

*Effect of light sterile states on measurements
at long-baseline experiments*

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**joining soon..*

Oscillation probabilities: $\mathcal{F}(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, \Delta m_{21}^2, \Delta m_{31}^2)$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{bmatrix} \begin{bmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{-i\delta_{\text{CP}}} & 0 & \cos \theta_{13} \end{bmatrix} \begin{bmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

LBL superbeam +
Atmospheric exp.
 $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$
SBL reactor exp.
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$
Solar + LBL
reactor exp.
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$

Well-measured parameters:
(best-fit $\pm 1\sigma$)

$$\theta_{12}(^{\circ}) = 34.3 \pm 1.0$$

$$\Delta m_{21}^2 = 7.50^{+0.22}_{-0.20} \times 10^{-5} \text{ eV}^2$$

$$\theta_{13}(^{\circ}) = 8.53^{+0.13}_{-0.12}$$

$$|\Delta m_{31}^2| = 2.55^{+0.02}_{-0.03} \times 10^{-3} \text{ eV}^2$$

Not so well measured:

$$\theta_{23}(^{\circ}) \approx 49.0$$

$$3\sigma \text{ range } (^{\circ}) : 41.20 - 51.33$$

What is the neutrino mass ordering

- **normal i.e.** $m_3 \gg m_2 > m_1$ **or**
- **inverted i.e.** $m_2 > m_1 \gg m_3$?

Do neutrinos violate CP?

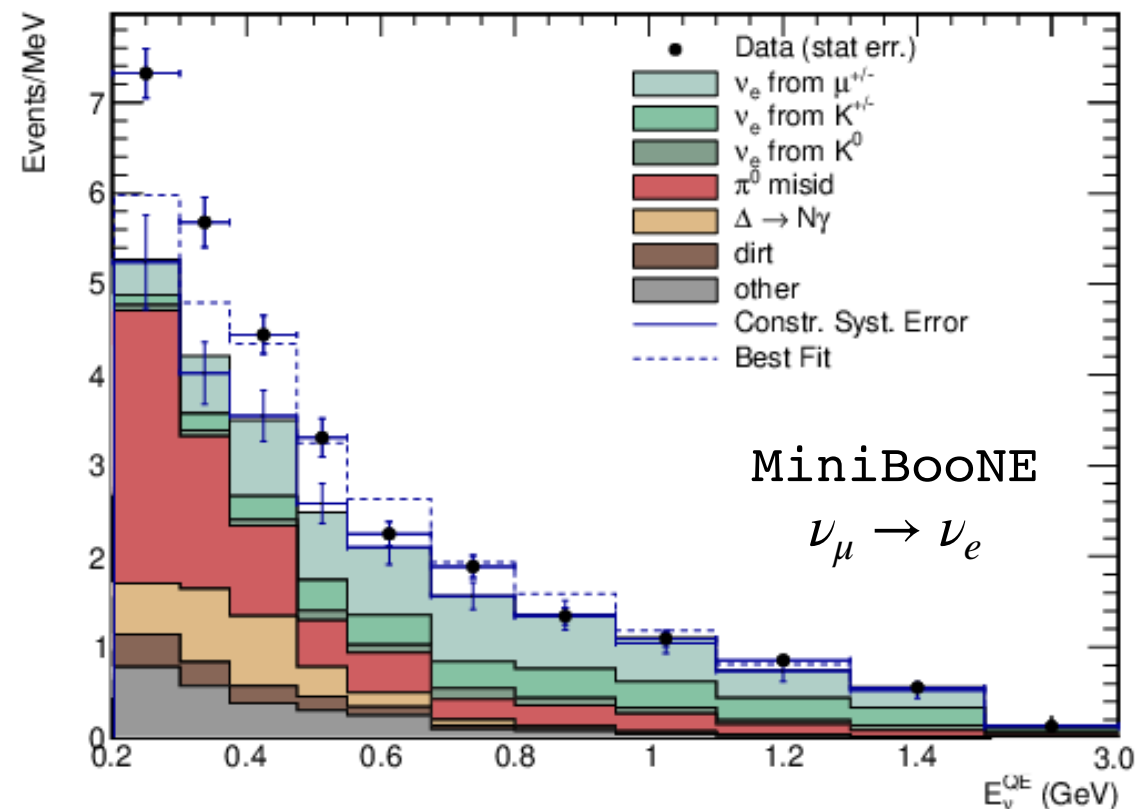
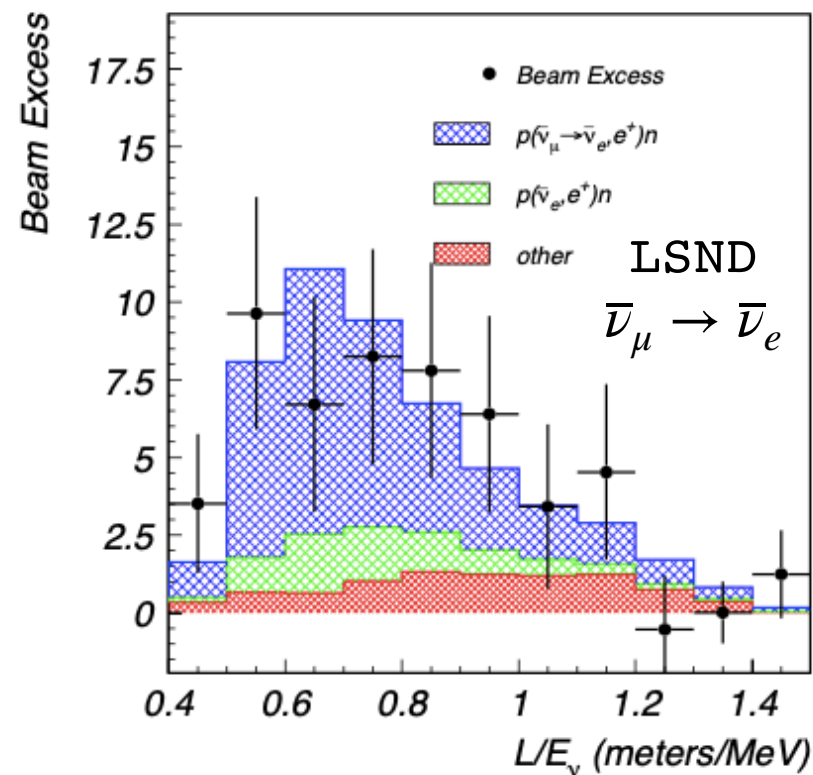
- **Is** $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$?
- $\delta_{\text{CP}}(^{\circ}) = ??$

Short-baseline anomalies

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A bunch of unexplained anomalies when neutrinos travel very short distances !!

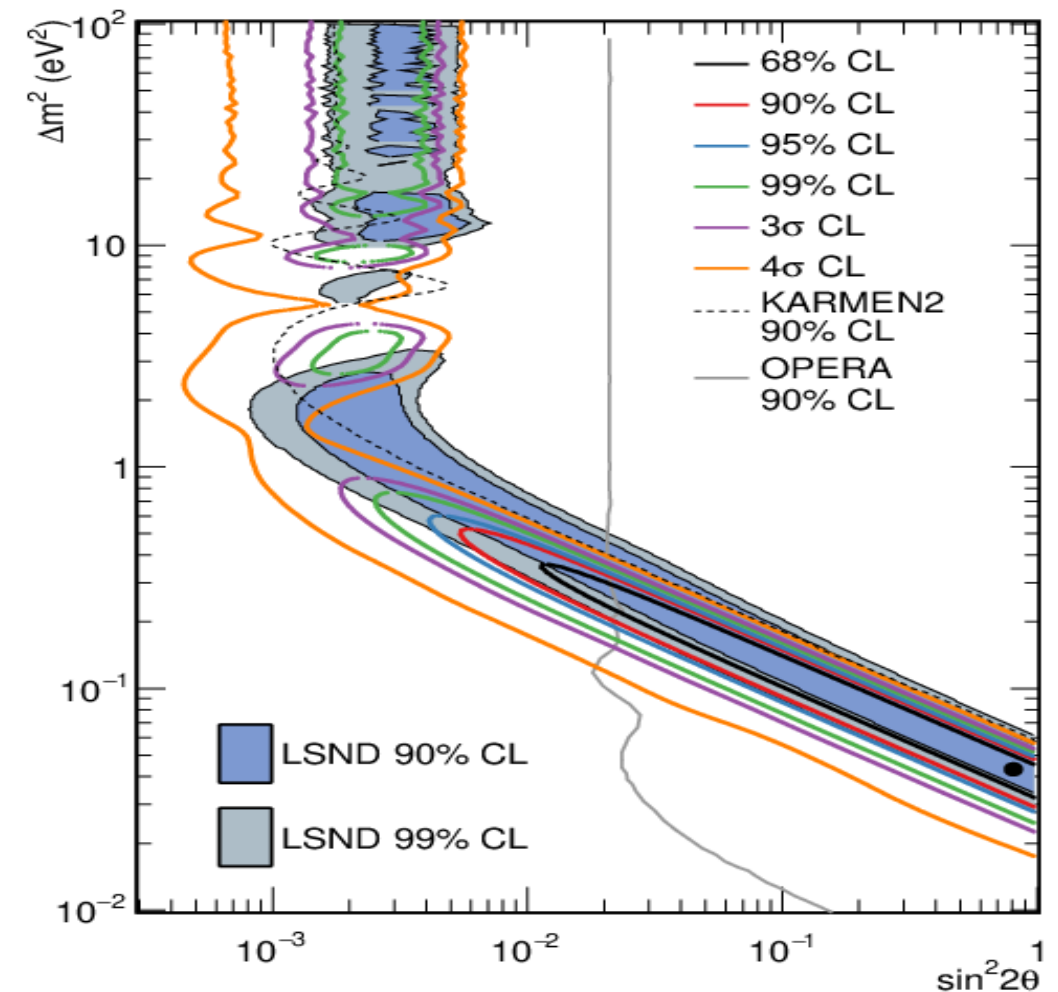
Phys.Rev.D64(2001)112007



Two flavor oscillation fit:

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

$$(\sin^2 2\theta, \Delta m^2) = (0.807, 0.043 \text{ eV}^2)$$



Phys.Rev.Lett121(2018)221801

Combined significance of LSND and MiniBooNE excess is $\sim 6\sigma$!

