

Towards a precise calculation of Neff in the SM

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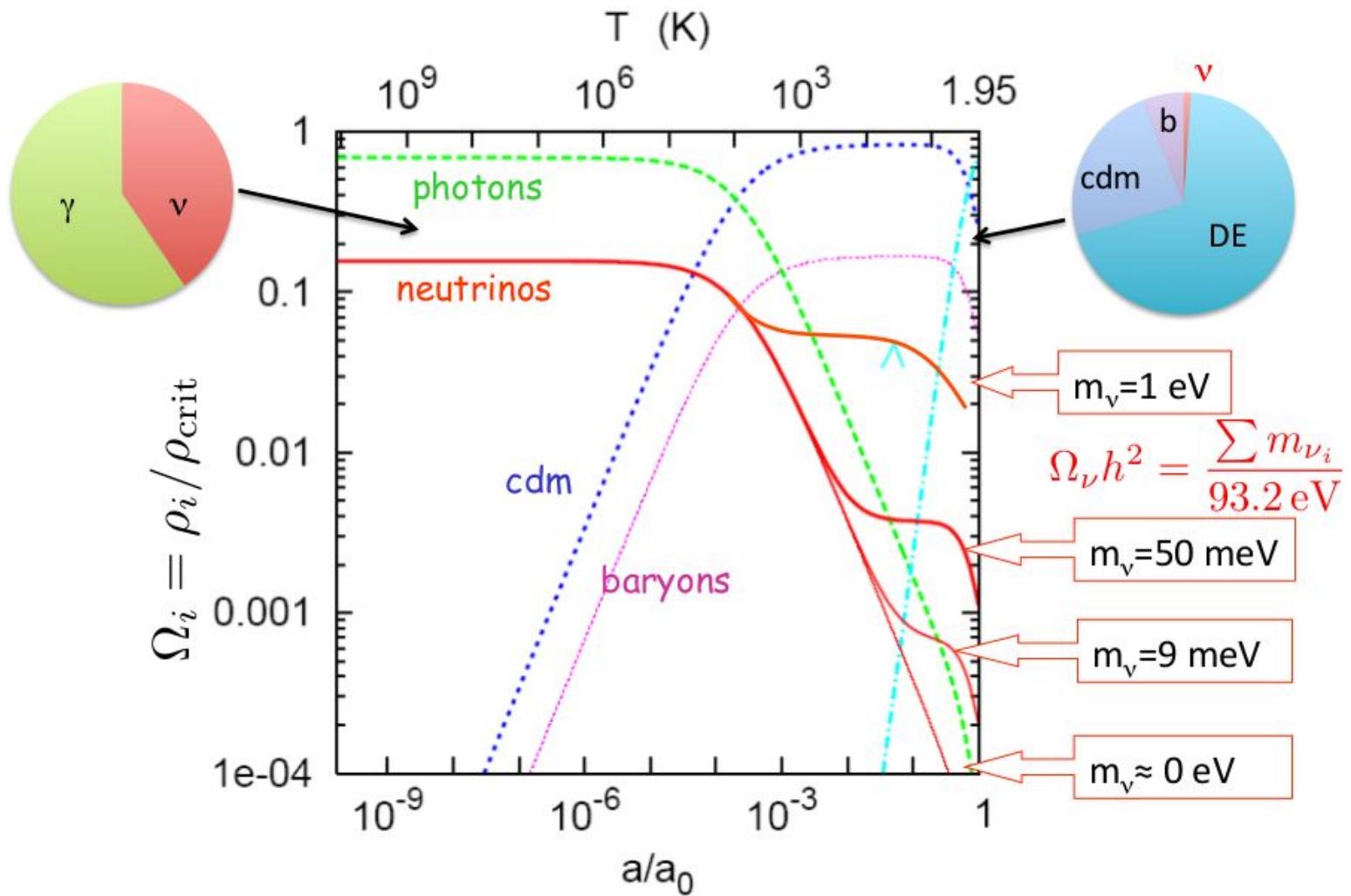
NuCo 2021: Neutrinos en Colombia
2021 – 30th July

Talk based on [arXiv:2012.02706](https://arxiv.org/abs/2012.02706),
in collaboration with

**J.J. Bennett, G. Buldgen, M. Drewes,
S. Gariazzo, S. Pastor and Y.Y.Y. Wong**



Neutrinos in cosmology



Effective number of neutrinos

N_{eff} accounts for any contribution to radiation other than photons

$$\rho_r = \rho_\gamma + \rho_\nu + \rho_X$$

.....

counted as standard neutrinos decoupled in an **ideal scenario**:

- All particle species behave as ideal gases
- Neutrinos decouple instantaneously
- Electrons and positrons are ultra-relativistic at neutrino decoupling

$$\rho_r = \left(1 + N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right) \rho_\gamma$$

In this ideal scenario, **N_{eff} = 3**

Effective number of neutrinos

Planck 2018, 95% CL [arXiv:1807.06209]

$$N_{\text{eff}} = 3.00^{+0.57}_{-0.53} \quad \text{TT + lowE}$$

$$N_{\text{eff}} = 2.92^{+0.36}_{-0.37} \quad \text{TT, TE, EE + lowE}$$

$$N_{\text{eff}} = 2.89^{+0.36}_{-0.38} \quad \text{TT, TE, EE + lowE + lensing}$$

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \quad \text{TT, TE, EE + lowE + lensing + BAO}$$

In the **standard scenario** (not ideal, but only neutrinos), **N_{eff} ≈ 3**

Neff: standard theoretical value

Long history of works towards a precise calculation of Neff.

