

Radiative neutrino mass models.

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Based on: AECH, J. W. F. Valle and C. A. Vaquera-Araujo, Phys. Lett. B **809**, 135757 (2020).

C. Arbeláez, AECH, R. Cepedello, S. Kovalenko and I. Schmidt, JHEP **06** (2020), 043

AECH, S. Kovalenko, M. Maniatis and I. Schmidt, 2104.07047
AECH, I. de Medeiros Varzielas, M. L. López-Ibáñez and A. Melis,
JHEP **05**, 215 (2021)

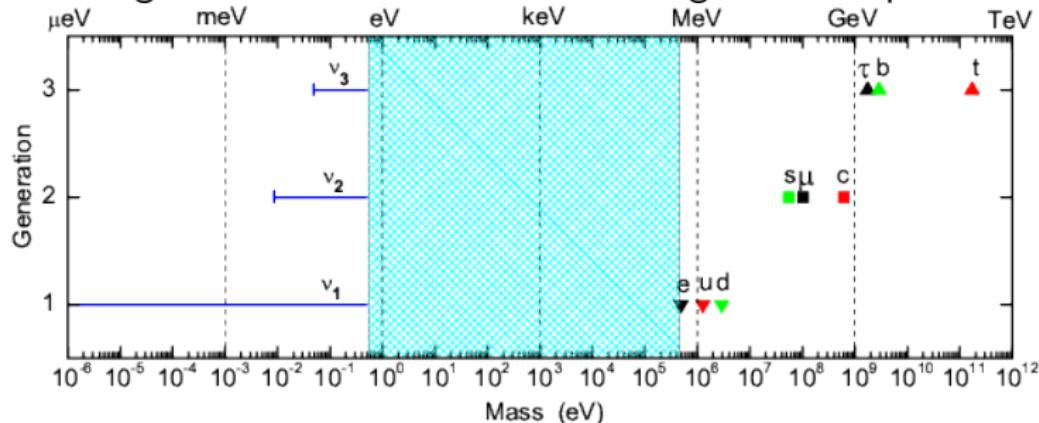
AECH, S. Kovalenko, F. S. Queiroz and Y. S. Villamizar, 2105.01731
A. Abada, N. Bernal, AECH, X. Marcano and G. Piazza, 2107.02803.

Overview

- 1 Introduction
- 2 Simple theory for scotogenic DM with residual symmetry
- 3 An extended 3-3-1 model with radiative linear seesaw mechanism.
- 4 Gauged Inverse Seesaw from Dark Matter.
- 5 The Cárcamo Hernández-Kovalenko-Schmidt (CKS) mechanism
- 6 Sequentially loop suppressed fermion masses at renormalizable level
- 7 An extended 3HDM Model with the CKS mechanism.
- 8 $\Delta(27)$ 3+1 Higgs Doublet Model.
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Introduction

The origin of fermion masses and mixings is not explained by the SM.

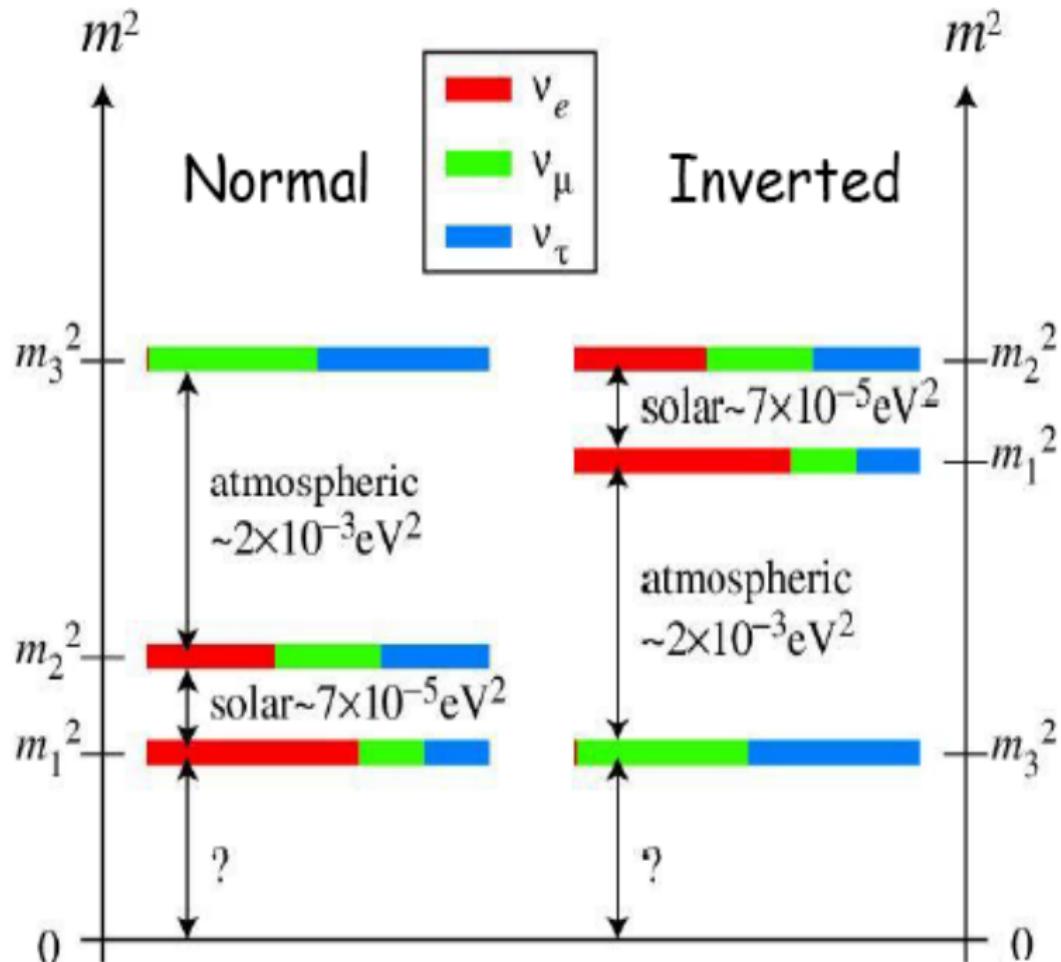


CKM

$$|V| = \begin{bmatrix} d & s & b \\ u & c & t \end{bmatrix}$$

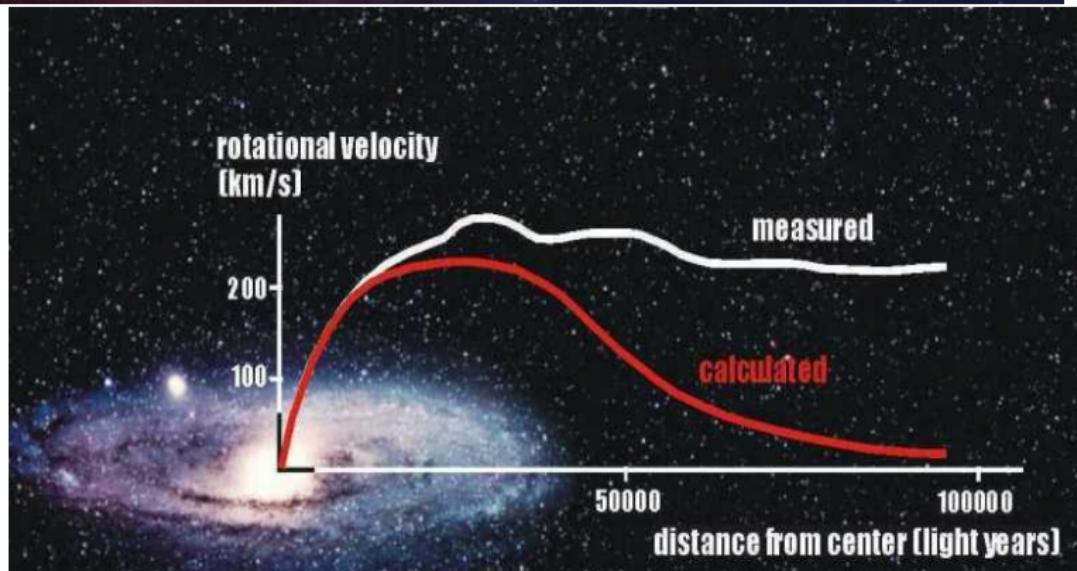
PMNS

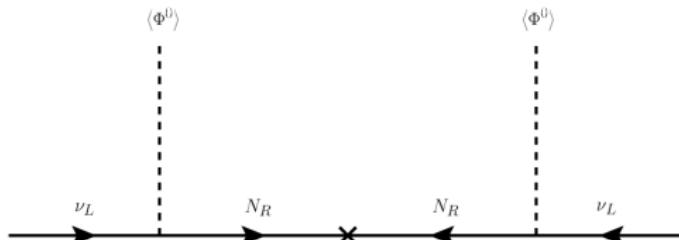
$$|U| = \begin{bmatrix} 1 & 2 & 3 \\ e & \mu & \tau \end{bmatrix}$$





Source: NASA





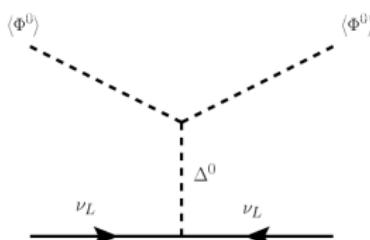
Type I seesaw

$$LHN \quad 2 \otimes 2 \otimes 1$$

Minkowski 1977, Gellman, Ramond, Slansky 1980

Glashow, Yanagida 1979, Mohapatra, Senjanovic 1980

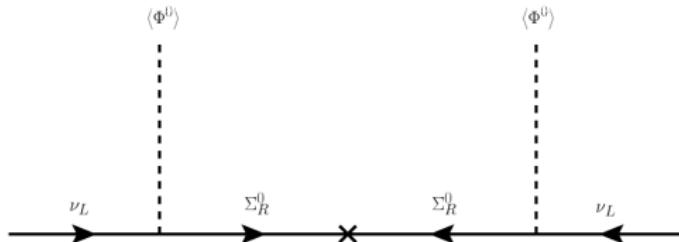
Lazarides Shafi Weterrick 1981, Schechter-Valle 1980 and 1982



Type II seesaw

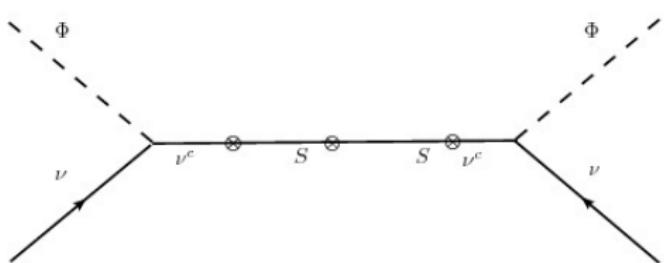
$$L\Delta L \quad 2 \otimes 3 \otimes 2$$

Schechter-Valle 1980 and 1982



Type III seesaw

$$LH\Sigma \quad 2 \otimes 2 \otimes 3$$



Inverse seesaw

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2} \left(\begin{array}{ccc} \overline{\nu_L^C} & \overline{N_R} & \overline{S_R} \end{array} \right) \mathbf{M}_\nu \left(\begin{array}{c} \nu_L \\ N_R^C \\ S_R^C \end{array} \right) + H.c$$

$$\mathbf{M}_\nu = \left(\begin{array}{ccc} 0_{3 \times 3} & \mathbf{M}_1 & \mathbf{M}_L \\ \mathbf{M}_1^T & 0_{3 \times 3} & \mathbf{M}_2 \\ \mathbf{M}_L^T & \mathbf{M}_2^T & \mu \end{array} \right)$$

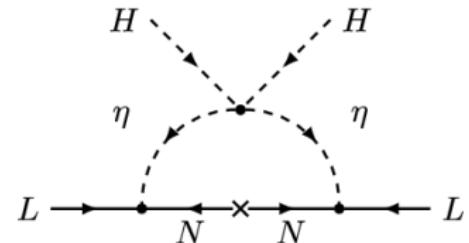
$$\mathbf{M}_L = 0_{3 \times 3}$$

$$Q_{\nu_L}^{U(1)_L} = Q_{S_R}^{U(1)_L} = -Q_{N_R}^{U(1)_L} = 1$$

$$\tilde{\mathbf{M}}_\nu = \mathbf{M}_1 (\mathbf{M}_2^T)^{-1} \mu \mathbf{M}_2^{-1} \mathbf{M}_1^T$$

$$\mathbf{M}_\nu^{(1)} = -\frac{1}{2} (\mathbf{M}_2 + \mathbf{M}_2^T) + \frac{1}{2} \mu$$

$$\mathbf{M}_\nu^{(2)} = \frac{1}{2} (\mathbf{M}_2 + \mathbf{M}_2^T) + \frac{1}{2} \mu$$