- * For the baselines and neutrino energies of the short-baseline experiments, the data cannot be explained by oscillations due to the solar or atmospheric mass-squared differences.
- * The measurement of the invisible decay width of the Z boson limits the number of light SM active neutrinos to $N_\nu = 2.9840 \pm 0.0082$.
- * The simplest way to explain the SBL anomalies is to assume the existence of new light neutrinos which are similar to active neutrinos in masses, mixings except that they do not undergo weak interactions.
- * Conventionally, these new neutrinos have been called “Sterile”. Sterile neutrinos are SM singlets.
- * SBL anomalies can be explained if Active and Sterile neutrinos are mixed. In the simplest case of the 3+1 model, a sterile neutrino of mass $\sim 1\text{eV}$ is needed along with the 3 active neutrinos.

Deep Underground Neutrino Experiment

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hep-ex/arXiv:2002.03005

1.2 MW neutrino beam

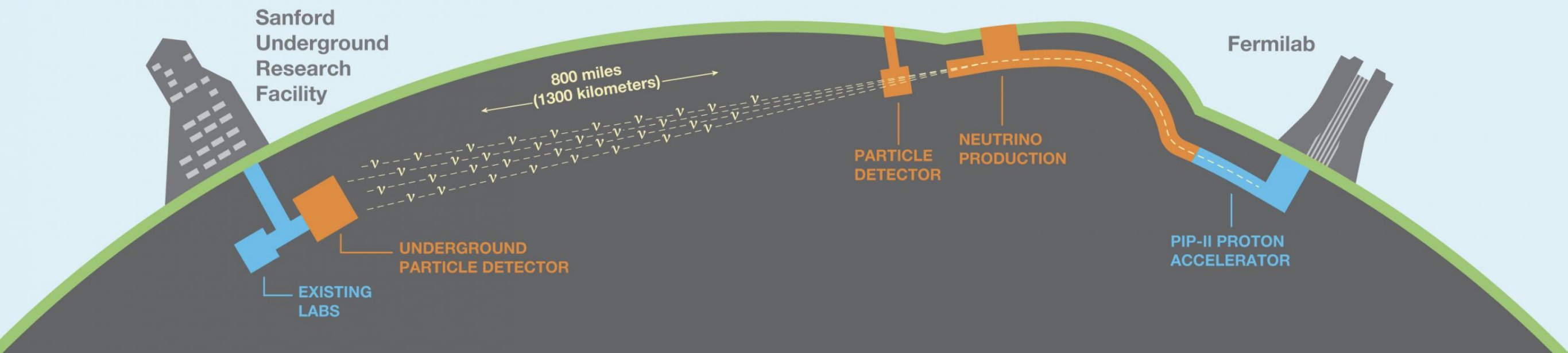
1.1×10^{21} protons-on-target/year

3.5 year ν + 3.5 year $\bar{\nu}$

Detector: Liquid Argon TPC

$E_\nu \in (0.5 \text{ GeV}, 10 \text{ GeV})$

Baseline: 1300 km



The primary science goals of DUNE are to measure:

- CP phase
- Neutrino mass ordering
- Octant of 2-3 mixing angle
- Precision measurements

Can $\Delta m^2 \sim 1 \text{ eV}^2$ oscillations impact LBL far-detector measurements?
If yes, how exactly?

Gandhi, Kayser, Masud, Prakash JHEP11(2015)039
Dutta, Gandhi, Kayser, Masud, Prakash JHEP11(2016)122

The 3+1 formalism

- * We augment the standard neutrino theory (3+0) by a new heavier mass eigenstate ν_4 (3+1).
- * This new state is sterile as far as the weak interactions are concerned but it can mix with the active neutrinos.
- * The mass of ν_4 is around 1 eV such that $\Delta m_{41}^2 = m_4^2 - m_1^2 = 1 \text{ eV}^2$.
- * $\Delta m_{41}^2 \sim \Delta m_{42}^2 \sim \Delta m_{43}^2 \gg |\Delta m_{31}^2| \sim |\Delta m_{32}^2| \gg \Delta m_{21}^2$
- * The Δm_{4i}^2 - driven oscillations at the far detector with $L/E \sim 500 \text{ km/GeV}$ will be too fast to be resolved by the detector.
- * The oscillations in 4x4 mixing matrix are characterized by **6 mixing angles**, **3 CP-violating phases** and **3 mass-squared differences**.

$$U_{\text{PMNS}}^{3+1} = \underbrace{O(\theta_{34}, \delta_{34}) \ O(\theta_{24}, \delta_{24}) \ O(\theta_{14})}_{\text{new mixing angles and phases}} \underbrace{O(\theta_{23}) \ O(\theta_{13}, \delta_{13}) \ O(\theta_{12})}_{\text{standard PMNS}}$$

(Here O's are 4x4 Euler rotation matrices)

Constraints on the active-sterile mixings

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- * Constraints derived are calculated in the region of $\Delta m_{41}^2 \sim 1 \text{ eV}^2$. At such values, **the most stringent constraints come from the disappearance experiments.**
- * Consistent parameterisation.
- * We assume the lower limits on the active-sterile mixings to be 0.
- * ν_e and $\bar{\nu}_e$ disappearance searches probe $|U_{e4}| = \sin \theta_{14}$.
- * ν_μ , $\bar{\nu}_e$ and NC disappearance searches probe
$$|U_{\mu 4}| = \cos \theta_{14} \sin \theta_{24} \text{ and}$$
$$|U_{\tau 4}| = \cos \theta_{14} \cos \theta_{24} \sin \theta_{34}.$$
- *

Daya Bay:	$\theta_{14} \in [0, 13^\circ]$	95% C.L.	PRL113,141802(2014)
IceCube:	$\theta_{24} \in [0, 7^\circ]$	99% C.L.	PRL117,071801(2016)
MINOS(+):	$\theta_{34} \in [0, 26^\circ]$	90% C.L.	PRL107,011802(2011)
- * The CP phases remain unconstrained.
- * LSND: $\sin^2 2\theta_{\mu e} < 0.008$ for $\Delta m_{41}^2 \sim 1 \text{ eV}^2$.

$\nu_\mu \rightarrow \nu_e$ in 3+1 in vacuum

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= 4 |U_{\mu 4} U_{e 4}|^2 \times 0.5 - 4 \sum_{j < 4, i < j} \text{Re}(U_{\mu i} U_{e i}^* U_{\mu j}^* U_{e j}) \sin^2 \Delta m_{ji}^2 + 2 \sum_{j < 4, i < j} \text{Im}(U_{\mu i} U_{e i}^* U_{\mu j}^* U_{e j}) \sin 2\Delta m_{ji}^2 \\
 &= \frac{1}{2} \sin^2 2\theta_{\mu e}^{4\nu} + (a^2 \sin^2 2\theta_{\mu e}^{3\nu} - \frac{1}{4} \sin^2 2\theta_{13} \sin^2 2\theta_{\mu e}^{4\nu}) [\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}] \\
 &\quad + \cos(\delta_{13}) b a^2 \sin 2\theta_{\mu e}^{3\nu} [\cos 2\theta_{12} \sin^2 \Delta_{21} + \sin^2 \Delta_{31} - \sin^2 \Delta_{32}] \\
 &\quad + \cos(\delta_{24}) b a \sin 2\theta_{\mu e}^{4\nu} [\cos 2\theta_{12} \cos^2 \theta_{13} \sin^2 \Delta_{21} - \sin^2 \theta_{13} (\sin^2 \Delta_{31} - \sin^2 \Delta_{32})] \\
 &\quad + \cos(\delta_{13} + \delta_{24}) a \sin 2\theta_{\mu e}^{3\nu} \sin 2\theta_{\mu e}^{4\nu} \left[-\frac{1}{2} \sin^2 2\theta_{12} \cos^2 \theta_{13} \sin^2 \Delta_{21} \right. \\
 &\quad \left. + \cos 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \right] \\
 &\quad - \frac{1}{2} \sin(\delta_{13}) b a^2 \sin 2\theta_{\mu e}^{3\nu} [\sin 2\Delta_{21} - \sin 2\Delta_{31} + \sin 2\Delta_{32}] \\
 &\quad + \frac{1}{2} \sin(\delta_{24}) b a \sin 2\theta_{\mu e}^{4\nu} [\cos^2 \theta_{13} \sin 2\Delta_{21} + \sin^2 \theta_{13} (\sin 2\Delta_{31} - \sin 2\Delta_{32})] \\
 &\quad + \frac{1}{2} \sin(\delta_{13} + \delta_{24}) a \sin 2\theta_{\mu e}^{3\nu} \sin 2\theta_{\mu e}^{4\nu} [\cos^2 \theta_{12} \sin 2\Delta_{31} + \sin^2 \theta_{12} \sin 2\Delta_{32}] \\
 &\quad + (b^2 a^2 - \frac{1}{4} a^2 \sin^2 2\theta_{12} \sin^2 2\theta_{\mu e}^{3\nu} - \frac{1}{4} \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 2\theta_{\mu e}^{4\nu}) \sin^2 \Delta_{21}
 \end{aligned}$$

Here:

$$\Delta_{ji} = \Delta m_{ji}^2$$

$$\sin 2\theta_{\mu e}^{3\nu} = \sin 2\theta_{13} \sin \theta_{23}$$

$$\sin 2\theta_{\mu e}^{4\nu} = \sin 2\theta_{14} \sin \theta_{24}$$

$$a = \cos \theta_{14} \cos \theta_{24}$$

$$b = \cos \theta_{13} \cos \theta_{23} \sin 2\theta_{12}$$

- * In vacuum, $P(\nu_\mu \rightarrow \nu_e)$ does not depend on θ_{34} and δ_{34} .
- * In vacuum, there are only two effective CP-violating phases: δ_{24} and $(\delta_{13} + \delta_{24})$.
- * Terms marked in **red** are the ones which lead to substantial differences between 3+0 and 3+1.
- * **These are interference terms which depend on the sines and cosines of the CP-violating phases.**