Updated standard theoretical value

$$N_{\text{eff}} = 3.0440 \pm 0.0002$$

...

M. Escudero, arXiv:2001.04466

K. Akita and M. Yamaguchi, arXiv:2005.07047

J. Froustey, C. Pitrou and M.C. Volpe, arXiv:2008.01074

J.J. Bennett, G. Buldgen, PFdS, M. Drewes, S. Gariazzo, S. Pastor and Y.Y.Y. Wong, arXiv:2012.02706

R.S.L. Hansen, S. Shalgar and I. Tamborra, arXiv:2012.03948

(and a long, long list of other studies that can be found in the reference lists within these)

Deviation from $N_{\text{eff}} = 3$ with only neutrinos

- Neutrino decoupling and e^+e^- annihilation
- Finite temperature QED corrections
- Neutrino oscillations

STANDARD

- Non-standard neutrino-electron interactions (NSI)
- Very low reheating scenarios ($N_{\text{eff}} < 3$)
- Extra (sterile) neutrino species
- Neutrinos without a Fermi-Dirac distribution

NON-STANDARD

Neutrino decoupling and e^+e^- annihilations

Instantaneous decoupling approximation

$$T = T_\gamma = T_\nu$$

$$f_\nu = \frac{1}{\exp(p/T) + 1}$$

10 MeV

ν decoupling

1 MeV

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

$$f_\nu = \frac{1}{\exp(p/T_\nu) + 1}$$

nucleosynthesis

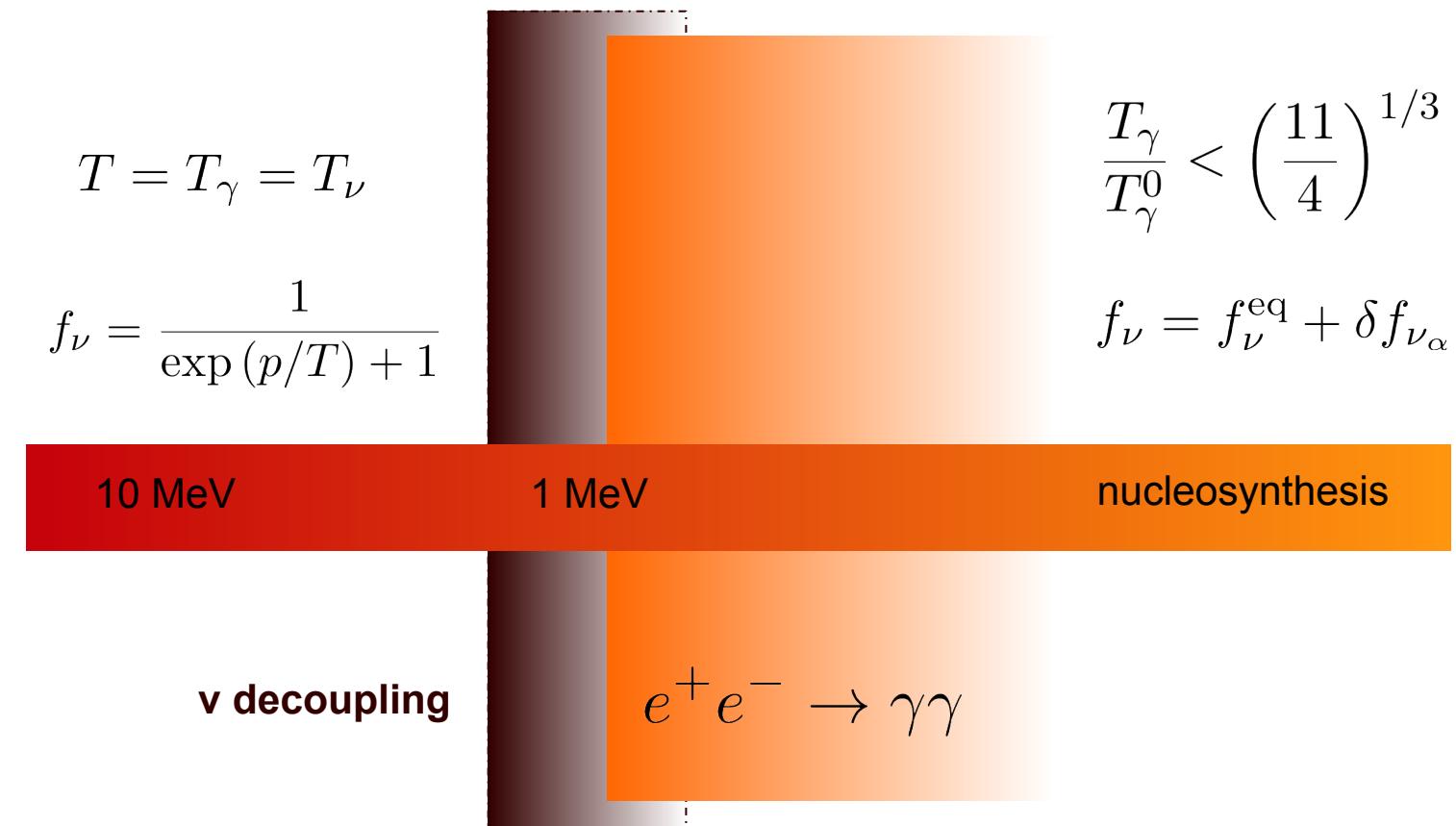


$$\rho_{\text{BE}} = g \frac{\pi^2}{30} T^4$$

$$\rho_{\text{FD}} = \frac{7}{8} g \frac{\pi^2}{30} T^4$$

Neutrino decoupling and e^+e^- annihilation

- e^+e^- are not ultra-relativistic at neutrino decoupling
- Neutrino decoupling is not instantaneous