One loop Ma radiative seesaw model

η and N are odd under a preserved Z_2

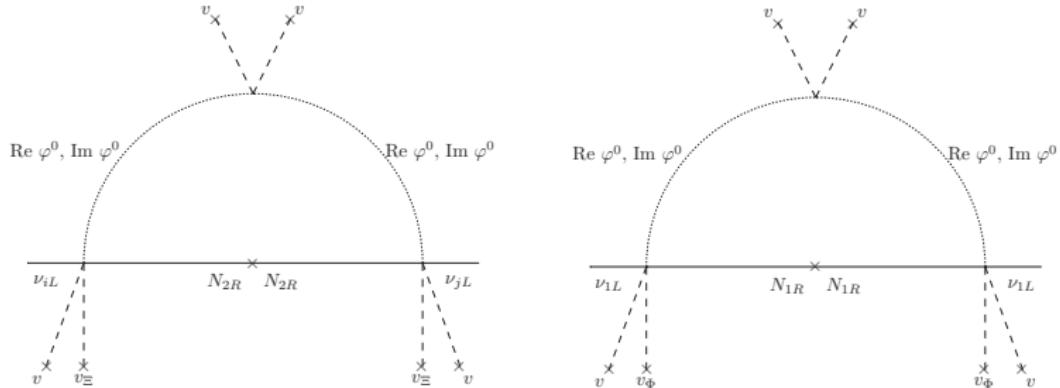
$$L \tilde{\eta} N, \frac{\lambda_5}{2} (H^\dagger \cdot \eta) + h.c$$



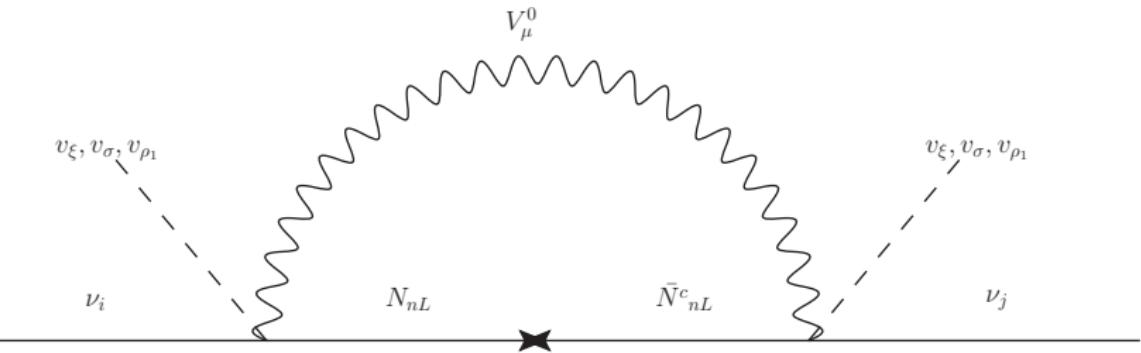
Linear seesaw:

$$\mu = 0_{3 \times 3}$$

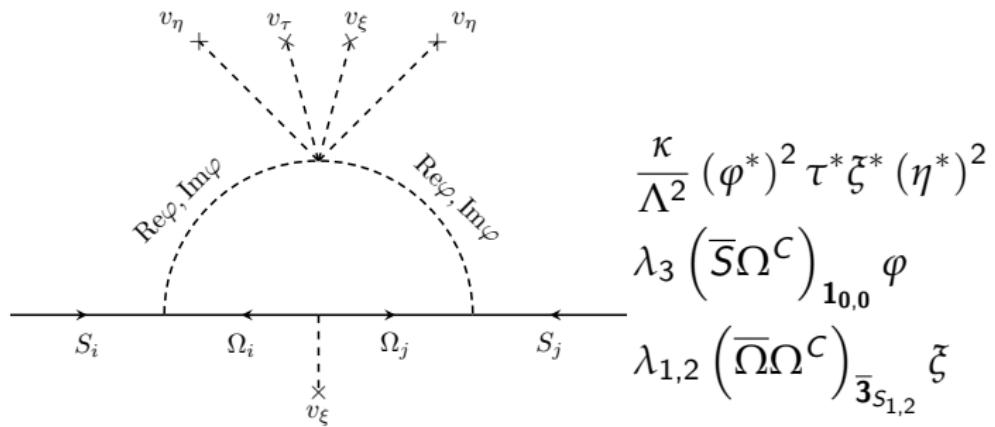
$$\tilde{\mathbf{M}}_\nu = -\mathbf{M}_L \mathbf{M}_2^{-1} \mathbf{M}_1^T - \mathbf{M}_1 (\mathbf{M}_2^T)^{-1} \mathbf{M}_L^T$$



One loop radiative seesaw with non renormalizable Dirac Yukawa terms,
 N. Bernal, A. E. Cárcamo Hernández, Ivo de Medeiros Varzielas and
 S. Kovalenko, JHEP **1805**, 053 (2018)

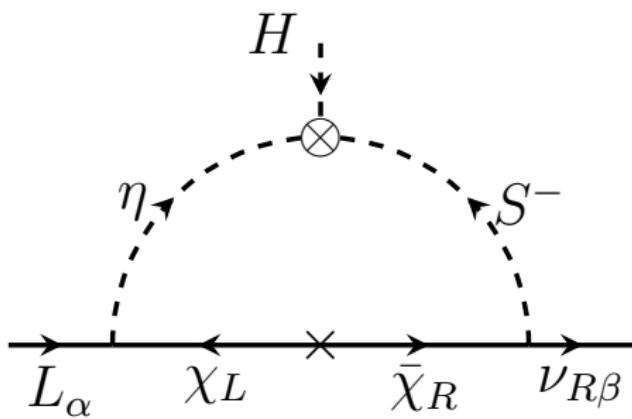


One loop radiative seesaw with non renormalizable interactions of the heavy vectors with Majorana neutrinos and scalar singlets, A. E. Cárcamo Hernández, J. Vignatti and A. Zerwekh, J. Phys. G **46**, no. 11, 115007 (2019)



One loop radiative inverse seesaw with non renormalizable scalar interactions, A. E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle and C. A. Vaquera-Araujo, arxiv:hep-ph/1705.06320, JHEP **07**, 118 (2017)

Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_4	Residual Z_2
L	(1, 2, -1/2)	1	1
e^c	(1, 1, 1)	1	1
ν_R	(1, 1, 0)	-1	1
H	(1, 2, 1/2)	1	1
(χ_L, χ_R)	(1, 1, 1)	(i, i)	(-1, -1)
η	(1, 2, 1/2)	i	-1
S^-	(1, 1, -1)	i	-1



Radiative type I seesaw, C. Arbeláez, AECH, R. Cepedello, M. Hirsch and S. Kovalenko, Phys. Rev. D **100**, no.11, 115021 (2019).

Simple theory for scotogenic DM with residual symmetry

Field	$SU(3)_c$	$SU(3)_L$	$U(1)_X$	$U(1)_N$	Q	$M_P = (-1)^{3(B-L)+2s}$
q_{iL}	3	$\bar{3}$	0	0	$(-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3})^T$	$(+-+)^T$
q_{3L}	3	3	$\frac{1}{3}$	$\frac{2}{3}$	$(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3})^T$	$(++-)^T$
u_{aR}	3	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	+
d_{aR}	3	1	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	+
U_{3R}	3	1	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	-
D_{iR}	3	1	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	-
l_{aL}	1	3	$-\frac{1}{3}$	$-\frac{2}{3}$	$(0, -1, 0)^T$	$(++-)^T$
e_{aR}	1	1	-1	-1	-1	+
ν_{iR}	1	1	0	-4	0	-
ν_{3R}	1	1	0	5	0	+
N_{aR}	1	1	0	0	0	-
η	1	3	$-\frac{1}{3}$	$\frac{1}{3}$	$(0, -1, 0)^T$	$(++-)^T$
ρ	1	3	$\frac{2}{3}$	$\frac{1}{3}$	$(1, 0, 1)^T$	$(++-)^T$
χ	1	3	$-\frac{1}{3}$	$-\frac{2}{3}$	$(0, -1, 0)^T$	$(- - +)^T$
ϕ	1	1	0	2	0	+
σ	1	1	0	1	0	-

Table: 3311 model particle content ($a = 1, 2, 3$ and $i = 1, 2$ represent generation indices). Note the non-standard charges of “right handed neutrinos” ν_R .

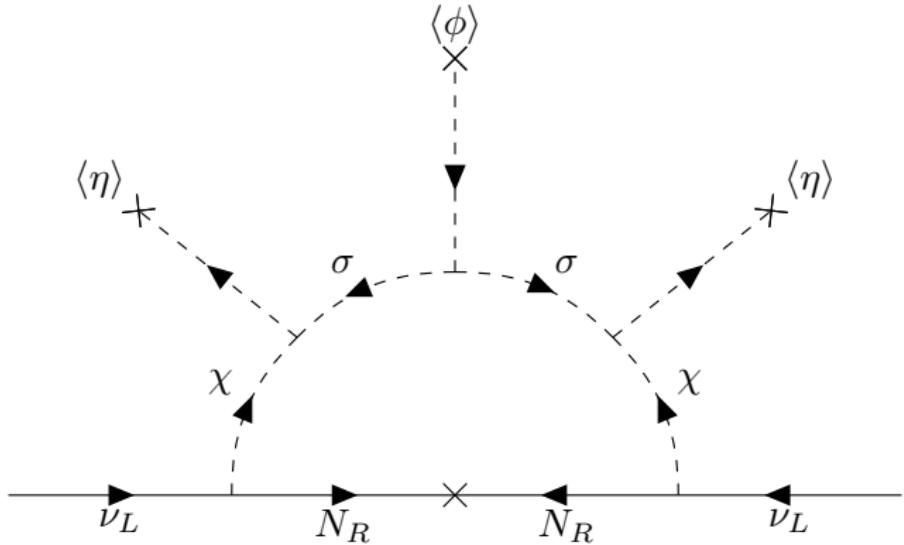


Figure: Feynman-loop diagram contributing to the light active Majorana neutrino mass matrix.

$$Q = T_3 - \frac{T_8}{\sqrt{3}} + X, \quad B - L = -\frac{2}{\sqrt{3}} T_8 + N, \quad (1)$$

$$q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ D_i \end{pmatrix}_L \quad q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ U_3 \end{pmatrix}_L \quad l_{aL} = \begin{pmatrix} \nu_a \\ e_a \\ N_a \end{pmatrix}_L, \quad (2)$$

The gauged $B - L$ symmetry is spontaneously broken leaving a discrete remnant symmetry $M_P = (-1)^{3(B-L)+2s}$.

$$\begin{aligned} \langle \eta \rangle &= \frac{1}{\sqrt{2}}(\nu_1, 0, 0)^T, & \langle \rho \rangle &= \frac{1}{\sqrt{2}}(0, \nu_2, 0)^T, & \langle \chi \rangle &= (0, 0, w)^T, \\ \langle \phi \rangle &= \frac{1}{\sqrt{2}}\Lambda, & \langle \sigma \rangle &= 0. \end{aligned} \quad (3)$$

We assume $w, \Lambda \gg \nu_1, \nu_2$, such that the SSB pattern of the model is

$$\begin{array}{c} SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_N \\ \downarrow w, \Lambda \\ SU(3)_C \times SU(2)_L \times U(1)_Y \times M_P \\ \downarrow \nu_1, \nu_2 \\ SU(3)_C \times U(1)_Q \times M_P. \end{array} \quad (4)$$

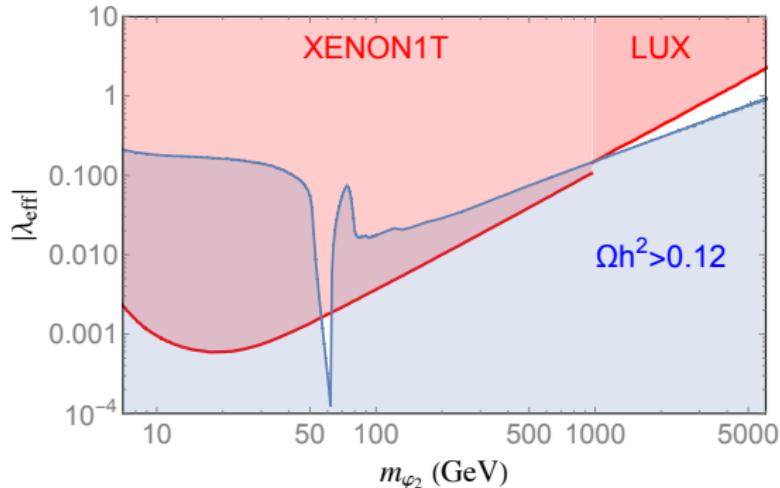


Figure: The direct detection and relic abundance constraints on the dark matter mass m_{φ_2} for the simplified scenario described in the text. The red shaded regions are ruled out by direct detection experiments, XENON1T (below 1 TeV) and LUX (above 1 TeV). The blue shaded region is not compatible with the measured dark matter relic abundance

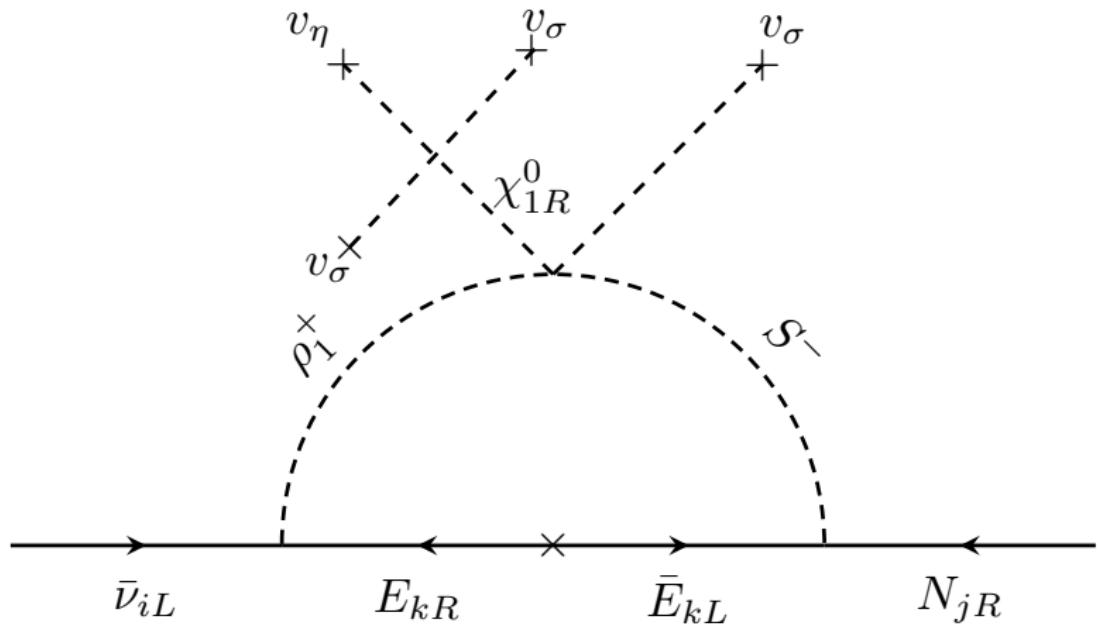
3-3-1 model with radiative linear seesaw mechanism.