$\nu_\mu \rightarrow \nu_e$ in 3+1 in matter

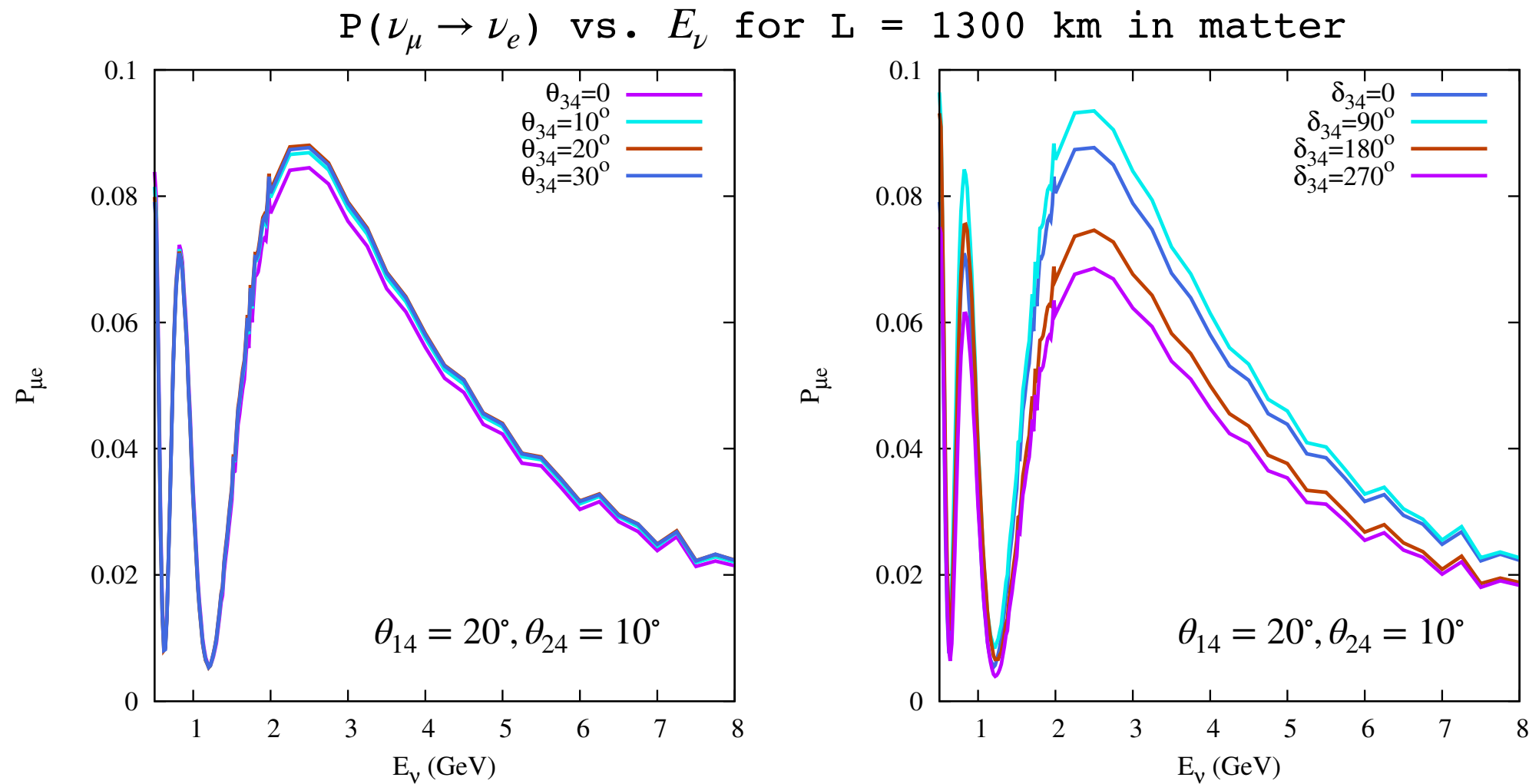
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We calculate exact oscillation probabilities in matter using GLoBES.

Comput.Phys.Commun.167(2005)195

Left plot: CP-violating phases are equal to 0, θ_{34} is varied $\in [0, 30^\circ]$.

Right plot: θ_{34} is fixed equal to 30° , CP-violating phases are varied $\in [0, 360^\circ]$.



- * θ_{34} and δ_{34} play significant roles in matter.
- * These parameters play no role in vacuum or at short-baselines
- * **Considerations for vacuum/SBL do not carry over to matter/LBL!**

$\nu_\mu \rightarrow \nu_e$ in 3+1 in matter

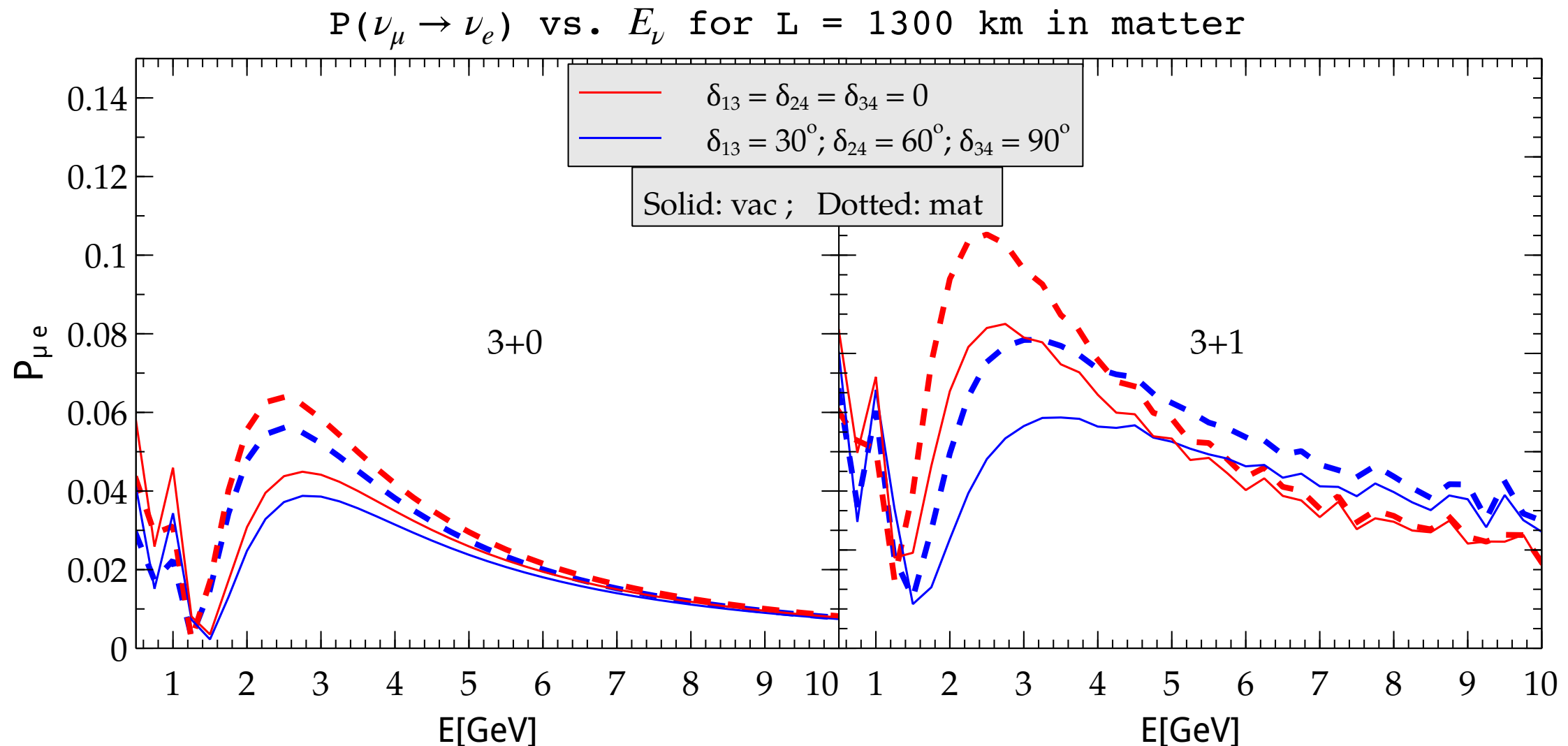
10

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Left plot: CP-violating phases are equal to 0, θ_{34} is varied $\in [0, 30^\circ]$.

Right plot: θ_{34} is fixed equal to 30° , CP-violating phases are varied $\in [0, 360^\circ]$.



- * θ_{34} and δ_{34} play significant roles in matter.
- * These parameters play no role in vacuum or at short-baselines
- * Considerations for vacuum/SBL do not carry over to matter/LBL!

- * The measure of CP violation (in vacuum) can be given by $\Delta P_{\alpha\beta} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$
- * $\Delta P_{\alpha\beta} = -\Delta P_{\beta\alpha}$ due to CPT invariance $\implies \Delta P_{\alpha\alpha} = 0$ i.e. no CPV in disappearance channels.
- * In 3+0, we can have 3 independent CP violating differences: $\Delta P_{\mu e}, \Delta P_{\mu\tau}, \Delta P_{e\tau}$ while in 3+1, we can have 6 independent CP violating differences: $\Delta P_{\mu e}, \Delta P_{\mu\tau}, \Delta P_{e\tau}, \Delta P_{e\mu}, \Delta P_{\mu s}, \Delta P_{\tau s}$.
- * From the conservation of probability, we have

$$P_{\mu e} + P_{\mu\mu} + P_{\mu\tau} = \bar{P}_{\mu e} + \bar{P}_{\mu\mu} + \bar{P}_{\mu\tau} = 1$$

$$\implies \Delta P_{\mu e} + \Delta P_{\mu\tau} = 0$$

Similarly, we get: $\Delta P_{\tau e} + \Delta P_{\tau\mu} = 0$

This means, $\Delta P_{\mu e} = -\Delta P_{\mu\tau} = \Delta P_{e\tau}$

- * Thus, in 3+0, the extent of CP violation is the same across all channels. If we find CP to be conserved in one channel, it will also be conserved in the other ones.

However, in 3+1, using the same logic, we get relations such as: $\Delta P_{\mu e} = \Delta P_{\tau\mu} - \Delta P_{\mu s}$

Thus, extent of CP-violation across all channels in 3+1 may not be equal. We may not be able to conclude definitively by measuring only one CP-violating channel.