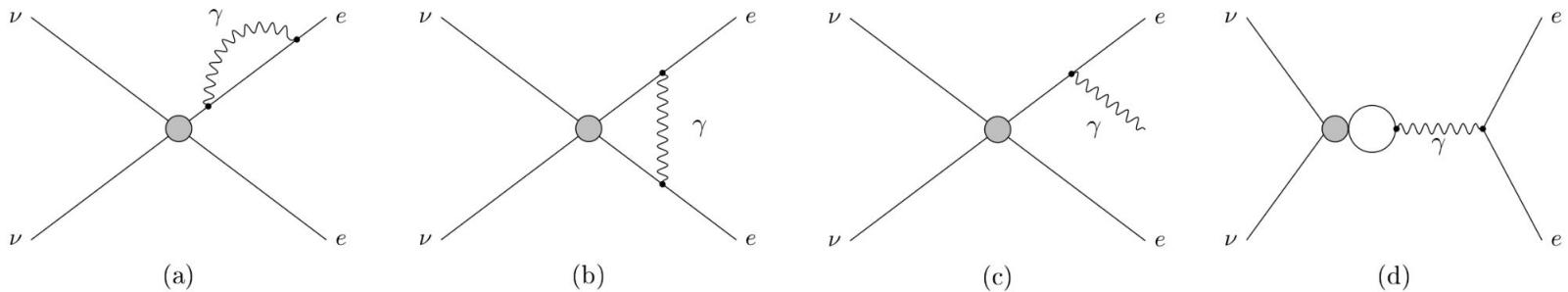


Finite-temperature QED corrections

- Altered equation of state for the QED plasma

$$\ln Z^{(2)} + \ln Z^{(3)} = -\frac{1}{2} \text{ (loop diagram)} + \frac{1}{2} \left[\frac{1}{2} \text{ (loop diagram)} - \frac{1}{3} \text{ (loop diagram)} + \frac{1}{4} \text{ (loop diagram)} + \dots \right]$$

- Correction to the scattering rates of QED particles with neutrinos



G. Mangano et al, PLB 534 (2002) 8-16

A. Heckler, PRD 49 (1994) 611-617

N. Fornengo et al, PRD 56 (1997) 5123

O. Tomalak et al, PRD 101 (2020) 033006

J.J. Bennett et al, JCAP 03 (2020) 003

Finite-temperature QED corrections

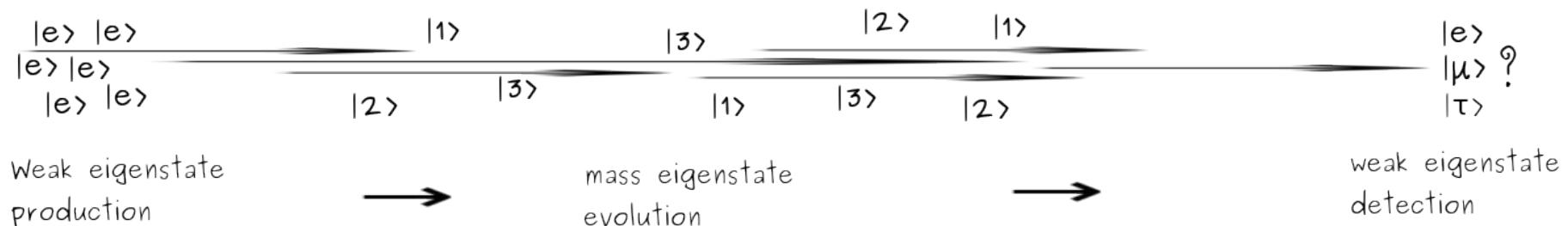
Main effects

- Alter equation of state (EoS) for the QED plasma $\rho_{\text{QED}}, P_{\text{QED}}$
- Correction to the scattering rates of QED particles with neutrinos
 $\mathbb{H}_{\text{QED}} \text{ matter effects}, \mathcal{I}_{\nu e}[\varrho]$

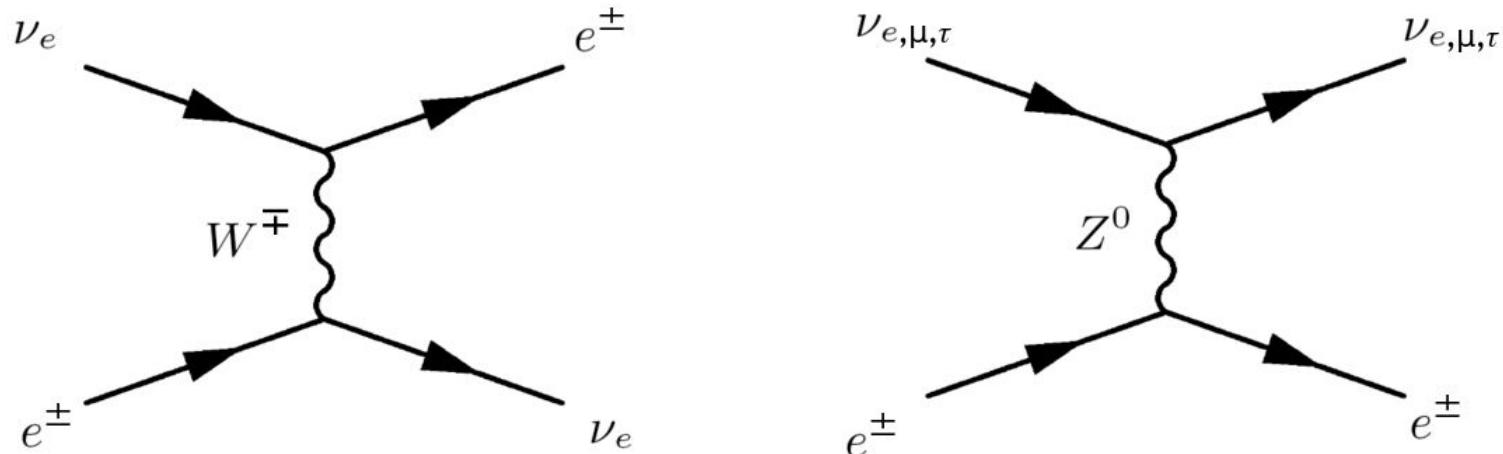
Main additional corrections included for the EoS