	$SU(3)_C$	$SU(3)_L$	$U(1)_X$	$U(1)$
χ	1	3	$-\frac{1}{3}$	0
η	1	3	$-\frac{1}{3}$	4
ρ	1	3	$\frac{2}{3}$	-2
S^-	1	1	-1	0
σ	1	1	0	-2

Table: Scalar assignments under $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)$.

	$SU(3)_C$	$SU(3)_L$	$U(1)_X$	$U(1)$
L_{iL}	1	3	$-\frac{1}{3}$	1
e_{iR}	1	1	-1	1
N_{iR}	1	1	0	1
E_{iL}	1	1	-1	3
E_{iR}	1	1	-1	3

Table: Lepton assignments under $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)$. Here $i = 1, 2, 3$.



The neutrino mass matrix in the basis $(\nu_L, \nu_R^C, N_R^C)^T$ is:

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & m_{\nu D} & \varepsilon \\ m_{\nu D}^T & 0_{3 \times 3} & M \\ \varepsilon^T & M^T & 0_{3 \times 3} \end{pmatrix}, \quad (5)$$

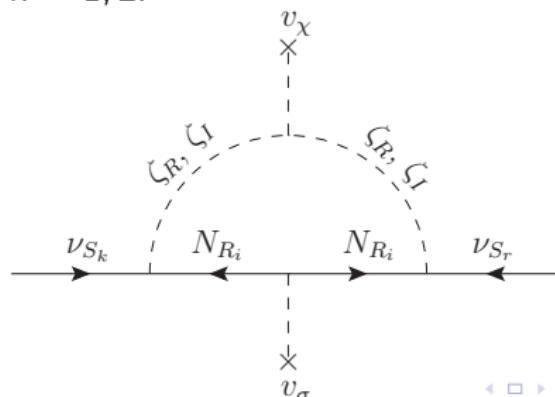
	$SU(3)_C$	$SU(3)_L$	$U(1)_X$	$U(1)$
Q_{nL}	3	3	0	0
Q_{3L}	3	3	$\frac{1}{3}$	0
u_{iR}	3	1	$\frac{2}{3}$	-4
d_{iR}	3	1	$-\frac{1}{3}$	2
J_{1R}	3	1	$\frac{2}{3}$	0
J_{nR}	3	1	$-\frac{1}{3}$	0
T_{nL}	3	1	$\frac{2}{3}$	-2
T_{nR}	3	1	$\frac{2}{3}$	-2
B_{nL}	3	1	$-\frac{1}{3}$	4
B_{nR}	3	1	$-\frac{1}{3}$	4

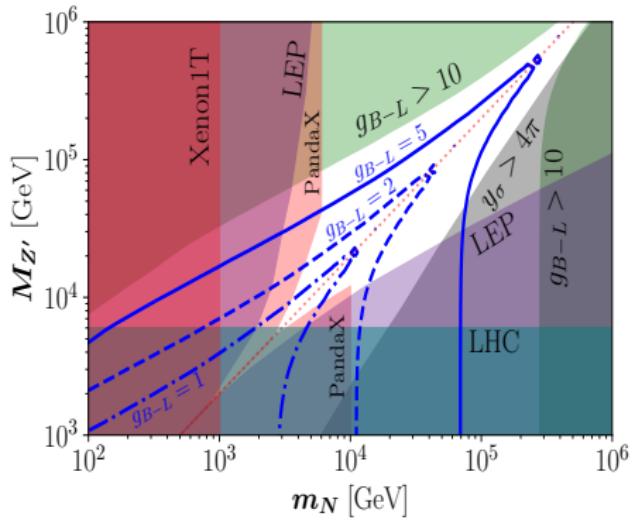
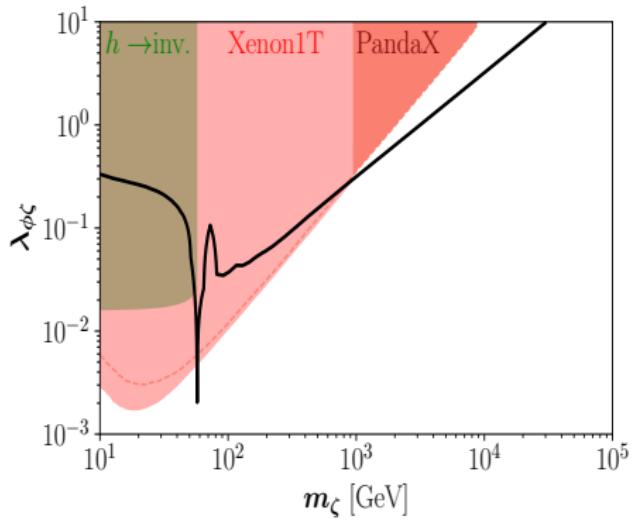
Table: Quark assignments under $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)$. Here $i = 1, 2, 3$ and $n = 1, 2$.

Gauged Inverse Seesaw from Dark Matter.

Field	q_{iL}	u_{iR}	d_{iR}	ℓ_{iL}	ℓ_{iR}	ν_{R_k}	ν_{S_k}	N_{R_i}	ϕ	ζ	σ	χ
$SU(3)_C$	3	3	3	1	1	1	1	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	1	1	2	1	1	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	0	0	$\frac{1}{2}$	0	0	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1	1	-1	0	0	-2	0
\mathbb{Z}_4	1	1	1	1	1	1	-1	i	1	$-i$	-1	-1

Table: Particle charge assignments under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \otimes \mathbb{Z}_4$ symmetry. The indices run as follows: $i = 1, 2, 3$ and $k = 1, 2$.





Cárcamo Hernández-Kovalenko-Schmidt (CKS) mechanism

In the CKS mechanism (A. E. Cárcamo Hernández, S. Kovalenko and I. Schmidt, JHEP **02**, 125 (2017)), the SM fermion mass hierarchy is explained by a sequential loop suppression, so that the masses are generated according to:

t-quark \rightarrow tree-level mass from $\bar{q}_{jL} \tilde{\phi} u_{3R}$, (6)

b, c, τ, μ \rightarrow 1-loop mass; tree-level (7)

suppressed by a symmetry.

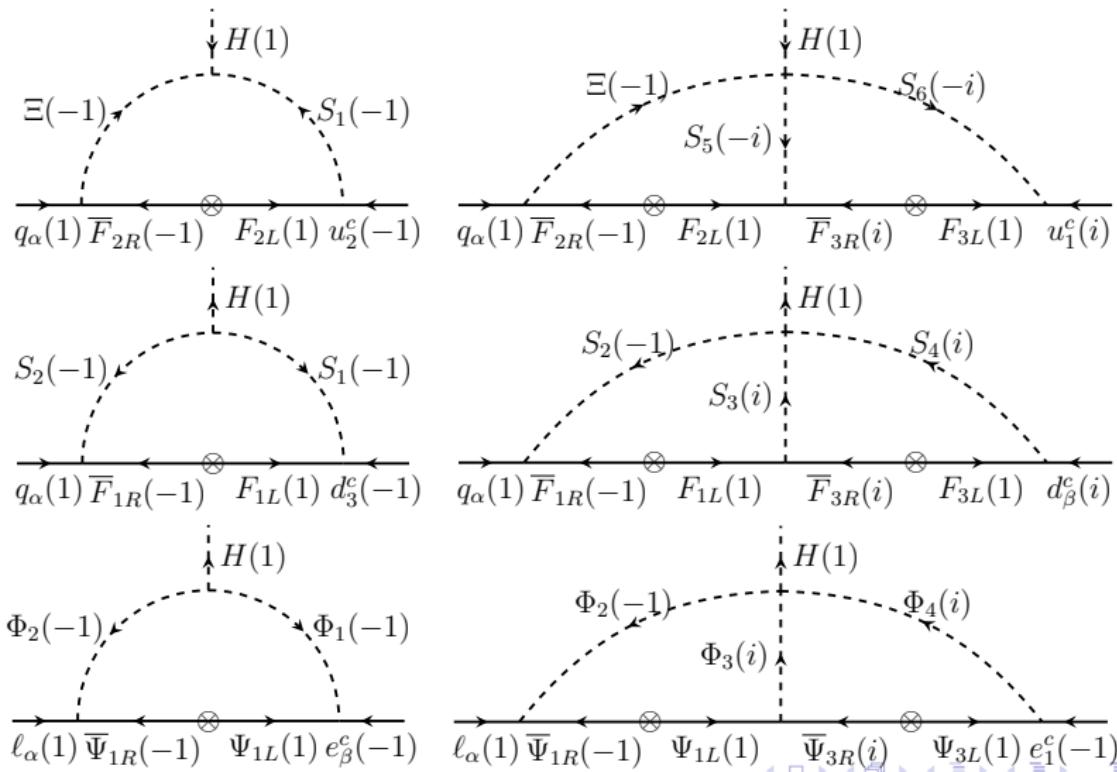
s, u, d, e \rightarrow 2-loop mass; tree-level & 1-loop (8)

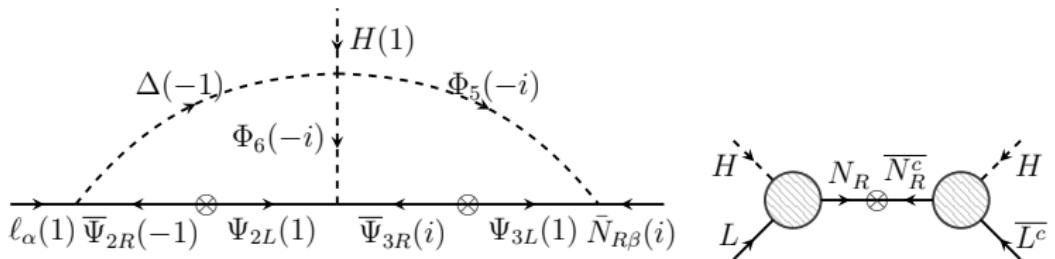
suppressed by a symmetry.

ν_i \rightarrow 4-loop mass; tree-level & lower loops (9)

suppressed by a symmetry.

Sequentially loop suppressed fermion masses at renormalizable level





$$m_D \sim \left(\frac{1}{16\pi^2} \right)^2 \kappa^{(\nu)} v Y^3 I_{loop}(m_S/M_R), \quad (10)$$

$$m_\nu \simeq m_D^T M_R^{-1} m_D, \quad (11)$$

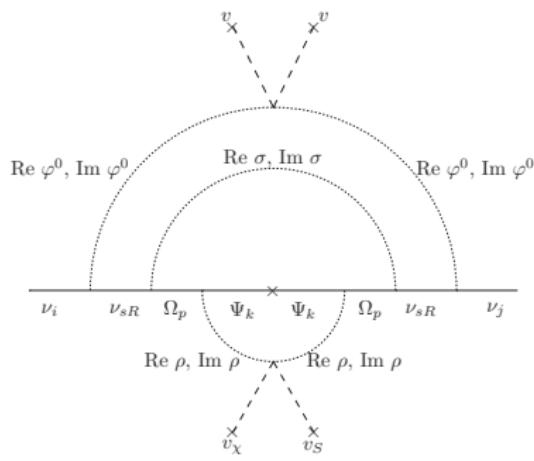
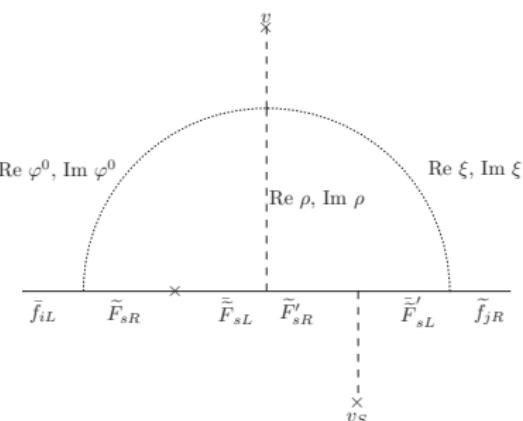
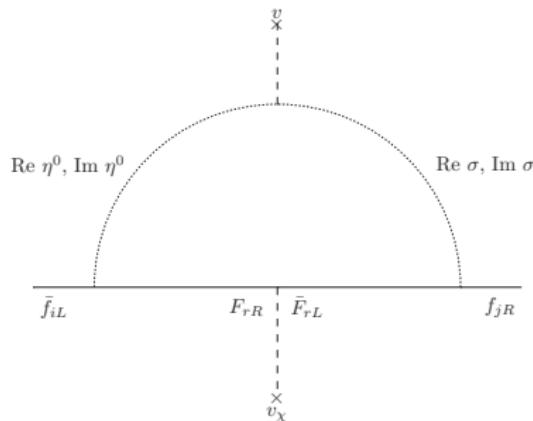
$$m_\nu \sim \left(\frac{1}{16\pi^2} \right)^4 (\kappa^{(\nu)})^2 Y^6 \frac{v^2}{M_R} [I_{loop}(m_S/M_R)]^2. \quad (12)$$

An extended 3HDM Model with the CKS mechanism.

