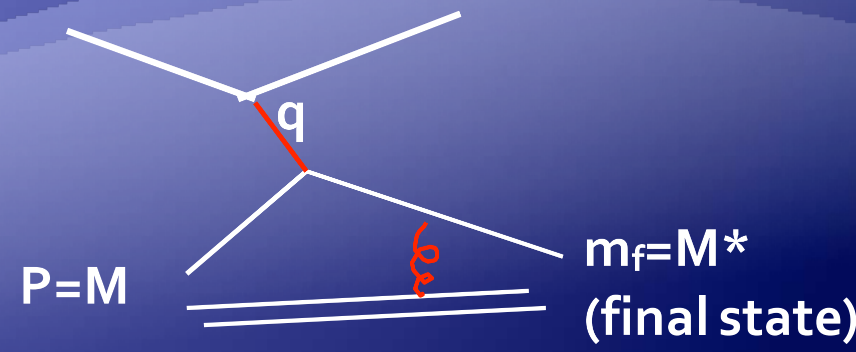


- Very high x data is well described by the pQCD+TM+HT
- Extraction of the high x PDF is promising

PRL 82, 2467 (1999)

Modeling neutrino cross sections

- NNLO pQCD +TM approach:
  describes the DIS region
  and resonance data very well



- Bodek-Yang LO approach: (pseudo NNLO)

• Use effective LO PDFs with a new scaling variable, ξw to absorb target mass, higher twist, missing QCD higher orders

$$x_{Bj} = \frac{Q^2}{2M\nu} \Rightarrow \xi_W = \frac{Q^2 + B}{\{M\nu[1+\sqrt{(1+Q^2/\nu^2)}] + A\}}$$

• Multiply all PDFs by K factors for photo prod. limit and higher twist

$$F_2(x, Q^2) \rightarrow \frac{Q^2}{Q^2 + C} F_2(\xi_w, Q^2)$$

B to be able to qo to Q2=0 , and quark PT

A an enhanced target mass term

Bodek-Yang Effective LO PDFs Model

1. Start with GRV98 LO (Q^2_{min}=0.80)
2. Replace x_{bj} with a new scaling, ξ_w
3. Multiply all PDFs by K factorfor photo prod. limit and higher twist [$\sigma(\gamma)= 4\pi\alpha/Q^2 * F_2(x, Q^2)$]

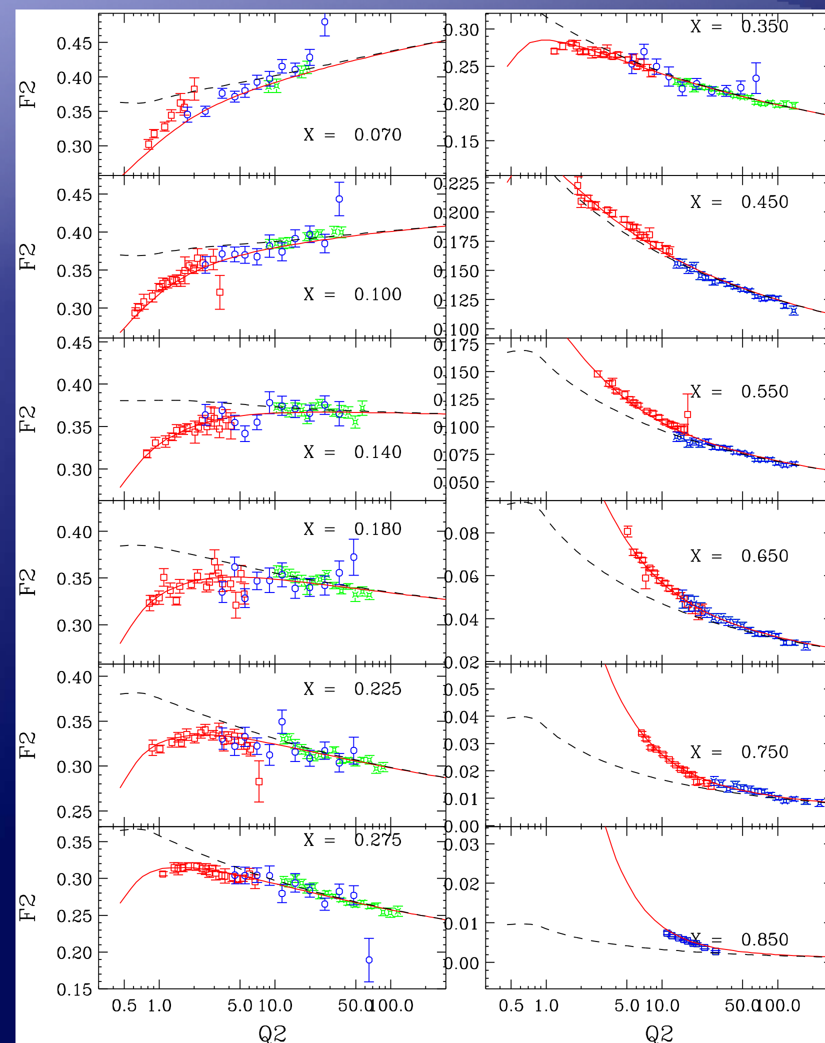
   Ksea = $Q^2/[Q^2+C_{sea}]$

   Kval = $[1- G_D{}^2(Q^2)]$
   $* [Q^2+C_{2V}] / [Q^2+C_{1V}]$

   motivated by Adler Sum rule

   where $G_D{}^2(Q^2) = 1/ [1+Q^2 / 0.71]^4$

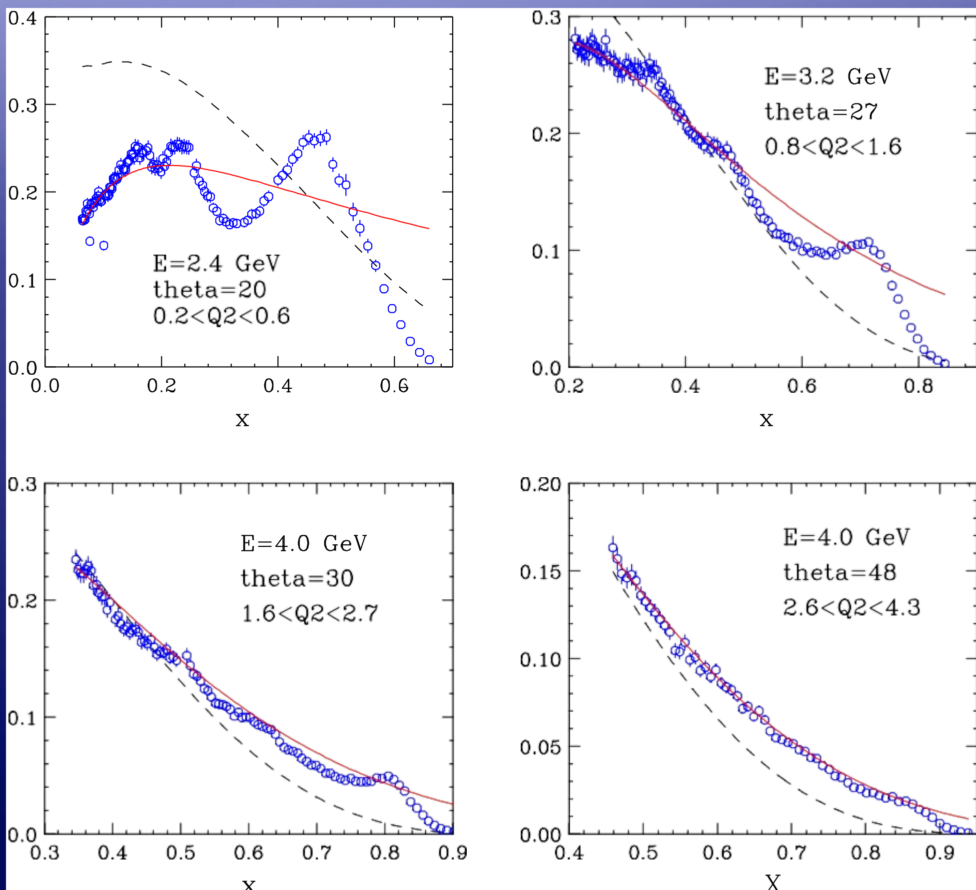
4. Freeze the evolution at $Q^2 = Q^2_{min}$
   - $F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(\xi w, Q^2=0.8)$
5. Fit all DIS F_2(p/D) data: with *W>2 GeV* SLAC/BCDMS/NMC/HERA data