- $\mathcal{O}(e^2)$ momentum-dependent logarithmic term
- $\mathcal{O}(e^3)$ correction

Neutrino oscillations



N_{eff} affected by oscillations because **electron neutrinos** interact via **CC** apart from **NC**



Leading-digit contributions

Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Important note: non-instantaneous decoupling is not the same as non-relativistic (m_e/T_d) correction

Standard value of Neff

Key aspects of our updated calculation:

- Subdominant finite-temperature QED corrections up to $\mathcal{O}(e^3)$
[J.J. Bennett, G. Buldgen, M. Drewes, and Y. Y. Y. Wong, arXiv:1911.04504]
- Complete neutrino-neutrino integrals in the presence of flavour oscillations
- Thorough assessment of uncertainties
- FortEPiano https://bitbucket.org/ahep_cosmo/fortepiano_public
[S. Gariazzo, PFdS, S. Pastor, arXiv:1905.11290]

$$N_{\text{eff}} = 3.0440 \pm 0.0002$$

[J.J. Bennett, G. Buldgen, PFdS, M. Drewes, S. Gariazzo, S. Pastor, and Y.Y.Y. Wong, arXiv:2012.02706]

The resulting Neff coincides with that of other recent studies within the respective estimated uncertainties

Assessment of uncertainties

- Measurement uncertainties of physical parameters
- Numerical errors (discretisation and initialisation)
- Error of using an (optional) approximate modelling of collision integrals

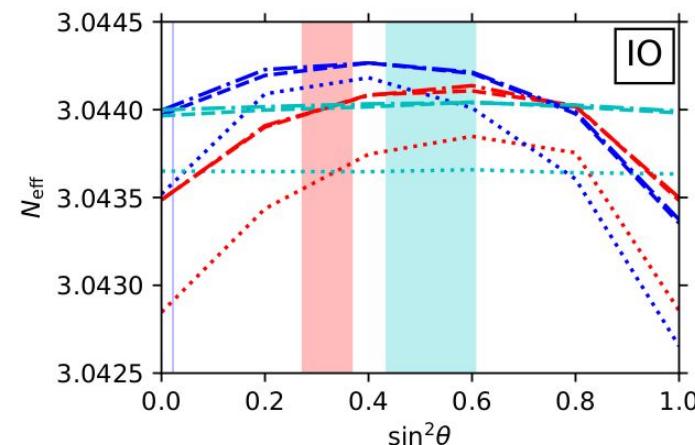
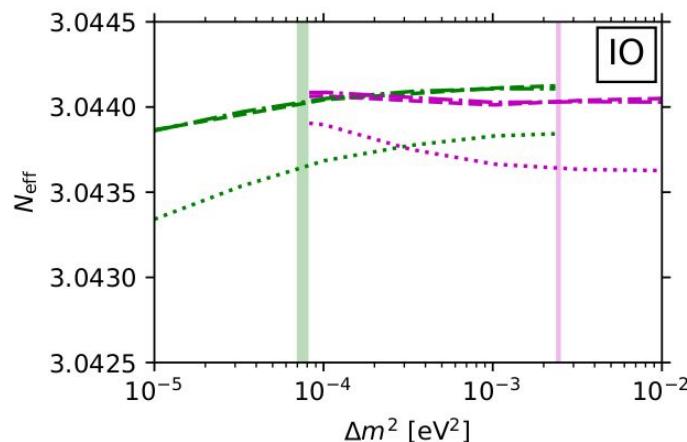
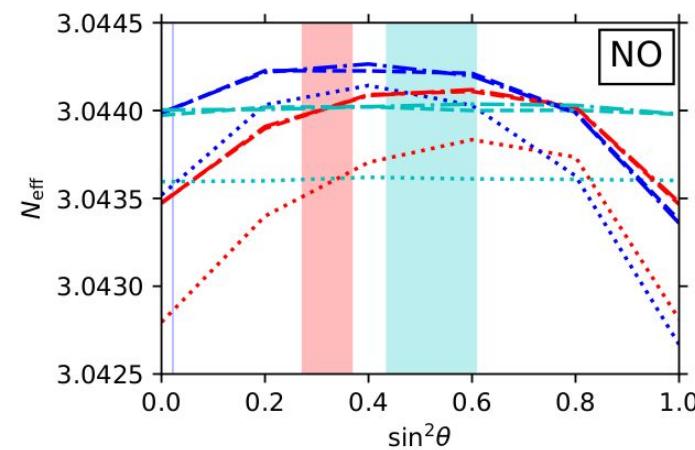
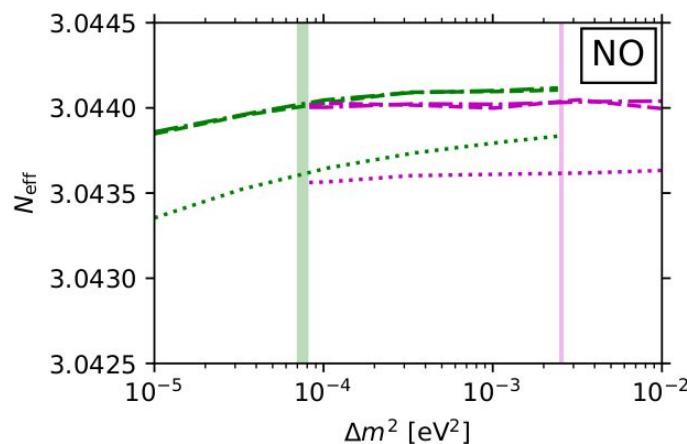
Oscillation parameters