	ϕ	η	φ	σ	ρ	ξ	χ	ζ
SU_{3c}	1	1	1	1	1	1	1	1
SU_{2L}	2	2	2	1	1	1	1	1
U_{1Y}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0
$Z_2^{(1)}$	1	1	1	1	1	1	-1	-1
$Z_2^{(2)}$	1	-1	-1	-1	-1	1	1	1
Z_4	1	1	-1	1	$-i$	$-i$	1	-1

Table: Scalar assignments under $SU_{3c} \times SU_{2L} \times U_{1Y} \times Z_2^{(1)} \times Z_2^{(2)} \times Z_4$.

ϕ is the SM Higgs doublet. χ and S are the only scalar singlets which do acquire VEVs.



$$m_\nu \sim l^3 y^6 \lambda \frac{v^2}{M},$$

$$y \sim 0.3, \lambda \sim 0.1$$

$$M \sim \mathcal{O}(13) \text{ TeV}$$

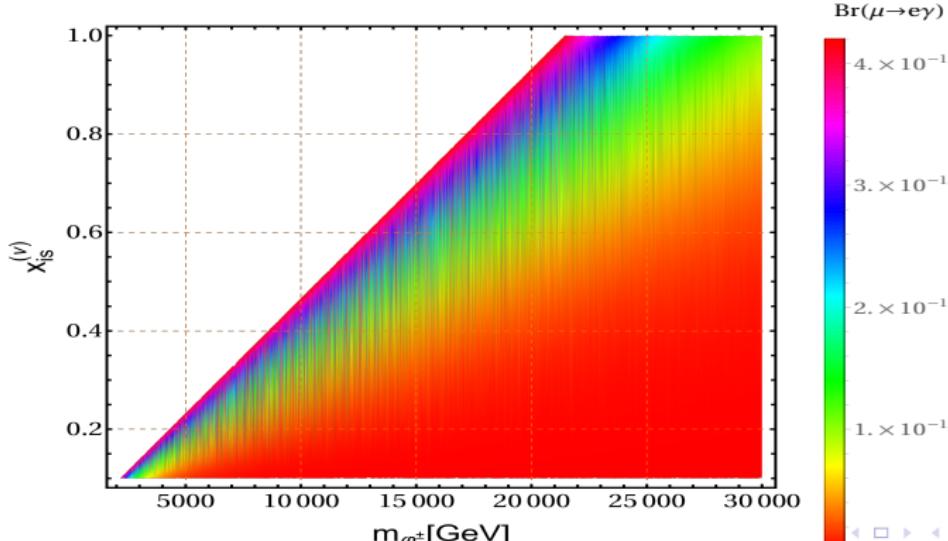
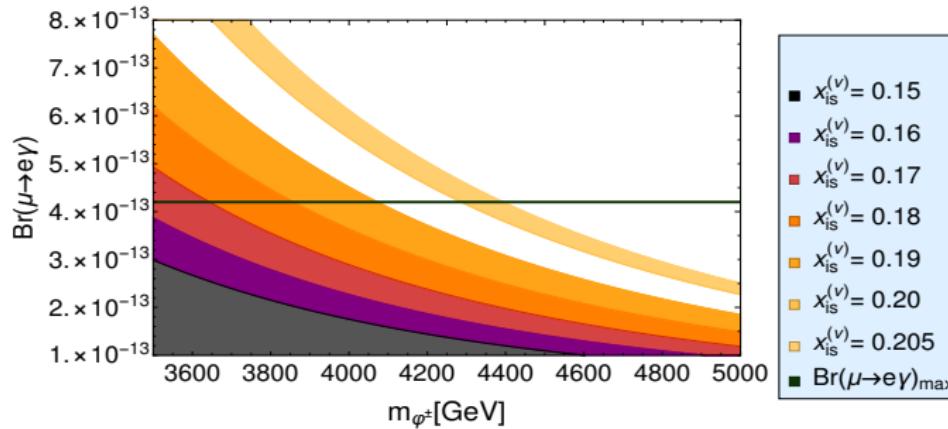
$$m_\nu \sim \mathcal{O}(0.1) \text{ eV}$$

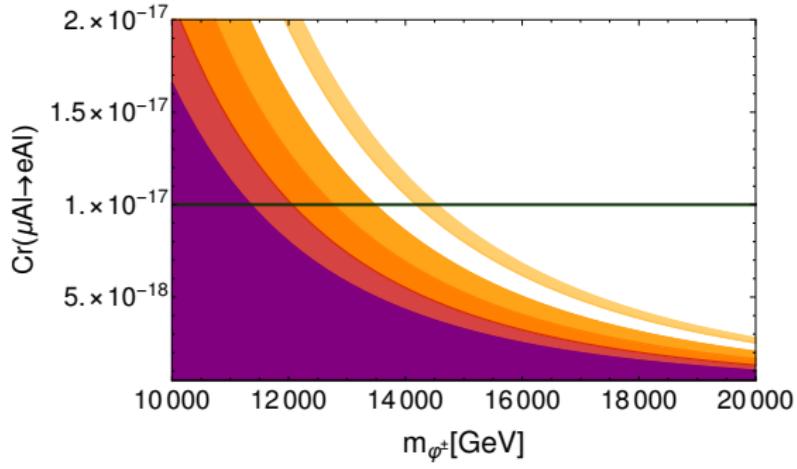
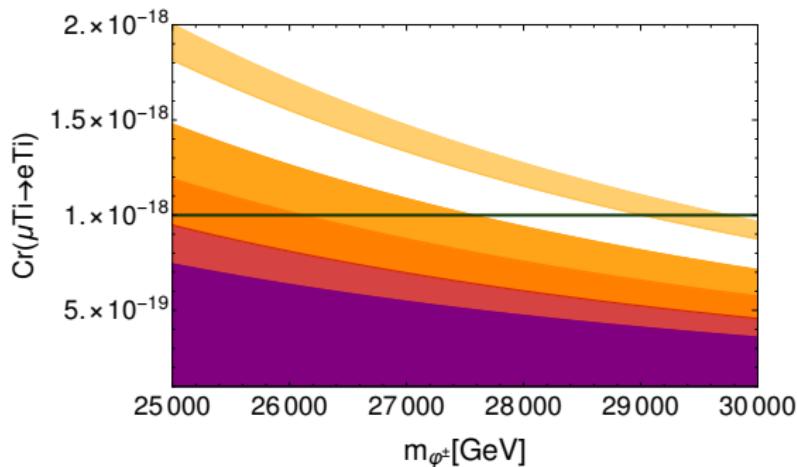
	q_{jL}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	T_L	T_R	\tilde{T}_L	\tilde{T}_R	\tilde{T}'_L	\tilde{T}'_R	B_L	B_R	\tilde{B}_{sL}	\tilde{B}'_{sL}	\tilde{B}_{sR}	\tilde{B}'_{sR}
SU_{3c}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
SU_{2L}	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
U_{1Y}	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	
$Z_2^{(1)}$	1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	1	1	1	-1	1	
$Z_2^{(2)}$	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	
Z_4	1	-1	1	1	-1	-1	1	1	1	-1	-1	-i	i	1	1	-1	-1	-i	

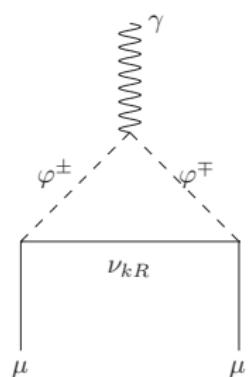
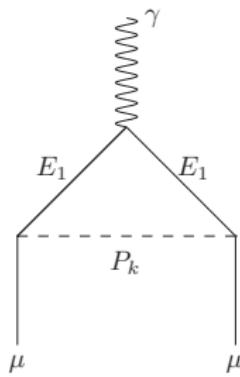
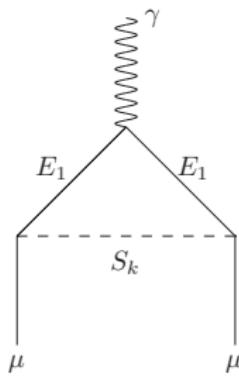
Table: Quark assignments under $SU_{3c} \times SU_{2L} \times U_{1Y} \times Z_2^{(1)} \times Z_2^{(2)} \times Z_4$. Here $j = 1, 2, 3$ and $s = 1, 2$.

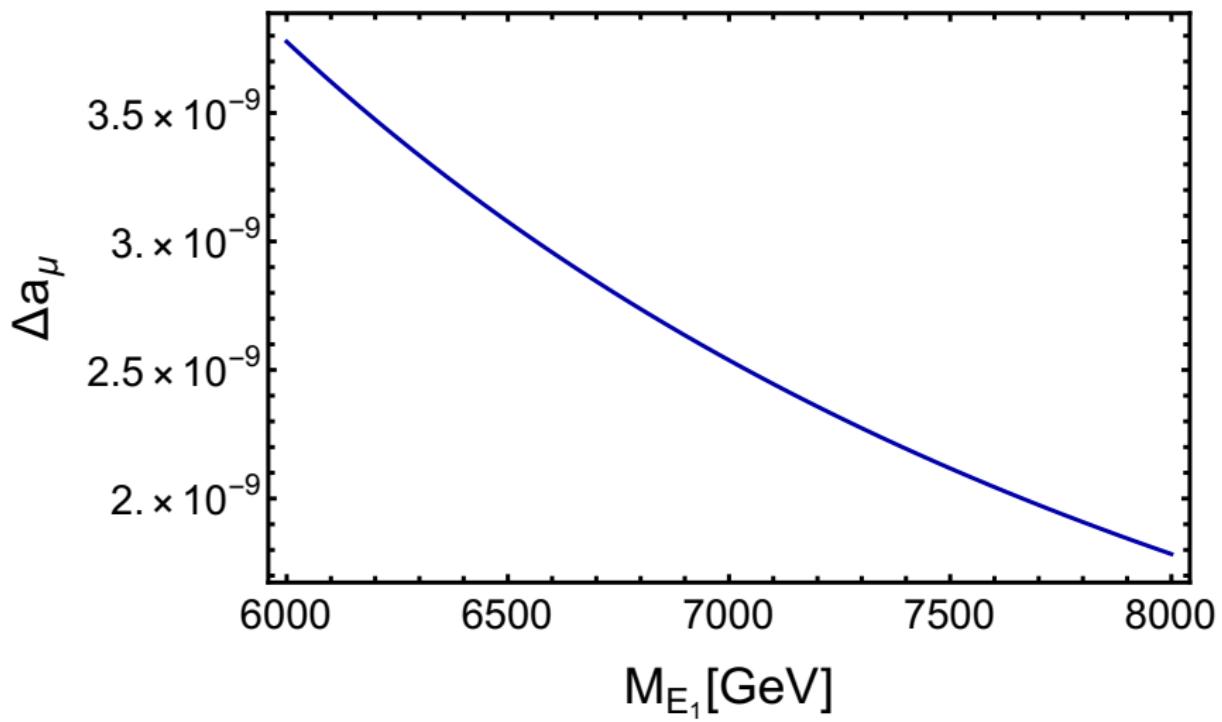
	l_{jL}	l_{1R}	l_{2R}	l_{3R}	E_{sL}	E_{sR}	\tilde{E}_L	\tilde{E}_R	\tilde{E}'_L	\tilde{E}'_R	ν_{sR}	Ω_{sR}	Ψ_{sR}
SU_{3c}	1	1	1	1	1	1	1	1	1	1	1	1	1
SU_{2L}	2	1	1	1	1	1	1	1	1	1	1	1	1
U_{1Y}	$-\frac{1}{2}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
$Z_2^{(1)}$	1	-1	-1	-1	-1	1	1	1	-1	1	1	1	1
$Z_2^{(2)}$	1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1
Z_4	i	$-i$	i	i	i	i	$-i$	$-i$	$-i$	-1	1	$-i$	i

Table: Lepton assignments under $SU_{3c} \times SU_{2L} \times U_{1Y} \times Z_2^{(1)} \times Z_2^{(2)} \times Z_4$. Here $j = 1, 2, 3$ and $s = 1, 2$.









$\Delta(27)$ 3+1 Higgs Doublet Model.