F2(p)

$$\chi^2/DOF = 1235/1200$$

Predictions for Resonance, Photo-production data



F_2(d) resonance

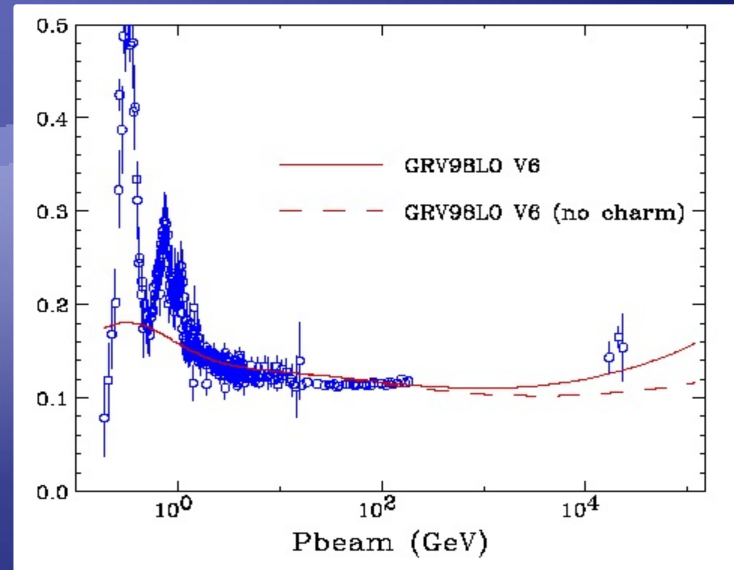


Photo-production (P)

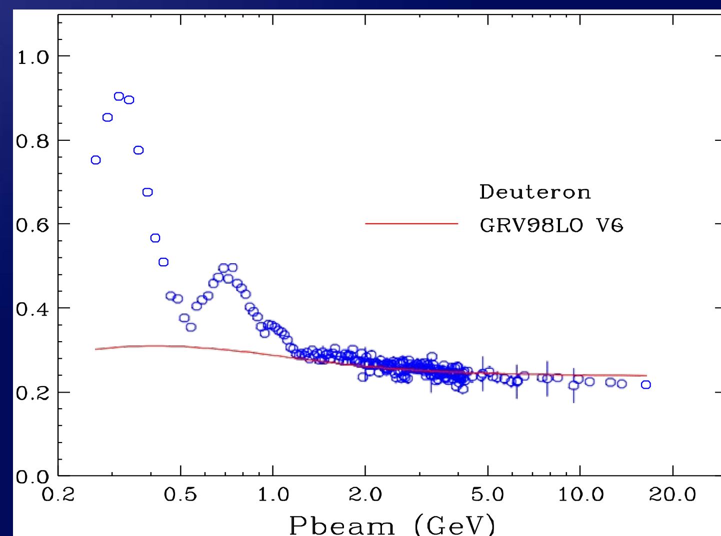


Photo-production (d)

Bodek-Yang Effective LO PDFs Model

- Include the photo-production data
- Use different K factors for up and down quark type separately

Kval (u,d) $= [1- G_D{}^2 (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

Ksea (u,d,s) $= Q^2/[Q^2+Csea]$

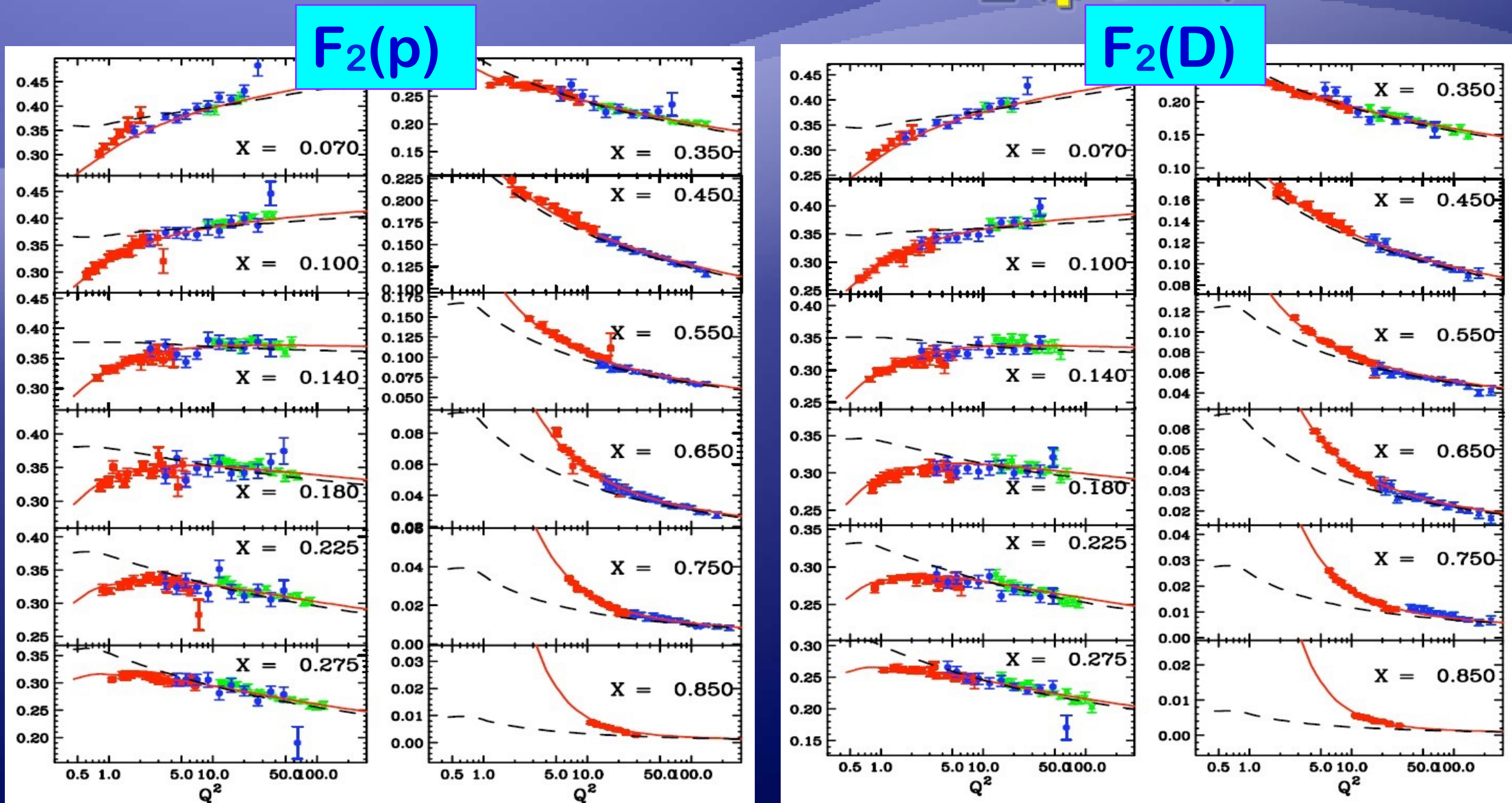
- Additional K^{LW} factor for valence quarks:

Kval $= K^{LW} * [1- G_D{}^2 (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

where $K^{LW} = (\nu^2+C^{\nu})/\nu^2$

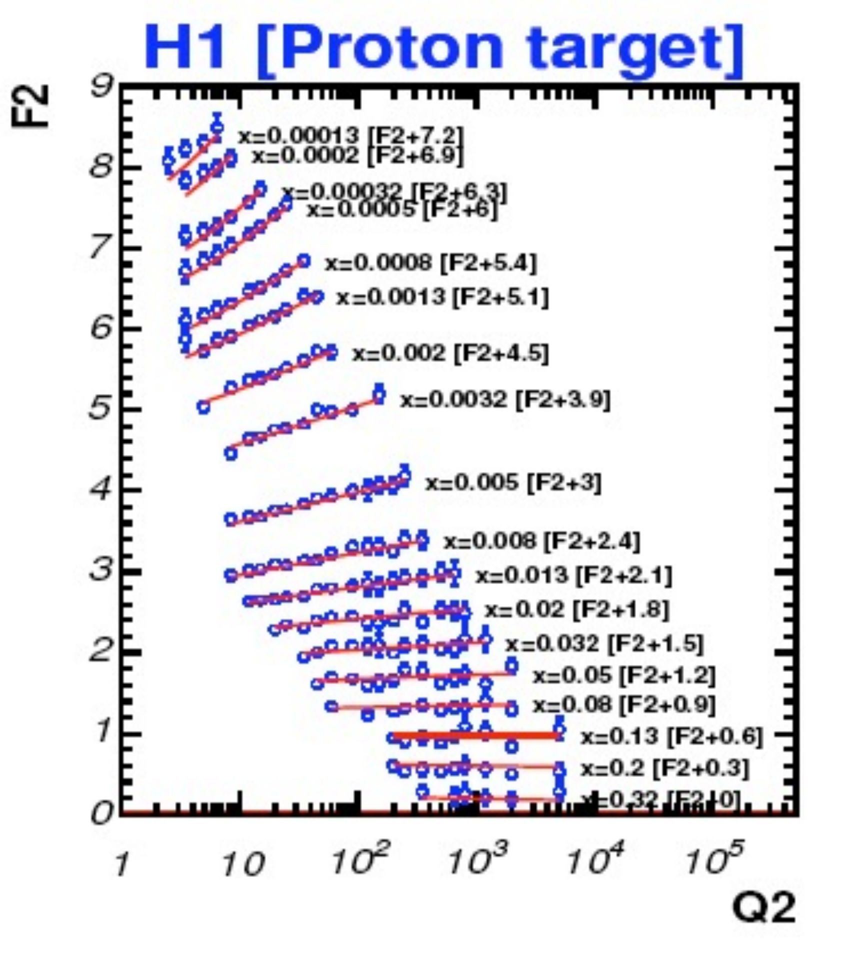
A 0.621	B 0.380	C_{v2d} 0.323	C_{v2u} 0.264
C^{down}_{sea} 0.561	C^{up}_{sea} 0.369	C_{v1d} 0.341	C_{v1u} 0.417
$C^{strange}_{sea}$ 0.561	$C^{low-\nu}$ 0.218	$\mathcal{F}_{valence}$ $[1-G_D^2(Q^2)]$	N 1.026