..... no $\nu\nu$
 - - - $\nu\nu$, GL
 - - - $\nu\nu$, NC

— $\sin^2\theta_{12}$
 — $\sin^2\theta_{13}$
 — $\sin^2\theta_{23}$

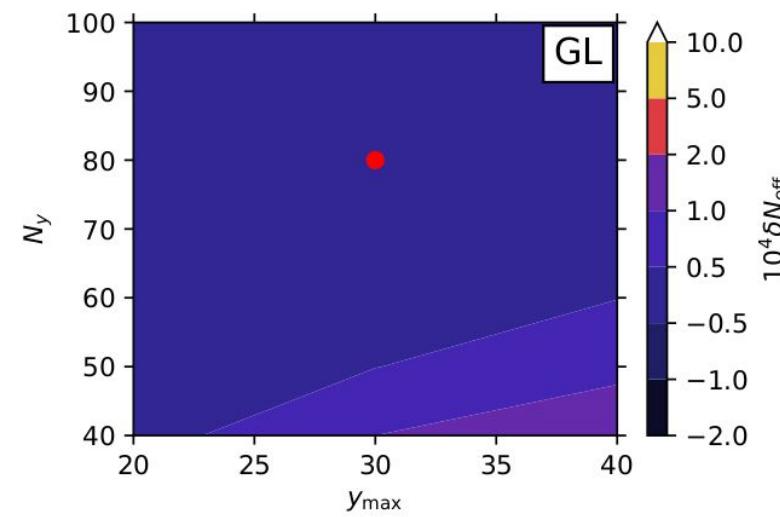
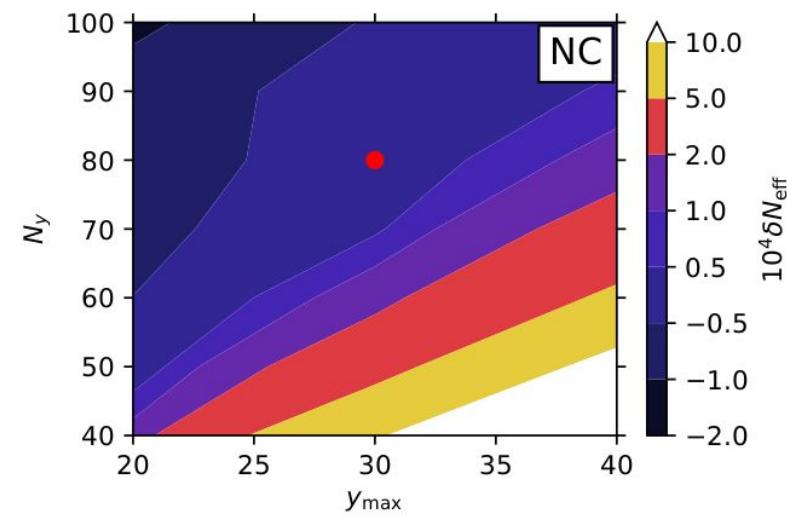
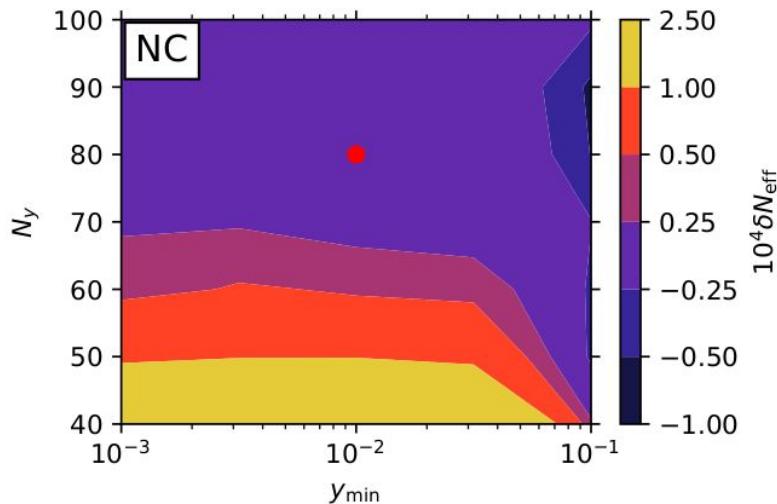
Δm^2_{21}
 Δm^2_{31}