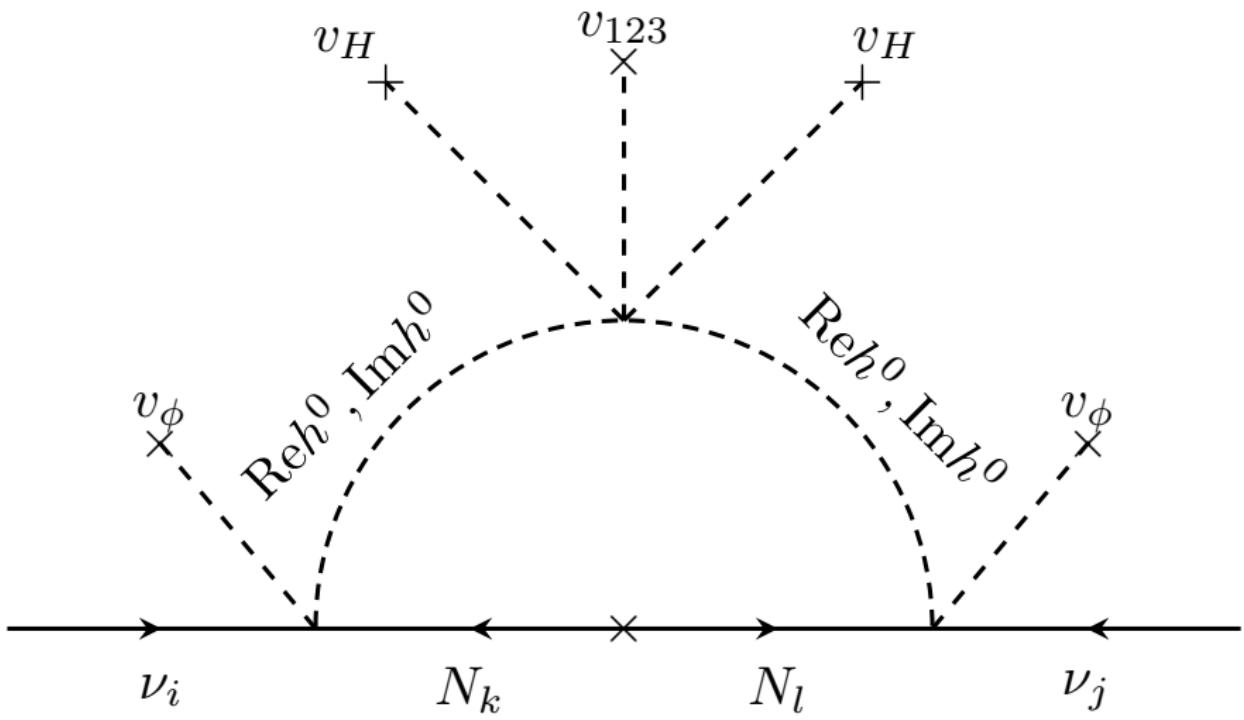
	H	h	σ	ϕ_{123}	ϕ_1	ϕ_{23}	ϕ_3
$\Delta(27)$	3̄	1_{0,0}	1_{0,0}	3	3	3	3
$Z_2^{(1)}$	0	1	0	0	0	0	0
$Z_2^{(2)}$	0	0	0	0	0	1	0
$Z_2^{(3)}$	0	0	0	0	0	0	1
Z_{18}	0	0	-1	0	0	0	0

Table: Scalar assignments under the $\Delta(27) \times Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)} \times Z_{18}$.

Here $H = (H_1, H_2, H_3)$. Furthermore H_i ($i = 1, 2, 3$) and h are $SU(2)$ scalar doublets whereas the remaining scalars are SM gauge singlets.

$$\begin{aligned} \langle H \rangle &= v_H (0, 0, 1), \quad \langle \phi_1 \rangle = v_1 (1, 0, 0), \quad \langle \phi_3 \rangle = v_3 (0, 0, 1), \\ \langle \phi_{123} \rangle &= v_{123} (1, \omega, \omega^2), \quad \langle \phi_{23} \rangle = v_{23} (0, 1, -1), \quad \langle \sigma \rangle = v_\sigma \sim \lambda \Lambda, \end{aligned}$$

with $v_H = \frac{v}{\sqrt{2}}$, being $v = 246$ GeV, and $\lambda \simeq 0.225$ The $Z_2^{(1)}$ symmetry is preserved, the other discrete symmetries are spontaneously broken.

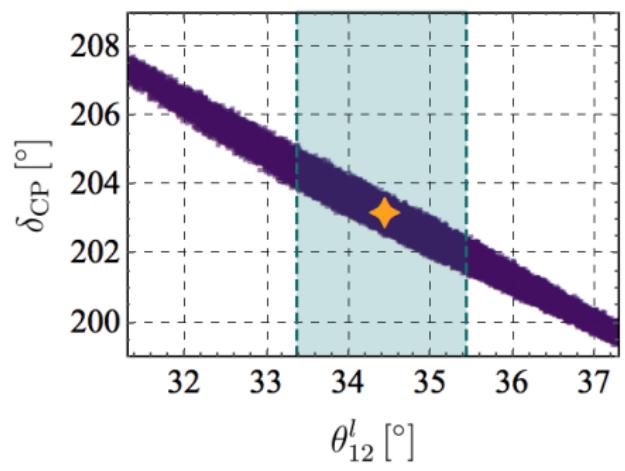
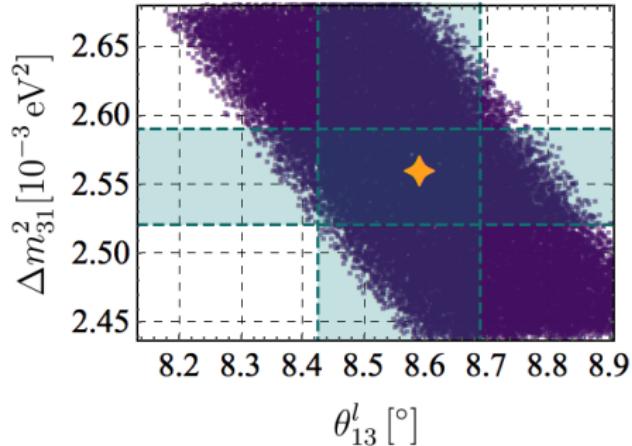
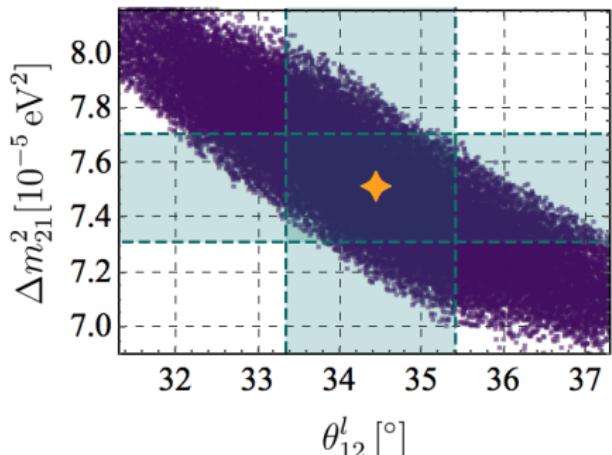


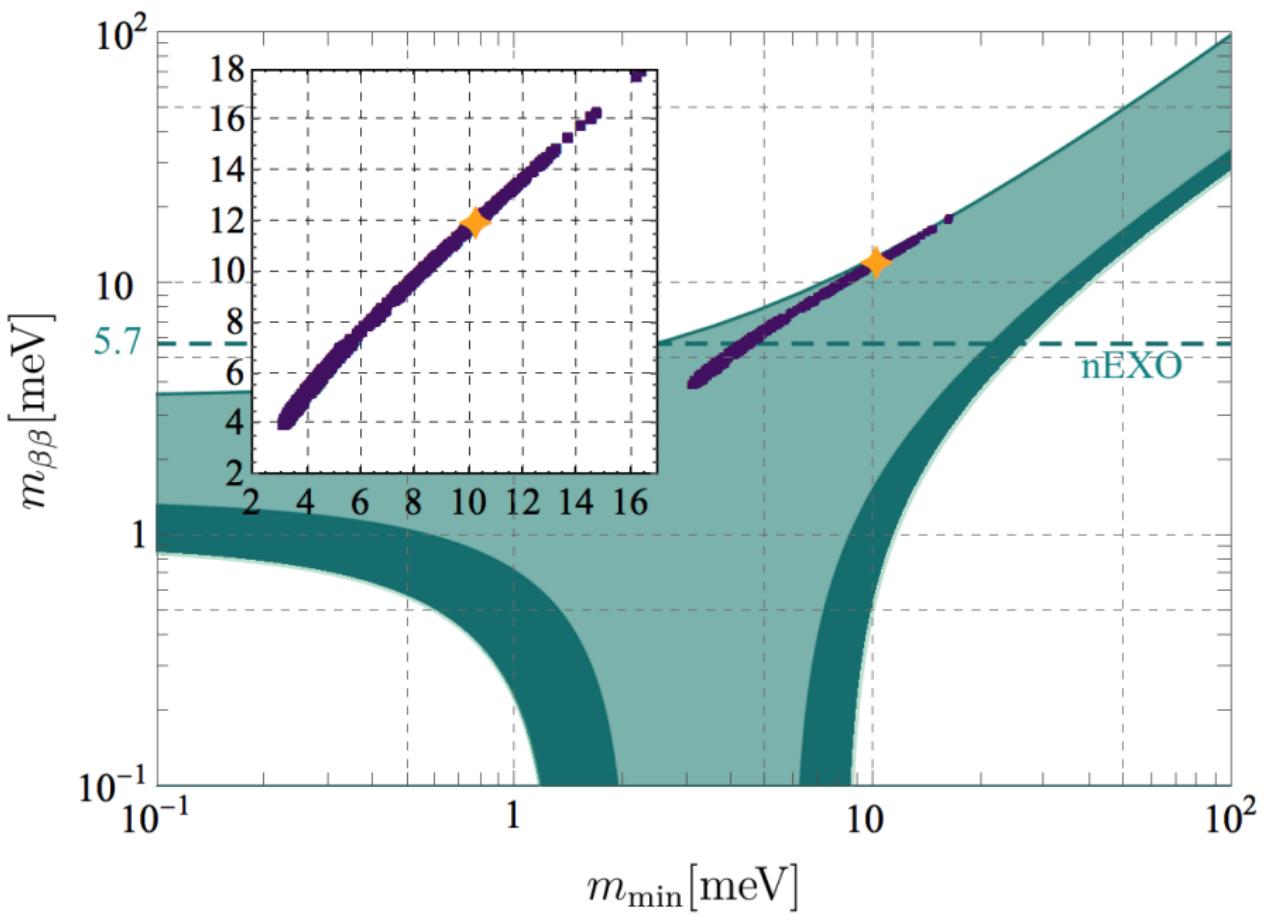
	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}
$\Delta(27)$	$\mathbf{1}_{0,0}$								
$Z_2^{(1)}$	0	0	0	0	0	0	0	0	0
$Z_2^{(2)}$	0	0	0	0	0	0	0	0	0
$Z_2^{(3)}$	0	0	0	1	1	1	1	1	1
Z_{18}	-4	-2	0	4	2	0	4	3	3

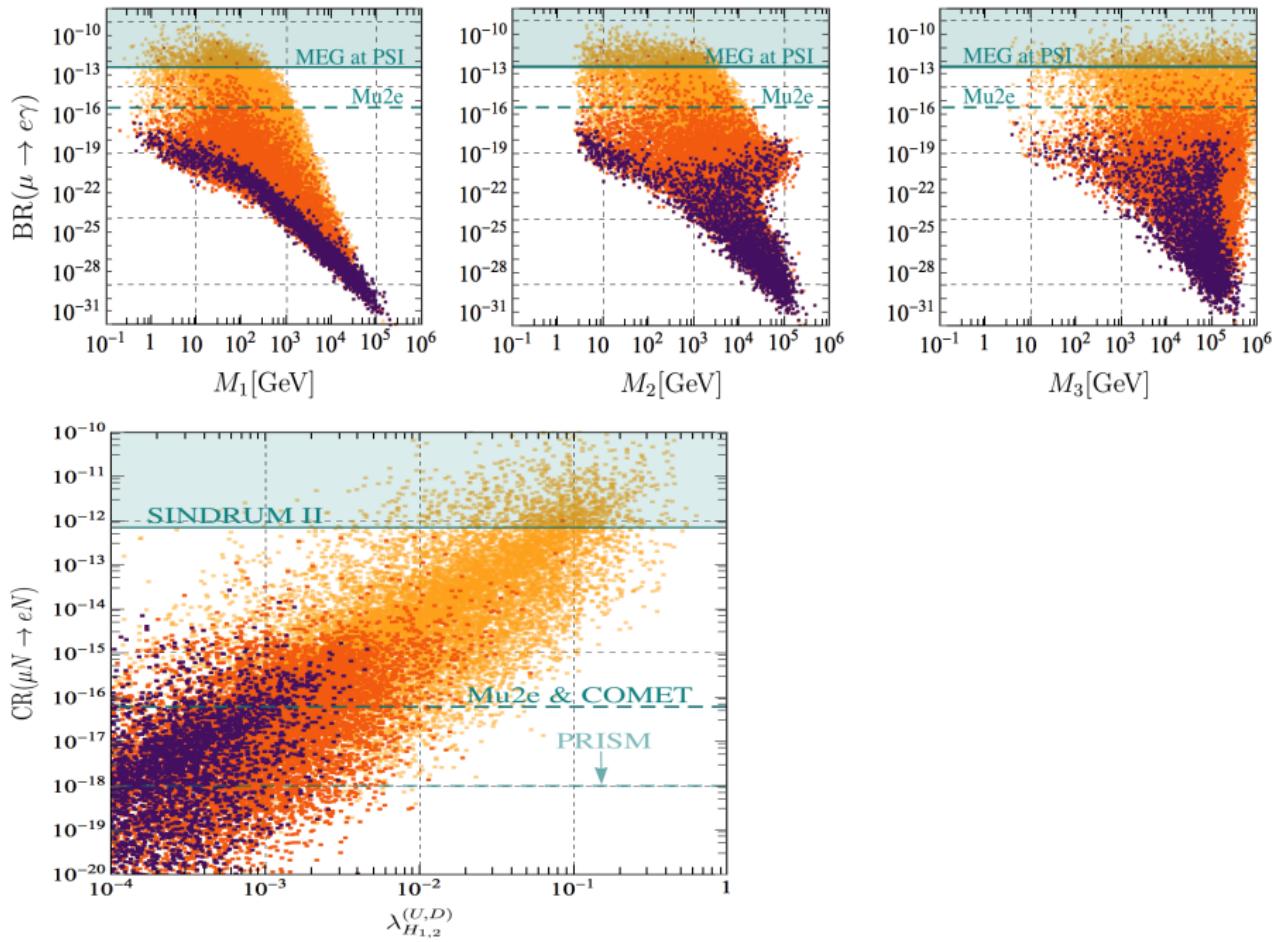
Table: Quark assignments under the $\Delta(27) \times Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)} \times Z_{18}$.