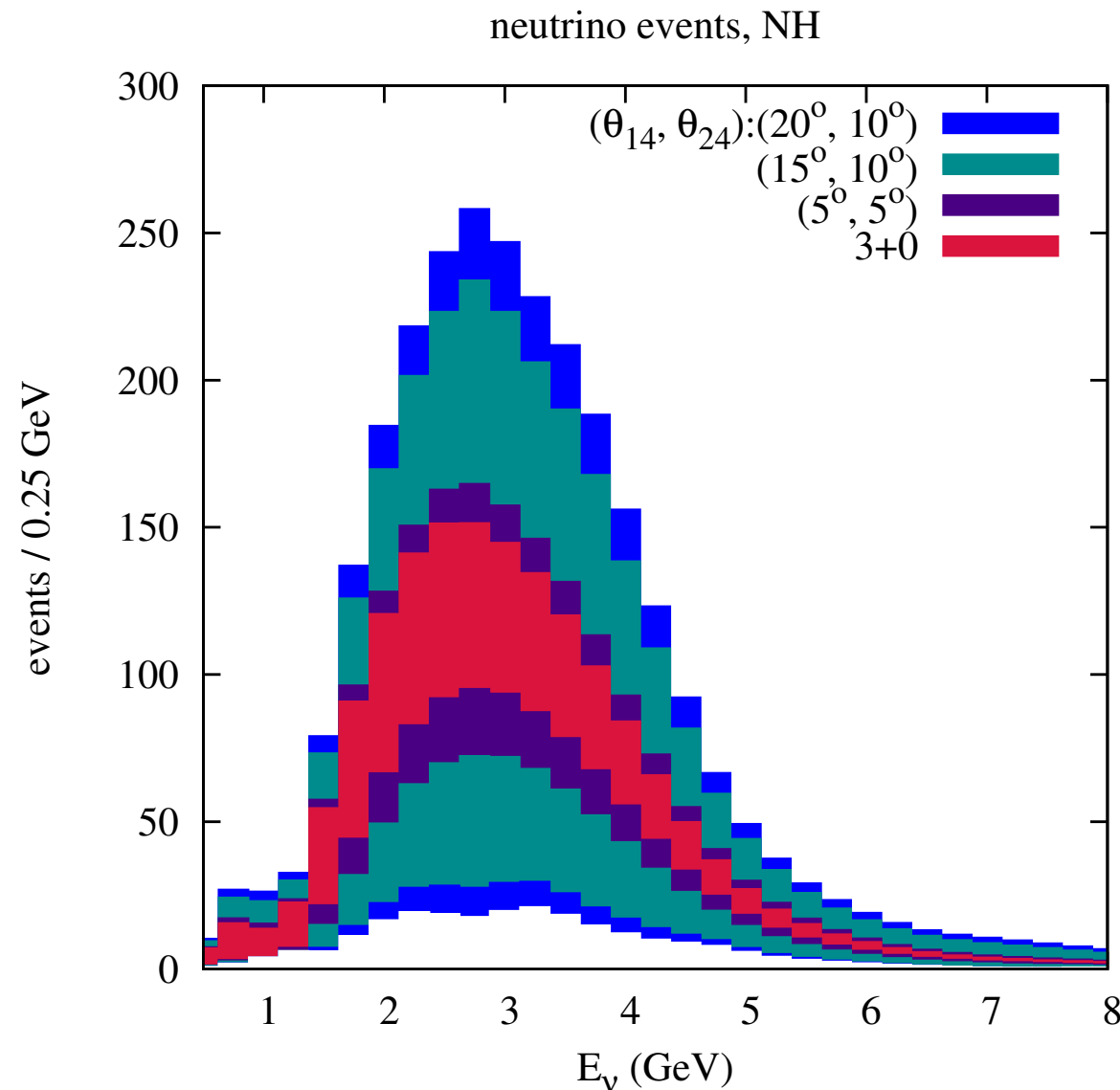
Event rates at DUNE

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We calculate events rates for DUNE by performing realistic simulations.

Red band: corresponds to 3+0 and variation of $\delta_{CP} \in [0, 360^\circ]$.

Other bands: correspond to 3+1 and variation of $\theta_{34} \in [0, 30^\circ]$ and $\delta_{13}, \delta_{24}, \delta_{34} \in [0, 360^\circ]$.



- * Too large or too small events rates may be pointers to sterile states
- * 3+0 encompassed by 3+1: leading to substantial degeneracy
- * If observed events in the red band then degenerate solutions possible!

Excluding CP-conserving values as a function of true oscillation parameters

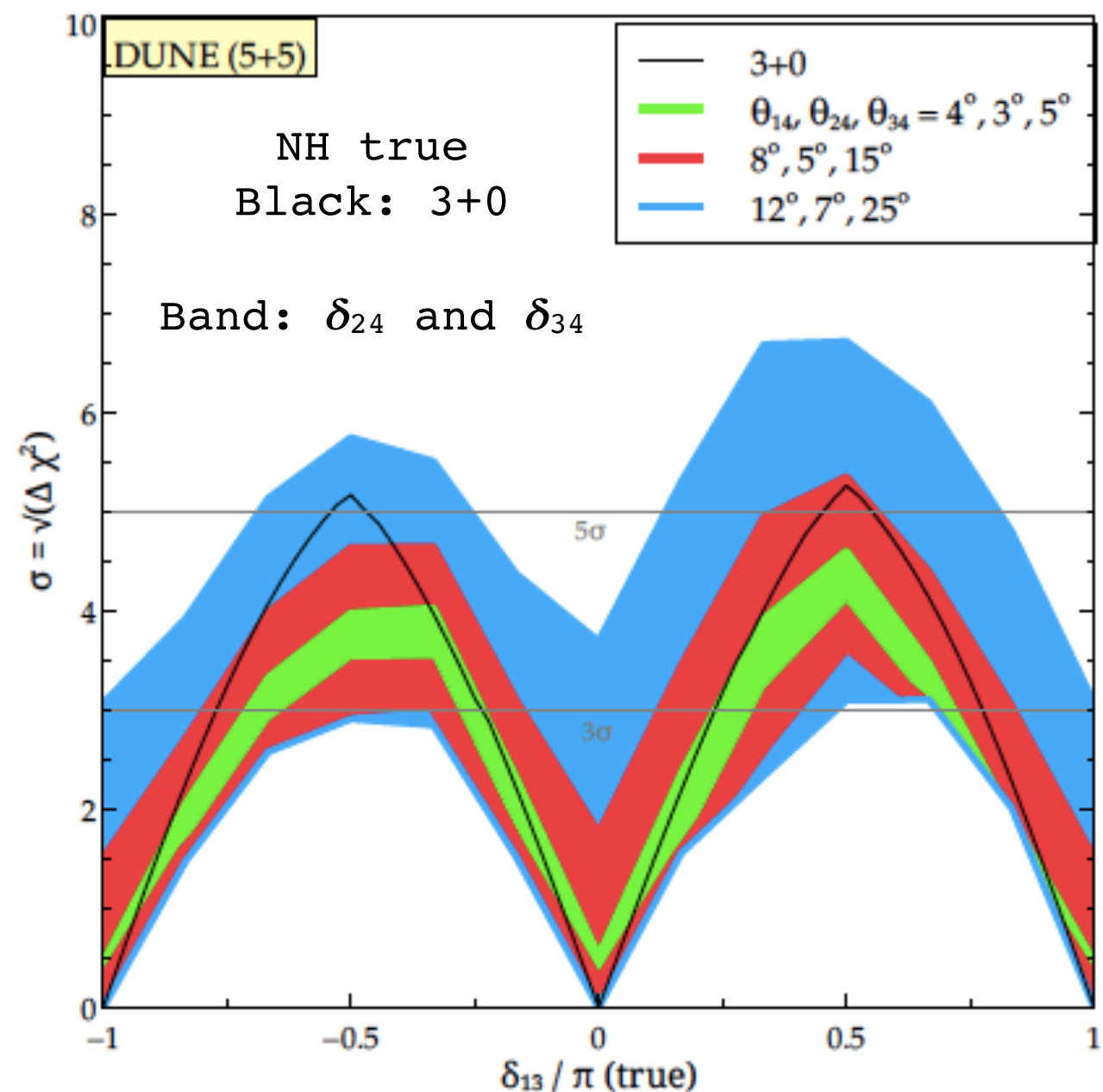
Results for the DUNE experiment:

- * We compare 3+0 and 3+1.
- * 3+0: test values: $\delta_{13} = 0, \pm \pi$.
- * 3+1: test values: $\delta_{13}, \delta_{24}, \delta_{34} = 0, \pm \pi$.
- * 3+0 oscillation parameters held fixed in fit.
- * χ^2 marginalized over CP-conserving values and in allowed ranges of test $\theta_{14}, \theta_{24}, \theta_{34}$.

Small mixings: Sensitivity decreases

Large mixings: Sensitivity spans on both sides of the 3+0 sensitivity

Significant amplification of sensitivity even when true $\delta_{CP} = 0$



Determining the phase responsible for CPV

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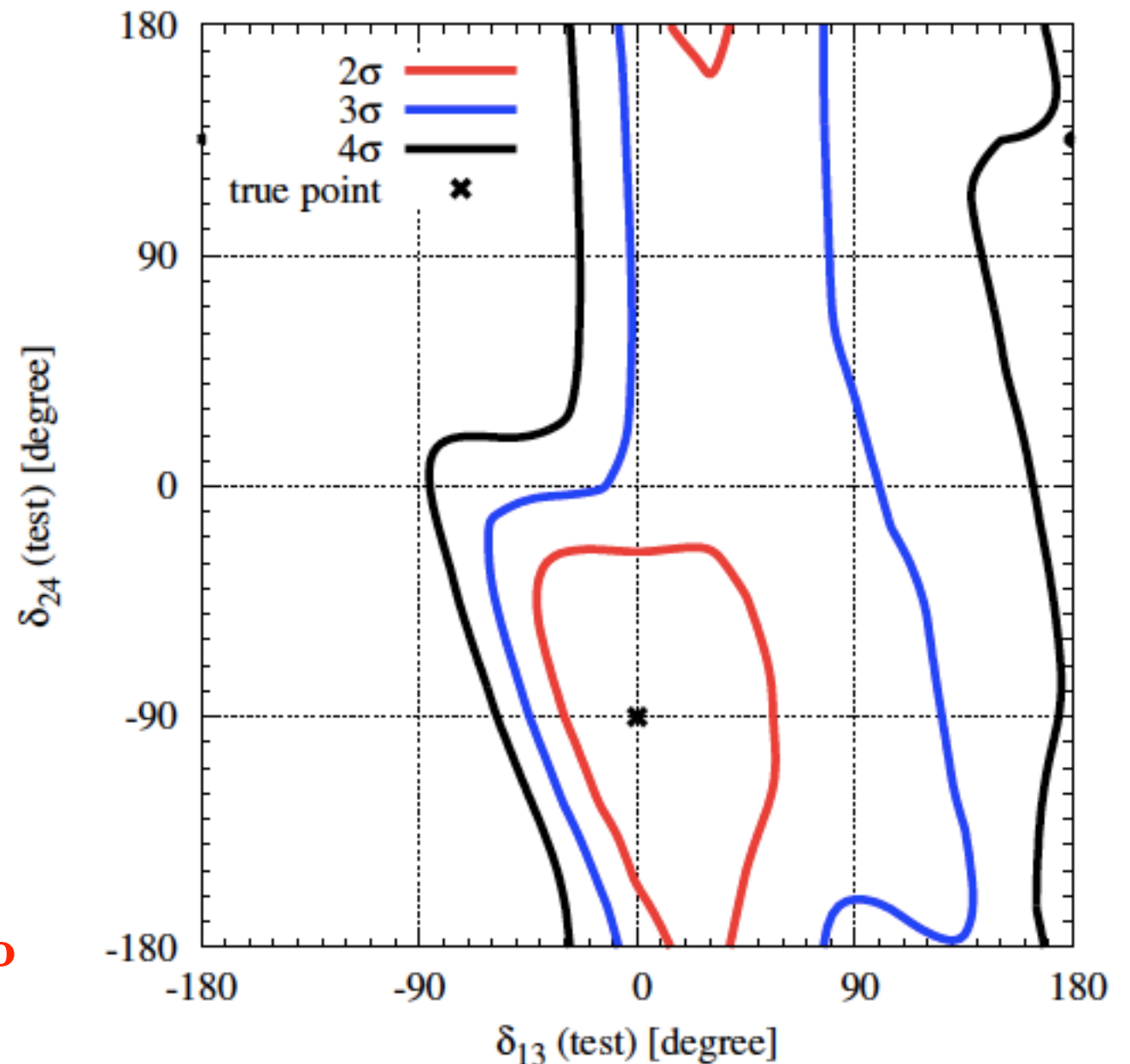
Assuming 3+1 is true and CPV has been observed, can we identify its source?