Bands are 3-sigma ranges from Valencia Global Fit, arXiv:2006.11237



Numerical convergence

Comoving neutrino momentum
 $y \equiv a p$



Quadrature method:
NC: Newton-Cotes
GL: Gauss-Laguerre

Standard value of N_{eff}

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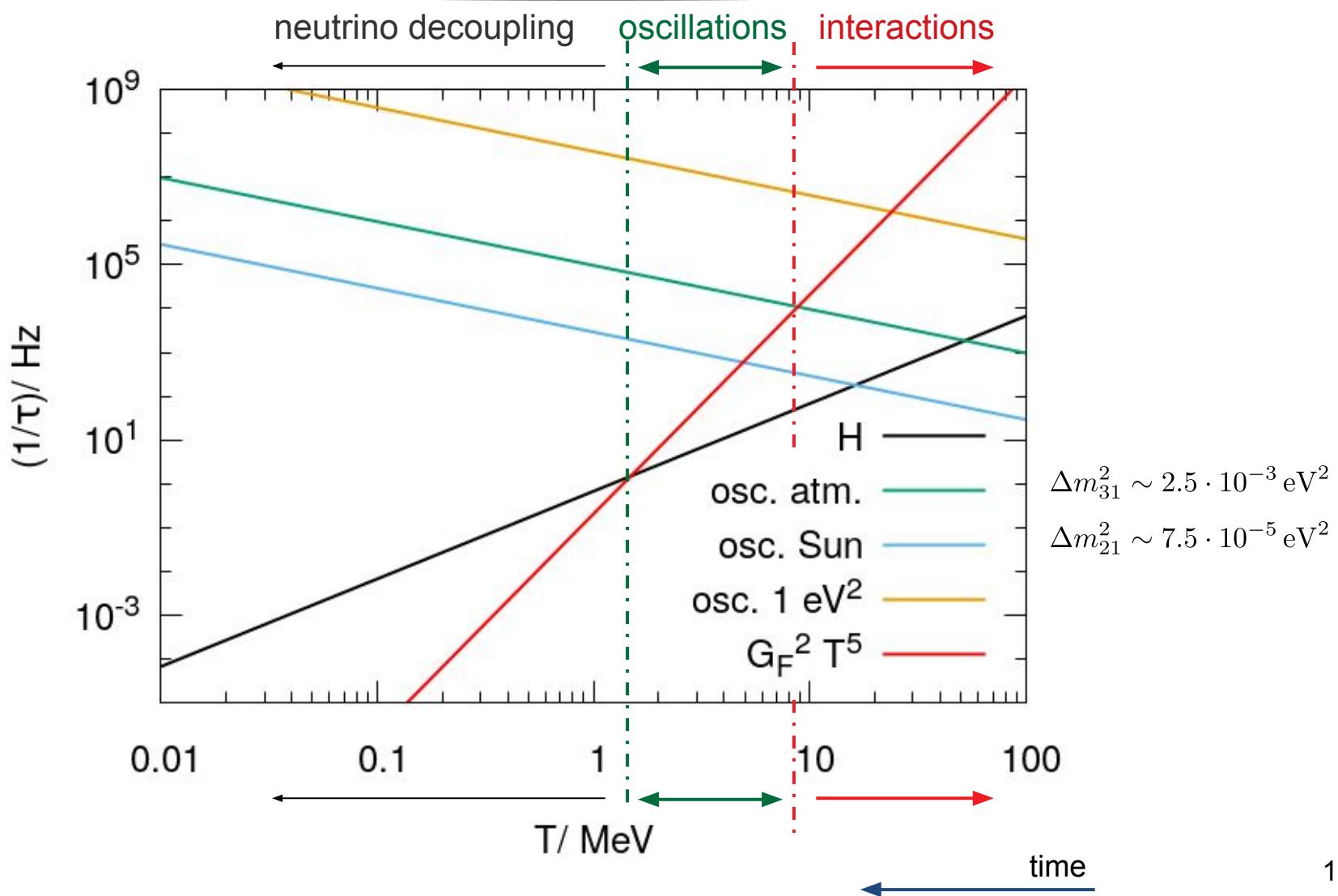
[J.J. Bennett, G. Buldgen, PFdS, M. Drewes, S. Gariazzo, S. Pastor, and Y.Y.Y. Wong, arXiv:2012.02706]

Error estimated from

- ~0.0001 numerical
- ~0.0001 from measurement of solar angle of neutrino oscillations

Additional content

Interactions + oscillations + expanding Universe



Calculating neutrino decoupling

Density matrix formalism

$$\varrho_p = \begin{pmatrix} f_{\nu_e} & A & B \\ A^* & f_{\nu_\mu} & C \\ B^* & C^* & f_{\nu_\tau} \end{pmatrix}$$

f_ν : occupational numbers

A, B, C : phase information, non-zero if oscillations are considered

Continuity equation

$$\frac{d\rho}{dt} = -3H(\rho + P)$$

Solve Boltzmann equations with $\mathbf{l}_{\text{coll}} \neq 0$

$$(\partial_t - H p \partial_p) \varrho_p = -i [(\mathbb{H}_{\text{vac}} + \mathbb{H}_{\text{mat}}), \varrho_p] + \mathcal{I}_{\text{coll}}[\varrho_p]$$

Diagram illustrating the components of the Boltzmann equation:

- Expanding Universe** (represented by a dashed orange circle)
- Vacuum** (represented by a dashed orange line)
- Matter** (represented by a dashed orange line)
- Oscillations** (represented by a double-headed arrow between the Vacuum and Matter regions)
- Interactions** (represented by a dashed blue rectangle)