	I_L	I_{1R}	I_{2R}	I_{3R}	N_{1R}	N_{2R}	N_{3R}
$\Delta(27)$	$\mathbf{\bar{3}}$	$\mathbf{1}_{0,1}$	$\mathbf{1}_{0,2}$	$\mathbf{1}_{0,1}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$
$Z_2^{(1)}$	0	0	0	0	1	1	1
$Z_2^{(2)}$	0	0	0	0	0	0	1
$Z_2^{(3)}$	0	0	0	0	0	0	0
Z_{18}	0	9	5	3	0	0	0

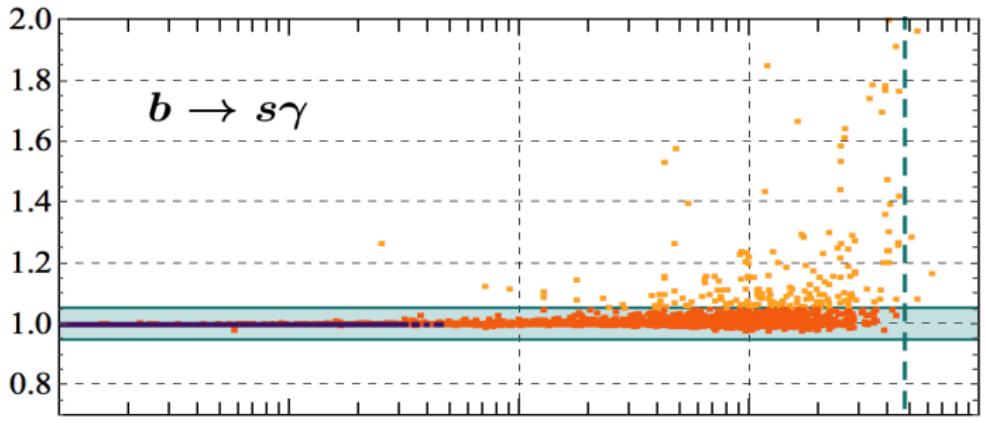
Table: Lepton assignments under the $\Delta(27) \times Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)} \times Z_{18}$.



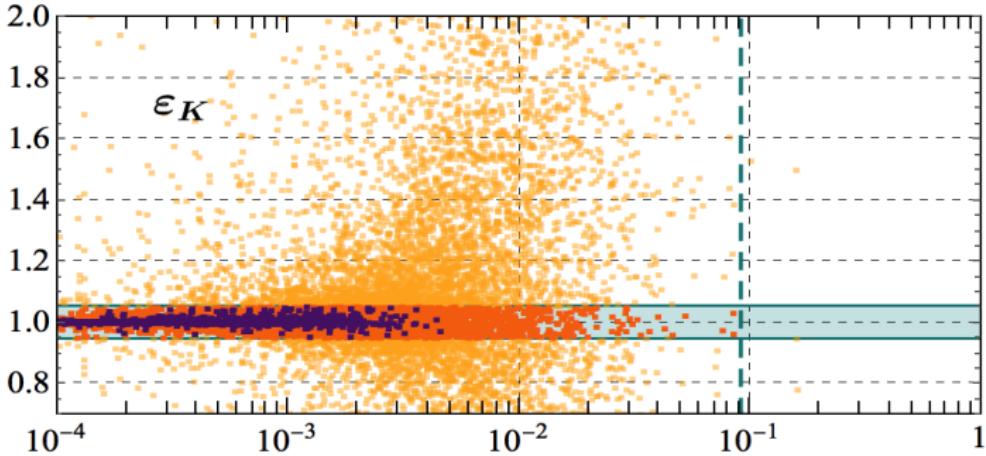




$\mathcal{O}/\mathcal{O}_{SM}$



$\mathcal{O}/\mathcal{O}_{SM}$



$$\lambda_{H_{1,2}^0}^{(U,D)}$$

Conclusions

- Dark matter stability can arise from a residual matter-parity symmetry.
- A Gauged Inverse seesaw mechanism can be implemented with few particle content and features TeV scale Dark matter candidates.
- Radiative fermion mass models have DM particle candidates mediating radiative mechanisms responsible for the SM fermion masses.
- Implementing the sequential loop suppression mechanism requires to consider vector like exotic fermions and extended scalar sectors.
- The extended 3HDM model predicts CLFV decays within the reach of the future experimental sensitivity
- $\Delta(27)$ 3+1 Higgs Doublet Model has a predictive lepton sector and is very useful for describing SM fermion masses and mixings.
- FCNC lead to strong constraints on the model parameter space of the $\Delta(27)$ 3+1 Higgs Doublet Model.

Acknowledgements

Thank you very much to all of you for the attention.

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Extra Slides

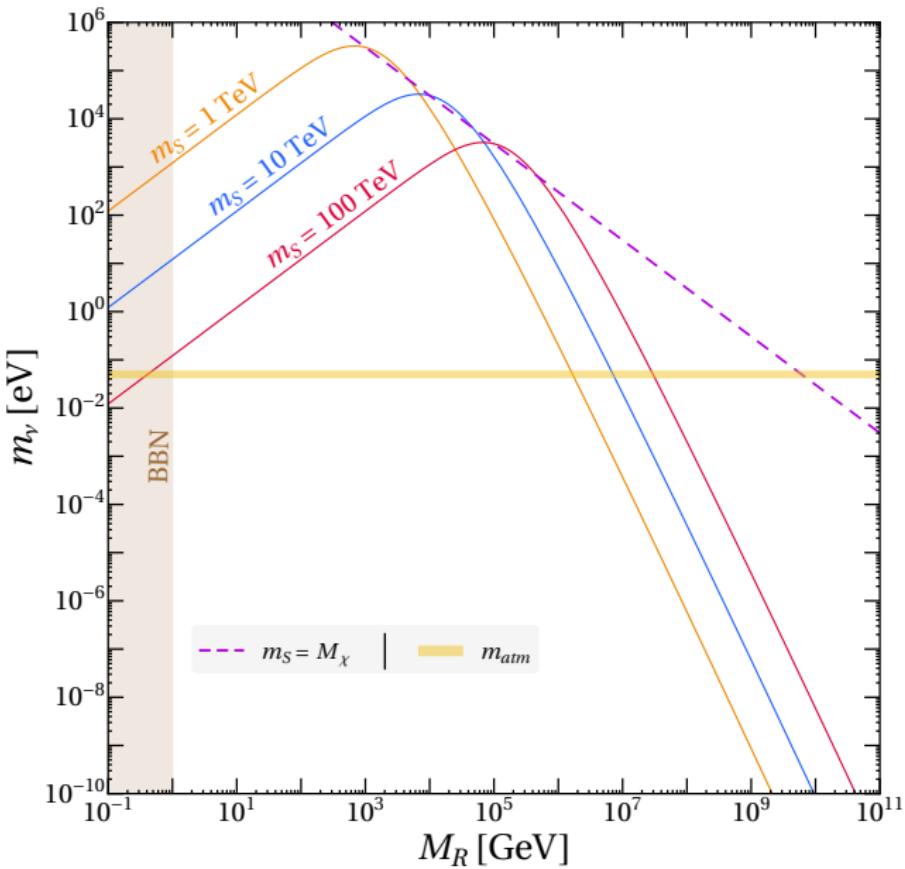


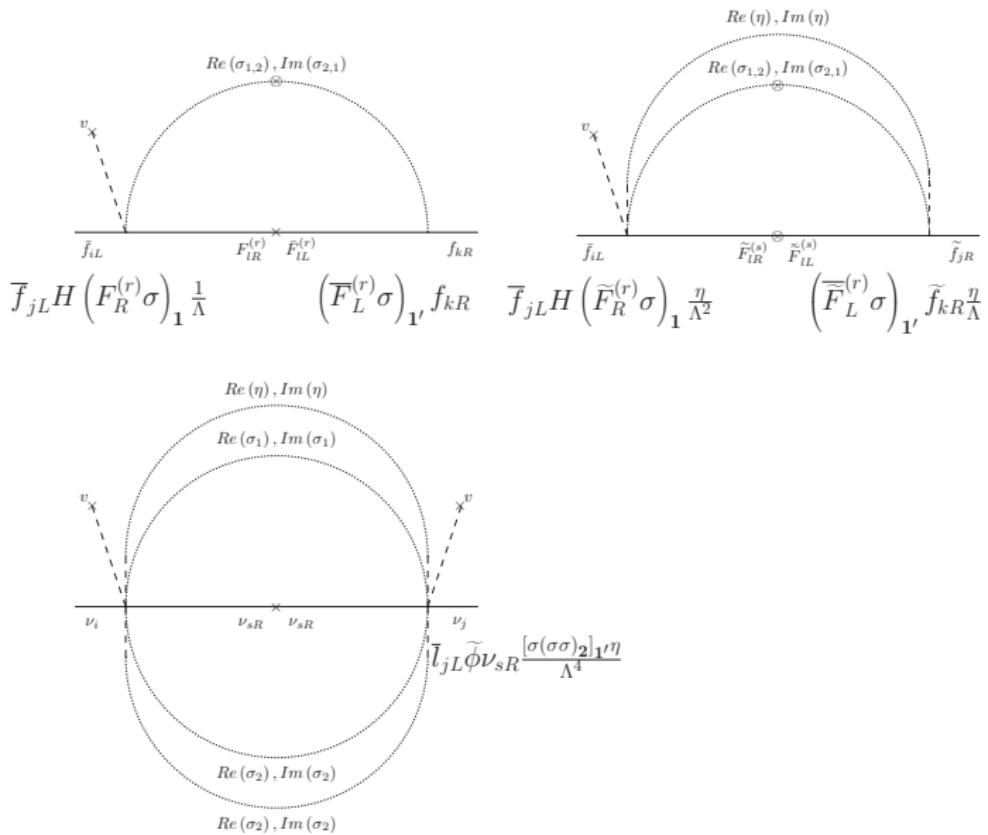
Figure: One-loop neutrino mass scale in the radiative type I seesaw model.

The S_3 discrete group

The S_3 is the smallest non-abelian group having a doublet and two singlet irreducible representations. The S_3 group has three irreducible representations: **1**, **1'** and **2**. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010):

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} + \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_2, \quad (13)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_2, \quad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \quad (14)$$



The $S_3 \times Z_2$ particle assignments of the model are:

	ϕ	σ	η
S_3	1	2	1
Z_2	1	1	-1

	q_{iL}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	l_{iL}	l_{1R}	l_{2R}	l_{3R}
S_3	1	1'	1'	1	1'	1'	1'	1	1'	1'	1'
Z_2	1	-1	1	1	-1	-1	1	1	-1	1	1

	ν_{sR}	T_L	T_R	\tilde{T}_L	\tilde{T}_R	B_L	B_R	$\tilde{B}_L^{(s)}$	$\tilde{B}_R^{(s)}$	$E_L^{(s)}$	$E_R^{(s)}$	\tilde{E}_L	\tilde{E}_R
S_3	1'	2	2	2	2	2	2	2	2	2	2	2	2
Z_2	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1

φ is the SM Higgs doublet.

The scalar fields σ and η and all exotic fermions are $SU(2)_L$ singlets.

The $S_3 \times Z_2$ discrete group is assumed to be softly broken.

