Shaken, not stirred: test particles in binary black hole mergers.

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Abstract

In 2015 the advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) detected the first ever gravitational event, gravitational wave event GW150914, with multiple new gravitational wave events, originating from both binary neutron stars and binary black hole (BBH) mergers, detected in subsequent years. In light of these detections, we simulate the dynamics of ambient test particles in the gravitational potential well of a BBH system close to its inspiral phase with the goal of simulating the associated electromagnetic radiation and resulting spectral energy density distribution of such a BBH system. This could shed light on possible detection ranges of electromagnetic counterparts to BBH mergers. The potentials are numerically calculated using finite difference methods, under the assumption of non-rotating black holes with the post-Newtonian Paczynski-Wiita potential approximation in tandem with retarded time concepts analogous to electrodynamics. We find that the frequencies of potential electromagnetic radiation produced by these systems (possibly reaching earth), range between a few kHz to a few $100kHz$. The bulk of radiation is distributed at frequencies below $100kHz$.

Keywords: Binary black hole merger, binary black hole, binary black hole merger simulation, particle acceleration, gravity.

1 The Paczynski-Wiita Potential

The Paczynski-Wiita potential is an important pseudo-Newtonian approximation to the general relativistic Schwarzschild geometry developed by B. Paczynski and P. Wiita while studying thick accretion disks and supercritical luminosities [1]:

$$\Phi_{PW}(r) = -\frac{GM}{r - r_s}, \quad r_s = \frac{2GM}{c^2}. \tag{1}$$

The Paczynski-Wiita potential is a Newton-like potential that (for the case of non-rotating BHs) exactly reproduces the marginally bound ($r_{mb} = 2r_s$) and marginally stable circular ($r_{ms} = 3r_s$) orbits of the Schwarzschild geometry, as well as the form of the Keplerian angular momentum.

2 Solving the effective one-body problem

The effective one-body problem entails treating two bodies, with masses M_1 and M_2, with a separation $\vec{r} = \vec{r}_2 - \vec{r}_1$, orbiting a common center of mass as a single body with mass $\mu = \frac{M_1 M_2}{M_1 + M_2}$ under the influence of an external potential due to mass $M = M_1 + M_2$. The analytical solution of the circularized, near-inspiral BBH orbit can be derived by using the classical Newtonian gravitational potential and Lagrangian mechanics. The position vectors of the two bodies as a function of time in the CoM frame is found to be

$$\vec{r}_1 = \begin{pmatrix} \frac{M_2 P_\phi^2}{\mu G M^2} \cos\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \\ \frac{M_2 P_\phi^2}{\mu G M^2} \sin\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \end{pmatrix}, \quad \text{and } \vec{r}_2 = -\begin{pmatrix} \frac{M_1 P_\phi^2}{\mu G M^2} \cos\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \\ \frac{M_1 P_\phi^2}{\mu G M^2} \sin\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \end{pmatrix}. \tag{2}$$

3 Calculating the acceleration of a charged particle in a near-inspiral phase BBH merger

Consider a charged particle, with mass m_e, position $\vec{r}_{m_e}$, and velocity $\vec{v}_{m_e}$ in the lab frame. If the particle is under the influence of a force $\vec{F}$ due to the Paczynski-Wiita potentials Φ_{PW} resulting from the BBHs, calculated at retarded position $\vec{r}_k$ (that is calculated numerically from the definition of retarded time, $t_r = t - \frac{1}{c}|\vec{r} - \vec{r}_k|$, and equation 2 using Newton's method), then we have a set of discritized equations

$$\begin{aligned} &\vec{p}_{i+1} = \vec{F}_i dt + \vec{p}_i, && \vec{v}_i = \frac{\vec{p}_i}{m_e}\sqrt{1 - \frac{p_i^2}{m_e^2 c^2}\left(1 + \frac{p_i^2}{m_e^2 c^2}\right)^{-1}}, \\ &\vec{r}_{i+1} = \vec{v}_i dt + \vec{r}_i, && \\ &\gamma_i = \frac{p_x}{m_e v_x} = \frac{p_y}{m_e v_y} = \frac{p_z}{m_e v_z}, \quad \text{and} && \vec{a}_i = \frac{1}{m_e \gamma}\left[\vec{F}_i - \frac{\left(\vec{v}_i \cdot \vec{F}_i\right)}{c^2}\vec{v}_i\right], \end{aligned} \tag{3}$$

that describes the evolution of the particle in a near-inspiral BBH system.

