g-2

K. Kampf, ÚČJF

Žižkov, 5. 5. 2021

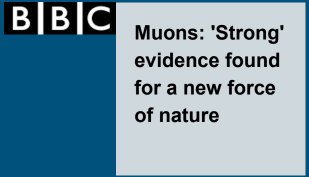
Žižkov 5.5.1945

(from ČTK)

outline

- introduction
- experiment
- theory
- our work
- conclusion (on page 33)

On April 7



BBC

Muons: 'Strong' evidence found for a new force of nature

Jako hodit šestkrát za sebou šestku

Byla při fyzikálním experimentu ve Fermilab skutečně objevena **tzv. pátá síla v přírodě**, jak naznačují některá data?

Ve středu 7. dubna obletěla svět zpráva o **významném objevu v americké laboratoři Fermilab** u Chicaga. Například britská televizní stanice BBC uvádí: „Fyzikové tvrdí, že našli možné signály páté fundamentální síly v přírodě.“ A to by mohla být hodně velká věc. Nejen pro svět fyziky.

JIŘÍ CHÝLA
částicový fyzik

del“ počítá s ještě základnějšími částicemi, z nichž se protony skládají. Podle něj jsou základními stavebními kameny veškeré nám známé hmoty ve vesmíru tři čtveřice částic, kterým říkáme kvarky a leptony.

V první čtveřici jsou dva kvarky, z nichž jsou složeny například protony a neutrony, a dva leptony: elektron, který spolu s protony a neutrony vytváří nám známě atomy, a elektronové neutrino, jež hraje klíčovou roli například v procesech spalování vodíku v nitru Slunce. Kdyby šlo jen o hmotu ve vesmíru, kterou známe, žádné další kvarky a leptony bychom nepotřebovali. Přesto další dvě „generace“ kvarků a leptonů existují. Tomu rozumíme jen s použitím antropického argumentu, tj. konstatování, že kdyby neexistovaly, neexistovali bychom ani my, protože by se vesmír ne-

né předpovědi pro všechny experimentálně měřitelné veličiny.

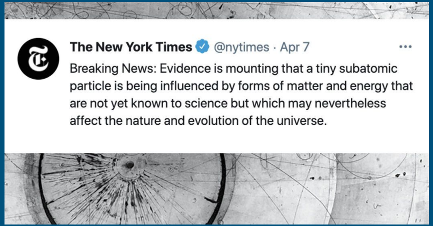
Standardní model je pozoruhodně úspěšný při popisu prakticky všech existujících experimentálních dat. Ve srážkách částic na urychlovačích tato data, které fyzici měří, představují primárně energie a úhly částic, jež při nich vznikají, a z nich pak usuzují na případnou existenci nových částic. V případě experimentu ve Fermilab se měří úhel a energie pozitronu vznikajícího v rozpadu kladně nabitého mionu.

A experimentátoři se standardní model ze všech sil snaží vyvrátit ve smyslu známé zásady: co mne nezabije, to mne posílí. Teoretici jsou s ním totiž nespokojení již delší dobu, protože tento model (který by se měl správně nazývat standardní teorie) má některé vady na kráse. Například obsahuje asi 25 volných para-

> Tehdy byla pravděpodobnost, že jde o statistickou fluktuaci, stejná, jako že hodíte šestku za sebou dokonce jedenáctkrát. A to už náhoda být nemohla. ”

Standardní model tedy zatím úsilí fyziků najít „novou fyziku“ úspěšně odolává. Ve srovnání jeho předpovědí s experimentálními daty existuje jen několik prasklin, z nichž ta nejzávažnější, byť ne

nou přesností byla změřena i v
mentu ve Fermilab. Tato dvě čísl
sebe liší o zhruba čtyřnásobek
chyb. Tak velký rozdíl je všeobe
važován za silnou indikaci, že
o projev statistické fluktuace exp
tálních dat, ale skutečný efekt: pr
dobnost, že tento rozdíl je důsledk
tistické fluktuace experimentální
je zhruba stejná jako pravděpod
že hodíte šestku šestkrát za sebou
k tomu totiž dojde, člověk usoudí
žiště kostky je pravděpodobně
blízko jedné ze stran. Ale jistota
není, ve fyzice je zvykem považo
za prokázaný až tehdy, pokud pra
dobnost, že jde o statistickou flu
odpovídá pravděpodobnosti, že
šestku za sebou osmkrát. Takže te
periment stále probíhá a jeho cíle
sáhnout během zhruba tří let pře



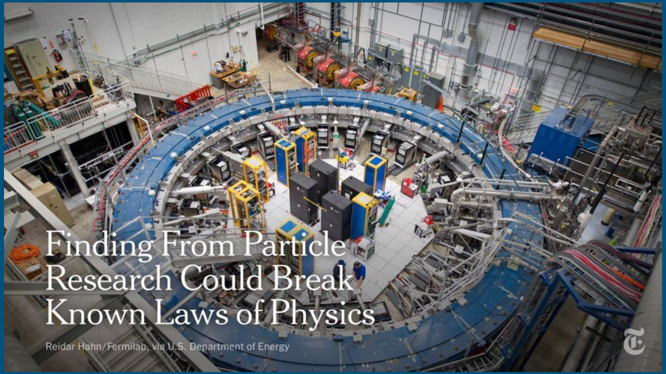
The New York Times @nytimes · Apr 7

Breaking News: Evidence is mounting that a tiny subatomic particle is being influenced by forms of matter and energy that are not yet known to science but which may nevertheless affect the nature and evolution of the universe.

CBC.ca

Suggestions of a new force echo the ancient quest for fundamental elements

Results from a particle physics experiment at Fermilab outside ... Empedocles, a philosopher from 5th century BCE Greece is thought to have ...



Finding From Particle Research Could Break Known Laws of Physics

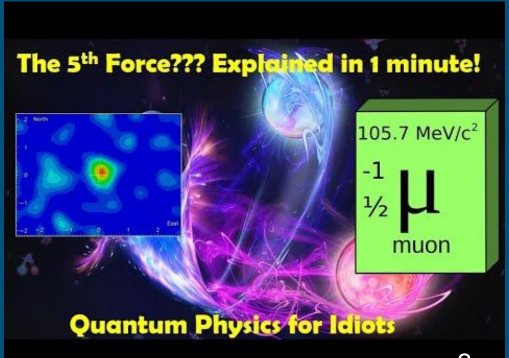
Reidar Hahn/Fermilab, via U.S. Department of Energy

Silný náznak novej fyziky. Nestabilná častica sa nespráva tak, ako by mala

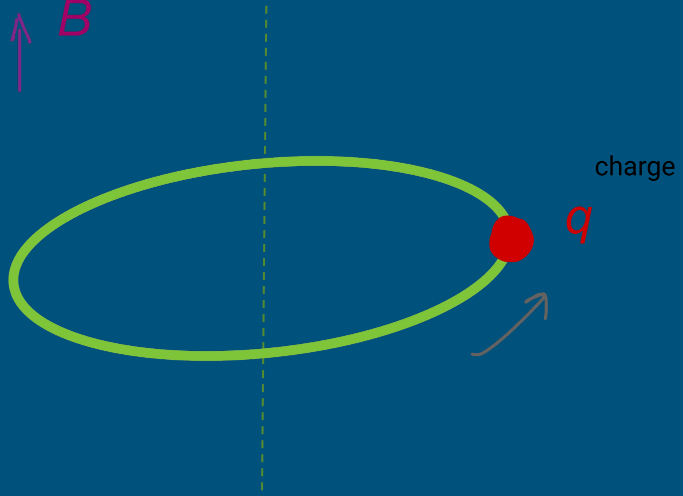
Na nové výsledky čakali dvadsať rokov.