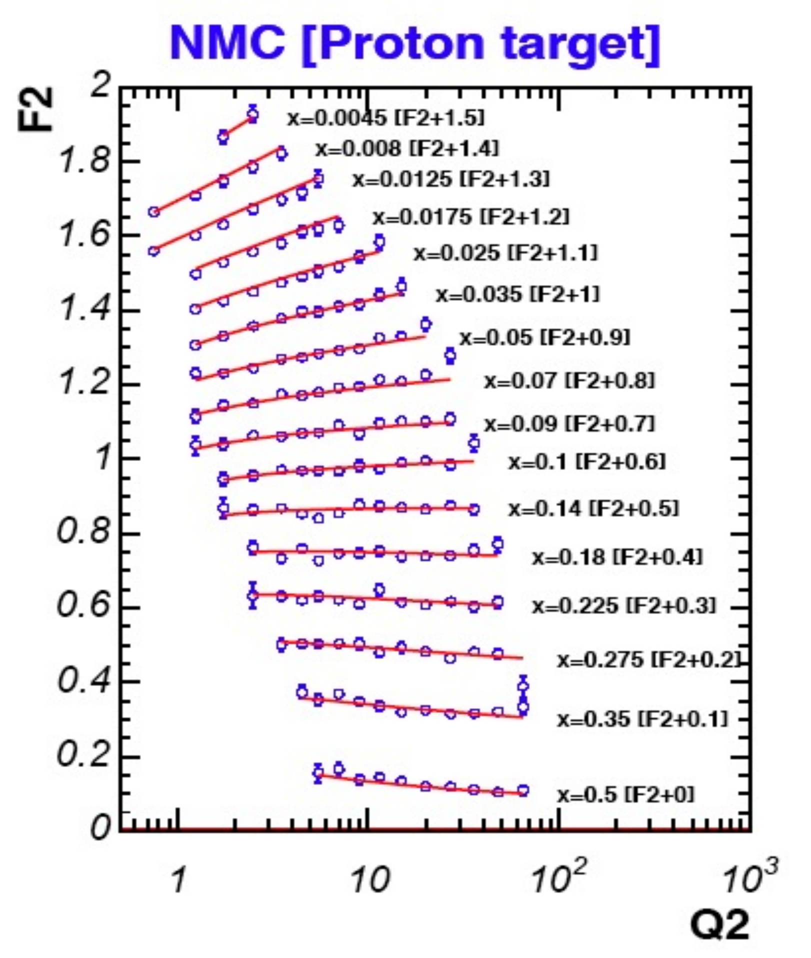
Fit Results on DIS $F_2(p/D)$ data



- Excellent Fits:
  - red solid line: effective LO using ξw
  - black dashed line: GRV98 with x_{bj}

Low x HERA and NMC data





- Fit works at low x

Photo-production data

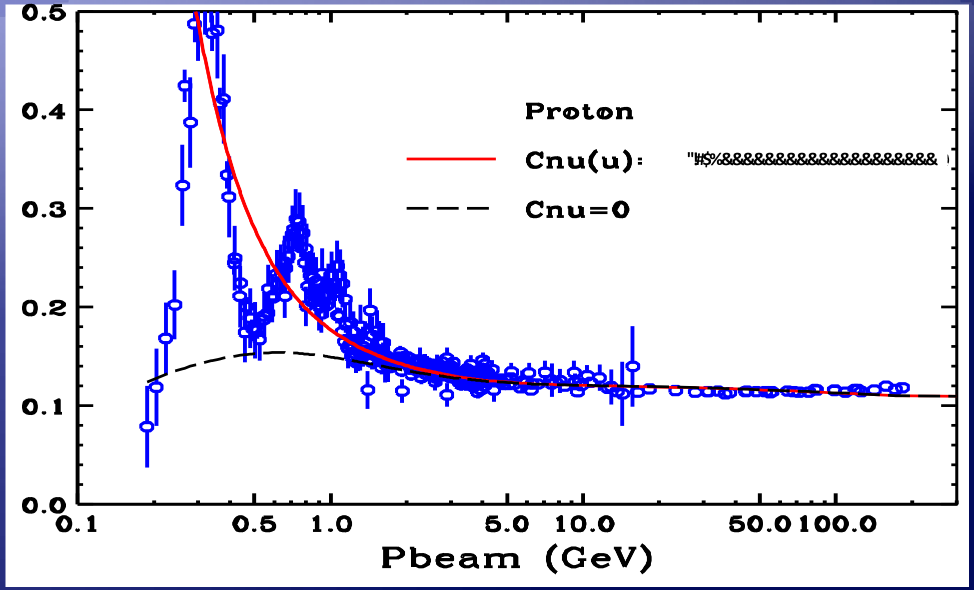
- Additional K^{LW} factor for valence quarks:

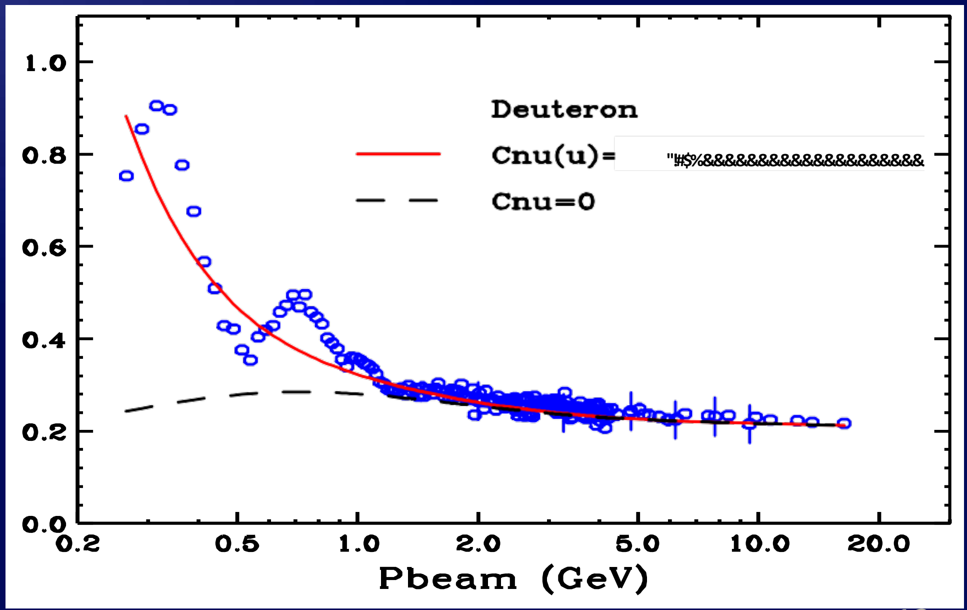
$$Kval = K^{LW} * [1 - G_D{}^2 (Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}]$$

$$K^{LW} = (\nu^2 + C^{\nu}) / \nu^2$$

This makes a duality work all the way down to Q2=0 (for charged leptons)

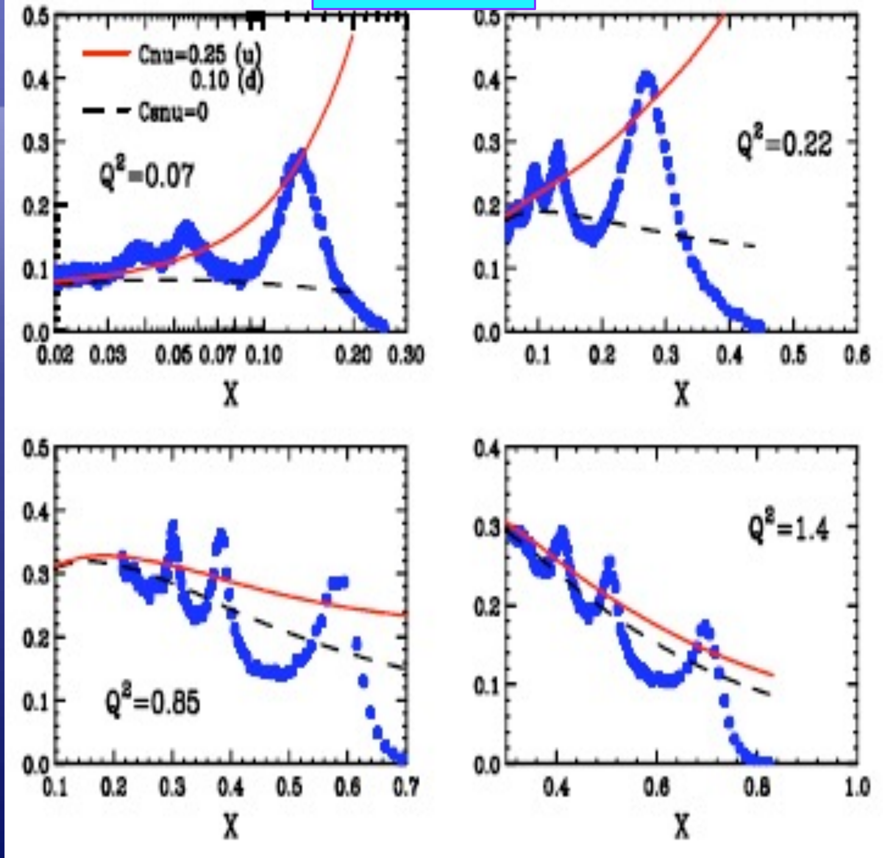
- Photo-production data with ν(Pbeam)>1 GeV included in the fitting