4 Calculating the SED of charged particles in a near-inspiral BBH merger

The spectral energy distribution (SED) of a single accelerated charged particle is given by

$$I(\nu) = \frac{\mu_0 q^2}{3\pi c}\left[\left|\gamma^2(\nu)\vec{a}_\parallel(\nu)\right|^2 + \left|\gamma^3(\nu)\vec{a}_\perp(\nu)\right|^2\right]. \tag{4}$$

From this, it is evident that $I_{\parallel,k} = \left|\gamma^2(\nu_k)\vec{a}_\parallel(\nu_k)\right|^2$, and $I_{\perp,k} = \left|\gamma^3(\nu_k)\vec{a}_\perp(\nu_k)\right|^2$ needs to be determined in frequency space. This is done by simply taking the discrete Fourier transform of the relevant products

$$I_{\parallel,k} = \sum_{j=0}^{N-1}\left|\gamma_i^2 \vec{a}_{\parallel,j}\right|^2 e^{-\frac{2\pi i}{N}kj}, \quad I_{\perp,k} = \sum_{j=0}^{N-1}\left|\gamma_j^3 \vec{a}_{\perp,j}\right|^2 e^{-\frac{2\pi i}{N}kj}, \tag{5}$$

with $i = \sqrt{-1}$ and N is the number of data points. From this

$$I(\nu_k) = \frac{\mu_0 q^2}{3\pi c}\left[I_{\parallel,k} + I_{\perp,k}\right], \tag{6}$$

where $\nu_k = kd\nu$. The total SED from the system is found by summing over all of the particles evolving through the system.

5 Implementing the model in code

The model described in the earlier sections of this chapter is implemented in two separate programs. The first program is written in C++, with the time evolution of each of the position, velocity and acceleration vectors, as well as that of the Lorentz factor, saved as separate output datasets. The second program is a *Python* program that takes the acceleration and Lorentz factor datasets of the previous program as input, and calculates the Fourier transform of each particle (as described in Section 4). This is done by using the pyFFTW package that gives a convenient way of implementing the FFTW C++ library within Python.

6 Results

The model, briefly described above, is applied to a BBH system with masses $M_1 = 30M_\odot$ and $M_2 = 35M_\odot$. The initial separation of the BHs are taken to be $r_0 = 1.0 \times 10^8$cm. A set of 50 particles are randomly distributed in the system, with a uniform probability distribution.

The top panel of figure 1 shows the components of the acceleration parallel and perpendicular to the velocity of a typical particle evolving through the system. The bottom panel shows the separation distance between the particle and event horizon of the two BHs. Figure 2 shows the total SED of the system calculated by summing over the SEDs of all 50 particles.