Comoving equations

Comoving variables

$$x \equiv a m_e, \quad y \equiv a p, \quad z \equiv a T_\gamma$$

Continuity equation

$$\frac{d}{dx} \bar{\rho}_{\text{tot}}(x, z(x)) = \frac{1}{x} [\bar{\rho}_{\text{tot}}(x, z(x)) - 3\bar{P}_{\text{tot}}(x, z(x))]$$

$$\bar{A} = (x/m_e)^4 A$$

$$\rho_{\text{tot}} = \rho_{\text{QED}} + \rho_\nu$$

$$P_{\text{tot}} = P_{\text{QED}} + P_\nu$$

Boltzmann equation

$$\frac{d\varrho(x, y)}{dx} = \frac{m_e^3}{x^4} \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\bar{\rho}_{\text{tot}}}} (-i [\mathbb{H}(x, y, z(x)), \varrho(x, y)] + \mathcal{I}[\varrho(x, y)])$$

Collision integrals

When oscillations are added we need to solve 9 collision integrals, one for each entry of the density matrix

$$\mathcal{I}_{\text{coll}} \propto \frac{1}{E_1} \int (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) F(f_{e^\pm}, \varrho, G^{R,L}) S |\mathcal{M}|_{12 \rightarrow 34}^2 \prod_{i=2}^4 \frac{d^3 \vec{p}_i}{2E_i (2\pi)^3}$$

Statistical factor

Strength of interactions

Procedure

- Analytically reduce the integrals from 9 to 2 dimensions
- Solve numerically with a grid on the incoming neutrino momentum

Uncertainties of physical parameters

	Parameter [Units]	Value $\pm 1\sigma$ uncertainty	Reference
QED	$\alpha/10^{-3}$	$7.2973525693 \pm 0.0000000011$	[49]
	m_e [MeV]	$0.51099895000 \pm 0.00000000015$	[49]
Weak	$\sin^2 \theta_W$	0.23871 ± 0.00009	[50, 51]
	g_L	0.727	[50, 51]
	\tilde{g}_L	-0.273	[50, 51]
	g_R	0.233	[50, 51]
	G_F [10^{-5} GeV $^{-2}$]	1.1663787 ± 0.0000006	[52]
	m_W [GeV]	80.379 ± 0.012	[52]
Neutrino	$\sin^2 \theta_{12}/10^{-1}$	3.18 ± 0.16	[33]
	$\sin^2 \theta_{13}/10^{-2}$	$2.200^{+0.069}_{-0.062}$	[33]
	$\sin^2 \theta_{23}/10^{-1}$	5.74 ± 0.14	[33]
	Δm_{21}^2 [10^{-5} eV 2]	$7.50^{+0.22}_{-0.20}$	[33]
	Δm_{31}^2 [10^{-3} eV 2] (NO)	$2.55^{+0.02}_{-0.03}$	[33]
	Δm_{31}^2 [10^{-3} eV 2] (IO)	$-2.45^{+0.03}_{-0.02}$	[33]

Table and references from Table 1 in

J.J. Bennett, G. Buldgen, PFdS, M. Drewes, S. Gariazzo, S. Pastor, and Y.Y.Y. Wong, arXiv:2012.02706

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The weak angle corresponds to $\sin^2 \theta_W(0)_{\overline{\text{MS}}}$ in the modified minimal subtraction scheme, established via SM renormalization group running from the measured value of $\sin^2 \theta_W(m_Z)_{\overline{\text{MS}}} = 0.23124 \pm 0.00006$ at the Z-pole.

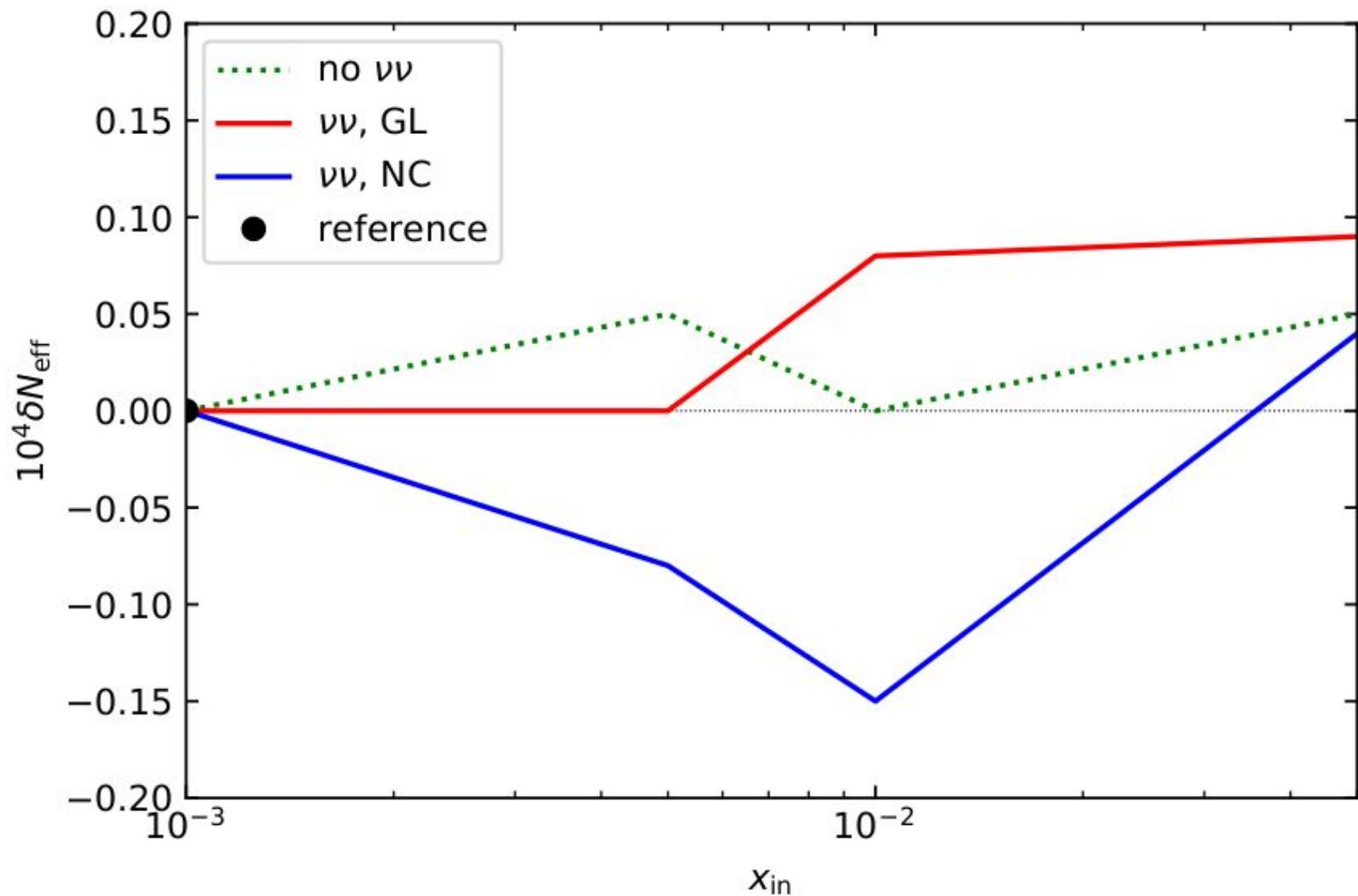
[49] E. Tiesinga et al., CODATA 2018, <https://physics.nist.gov/constants>