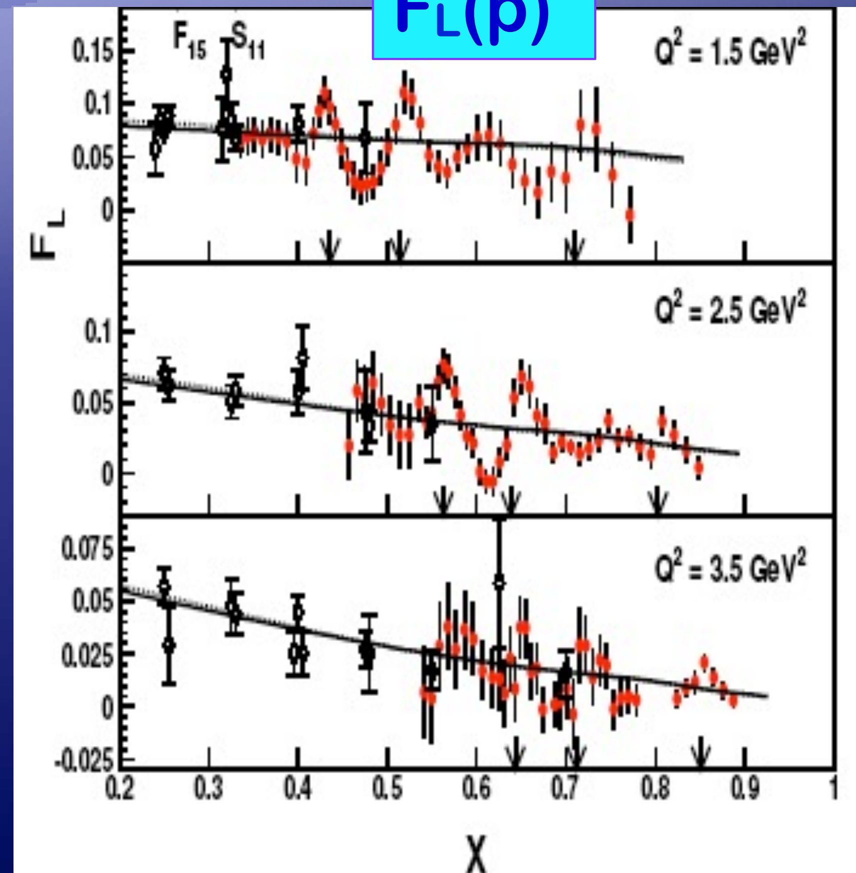


F2 & F_L Resonance data

$F_2(p)$



$F_L(p)$



$$F_L = F_2 (1 + 4M^2x^2/Q^2)\frac{R}{(1+R)}$$

- Predictions are in good agreement (not included in the fit) duality works
- F_L was calculated using F2 and R_{1998}

Neutrino cross sections

- Effective LO model with ξw describe all DIS and resonance F_2 data as well as photo-production data (Q^2=0 limit): vector contribution works well
- Neutrino Scattering:
  - Effective LO model works for xF_3?
  - Nuclear correction using e/μ scattering data
  - Axial vector contribution at low Q^2?
  - Use $R=R_{1998}$ to get $2xF_1$
  - Implement charm mass effect through ξw slow rescaling algorithm for F_2, $2xF_1$, and xF_3

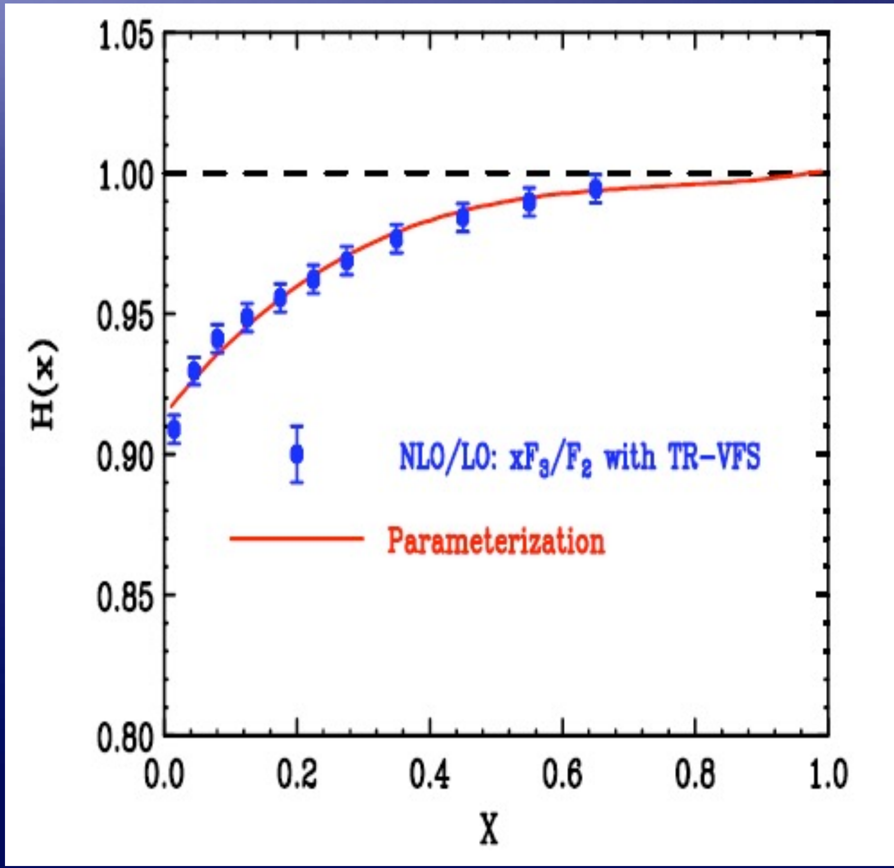
Effective LO model for xF_3?

- Scaling variable, ξw absorbs higher order effect for F_2, but the higher order effects for F_2 and xF_3 are not the same
- Use NLO QCD to get double ratio