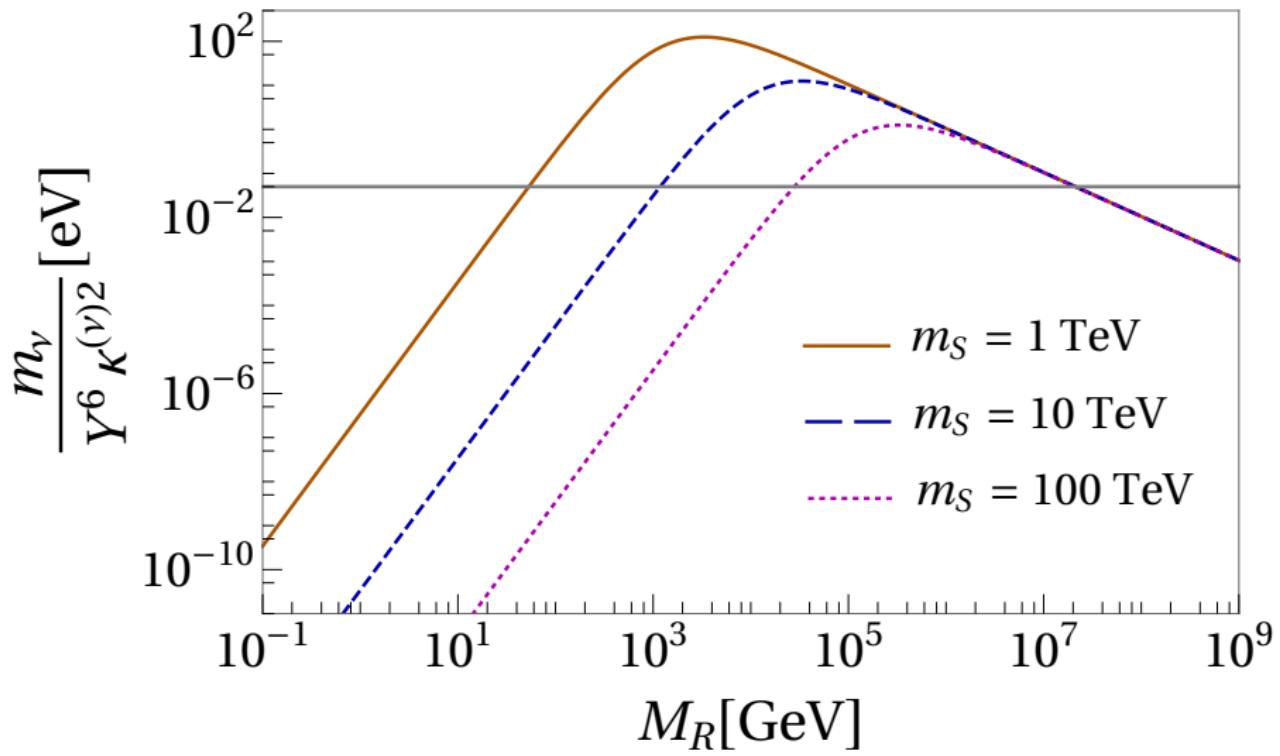
Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_4
q_α	(3, 2, 1/6)	(1, 1, 1)
u_α^c	(3, 1, -2/3)	(i , -1, 1)
d_α^c	(3, 1, 1/3)	(i , i , -1)
ℓ_α	(1, 2, -1/2)	(1, 1, 1)
e_α^c	(1, 1, 1)	(i , -1, -1)
$N_{R\beta}$	(1, 1, 0)	(i , i)
H	(1, 2, 1/2)	1

Table: Charge assignments for the SM fields and the right-handed field N_R , common to all the models. Not all the fields are charged under the extra global $U(1)_X$ symmetry. We consider the standard number of generations $\alpha = 1, 2, 3$, while $\beta = 1, 2$, to give masses to two active light neutrinos, which is the minimal phenomenologically viable choice. In the last column in the brackets we indicate Z_4 charges for the three generations.

	Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_4	$U(1)_X$	Comments
Fermions	(F_{1L}, F_{1R})	$(\mathbf{3}, \mathbf{1}, 2/3)$	$(1, -1)$	x	Vector-like u -quark
	(F_{2L}, F_{2R})	$(\mathbf{3}, \mathbf{1}, -1/3)$	$(1, -1)$	$-x$	Vector-like d -quark
	(F_{3L}, F_{3R})	$(\mathbf{3}, \mathbf{2}, 1/6)$	$(1, -i)$	$-x/2$	Vector-like doublet Q
	(Ψ_{1L}, Ψ_{1R})	$(\mathbf{1}, \mathbf{1}, 0)$	$(1, -1)$	x	Vector-like ν_R
	(Ψ_{2L}, Ψ_{2R})	$(\mathbf{1}, \mathbf{1}, 1)$	$(1, -1)$	$-x$	Vector-like e_R
	(Ψ_{3L}, Ψ_{3R})	$(\mathbf{1}, \mathbf{2}, -1/2)$	$(1, -i)$	$-x/2$	Vector-like doublet L
Scalars	S_1	$(\mathbf{1}, \mathbf{1}, -1)$	-1	$-x$	Scalar charged singlet
	S_2	$(\mathbf{1}, \mathbf{2}, 1/2)$	-1	x	Inert doublet
	S_3	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$3x/2$	Inert doublet
	S_4	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$-x/2$	Inert doublet
	S_5	$(\mathbf{1}, \mathbf{2}, 1/2)$	$-i$	$x/2$	Inert doublet

	Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_4	$U(1)_X$	Comments
Fermions	(F_{1L}, F_{1R})	$(\mathbf{3}, \mathbf{1}, 2/3)$	$(1, -1)$	x	Vector-like u -quark
	(F_{2L}, F_{2R})	$(\mathbf{3}, \mathbf{1}, -1/3)$	$(1, -1)$	$-x$	Vector-like d -quark
	(F_{3L}, F_{3R})	$(\mathbf{3}, \mathbf{2}, 1/6)$	$(1, -i)$	$-x/2$	Vector-like doublet Q
Scalars	S_1	$(\mathbf{1}, \mathbf{1}, -1)$	-1	$-x$	Scalar charged singlet
	S_2	$(\mathbf{1}, \mathbf{2}, 1/2)$	-1	x	Inert doublet
	S_3	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$3x/2$	Inert doublet
	S_4	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$-x/2$	Inert doublet
	S_5	$(\mathbf{1}, \mathbf{2}, 1/2)$	$-i$	$x/2$	Inert doublet
	Φ_1	$(\bar{\mathbf{3}}, \mathbf{1}, -5/3)$	-1	$-x$	-
	Φ_2	$(\mathbf{3}, \mathbf{2}, 7/6)$	-1	x	-
	Φ_3	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$3x/2$	Inert doublet
	Φ_4	$(\mathbf{3}, \mathbf{2}, 7/6)$	i	$-x/2$	-
	Φ_5	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	$-i$	$x/2$	Scalar Q
	Φ_6	$(\mathbf{1}, \mathbf{2}, 1/2)$	$-i$	$x/2$	Inert doublet
	Δ	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	-1	x	Scalar Q

	Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_4	$U(1)_X$	Comments
Fermions	(F_{1L}, F_{1R})	$(\mathbf{1}, \mathbf{1}, 0)$	$(1, -1)$	x	Vector-like ν_R
	(F_{2L}, F_{2R})	$(\mathbf{1}, \mathbf{1}, 1)$	$(1, -1)$	$-x$	Vector-like e_R
	(F_{3L}, F_{3R})	$(\mathbf{1}, \mathbf{2}, -1/2)$	$(1, -i)$	$-x/2$	Vector-like doublet L
Scalars	S_1	$(\mathbf{3}, \mathbf{1}, -1/3)$	-1	$-x$	Scalar d
	S_2	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	-1	x	Scalar Q
	S_3	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$3x/2$	Inert doublet
	S_4	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	i	$-x/2$	Scalar Q
	S_5	$(\mathbf{1}, \mathbf{2}, 1/2)$	$-i$	$x/2$	Inert doublet
	S_6	$(\mathbf{3}, \mathbf{2}, -5/6)$	$-i$	$x/2$	-
	Ξ	$(\mathbf{3}, \mathbf{2}, -5/6)$	-1	x	-
	Φ_1	$(\mathbf{1}, \mathbf{1}, -1)$	-1	$-x$	Scalar charged singlet
	Φ_2	$(\mathbf{1}, \mathbf{2}, 1/2)$	-1	x	Inert doublet
	Φ_3	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$3x/2$	Inert doublet
	Φ_4	$(\mathbf{1}, \mathbf{2}, 1/2)$	i	$-x/2$	Inert doublet
	Φ_5	$(\mathbf{1}, \mathbf{2}, 1/2)$	$-i$	$x/2$	Inert doublet



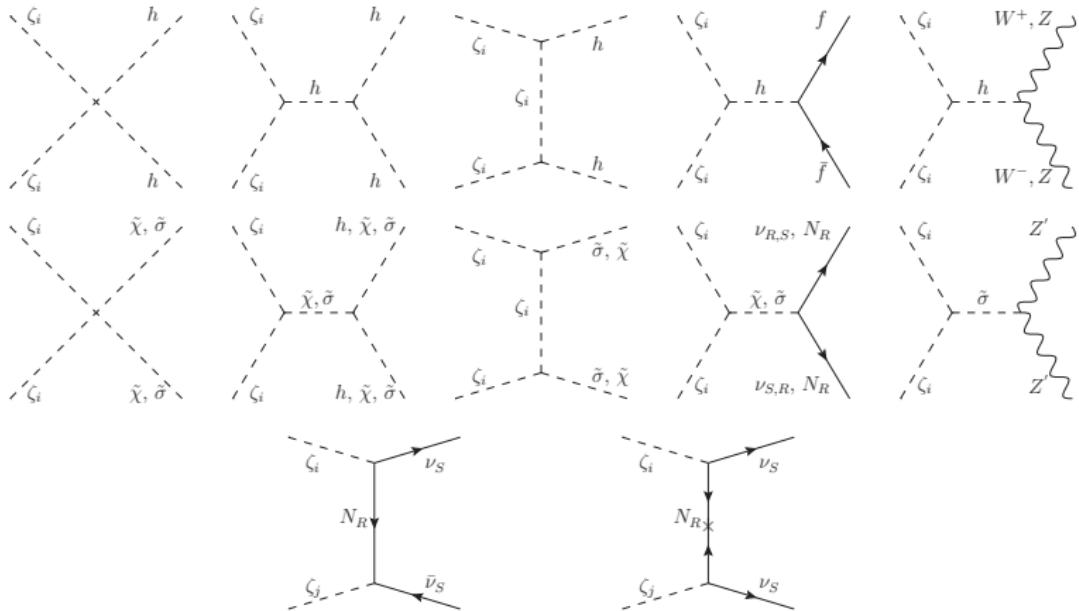


Figure: Main channels contributing to the production of scalar DM ζ_i , with i identifying the lightest state between the real and imaginary part of ζ , and $j = R, I$. Crossed diagrams are not shown, although taken into account in the analysis. f labels all the fermion fields coupled to the Higgs.

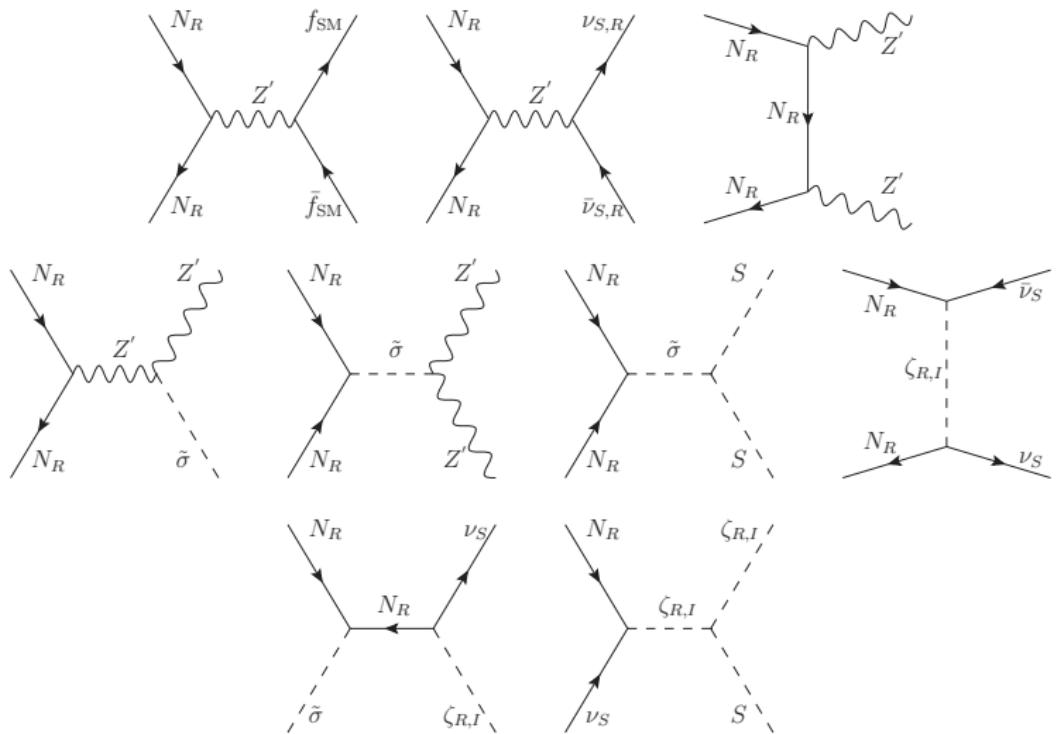


Figure: Main channels contributing to the production of *fermionic DM*. f_{SM} indicates all the SM fermions coupled to the Z' and $S = h, \tilde{\sigma}, \tilde{\chi}$, and $\zeta_{R,I}$.