7. apr 2021 o 17:46 RENÁTA ZELNÁ

SME



Magnetic moment: introduction



$$U = -\vec{\mu} \cdot \vec{B}$$

classically:

$$\vec{\mu} = \frac{q}{2m}\vec{L}$$

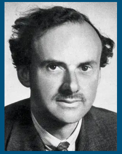
i.e. the magnetic dipole moment is proportional to the angular momentum of charged particles

QM:

$$\vec{\mu} = g\frac{q}{2m}\vec{S}$$

for electron, using
Dirac equation

$$g = 2$$

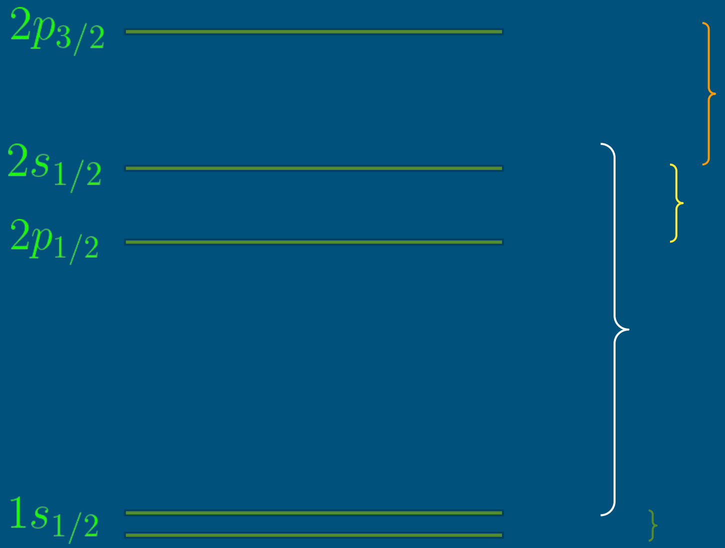


QED ‘28

first evidence of the electron *g*-2

nl_j

energy levels in hydrogen atom



$\Delta j \neq 0$	fine structure	Dirac equation
$\Delta l \neq 0$	Lamb shift	QED
$\Delta n \neq 0$	gross structure	QM (Bohr model)
$\sim (1 + \alpha/2\pi)$	hyperfine structure	QED

The muon *g-2* experiment

First we need to create polarized muons and measure them

shooting protons on a target we get pions

$$\pi \rightarrow \mu + \nu_\mu$$

$$\llcorner\!\!\longrightarrow \; e + \nu_e + \nu_\mu$$

measuring the direction of the electron momentum
provides the direction of the muon spin

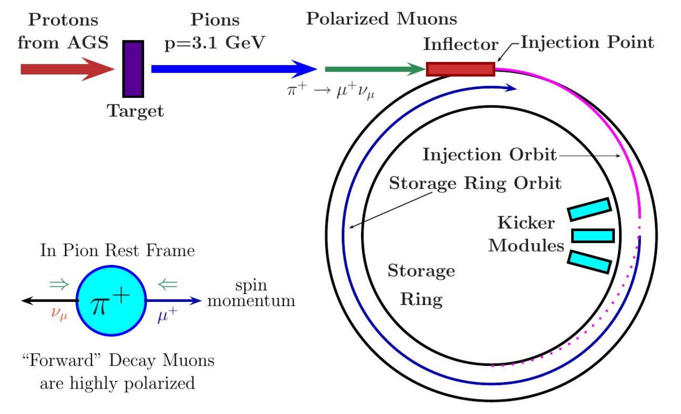
The muon *g-2* experiment

cyclotron frequency

$$\omega_c = \frac{eB}{m\gamma}$$

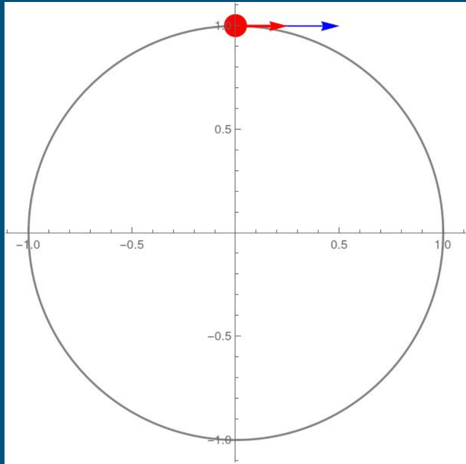
muon spin precession

$$\omega_s = \frac{eB}{m\gamma} + \overbrace{a_\mu \frac{eB}{m}}^{\omega_a}$$



[from Jegerlehner,Nyffeler '09]

$$\omega_a = \omega_s - \omega_c$$



The muon *g-2* experiment

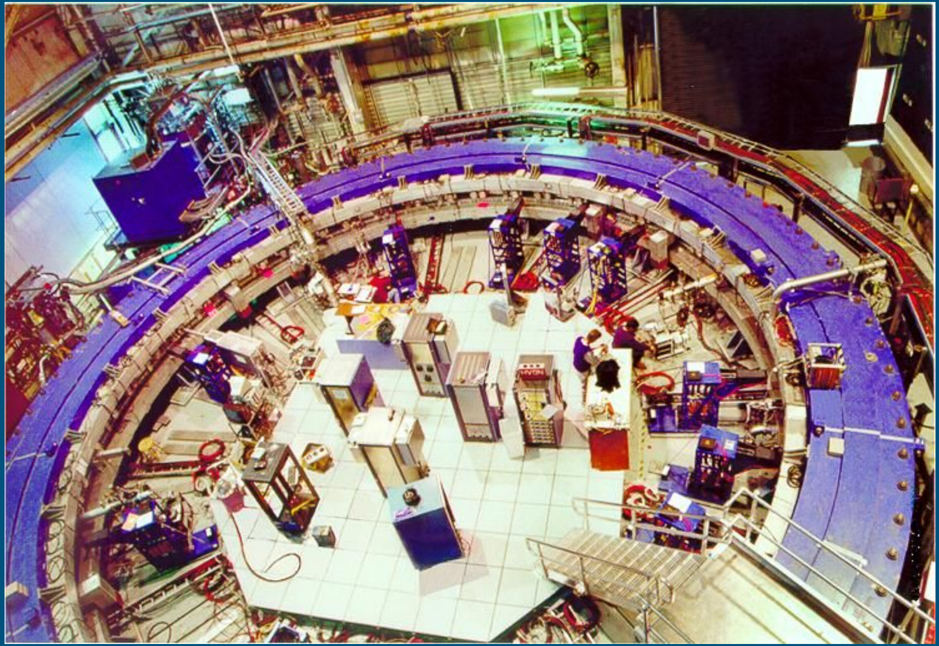
electrostatic focusing system needed:

$$\vec{\omega}_a = a_\mu \frac{e\vec{B}}{m} \longrightarrow \vec{\omega}_a = a_\mu \frac{e\vec{B}}{m} - \frac{e}{m}\underbrace{\left[a_\mu - \frac{1}{\gamma^2 - 1}\right]}_{=0}\vec{v} \times \vec{E}$$