$$H(x) = \frac{xF_3(NLO)}{xF_3(LO)} / \frac{F_2(NLO)}{F_2(LO)}$$

not 1 but almost indep. of Q^2

- Enhance anti-neutrino cross section by 3%

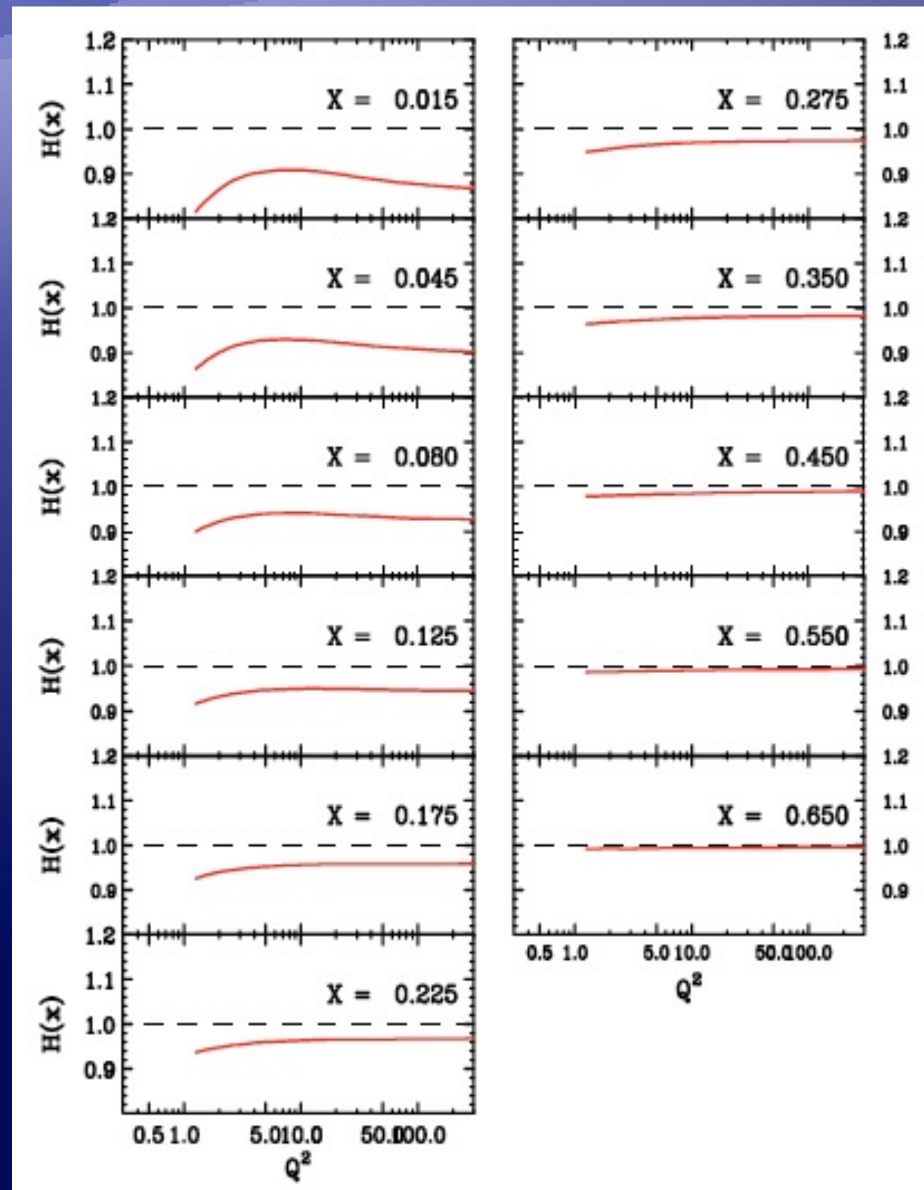


Effective LO model for xF_3?

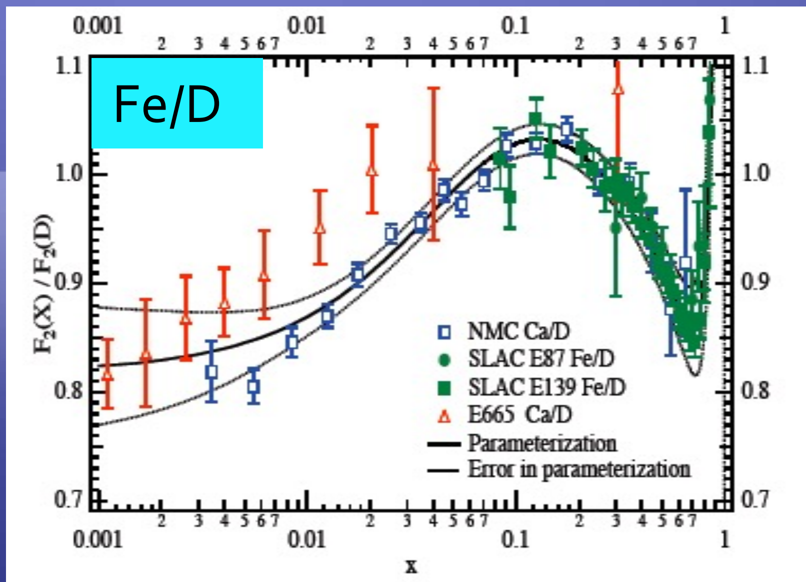
- $H(x,Q^2)$?

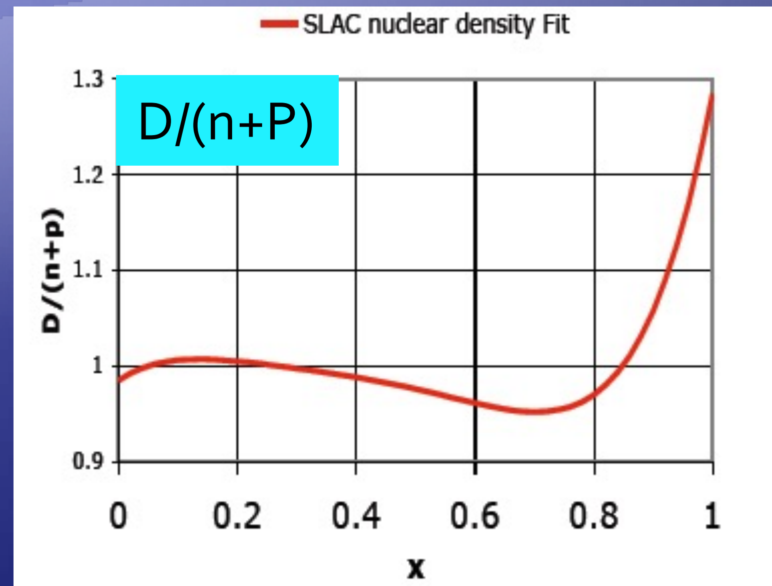
$$H(x) = \frac{xF_3(NLO)}{xF_3(LO)} \Big/ \frac{F_2(NLO)}{F_2(LO)}$$

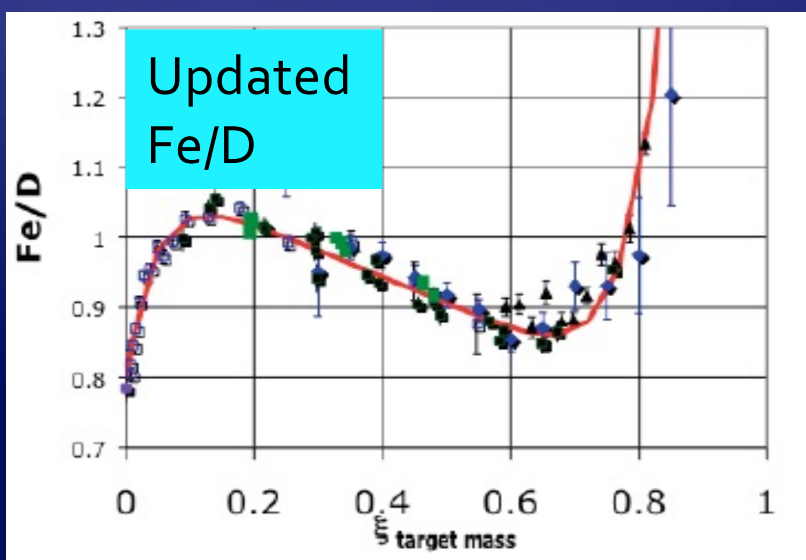
- $H(x,Q^2)$ is almost independent of Q^2

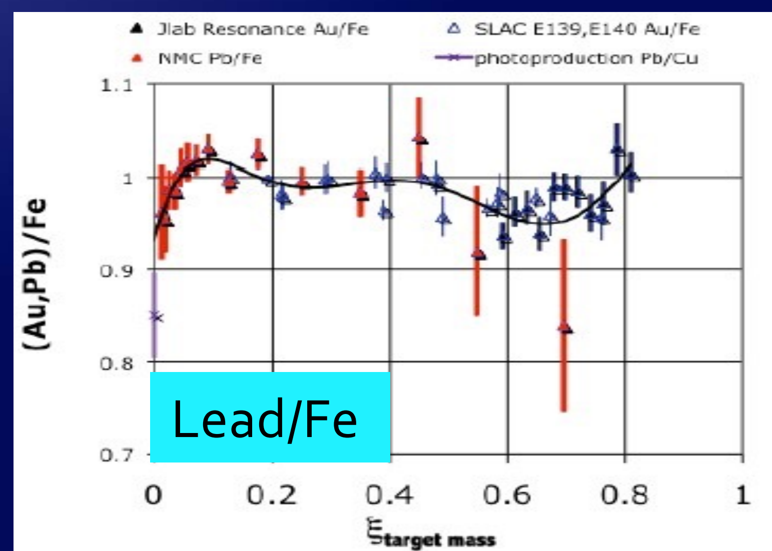


Nuclear Effects: use e/μ data









Axial Vector Structure Functions

- At high Q^2, vector and axial vector contribution are same, but not at low Q2. Previous assumption **Type I (axial=vector)**
- New: K factors for axial contributions: type Ⅱ **(Axial>Vector)**

$$K_{sea}^{vector} = \frac{Q^2}{Q^2 + C} \bullet K_{sea}^{axial} = \frac{Q^2 + 0.55C_{sea}^{axial}}{Q^2 + C_{sea}^{axial}} \qquad K_{val}^{axial} = \frac{Q^2 + 0.3C_{val}^{axial}}{Q^2 + C_{val}^{axial}}$$

$$\text{where } C_{sea}^{axial} = 0.75, \quad C_{val}^{axial} = 0.18$$

- 0.55 was chosen to satisfy the prediction from PCAC by Kulagin, agrees with CCFR data for F_2 extrapolation to (Q^2=0)
- But, the non-zero PCAC component of F_2^{axial} at low Q^2: mostly longitudinal