$$\begin{aligned}
\mathbf{3} \otimes \mathbf{3} &= \bar{\mathbf{3}}_{S_1} \oplus \bar{\mathbf{3}}_{S_2} \oplus \bar{\mathbf{3}}_A \\
\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} &= \mathbf{3}_{S_1} \otimes \mathbf{3}_{S_2} \oplus \mathbf{3}_A \\
\mathbf{3} \otimes \bar{\mathbf{3}} &= \sum_{r=0}^2 \mathbf{1}_{r,0} \oplus \sum_{r=0}^2 \mathbf{1}_{r,1} \oplus \sum_{r=0}^2 \mathbf{1}_{r,2} \\
\mathbf{1}_{k,\ell} \otimes \mathbf{1}_{k',\ell'} &= \mathbf{1}_{k+k' \text{mod}3, \ell+\ell' \text{mod}3} \tag{15} \\
(\mathbf{3} \otimes \mathbf{3})_{\bar{\mathbf{3}}_{S_1}} &= (x_1 y_1, x_2 y_2, x_3 y_3), \\
(\mathbf{3} \otimes \mathbf{3})_{\bar{\mathbf{3}}_{S_2}} &= \frac{1}{2} (x_2 y_3 + x_3 y_2, x_3 y_1 + x_1 y_3, x_1 y_2 + x_2 y_1), \\
(\mathbf{3} \otimes \mathbf{3})_{\bar{\mathbf{3}}_A} &= \frac{1}{2} (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1), \\
(\mathbf{3} \otimes \bar{\mathbf{3}})_{\mathbf{1}_{r,0}} &= x_1 y_1 + \omega^{2r} x_2 y_2 + \omega^r x_3 y_3, \\
(\mathbf{3} \otimes \bar{\mathbf{3}})_{\mathbf{1}_{r,1}} &= x_1 y_2 + \omega^{2r} x_2 y_3 + \omega^r x_3 y_1, \\
(\mathbf{3} \otimes \bar{\mathbf{3}})_{\mathbf{1}_{r,2}} &= x_1 y_3 + \omega^{2r} x_2 y_1 + \omega^r x_3 y_2, \tag{16}
\end{aligned}$$

where $r = 0, 1, 2$ and $\omega = e^{i\frac{2\pi}{3}}$.

$$M_U = \frac{\nu}{\sqrt{2} \Lambda} \begin{pmatrix} y_{11}^{(U)} \lambda^8 & y_{12}^{(U)} \lambda^6 & y_{13}^{(U)} \lambda^4 \\ y_{21}^{(U)} \lambda^6 & y_{22}^{(U)} \lambda^4 & y_{23}^{(U)} \lambda^2 \\ y_{31}^{(U)} \lambda^4 & y_{32}^{(U)} \lambda^2 & y_{33}^{(U)} \end{pmatrix},$$

$$M_D = \frac{\nu}{\sqrt{2} \Lambda} \begin{pmatrix} y_{11}^{(D)} \lambda^7 & y_{12}^{(D)} \lambda^6 & y_{13}^{(D)} \lambda^6 \\ y_{21}^{(D)} \lambda^6 & y_{22}^{(D)} \lambda^5 & y_{23}^{(D)} \lambda^5 \\ y_{31}^{(D)} \lambda^4 & y_{32}^{(D)} \lambda^3 & y_{33}^{(D)} \lambda^3 \end{pmatrix}, \quad (17)$$

$$M_I = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad M_\nu = \begin{pmatrix} a & d\omega^2 & d\omega \\ d\omega^2 & b e^{i\theta} & c \\ d\omega & c & b e^{-i\theta} \end{pmatrix}, \quad (18)$$

$$c = b(\sin \theta - \sqrt{3} \cos \theta) / \sqrt{3} \quad (19)$$

$$m_e = y_1^{(I)} \frac{\nu v_\sigma^9}{\sqrt{2} \Lambda^9} = y_1^{(I)} \lambda^9 \frac{\nu}{\sqrt{2}}, \quad m_\mu = y_2^{(I)} \frac{\nu v_\sigma^5}{\sqrt{2} \Lambda^5} = y_2^{(I)} \lambda^5 \frac{\nu}{\sqrt{2}},$$

$$m_\tau = y_3^{(I)} \frac{\nu v_\sigma^3}{\sqrt{2} \Lambda^3} = y_3^{(I)} \lambda^3 \frac{\nu}{\sqrt{2}}, \quad (20)$$

$$\begin{aligned}
M_U &= \frac{v}{\sqrt{2}} \frac{v_3}{\Lambda} \begin{pmatrix} y_{11}^{(U)} \lambda^8 & y_{12}^{(U)} \lambda^6 & y_{13}^{(U)} \lambda^4 \\ y_{21}^{(U)} \lambda^6 & y_{22}^{(U)} \lambda^4 & y_{23}^{(U)} \lambda^2 \\ y_{31}^{(U)} \lambda^4 & y_{32}^{(U)} \lambda^2 & y_{33}^{(U)} \end{pmatrix}, \quad M_D = \frac{v}{\sqrt{2}} \frac{v_3}{\Lambda} \begin{pmatrix} y_{11}^{(D)} \lambda^7 & y_{12}^{(D)} \lambda^6 & y_{13}^{(D)} \lambda^6 \\ y_{21}^{(D)} \lambda^6 & y_{22}^{(D)} \lambda^5 & y_{23}^{(D)} \lambda^5 \\ y_{31}^{(D)} \lambda^4 & y_{32}^{(D)} \lambda^3 & y_{33}^{(D)} \lambda^3 \end{pmatrix}, \\
Y_{H_1^0}^{(U)} &= \omega \lambda_{H_1^0}^{(U)} \begin{pmatrix} x_{11}^{(U)} \lambda^8 & x_{12}^{(U)} \lambda^6 & x_{13}^{(U)} \lambda^4 \\ x_{21}^{(U)} \lambda^6 & x_{22}^{(U)} \lambda^4 & x_{23}^{(U)} \lambda^2 \\ x_{31}^{(U)} \lambda^4 & x_{32}^{(U)} \lambda^2 & x_{33}^{(U)} \end{pmatrix}, \quad Y_{H_1^0}^{(D)} = \omega^2 \lambda_{H_1^0}^{(D)} \begin{pmatrix} x_{11}^{(D)} \lambda^7 & x_{12}^{(D)} \lambda^6 & x_{13}^{(D)} \lambda^6 \\ x_{21}^{(D)} \lambda^6 & x_{22}^{(D)} \lambda^5 & x_{23}^{(D)} \lambda^5 \\ x_{31}^{(D)} \lambda^4 & x_{32}^{(D)} \lambda^3 & x_{33}^{(D)} \lambda^3 \end{pmatrix}, \\
Y_{H_2^0}^{(U)} &= \lambda_{H_2^0}^{(U)} \begin{pmatrix} x_{11}^{(U)} \lambda^8 & x_{12}^{(U)} \lambda^6 & x_{13}^{(U)} \lambda^4 \\ x_{21}^{(U)} \lambda^6 & x_{22}^{(U)} \lambda^4 & x_{23}^{(U)} \lambda^2 \\ x_{31}^{(U)} \lambda^4 & x_{32}^{(U)} \lambda^2 & x_{33}^{(U)} \end{pmatrix}, \quad Y_{H_2^0}^{(D)} = \lambda_{H_2^0}^{(D)} \begin{pmatrix} x_{11}^{(D)} \lambda^7 & x_{12}^{(D)} \lambda^6 & x_{13}^{(D)} \lambda^6 \\ x_{21}^{(D)} \lambda^6 & x_{22}^{(D)} \lambda^5 & x_{23}^{(D)} \lambda^5 \\ x_{31}^{(D)} \lambda^4 & x_{32}^{(D)} \lambda^3 & x_{33}^{(D)} \lambda^3 \end{pmatrix}, \\
\delta Y_{H_3^0}^{(U)} &= \omega^2 \lambda_{H_3^0}^{(U)} \begin{pmatrix} x_{11}^{(U)} \lambda^8 & x_{12}^{(U)} \lambda^6 & x_{13}^{(U)} \lambda^4 \\ x_{21}^{(U)} \lambda^6 & x_{22}^{(U)} \lambda^4 & x_{23}^{(U)} \lambda^2 \\ x_{31}^{(U)} \lambda^4 & x_{32}^{(U)} \lambda^2 & x_{33}^{(U)} \end{pmatrix}, \quad \delta Y_{H_3^0}^{(D)} = \omega \lambda_{H_3^0}^{(D)} \begin{pmatrix} x_{11}^{(D)} \lambda^7 & x_{12}^{(D)} \lambda^6 & x_{13}^{(D)} \lambda^6 \\ x_{21}^{(D)} \lambda^6 & x_{22}^{(D)} \lambda^5 & x_{23}^{(D)} \lambda^5 \\ x_{31}^{(D)} \lambda^4 & x_{32}^{(D)} \lambda^3 & x_{33}^{(D)} \lambda^3 \end{pmatrix},
\end{aligned}$$

where it is convenient to introduce the global effective couplings as

$$\begin{aligned}
\lambda_{H_1^0}^{(U,D)} &= \frac{v_3 v_{123}}{\sqrt{2}\Lambda^2} \left(c_{S_2}^{(U,D)} - c_A^{(U,D)} \right), \quad \lambda_{H_2^0}^{(U,D)} = \frac{v_3 v_{123}}{\sqrt{2}\Lambda^2} \left[\left(c_{S_2}^{(U,D)} + c_A^{(U,D)} \right) + \frac{v_1}{v_{123}} \right] \\
\lambda_{H_3^0}^{(U,D)} &= \frac{v_3 v_{123}}{\sqrt{2}\Lambda^2} c_{S_1}^{(U,D)}.
\end{aligned}$$

$$a \simeq 10.64 \text{ meV}, \quad b \simeq 30.89 \text{ meV}, \\ c \simeq -19.79 \text{ meV}, \quad d \simeq (1.59 + i 5.83) \text{ meV}, \quad \theta \simeq 26.29^\circ. \quad (21)$$

Observable	Model value	Neutrino oscillation global fit values (NH)			
		Best fit $\pm 1\sigma$	Best fit $\pm 1\sigma$	3σ range	3σ range
$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	7.51	$7.50^{+0.22}_{-0.20}$	$7.42^{+0.21}_{-0.20}$	$6.94 - 8.14$	$6.82 - 8.04$
$\Delta m_{31}^2 [10^{-3} \text{ eV}^2]$	2.56	$2.56^{+0.03}_{-0.04}$	$2.517^{+0.026}_{-0.028}$	$2.46 - 2.65$	$2.435 - 2.598$
$\theta_{12}^l [^\circ]$	34.45	34.3 ± 1.0	$33.44^{+0.77}_{-0.74}$	$31.4 - 37.4$	$31.27 - 35.86$
$\theta_{13}^l [^\circ]$	8.59	$8.58^{+0.11}_{-0.15}$	8.57 ± 0.12	$8.16 - 8.94$	$8.20 - 8.93$
$\theta_{23}^l [^\circ]$	44.89	$48.79^{+0.93}_{-1.25}$	$49.2^{+0.9}_{-1.2}$	$41.63 - 51.32$	$40.1 - 51.7$
$\delta_{\text{CP}} [^\circ]$	203.15	216^{+41}_{-25}	197^{+27}_{-24}	$144 - 360$	$120 - 369$

Table: Model and experimental values of the neutrino mass squared splittings, leptonic mixing angles, and CP-violating phase.

$$M_U = \begin{pmatrix} -0.00111287 & 0.00224708 & 0.276781 \\ 0.00214193 & -0.621473 & -0.860806 \\ 0.0434745 & 0.849889 & 173.079 \end{pmatrix} \text{GeV},$$

$$M_D = \begin{pmatrix} 0.00396153 & 0.0120505 & -0.000101736 - 0.0100846i \\ 0.00331057 & 0.0648106 & 0.0952388 \\ -0.0480789 & 0.325728 & 2.82949 \end{pmatrix} \text{GeV}$$

Observable	Model value	Experimental value
m_u [MeV]	1.52	1.24 ± 0.22
m_c [GeV]	0.63	0.63 ± 0.02
m_t [GeV]	172.7	172.9 ± 0.4
m_d [MeV]	2.88	2.69 ± 0.19
m_s [MeV]	55.2	53.5 ± 4.6
m_b [GeV]	2.86	2.86 ± 0.03
$\sin \theta_{12}^q$	0.22627	0.22650 ± 0.00048
$\sin \theta_{23}^q$	0.04077	$0.04053^{+0.00083}_{-0.00061}$
$\sin \theta_{13}^q$	0.00369	$0.00361^{+0.00009}_{-0.00011}$
J_q	3.05×10^{-5}	$(3.00^{+0.15}_{-0.09}) \times 10^{-5}$

Table: Model and experimental values of the quark masses and CKM parameters.

effective parameters : $\frac{v_3}{\Lambda} \in [0.2, 0.5]$, $\frac{v_{i \neq 3}}{\Lambda} \in [10^{-3}, 0.5]$,

scalar potential : $\mu_h^2 \in [0, 10^6] \text{ GeV}^2$, $r_{1,2}$, d , s , $\alpha_{1,2} \in [0.05, 2]$, $\beta_{1,2} \in [0.05, 2]$ $\frac{v_{123}}{\Lambda}$,

quark sector : $x_{ij}^{(U,D)}$, $c_{S_2,A}^{(U,D)} \in \pm [0.5, 1.5]$,

neutrino sector : $y_{1,2,3,4,5}^{(\nu)} \in [0.5, 1.5]$, $m_{N_{1,2,3,4}} \in [10^{-1}, 10^6] \text{ GeV}$