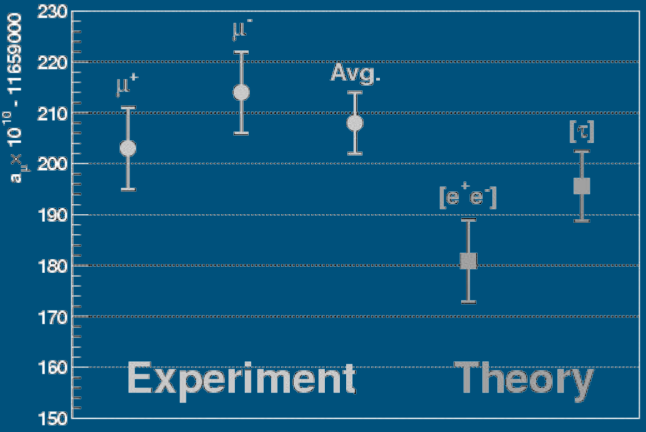
$$\gamma = \sqrt{1 + 1/a_\mu} = 29.3$$

magic gamma

BNL-E821 (1997-2001)



from www.g-2.bnl.gov



$$\times 10^{-11}$$

$$a_\mu^{\rm BNL} = 116\,592\,089 \pm 63\,(540\,\text{ppb})$$

3.7 σ

$$a_\mu^{\rm SM} = 116\,591\,810 \pm 43\,(368\,\text{ppb})$$


Fermilab
Brookhaven
OVERSIZE LOAD
Big Move
OVERSIZE LOAD

The magnet on the move in 2013. Cindy Arnold/Fermilab, via US Department of Energy

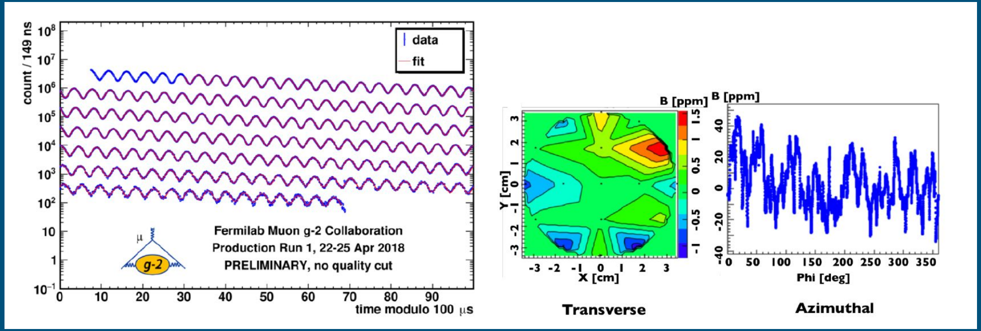
A new campus at the Fermilab was built in 2013 to study muons. Reidar Hahn/Fermilab, via US Department of Energy

Extracting a_μ

$$B = \frac{\omega_p}{2\mu_p} \qquad \mu_e = g_e \frac{e}{2m_e}$$

$$a_\mu = \frac{\omega_a}{B}\frac{m_\mu}{e} = \frac{\omega_a}{\bar{\omega}_p}\underbrace{\frac{m_\mu}{m_e}\frac{\mu_p}{\mu_e}\frac{g_e}{2}}_{\text{total uncertainty from 3 external sources 24 ppb}}$$

Extracting a_μ



from muon g-2 col.

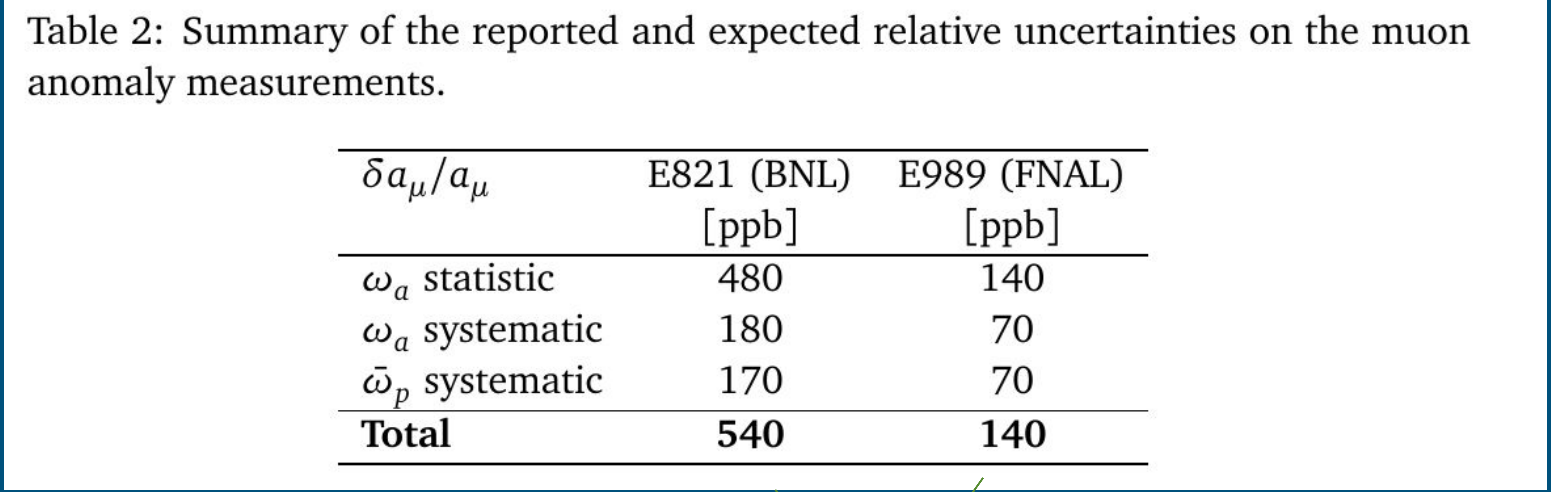
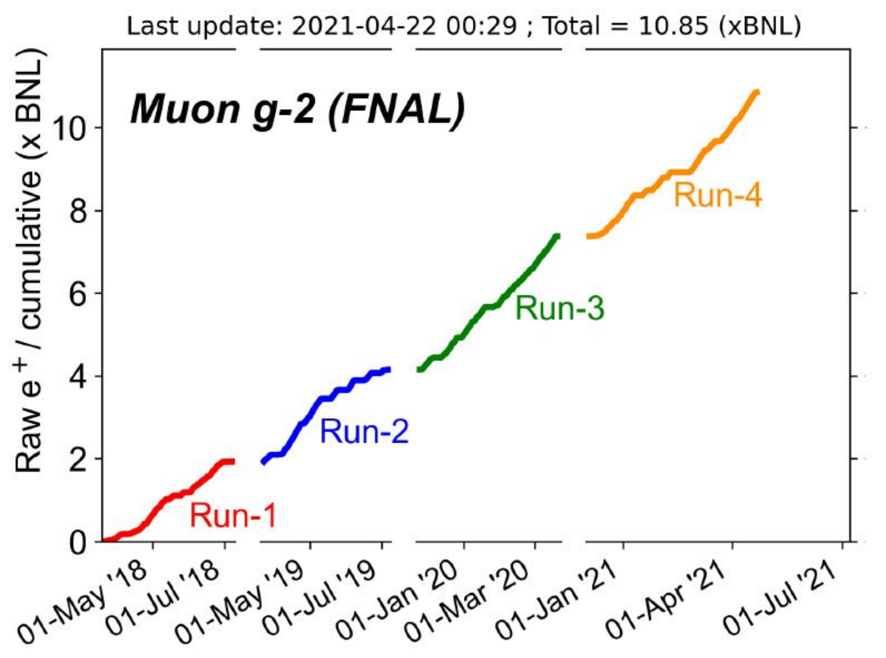