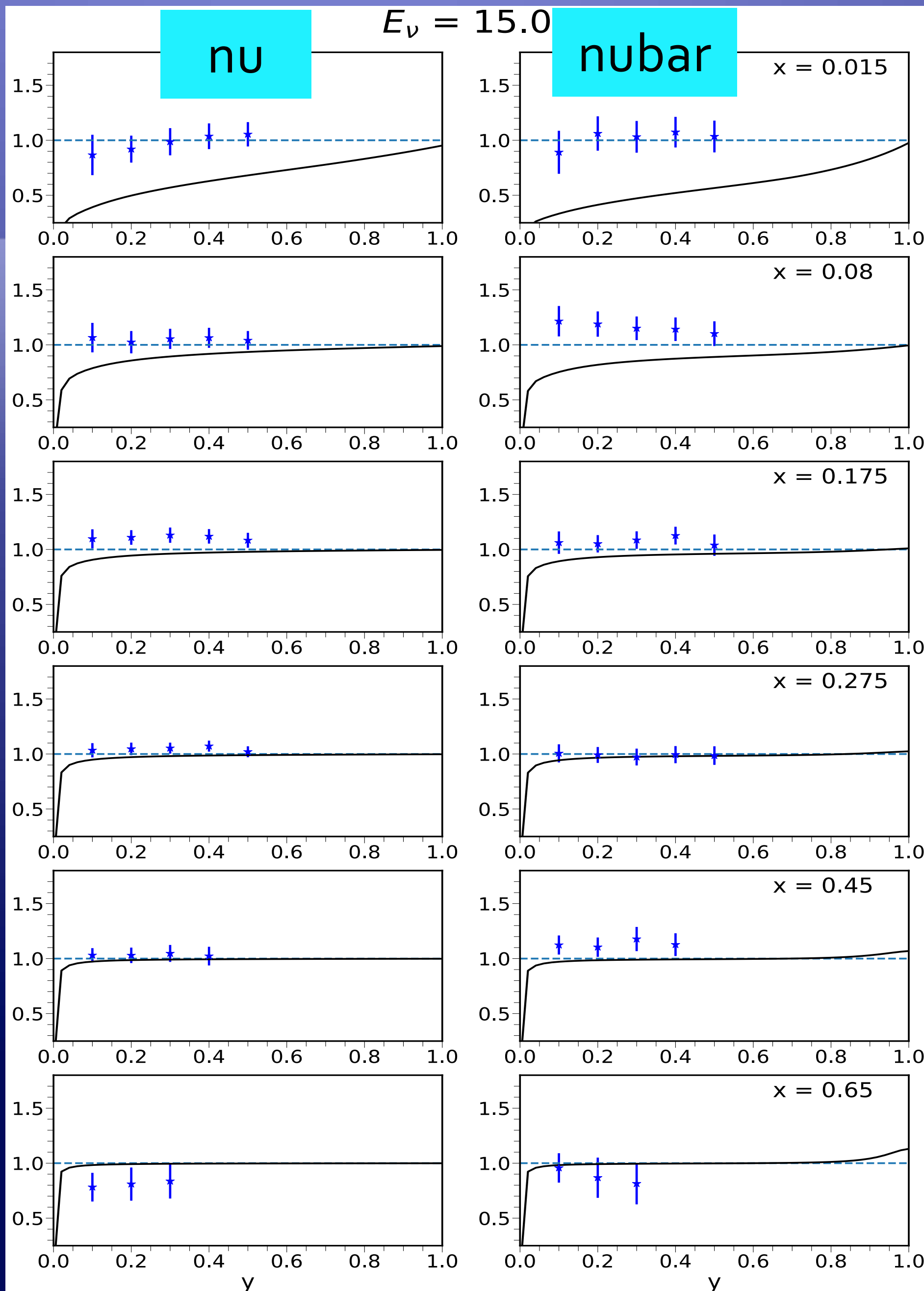
$$2xF_1^{axial} = 2xF_1^{vector}$$

Small modification to GRV98 u and d quark sea

To better describe ratio of antineutrino and neutrino cross sections increase GRV98 u and d sea by 5% and decrease valence quarks by same amount, thus leaving $F_2(x, Q^2)$ unchanged, but slightly increasing antineutrino cross sections.

$$
\begin{aligned}
d_{sea} &= 1.05\ d_{sea}^{grv98} \\
\bar{d}_{sea} &= 1.05\ \bar{d}_{sea}^{grv98} \\
u_{sea} &= 1.05\ u_{sea}^{grv98} \\
\bar{u}_{sea} &= 1.05\ \bar{u}_{sea}^{grv98} \\
d_{valence} &= d_{valence}^{grv98} - 0.05\ (d_{sea}^{grv98} + \bar{d}_{sea}^{grv98}) \\
u_{valence} &= u_{valence}^{grv98} - 0.05\ (u_{sea}^{grv98} + \bar{u}_{sea}^{grv98})
\end{aligned}
$$

Comparison with CCFR (Fe), CHORUS (Pb) data



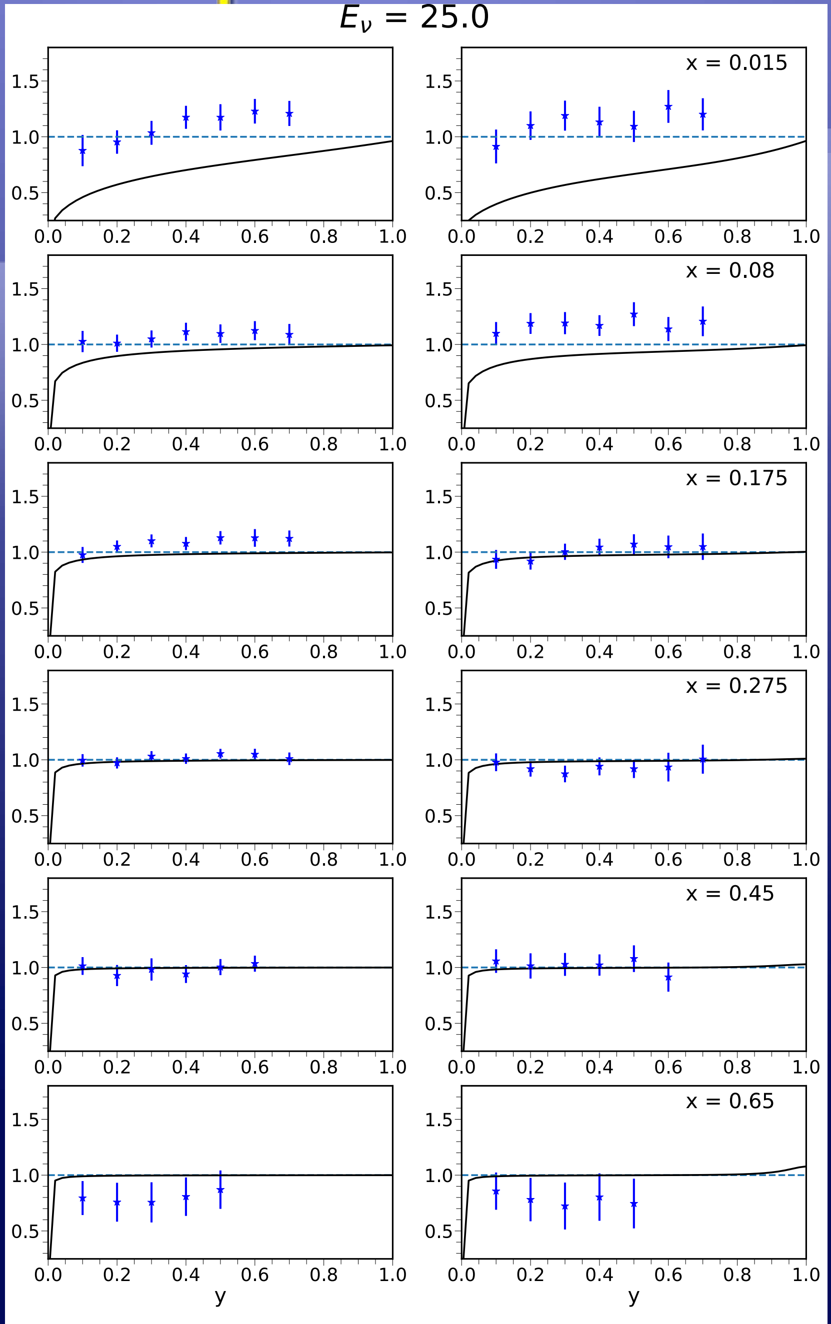
- Blue point: CHORUS/theory (type II)
- Solid line: **theory (type I V=A)/(type II A>V)**
- Red point: CCFR/theory (type II)