Results for the DUNE experiment:

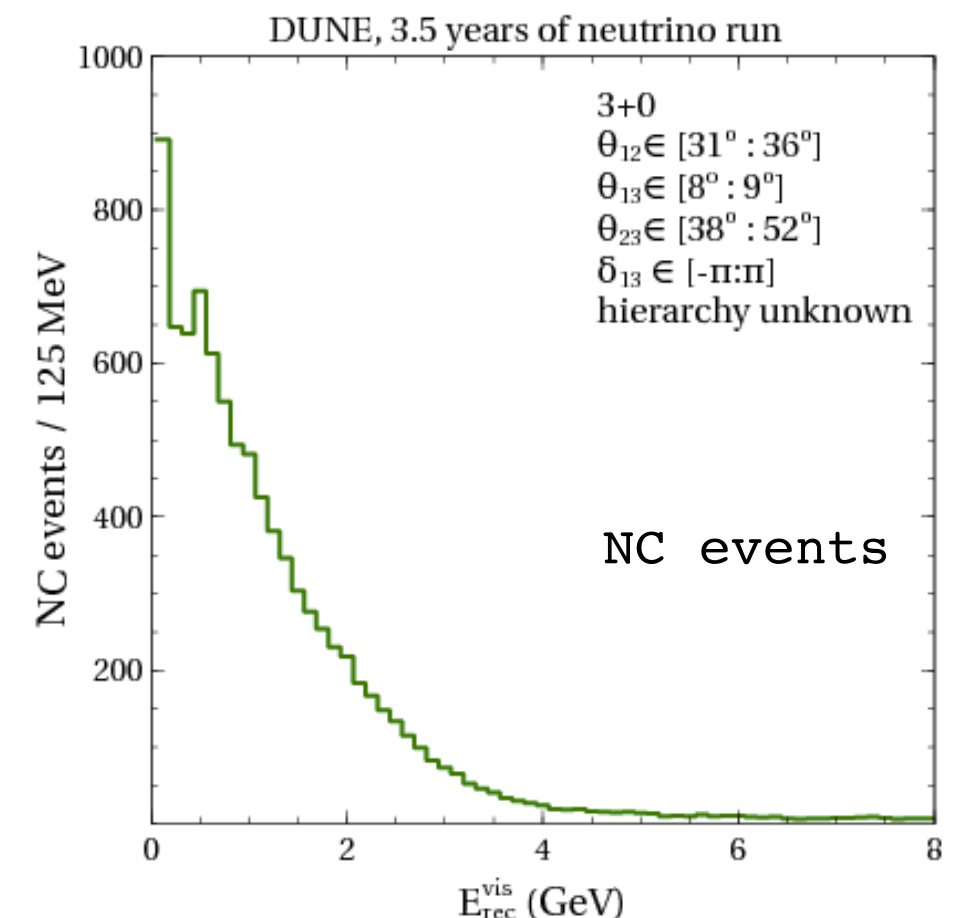
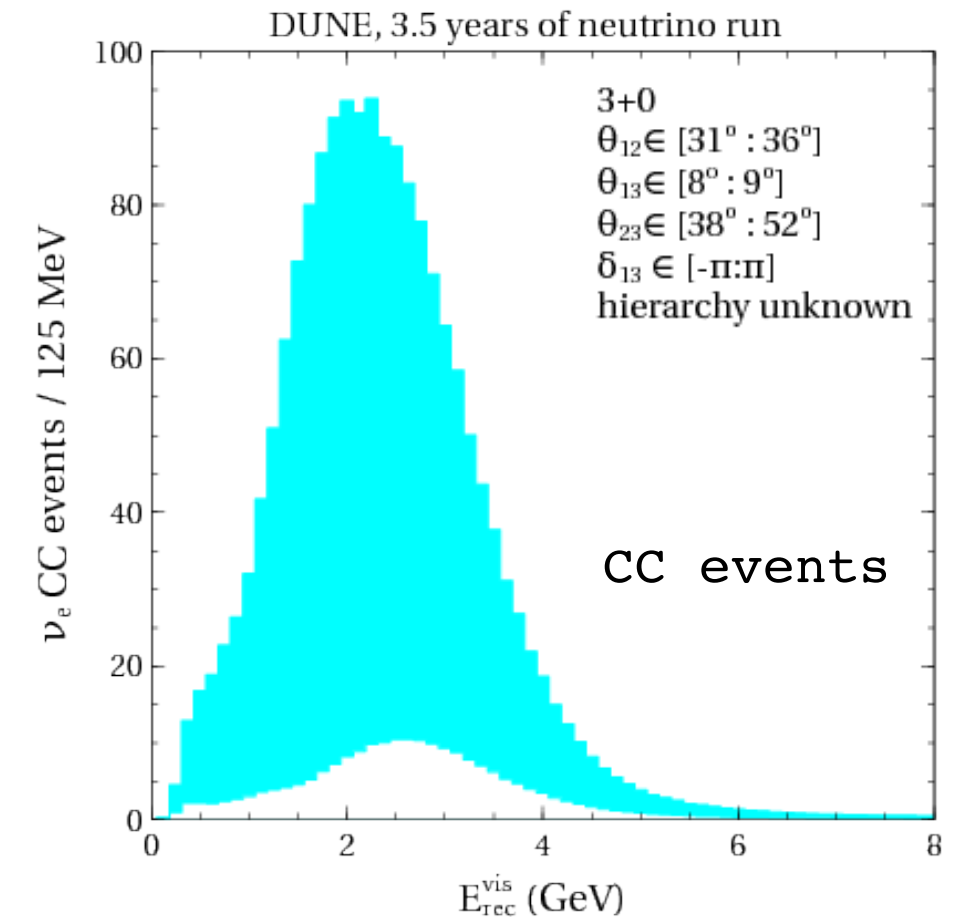
- * We assume 3+1 to be the true case.
- * 3+1: true values: $\theta_{14}, \theta_{24}, \theta_{34} = 12^\circ, 7^\circ, 25^\circ$.
- * 3+1: true values: $\delta_{13}, \delta_{24}, \delta_{34} = 0, -90^\circ, 0$.
- * 3+0 oscillation parameters held fixed in fit.
- * χ^2 marginalized over test δ_{34} values and in allowed ranges of test $\theta_{14}, \theta_{24}, \theta_{34}$.

**Test values close to $(90^\circ, 0)$
and $(90^\circ, 180^\circ)$ allowed at 3σ**

**Attributing CP violation unambiguously to
any one phase may not be possible at 3σ**



- * We investigate how the NC measurements made at DUNE far-detector can help in the search of new physics scenarios.
- * NC events are insensitive to oscillations and therefore the uncertainties related to them. Any deviation from the expected events rates may be pointer to new physics.
- * NC events are statistically very rich. At DUNE near detector, events in surplus of 400,000 per year are expected and at the far detector too, NC events are much more than what is observed in CC.
- * As we show in the following, NC events are also less prone to matter effects and may provide qualitatively different and complementary measurements to the CC events.



NC probability in 3+1

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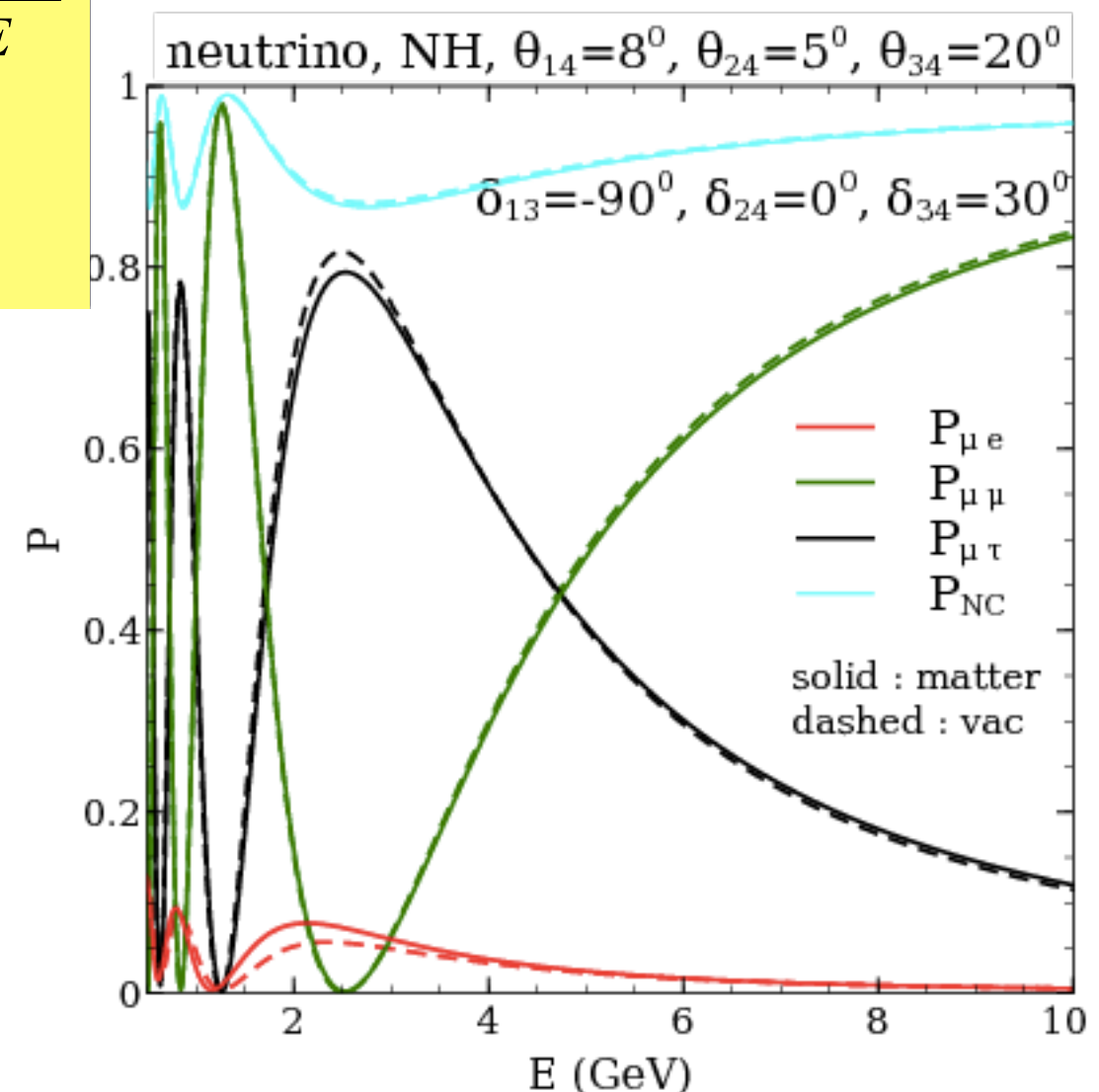
For a $(\bar{\nu})_\mu$ beam, we define $\bar{P}_{\text{NC}} = \bar{P}_{\mu e} + \bar{P}_{\mu\mu} + \bar{P}_{\mu\tau}$