Table 2: Summary of the reported and expected relative uncertainties on the muon anomaly measurements.

$\delta a_\mu / a_\mu$	E821 (BNL) [ppb]	E989 (FNAL) [ppb]
ω_a statistic	480	140
ω_a systematic	180	70
$\bar{\omega}_p$ systematic	170	70
Total	**540**	**140**

reduction by factor of 4

Status and plans



- Run-1 results presented on April 7
  - 15% smaller uncertainty than BNL
  - 3.3 σ tension with theory
- Run-4 currently ongoing
- Run-5 will start in fall

Theory: short historical overview of muon

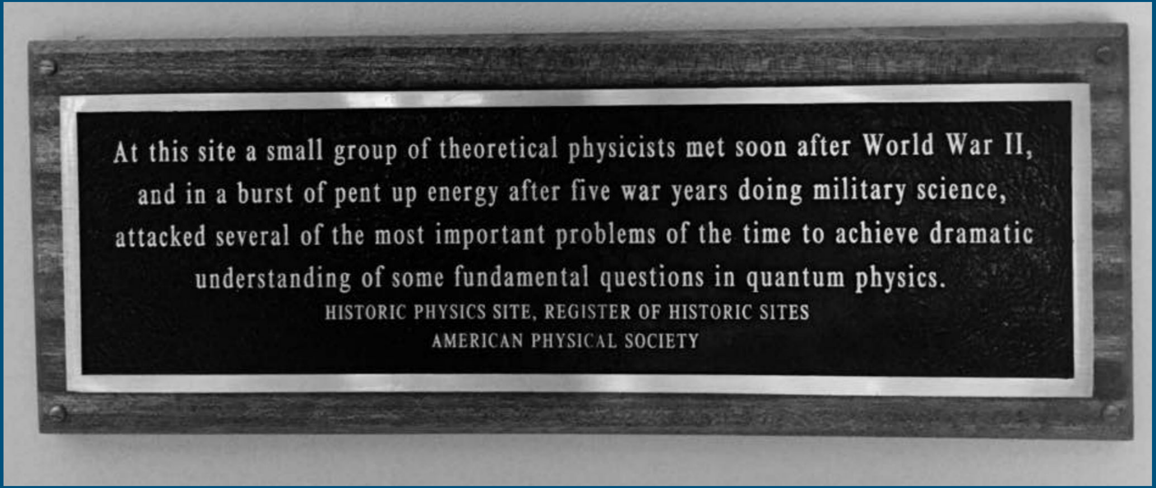
- 1936: discovery of “mesotron”, C.Anderson, S.Neddermeyer
- 1941-1947: it cannot be meson (Yukawa’s particle), but rather fermion → μ-on
- muons are about 200 times heavier than electron

$$m_e = 0.511\,\mathrm{MeV}\,, \qquad m_\mu = 105.66\,\mathrm{MeV}$$

- I. I. Rabi: “Who ordered that?”

QED: renormalization

June 1947: Shelter Island Conference (NY)

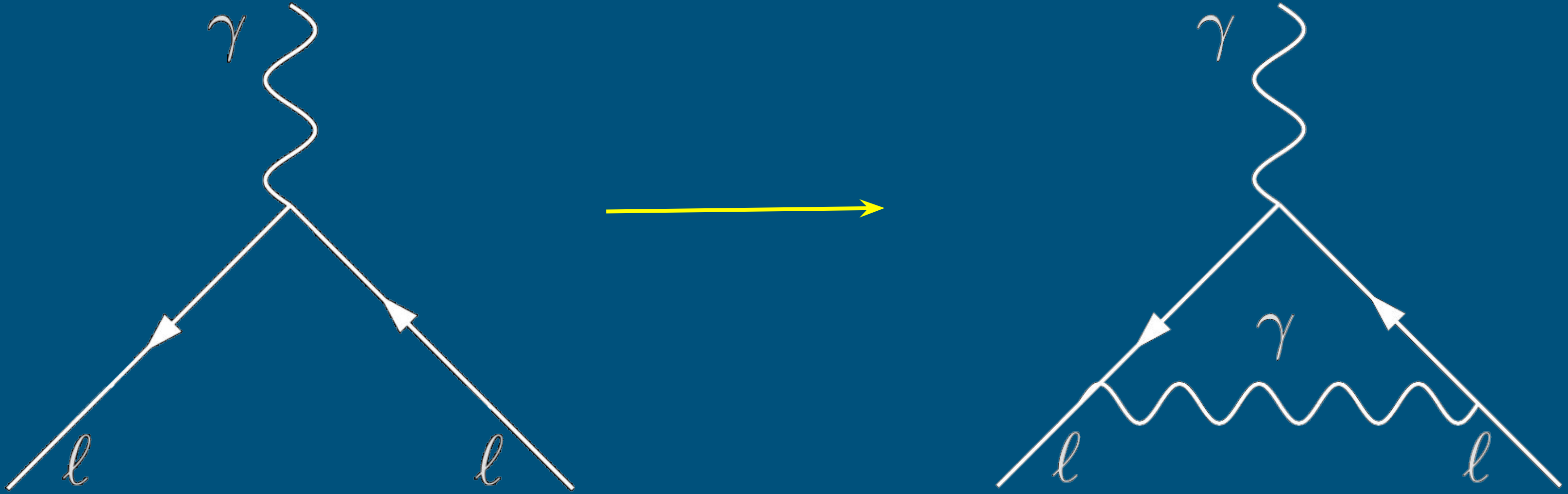


At this site a small group of theoretical physicists met soon after World War II, and in a burst of pent up energy after five war years doing military science, attacked several of the most important problems of the time to achieve dramatic understanding of some fundamental questions in quantum physics.