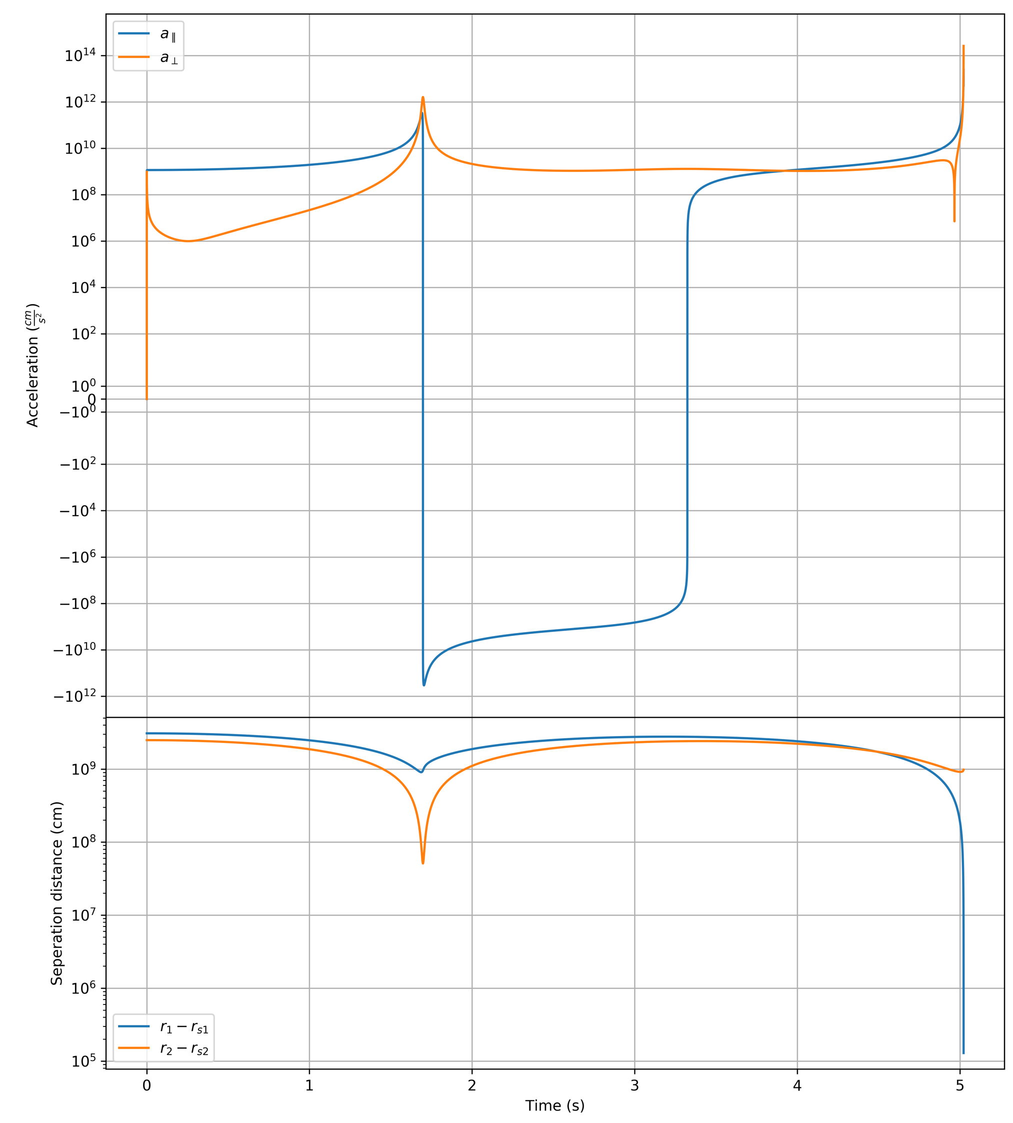

Figure 1: Acceleration components and separation distances as a function of time for particle 1, with an initial position of $\vec{r}_1 = \left(1.368 \times 10^9, 2.430 \times 10^9, 2.137 \times 10^9\right) cm$ and initial velocity $\vec{v}_1 = \left(5.68 \times 10^4, -1.77 \times 10^5, -8.09 \times 10^4\right) cm \cdot s^{-1}$.

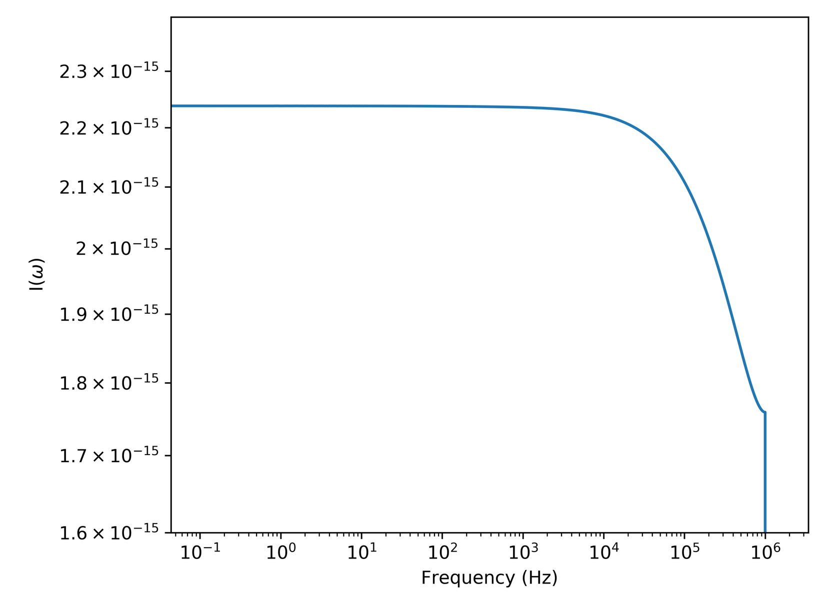

Figure 2: Total SED, determined by integrating over all 50 particles distributed in the system. There is a steep drop off that occurs at $\sim 10^5$Hz. In addition to the drop off, there is also a cut of at $\sim 10^6$Hz that occurs due to limitations on the numerical resolution.

7 Discussion and Conclusion

From the results given in the previous chapter, it becomes evident that the bulk of possible EM radiation that originates from charged particles, accelerated in near-inspiral BBH merger systems (determined from our model), is distributed at frequencies well below the operating ranges of current radio telescopes. The current operating range of the Low-Frequency Array (LOFAR) is $10 - 240MHz$ [2]. Figure 2 illustrates how the total SED calculated from the model drops off at $10^5 Hz$, which lies below the frequency range of LOFAR. If we now consider that the inter stellar medium (ISM) plasma frequency is $\backsim 2kHZ$ (for an assumed electron density of $0.03cm^{-3}$), we know that radiation emitted at frequencies below this threshold will be absorbed by the ISM, and, therefore, the bulk of radiation emitted by the system will be absorbed by the ISM. This means that only radiation in a range of $\backsim 3kHz$ to $\backsim 100kHz$ will emerge from the system and reach the earth.

8 Future Work

The model will be extended to include inspiral phase BBH orbits where the possible particle trajectories and resulting gravitational acceleration is expected to become much more chaotic for which the model and code implementation will necessarily include simulations with a larger number of particles and smaller step sizes, in order to probe possible higher frequency elements. For this reason, it will become necessary to implement more realistic, fully general relativistic equations of motion for charged particles distributed through the system. We have also assumed that neither of the BHs have a Magnetosphere, with only gravity acting in on the particles distributed within the system. If at least one of the BHs are significantly magnetized, particle dynamics within the system will likely be dominated by the resulting Lorentz forces, rather than the gravitational forces.

References

[1] B. Paczynsky and P.J. Wiita. Thick accretion disks and supercritical luminosities. *aap*, 500:203–211, August 1980.

[2] M. P. van Haarlem, M. W. Wise, A. W. Gunst, G. Heald, J. P. McKean, J. W. T. Hessels, A. G. de Bruyn, R. Nijboer, J. Swinbank, R. Fallows, and et al. Lofar: The low-frequency array. *Astronomy and Astrophysics*, 556:A2, Jul 2013.

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