In 3+0, $\bar{P}_{\text{NC}} = 1$

For 3+1, $\bar{P}_{\text{NC}} = 1 - \bar{P}_{\mu s} = \bar{P}_{\mu e} + \bar{P}_{\mu\mu} + \bar{P}_{\mu\tau} < 1$

$$\begin{aligned}
 P_{\mu s}^{\text{vac}} &\simeq \cos^4 \theta_{14} \cos^2 \theta_{34} \sin^2 2\theta_{24} \sin^2 \frac{\Delta m_{41}^2 L}{4E} \\
 &+ \left[\cos^4 \theta_{13} \cos^2 \theta_{24} \sin^2 \theta_{34} - \cos^2 \theta_{13} \cos^2 \theta_{24} \cos^2 \theta_{34} \sin^2 \theta_{34} \right. \\
 &+ \frac{1}{\sqrt{2}} \sin 2\theta_{13} \sin 2\theta_{34} \sin \theta_{14} \cos^3 \theta_{24} \cos(\delta_{13} + \delta_{34}) \left. \right] \sin^2 \frac{\Delta m_{31}^2 L}{4E} \\
 &+ \frac{1}{2} \cos^2 \theta_{13} \cos^2 \theta_{24} \sin 2\theta_{34} \sin \theta_{24} \sin(\delta_{34} - \delta_{24}) \sin \frac{\Delta m_{31}^2 L}{4E}
 \end{aligned}$$

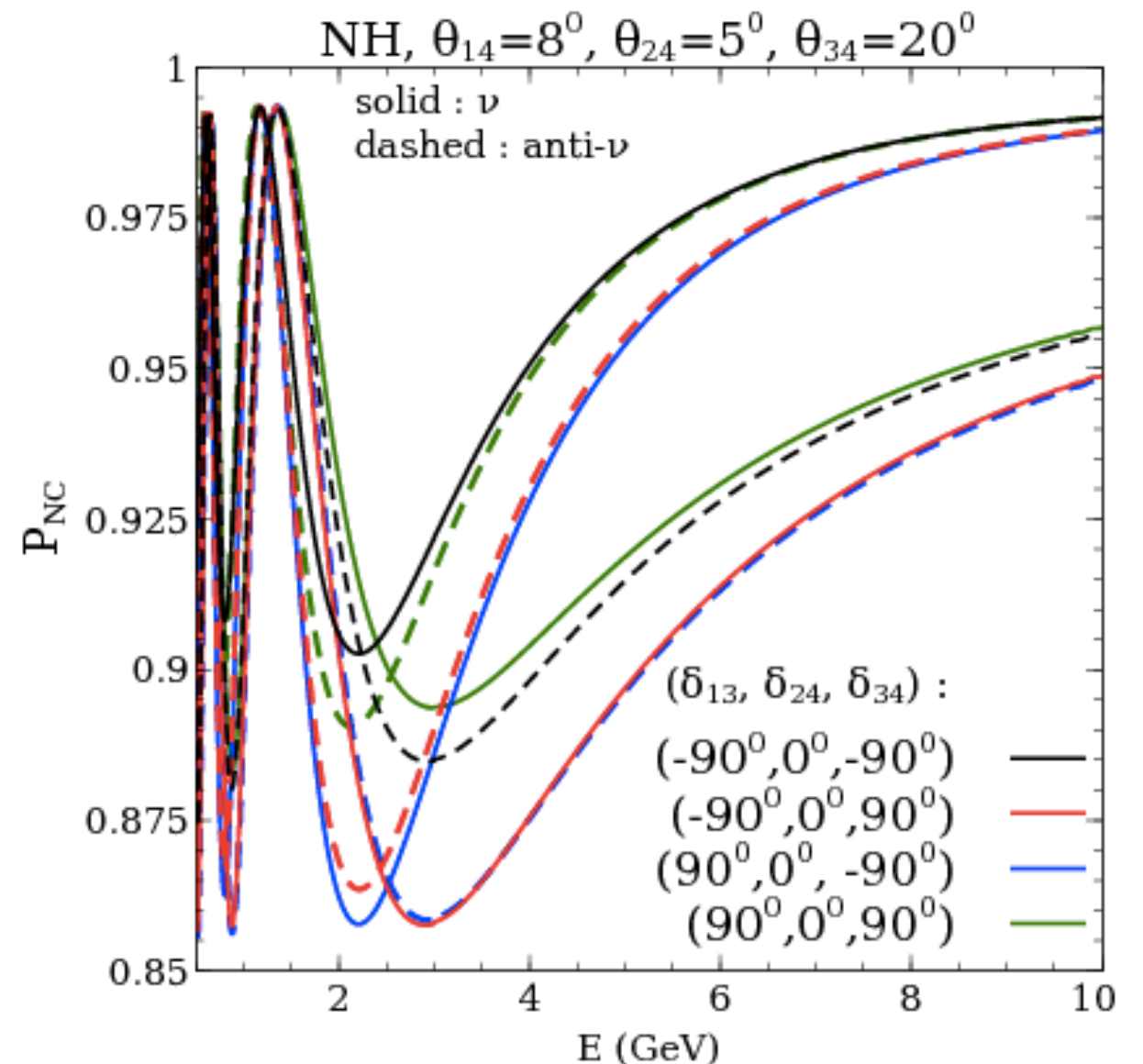
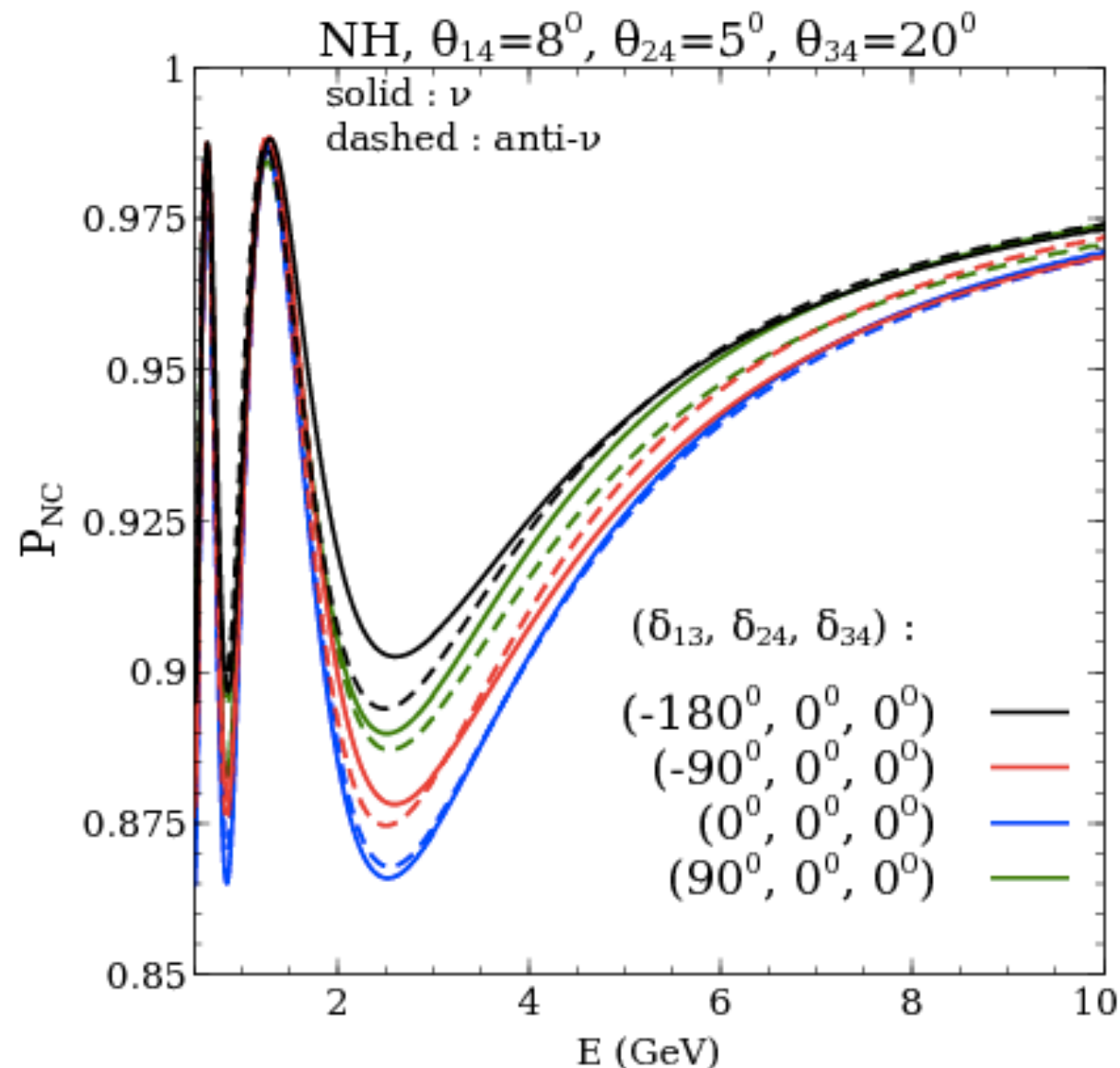
- * The first term is a rapidly oscillating term.
- * The dependence on CP phases only in linear combinations: $\delta_1 = \delta_{13} + \delta_{34}$ and $\delta_2 = \delta_{34} - \delta_{24}$.
- * There is very little matter effects in P_{NC} .
- * Very similar curves for anti-neutrino probabilities.