HISTORIC PHYSICS SITE, REGISTER OF HISTORIC SITES
AMERICAN PHYSICAL SOCIETY

QED: historical overview

it took 20 years to realize how to get from Dirac 1928 to Schwinger 1947



Theory prediction

Contributions to anomalous magnetic moment of lepton l = e, μ, τ

$$a_l \equiv \frac{g_l - 2}{2}$$

$$a_l^{\text{SM}} = a_l^{\text{QED}} + a_l^{\text{EW}} + a_l^{\text{had, VP}} + a_l^{\text{had, LbL}}$$

Theory prediction: QED

$$a_\mu^{\mathrm{QED}} = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e,\, m_\mu/m_\tau)$$

universal

mass dependence due to
different masses in loops

typically expanded in
fine structure constant:

$$A_1 = A_1^{(2)}\left(\frac{\alpha}{\pi}\right) + A_1^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + \ldots$$

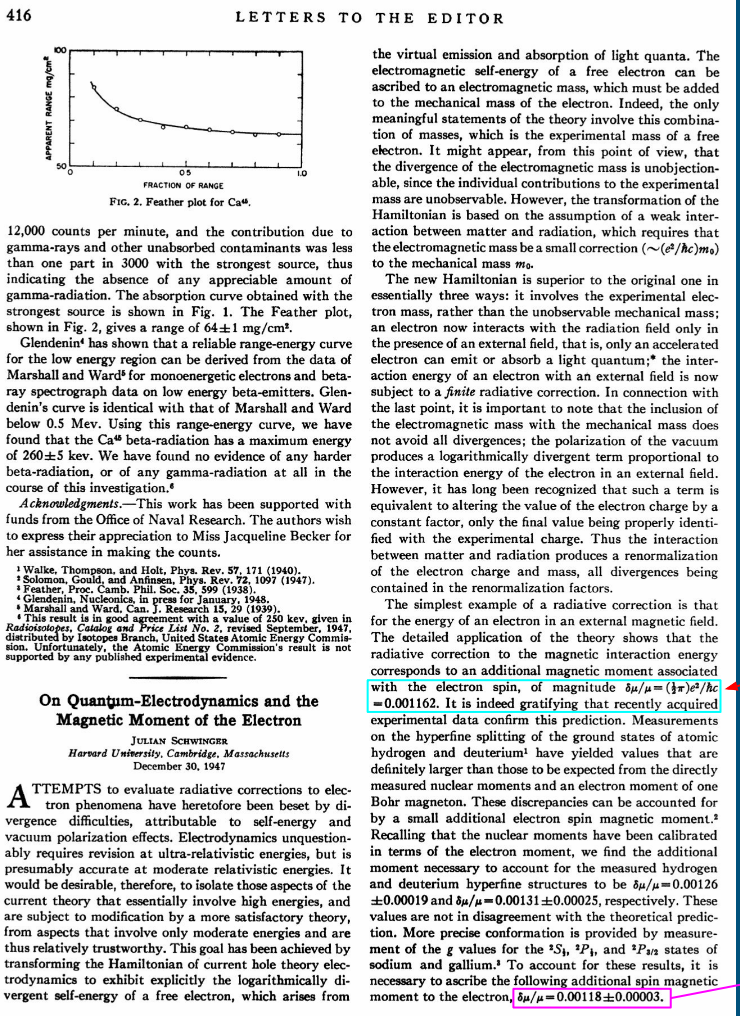


FIG. 2. Feather plot for Ca⁴⁵.

12,000 counts per minute, and the contribution due to gamma-rays and other unabsorbed contaminants was less than one part in 3000 with the strongest source, thus indicating the absence of any appreciable amount of gamma-radiation. The absorption curve obtained with the strongest source is shown in Fig. 1. The Feather plot, shown in Fig. 2, gives a range of 64±1 mg/cm².

Glendenin[4] has shown that a reliable range-energy curve for the low energy region can be derived from the data of Marshall and Ward[5] for monoenergetic electrons and beta-ray spectrograph data on low energy beta-emitters. Glendenin's curve is identical with that of Marshall and Ward below 0.5 Mev. Using this range-energy curve, we have found that the Ca^{45} beta-radiation has a maximum energy of 260±5 kev. We have found no evidence of any harder beta-radiation, or of any gamma-radiation at all in the course of this investigation.[6]

Acknowledgments.—This work has been supported with funds from the Office of Naval Research. The authors wish to express their appreciation to Miss Jacqueline Becker for her assistance in making the counts.

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[1] Walke, Thompson, and Holt, Phys. Rev. **57**, 171 (1940).
[2] Solomon, Gould, and Anfinsen, Phys. Rev. **72**, 1097 (1947).
[3] Feather, Proc. Camb. Phil. Soc. **35**, 599 (1938).
[4] Glendenin, Nucleonics, in press for January, 1948.
[5] Marshall and Ward, Can. J. Research **15**, 29 (1939).
[6] This result is in good agreement with a value of 250 kev, given in *Radioisotopes, Catalog and Price List No. 2*, revised September, 1947, distributed by Isotopes Branch, United States Atomic Energy Commission. Unfortunately, the Atomic Energy Commission's result is not supported by any published experimental evidence.

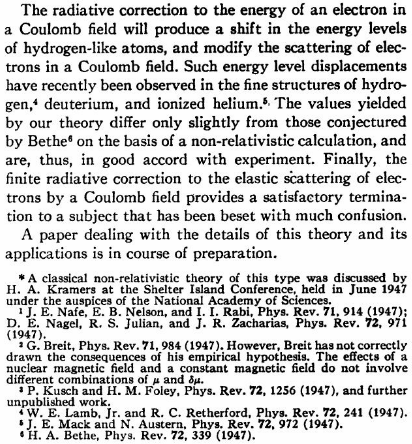
On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
December 30, 1947

ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction $(\sim(e^2/\hbar c)m_0)$ to the mechanical mass m_0.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;* the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu=(\frac{1}{2}\pi)e^2/\hbar c=0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu=0.00126\pm0.00019$ and $\delta\mu/\mu=0.00131\pm0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu=0.00118\pm0.00003$.



The radiative correction to the energy of an electron in a Coulomb field will produce a shift in the energy levels of hydrogen-like atoms, and modify the scattering of electrons in a Coulomb field. Such energy level displacements have recently been observed in the fine structures of hydrogen,[4] deuterium, and ionized helium.[5] The values yielded by our theory differ only slightly from those conjectured by Bethe[6] on the basis of a non-relativistic calculation, and are, thus, in good accord with experiment. Finally, the finite radiative correction to the elastic scattering of electrons by a Coulomb field provides a satisfactory termination to a subject that has been beset with much confusion.

A paper dealing with the details of this theory and its applications is in course of preparation.