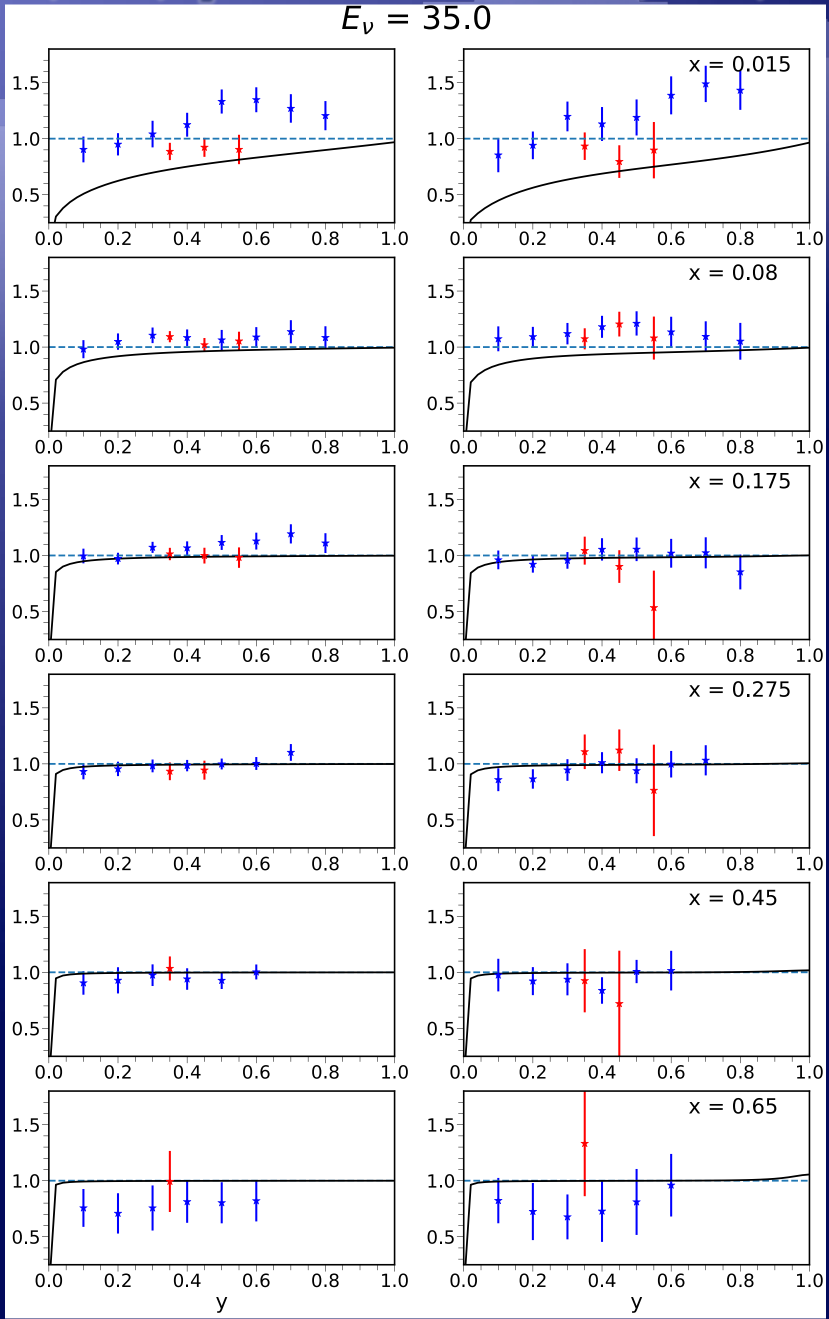
✓ Type I (Vector = Axial at low Q^2)
✓ Type II (Vector > Axial at low Q^2)
(Type II should be used)

Red point: CCFR/type II

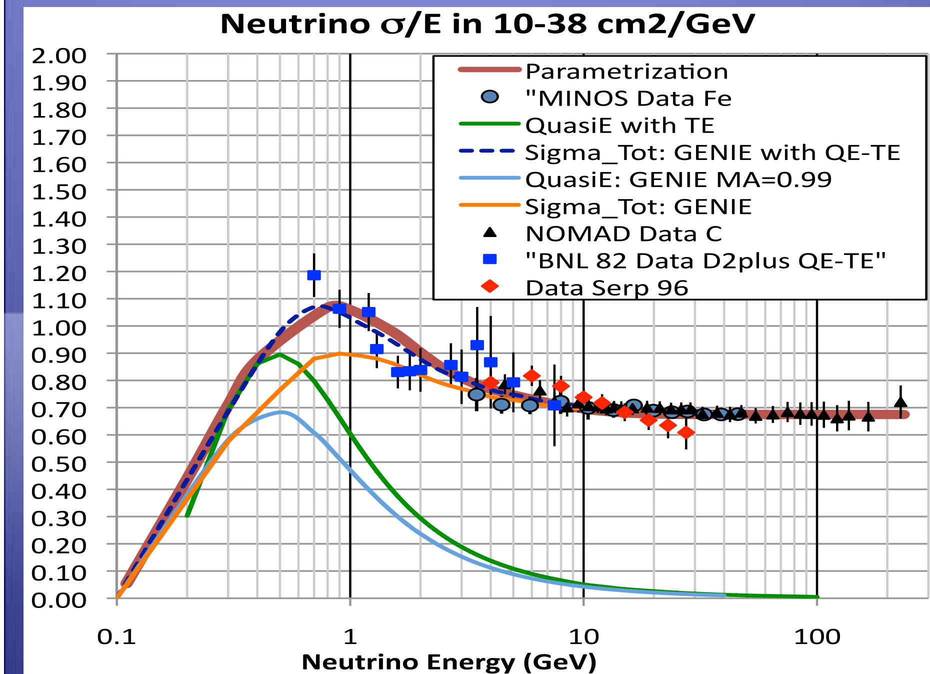
Blue point: CHORUS/type II

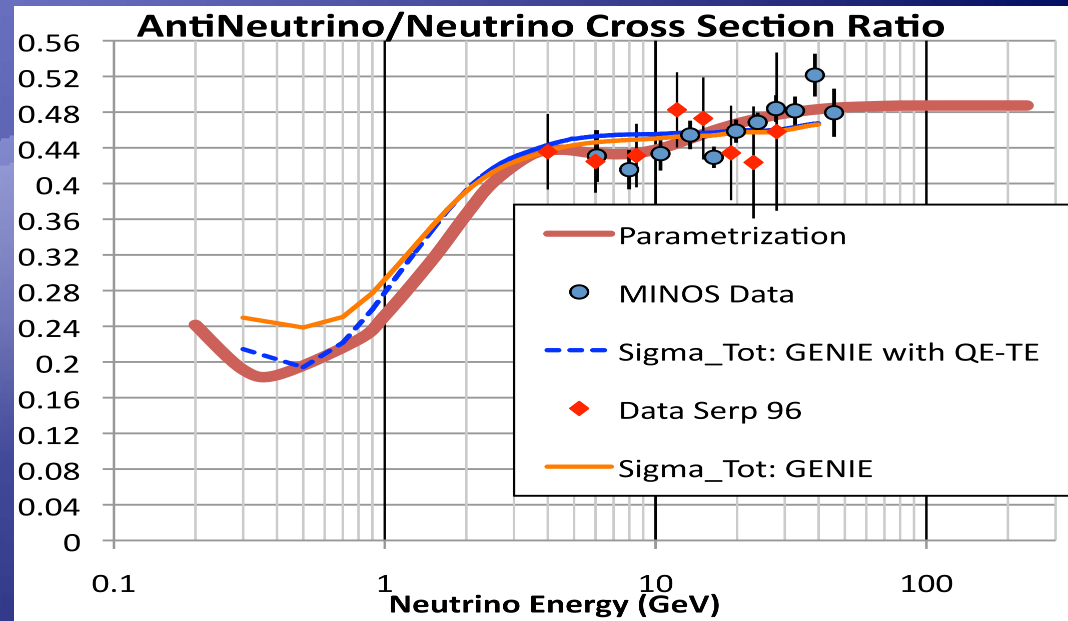
Comparison with CCFR(Fe) , CHORUS (Pb) data