Effects of CP phases on P_{NC}

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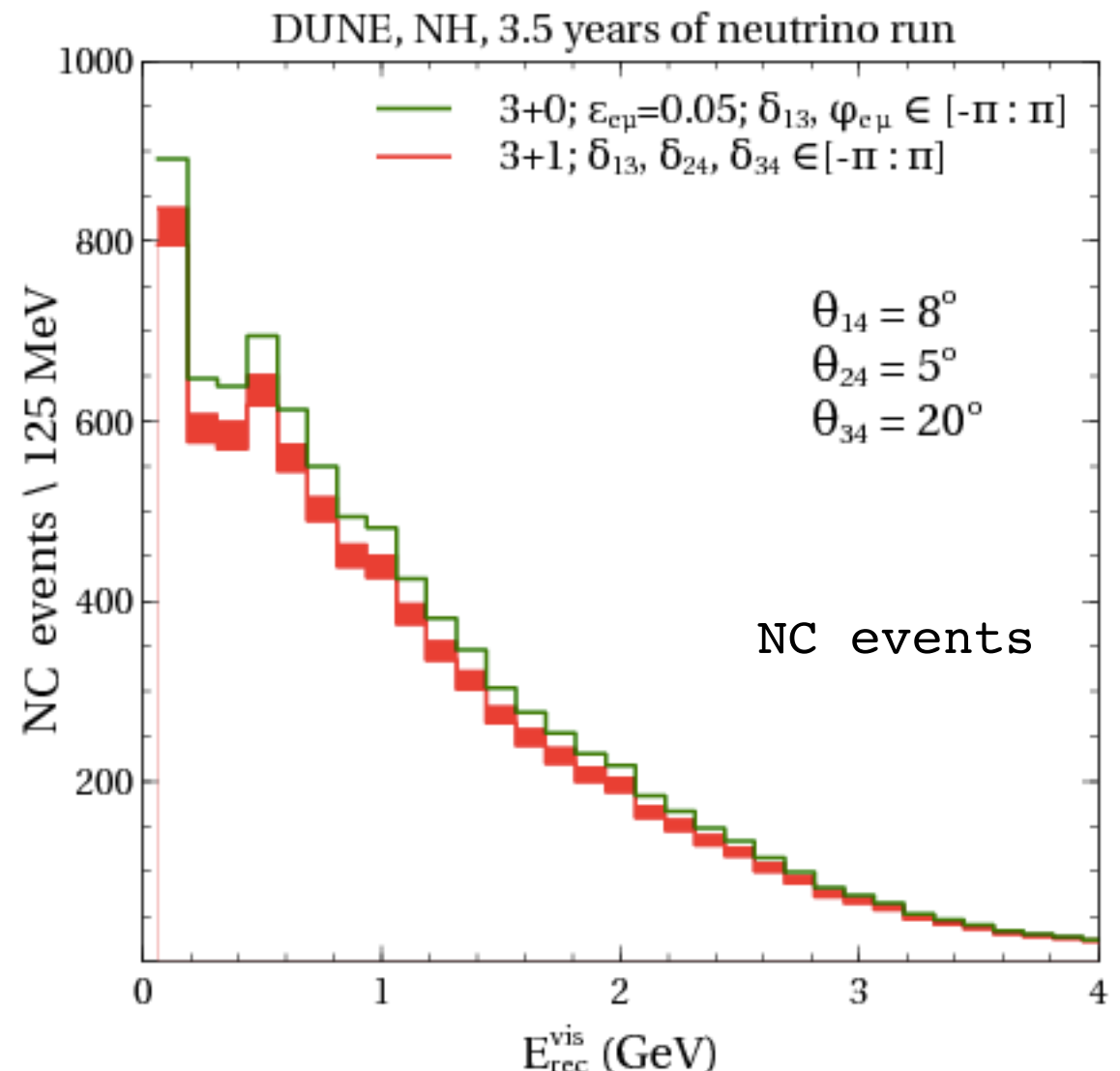
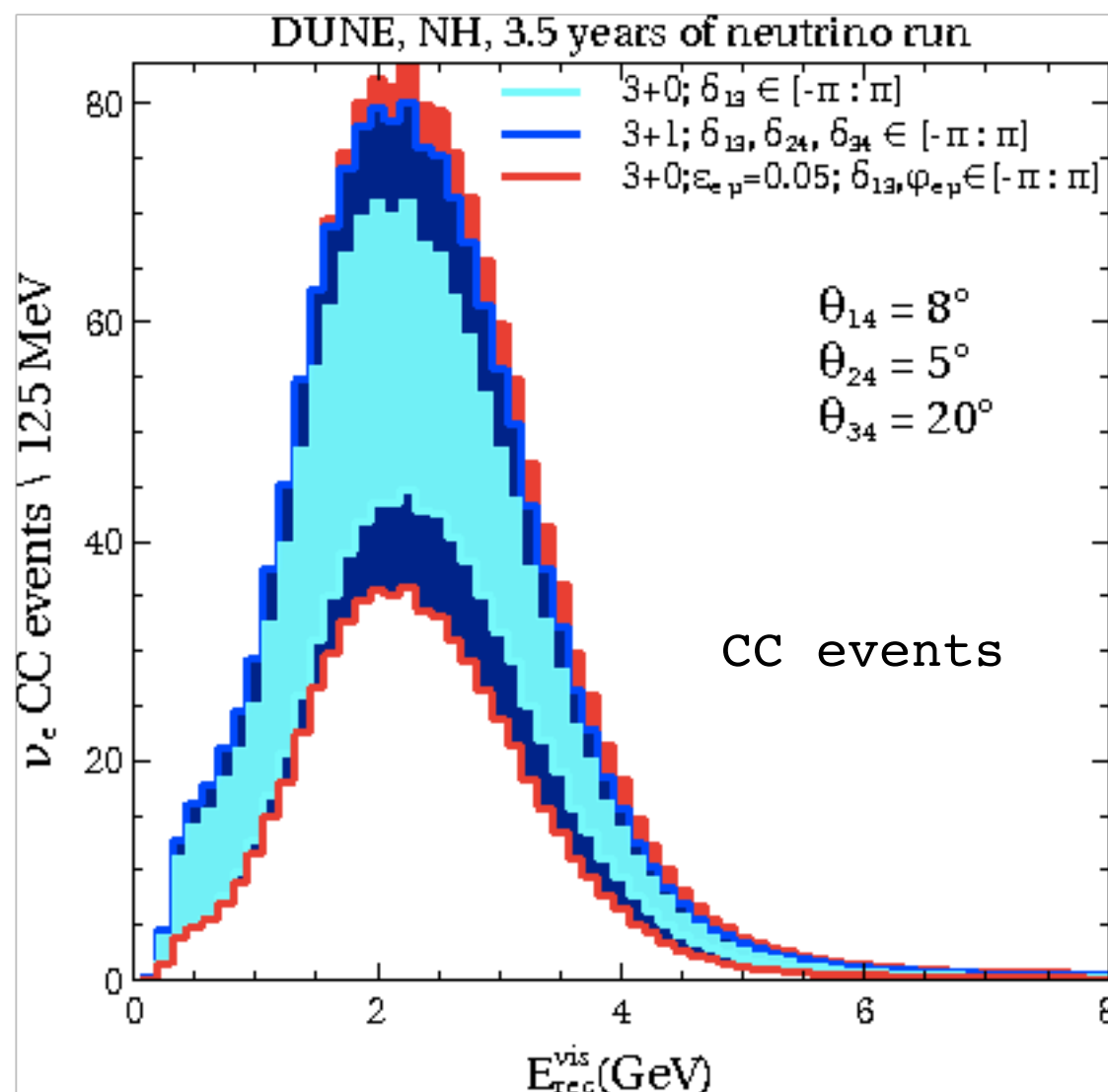
- * **Left panel:** The differences between P_{NC} and \bar{P}_{NC} are small. **Right panel:** δ_{34} induces larger differences between P_{NC} and \bar{P}_{NC}
- * Larger differences at higher energies. Large CP-asymmetry in NC events may point to a CP-violating value of δ_{34}