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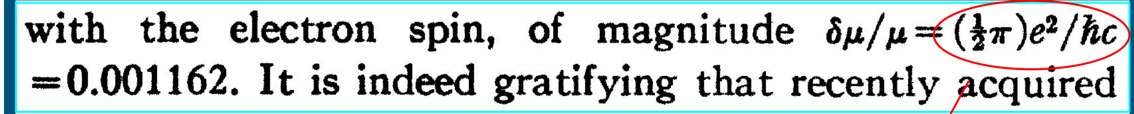
* A classical non-relativistic theory of this type was discussed by H. A. Kramers at the Shelter Island Conference, held in June 1947 under the auspices of the National Academy of Sciences.
[1] J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947); D. E. Nagel, R. S. Julian, and J. R. Zacharias, Phys. Rev. **72**, 971 (1947).
[2] G. Breit, Phys. Rev. **71**, 984 (1947). However, Breit has not correctly drawn the consequences of his empirical hypothesis. The effects of a nuclear magnetic field and a constant magnetic field do not involve different combinations of μ and $\delta\mu$.
[3] P. Kusch and H. M. Foley, Phys. Rev. **72**, 1256 (1947), and further unpublished work.
[4] W. E. Lamb, Jr. and R. C. Retherford, Phys. Rev. **72**, 241 (1947).
[5] J. E. Mack and N. Austern, Phys. Rev. **72**, 972 (1947).
[6] H. A. Bethe, Phys. Rev. **72**, 339 (1947).



with the electron spin, of magnitude $\delta\mu/\mu=(\frac{1}{2}\pi)e^2/\hbar c=0.001162$. It is indeed gratifying that recently acquired

Typo!

correct: $\frac{1}{2\pi}\frac{e^2}{\hbar c}=\frac{\alpha}{2\pi}$

Note theory vs. exp in 1947:
0.00116(1) vs. 0.00118(3)

i.e. only: 0.6σ in 1947!

No typo here, though:

$$\frac{\alpha}{2\pi}$$

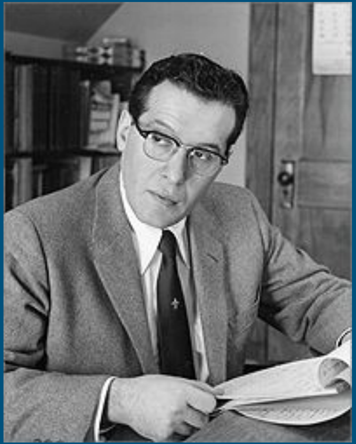
JULIAN SCHWINGER
2·12·1918 — 7·16·1994
CLARICE CARROL SCHWINGER
9·23·1917 — 1·9·2011

photo by K.A.Milton

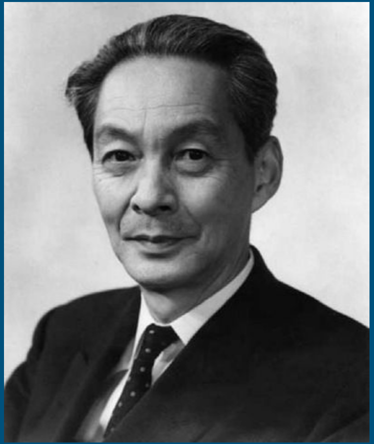
QED: renormalization

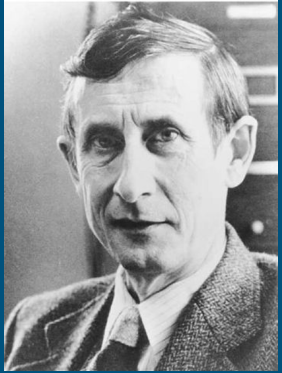
Feynman

Schwinger



Tomonaga



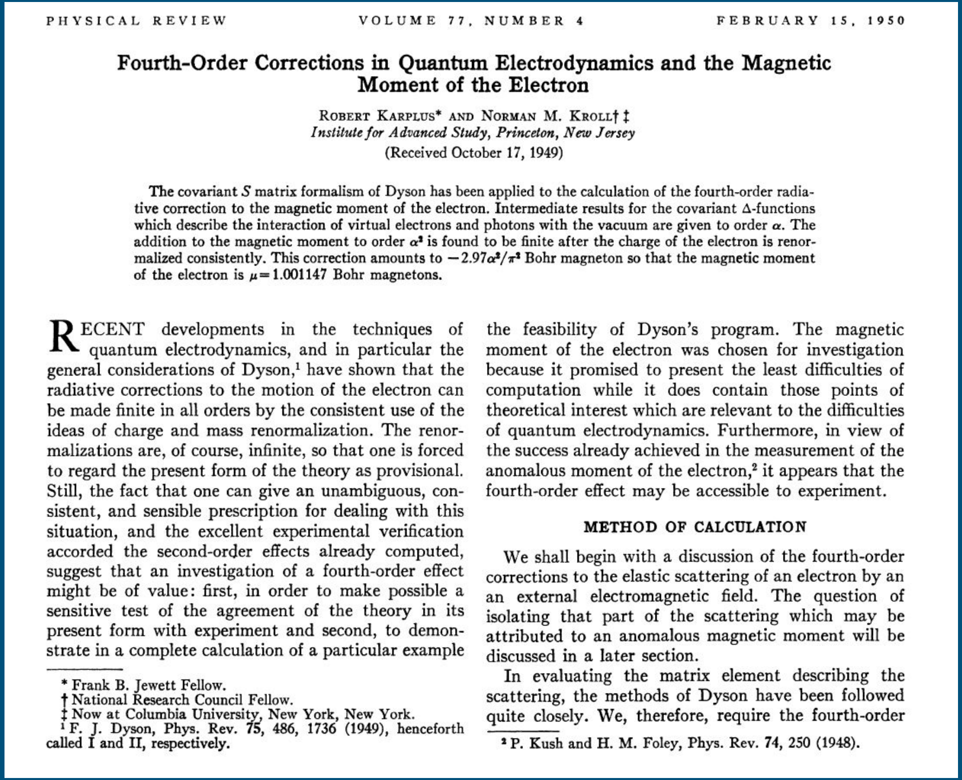


Dyson

Oppenheimer: “other people give talks to tell you how to do it, but Julian gives talks to tell you how only he can do it."

1949: QED theory as we know it

2-loop contributions: 2 years later



PHYSICAL REVIEW VOLUME 77, NUMBER 4 FEBRUARY 15, 1950

Fourth-Order Corrections in Quantum Electrodynamics and the Magnetic Moment of the Electron

ROBERT KARPLUS* AND NORMAN M. KROLL†‡
Institute for Advanced Study, Princeton, New Jersey
(Received October 17, 1949)

The covariant S matrix formalism of Dyson has been applied to the calculation of the fourth-order radiative correction to the magnetic moment of the electron. Intermediate results for the covariant Δ-functions which describe the interaction of virtual electrons and photons with the vacuum are given to order α. The addition to the magnetic moment to order α^2 is found to be finite after the charge of the electron is renormalized consistently. This correction amounts to $-2.97\alpha^2/\pi^2$ Bohr magneton so that the magnetic moment of the electron is $\mu=1.001147$ Bohr magnetons.