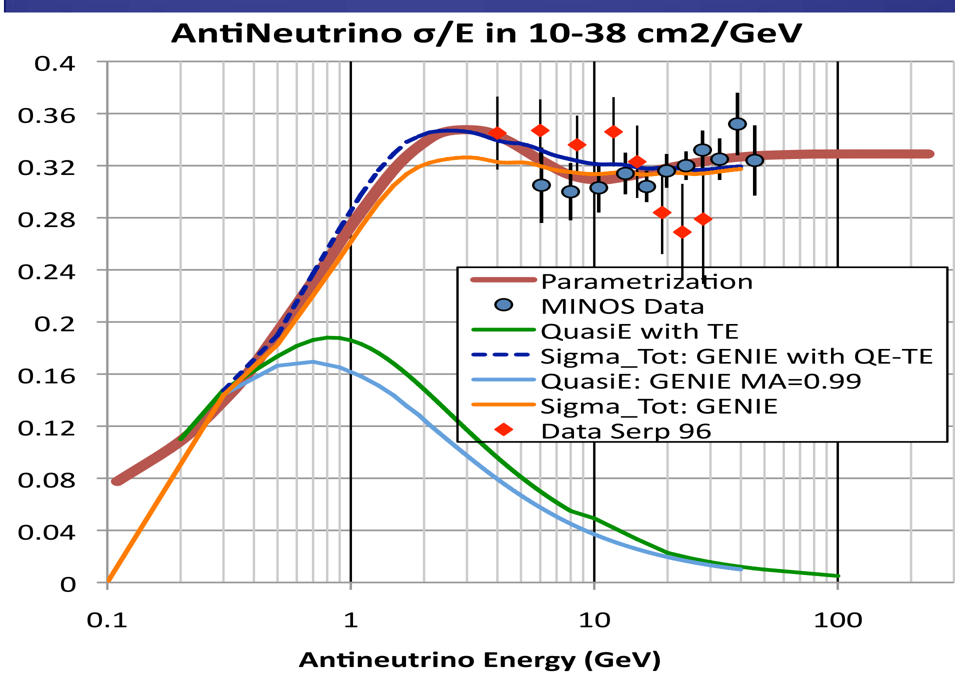




Neutrino and antineutrino total cross sections- Data

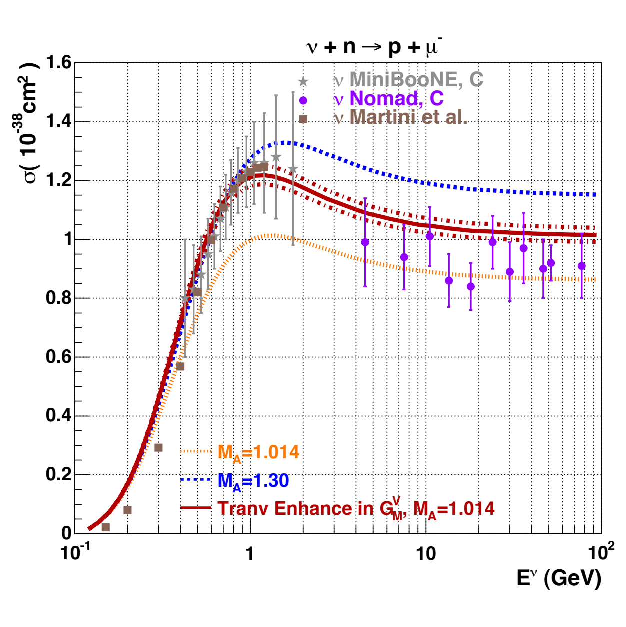


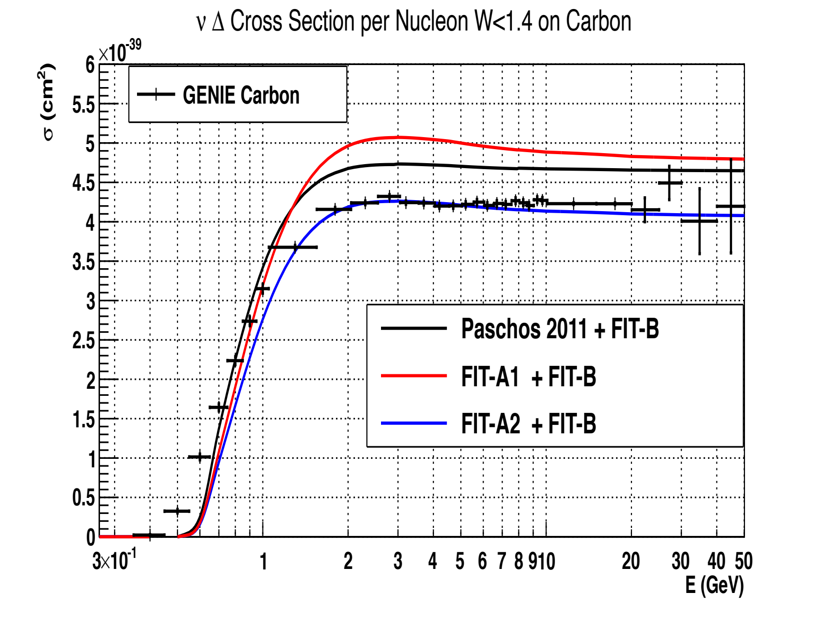


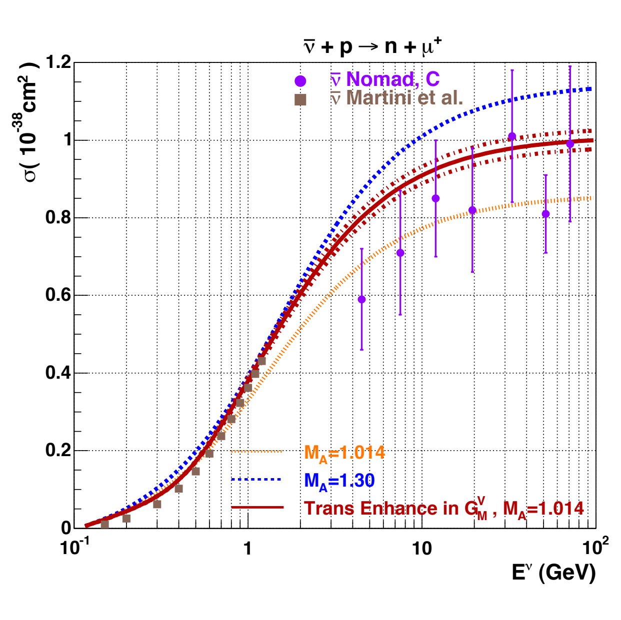


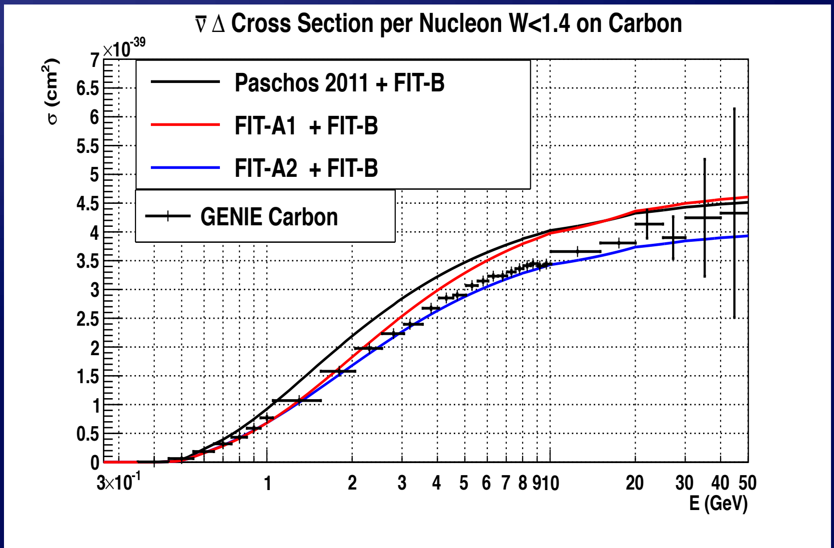
At 40 GeV the largest contribution to the total cross section comes from the $W > 1.4\ GeV$ region, with smaller contributions from resonance production and quasielastic scattering (≈3.5% for neutrinos and ≈7% for antineutrinos). Consequently, comparisons of our predicted cross section for $W > 1.4\ GeV$ (plus QE and Δ production cross sections) to total cross section data in this region provide a good test of the model.

To compare to total cross section data: use BY for W>1.4 GeV and add QE and Δ (W<1.4 GeV) cross section.









Total cross sections

- BY(DIS, W>1.4) + Q.E. + Resonance

At 40 GeV energy

	Type I (V=A)	Type II (A>V)	World Average
σ_ν /E	0.656 ±0.024	0.674 ±0.024	0.675 ±0.006
$\sigma_{\bar\nu}$/E	0.311 ± 0.016	0.327 ± 0.016	0.329 ± 0.011
$\sigma_{\bar\nu}/\sigma_\nu$	0.474 ± 0.012	0.487 ± 0.012	0.485 ± 0.005

source	change (error)	change in σ_ν	change in $\sigma_{\bar\nu}$	change in $\sigma_{\bar\nu}/\sigma_\nu$
R	+0.1	-1.3%	-2.7%	-1.4%
$f_{\overline{Q}}$	+5%	-0.4%	+0.9%	+1.4%
$K^{axial} - K^{vector}$	+ 50%	+1.3%	+2.4%	+1.1%
N	+3%	+3%	+3%	0
Total		±3.6%	±4.8%	±2.5%

Systematics

Summary & Discussions

- BY Effective LO model with ξw describe all e/μ DIS and resonance data as well as photo-production data (down to $Q^2=0$): provide a good reference for vector SF for neutrino cross section
- $d\sigma/dxdy$ data favor updated BY(DIS) type II model
- K factors for axial vectors in BY(DIS) type II model are based on PCAC and agree with CCFR F2 Q2=0 measurement.
- BY(DIS) type II model (low Q^2: axial>vector) provide a good reference for both neutrino and anti-neutrino cross sections (W>1.8).
- **Model also works on-average down to W>1.4 GeV, thus providing some overlap with resonance models (and should be used for W>1.8). It cannot be used for the Δ resonance since Δ has isospin 3/2 and quarks have isospin ½, so duality does not work for the Δ.**

Test of the Adler Sum Rule

- This sum rule should be valid at all values of Q^2

$$|F_V(Q^2)|^2 + \int_{\nu_0}^{\infty} \mathcal{W}_{2n-sc}^{\nu-vector}(\nu, Q^2)d\nu - \int_{\nu_0}^{\infty} \mathcal{W}_{2p-sc}^{\nu-vector}(\nu, Q^2)d\nu = 1$$

$$|\mathcal{F}_A(Q^2)|^2 + \int_{\nu_0}^{\infty} \mathcal{W}_{2n-sc}^{\nu-axial}(\nu, Q^2)d\nu - \int_{\nu_0}^{\infty} \mathcal{W}_{2p-sc}^{\nu-axial}(\nu, Q^2)d\nu = 1$$