NC events as new-physics discriminator

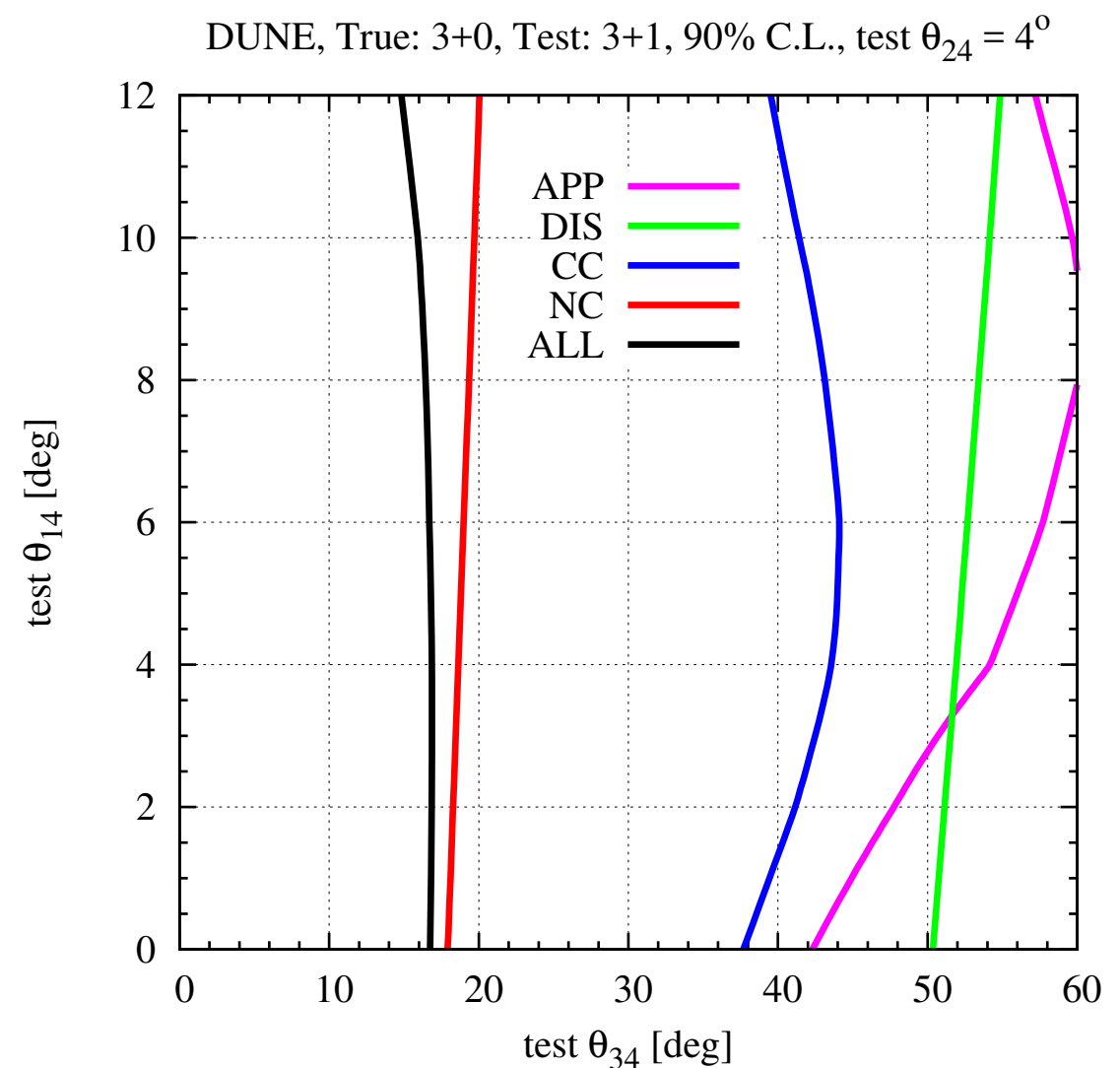
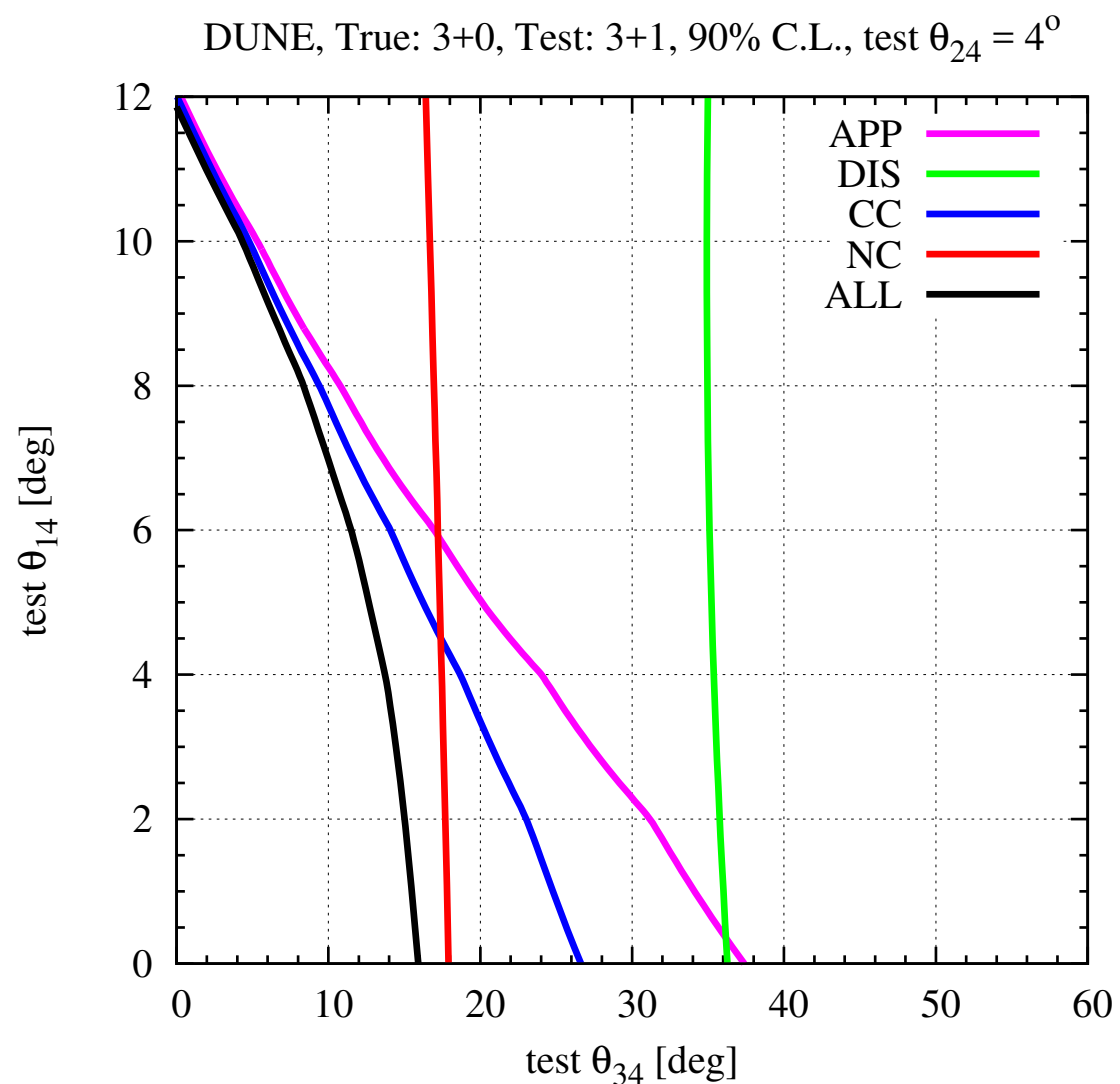
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- * **Left panel:** CC events with standard and new physics scenarios. **Right panel:** NC events with standard and new physics scenarios
- * **Left panel:** significant degeneracy between two new physics. **Right panel:** NC events discriminate between unitarity-violating and unitarity-preserving scenarios

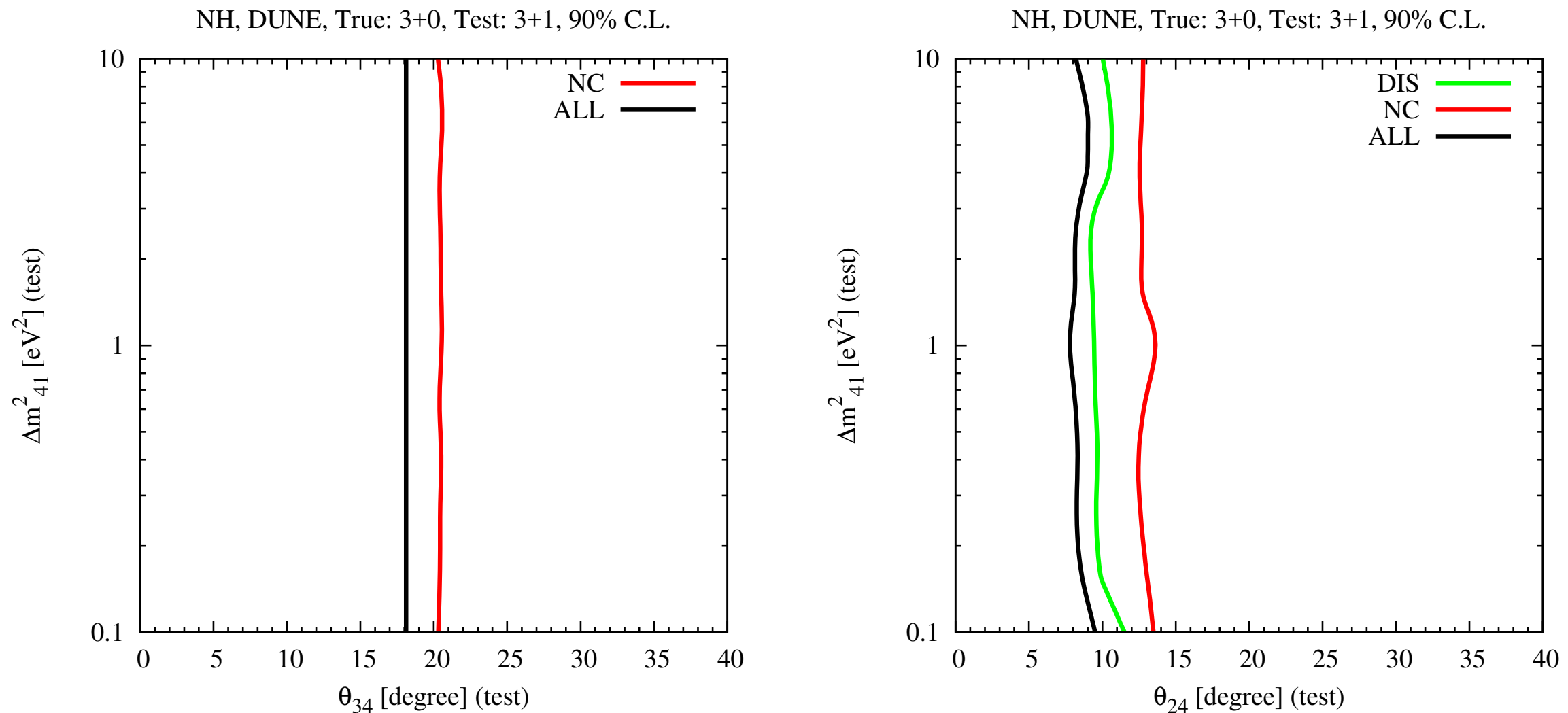


Left: CP phases fixed at true values

Right: CP phases varied in the fit



Constraints from CC depend a lot on the knowledge of CP phases
while those from NC seem to be immune to CP phases



- * **Left plot:** Sensitivity to exclusion of θ_{34} mainly from the NC data.
- * **Right panel:** Sensitivity to exclusion of θ_{24} mainly from the disappearance data.
- * **Both:** Sensitivity almost independent of the value of test Δm_{41}^2 . As a comparison, the sensitivity from IceCube reduces to $\theta_{24} \lesssim 45^\circ$ at 10 eV^2 [Phys.Rev.Lett.117\(2016\)071801](#)

- * From a probability and event rate analysis, we show that there are **large effects of the sterile oscillation parameters at long-baselines** especially via parameters which do not play a role in short-baseline experiments.
- * In the presence of even a single sterile neutrino, conclusions such as a) **CP is conserved or violated**, or, b) if the latter, whether the **violation is ascribable to the active neutrinos or the additional sterile neutrino**, or a combination of the two, are all **rendered significantly ambiguous**.
- * Our work stresses on the need to have **a dedicated short baseline program** which can put very stringent constraints on the existence of sterile neutrinos.
- * We show how **NC events can synergistically aid the search for new physics and CP violation** when combined with other measurements.
- * NC events offer a **window to CP phases and mixing angles** that is **complementary to** that accessed by **CC** measurements.
- * They can **break degeneracies** existing in CC measurements, allowing one to distinguish **between new physics** that violates $3+0$ unitarity and new physics that does not.

Thank you for your attention!