RECENT developments in the techniques of quantum electrodynamics, and in particular the general considerations of Dyson,[1] have shown that the radiative corrections to the motion of the electron can be made finite in all orders by the consistent use of the ideas of charge and mass renormalization. The renormalizations are, of course, infinite, so that one is forced to regard the present form of the theory as provisional. Still, the fact that one can give an unambiguous, consistent, and sensible prescription for dealing with this situation, and the excellent experimental verification accorded the second-order effects already computed, suggest that an investigation of a fourth-order effect might be of value: first, in order to make possible a sensitive test of the agreement of the theory in its present form with experiment and second, to demonstrate in a complete calculation of a particular example the feasibility of Dyson's program. The magnetic moment of the electron was chosen for investigation because it promised to present the least difficulties of computation while it does contain those points of theoretical interest which are relevant to the difficulties of quantum electrodynamics. Furthermore, in view of the success already achieved in the measurement of the anomalous moment of the electron,[2] it appears that the fourth-order effect may be accessible to experiment.

METHOD OF CALCULATION

We shall begin with a discussion of the fourth-order corrections to the elastic scattering of an electron by an an external electromagnetic field. The question of isolating that part of the scattering which may be attributed to an anomalous magnetic moment will be discussed in a later section.

In evaluating the matrix element describing the scattering, the methods of Dyson have been followed quite closely. We, therefore, require the fourth-order

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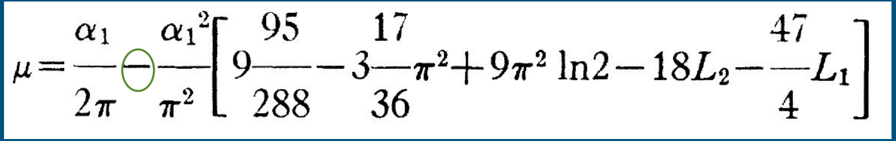
[1] F. J. Dyson, Phys. Rev. 75, 486, 1736 (1949), henceforth called I and II, respectively.

[2] P. Kush and H. M. Foley, Phys. Rev. 74, 250 (1948).

two independent calculations lead to:



$$\mu=\frac{\alpha_1}{2\pi}\ominus\frac{{\alpha_1}^2}{\pi^2}\left[9\frac{95}{288}-3\frac{17}{36}\pi^2+9\pi^2\ln 2-18L_2-\frac{47}{4}L_1\right]$$

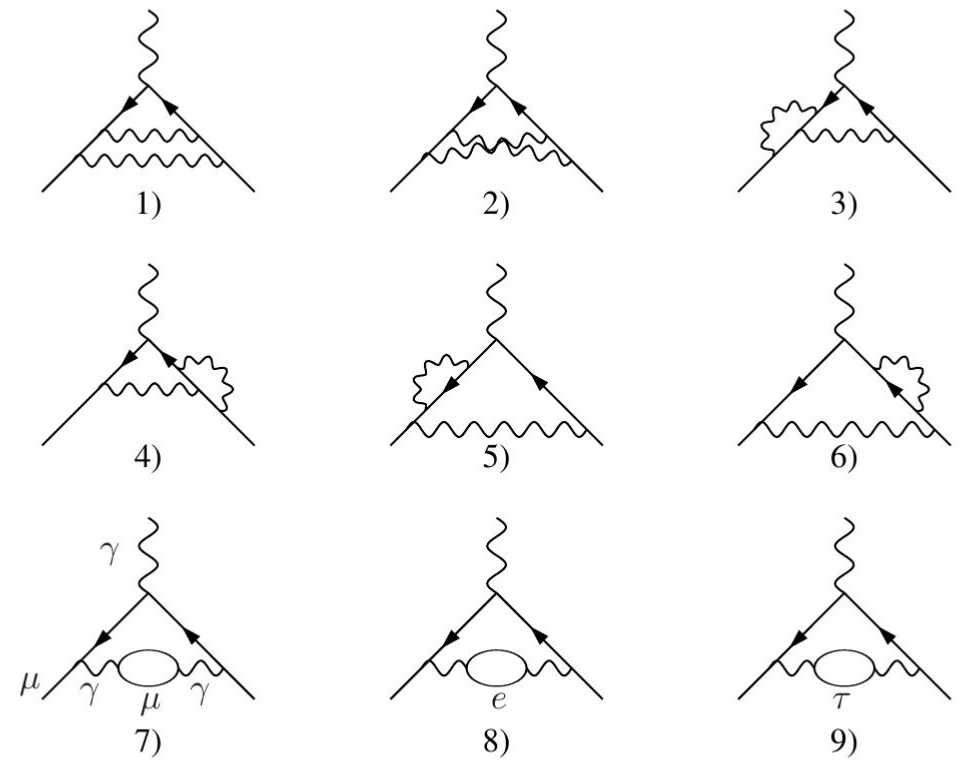
$$\xcancel{-\frac{2687}{288}+\frac{125}{36}\pi^2-9\pi^2\ln 2+28\zeta(3)}=-2.97$$

Schwinger's student

wrong! - corrected by Sommerfield '57, Petermann '57

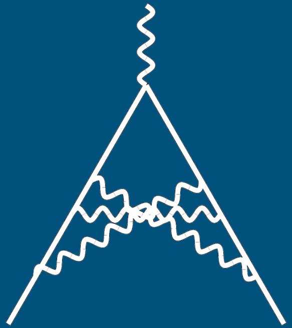
$$\frac{197}{144}+\frac{1}{12}\pi^2-\frac{1}{2}\pi^2\ln 2+\frac{3}{4}\zeta(3)=-0.33$$

2-loop Feynman diagrams



[from Jegerlehner,Nyffeler '09]

3-loop: 72 diagrams to calculate



analytical result in [Laporta, Remiddi ‘96] after cracking the triple cross diagram

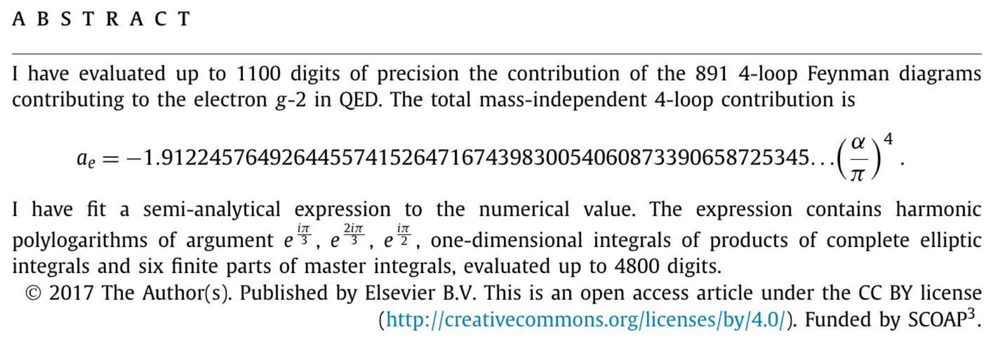
$$A_1^6 = \frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2 \ln 2 + \frac{139}{18}\zeta(3) - \frac{239}{2160}\pi^4 + \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left\{\mathrm{Li}_4(1/2) + \frac{1}{24}\ln^4 2 - \frac{1}{24}\pi^2 \ln^2 2\right\} = 1.181\,241\ldots$$

in agreement with numerical value [Kinoshita ‘95]: 1.181259(40),

however, note that it was revised from: 1.17611(42)