[50] K. Kumar et al., arXiv:1302.6263

[51] J. Erler & S. Su, arXiv:1303.5522

[52] PDG 2020

Numerical convergence



Numerical convergence

N_y grid	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha}$	N_y	$N_{\text{eff}}^{\text{SM}}$ (no osc)	$N_{\text{eff}}^{\text{SM}}$ (NO)
GL	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	40	3.0426	3.0436
		60	3.0426	3.0435
		80	3.0425	3.0435
	Diagonal ϱ	40	3.0434	3.0442
		60	3.0433	3.0441
		80	3.0433	3.0441
	Full	40	3.0434	3.0439
		60	3.0433	3.0439
		80	3.0433	3.0439
NC	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	40	3.0428	3.0438
		60	3.0426	3.0436
		80	3.0426	3.0436
	Diagonal ϱ	40	3.0436	3.0444
		60	3.0434	3.0442
		80	3.0433	3.0442
	Full	40	3.0436	3.0441
		60	3.0434	3.0439
		80	3.0433	3.0439

Numerical convergence

	$N_{\text{eff}}^{\text{SM}}$ (no osc)	$N_{\text{eff}}^{\text{SM}}$ (NO)	$N_{\text{eff}}^{\text{SM}}$ (IO)
Benchmark A — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$			
Assuming:			
<ul style="list-style-type: none"> • (2)ln + (2)ln + (3)+ type (a) weak rates • Damping for $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$ • $N_y = 60$, $y_{\max} = 20$, NC linearly spaced y_i 	3.04263	3.04360	3.04361
Alternative estimates			
Momentum grid			
$N_y = 40$, $y_{\max} = 20$, GL spacing of y_i nodes	3.04261	3.04355	3.04360
Integrals for off-diagonal $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$			
$N_y = 60$, $y_{\max} = 20$, NC linearly spaced y_i	3.04261	3.04357	3.04362
$N_y = 40$, $y_{\max} = 20$, GL spacing of y_i	3.04261	3.04357	3.04364
Finite-temperature QED corrections			
(2)ln	3.04361	3.04458	
(2)ln + (2)ln	3.04358	3.04452	
(2)ln + (3)	3.04264	3.04361	
(2)ln + (2)ln + (3)	3.04263	3.04360	

Numerical convergence

	$N_{\text{eff}}^{\text{SM}}$ (no osc)	$N_{\text{eff}}^{\text{SM}}$ (NO)	$N_{\text{eff}}^{\text{SM}}$ (IO)
Benchmark B — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$			
Assuming:			
<ul style="list-style-type: none"> • (2)lh + (2) ln + (3)+ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\max} = 30$, NC linearly spaced y_i 	3.04341	3.04398	3.04399
Alternative estimates			
Momentum grid			
$N_y = 80$, $y_{\max} = 30$, GL spacing of y_i	3.04334	3.04392	3.04392
$N_y = 80$, $y_{\max} = 20$, NC linearly spaced y_i	3.04334	3.04389	3.04391
$N_y = 80$, $y_{\max} = 20$, GL spacing of y_i	3.04334	3.04386	3.04393
Off-diagonal collision terms			
Damping terms, NC quadrature		3.04408	
Damping terms, GL quadrature		3.04399	
Neutrino–neutrino collision integral - $y_{\max} = 20$			
Diagonal ϱ	3.04333	3.04416	
Full ϱ , interpolate ϱ /FD only in diagonal	3.04334	3.04389	
Full ϱ , interpolate ϱ /FD also in off-diagonal	3.04334	3.04389	

Conservation tests