Higher order QED

4 loops: 891 diagrams to calculate → semi-analytic result: Laporta ‘17



A B S T R A C T

I have evaluated up to 1100 digits of precision the contribution of the 891 4-loop Feynman diagrams contributing to the electron g-2 in QED. The total mass-independent 4-loop contribution is

$$a_e = -1.912245764926445574152647167439830054060873390658725345\ldots\left(\frac{\alpha}{\pi}\right)^4 .$$

I have fit a semi-analytical expression to the numerical value. The expression contains harmonic polylogarithms of argument $e^{\frac{i\pi}{3}}$, $e^{\frac{2i\pi}{3}}$, $e^{\frac{i\pi}{2}}$, one-dimensional integrals of products of complete elliptic integrals and six finite parts of master integrals, evaluated up to 4800 digits.

in agreement with Kinoshita et al
−1.91298(84)

5 loops: 12672 diagrams to calculate → numerical result: Kinoshita’s group ‘15-’18

$$A_1^{(10)} = 6.678(192)$$

6 loops: 202770 diagrams to calculate → nobody

This was only QED. What about other contributions?

for magn.moment of electron not that important:

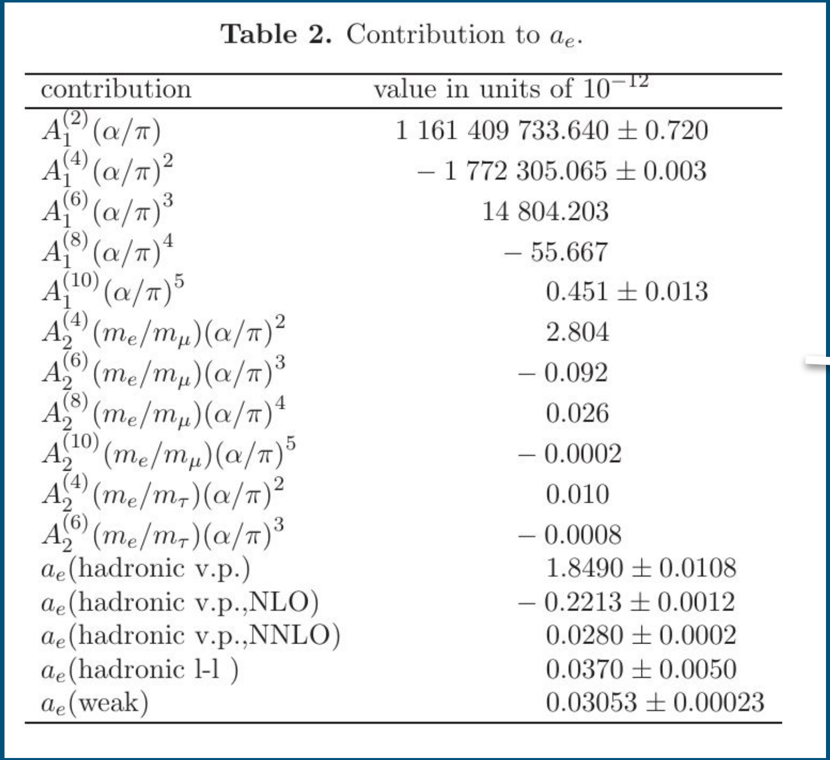


Table 2. Contribution to a_e.

contribution	value in units of 10^{-12}
$A_1^{(2)}(\alpha/\pi)$	$1\,161\,409\,733.640 \pm 0.720$
$A_1^{(4)}(\alpha/\pi)^2$	$-\,1\,772\,305.065 \pm 0.003$
$A_1^{(6)}(\alpha/\pi)^3$	$14\,804.203$
$A_1^{(8)}(\alpha/\pi)^4$	$-\,55.667$
$A_1^{(10)}(\alpha/\pi)^5$	0.451 ± 0.013
$A_2^{(4)}(m_e/m_\mu)(\alpha/\pi)^2$	2.804
$A_2^{(6)}(m_e/m_\mu)(\alpha/\pi)^3$	$-\,0.092$
$A_2^{(8)}(m_e/m_\mu)(\alpha/\pi)^4$	0.026
$A_2^{(10)}(m_e/m_\mu)(\alpha/\pi)^5$	$-\,0.0002$
$A_2^{(4)}(m_e/m_\tau)(\alpha/\pi)^2$	0.010
$A_2^{(6)}(m_e/m_\tau)(\alpha/\pi)^3$	$-\,0.0008$
a_e(hadronic v.p.)	1.8490 ± 0.0108
a_e(hadronic v.p.,NLO)	$-\,0.2213 \pm 0.0012$
a_e(hadronic v.p.,NNLO)	0.0280 ± 0.0002
a_e(hadronic l-l)	0.0370 ± 0.0050
a_e(weak)	0.03053 ± 0.00023

Most precise measurement:
Harvard group '11

$$\frac{g_{exp}}{2} = 1.001\,159\,652\,180\,73(28)\ [0.28\,\mathrm{ppt}]$$

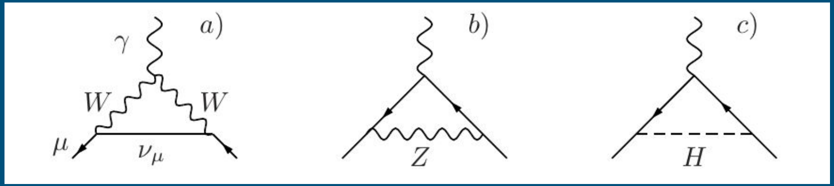
$$\frac{g_{th}}{2} = 1.001\,159\,652\,182\,032(13)(12)(720)$$

from Laporta '18

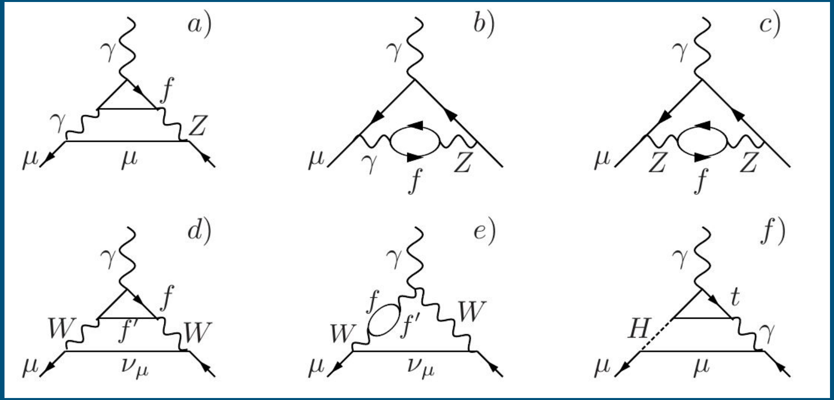
Electroweak corrections: muon g-2

Jegerlehner,Nyffeler ‘09]

$$\Delta a_\mu$$



$194.82(2) \times 10^{-11}$



$-42(2) \times 10^{-11}$

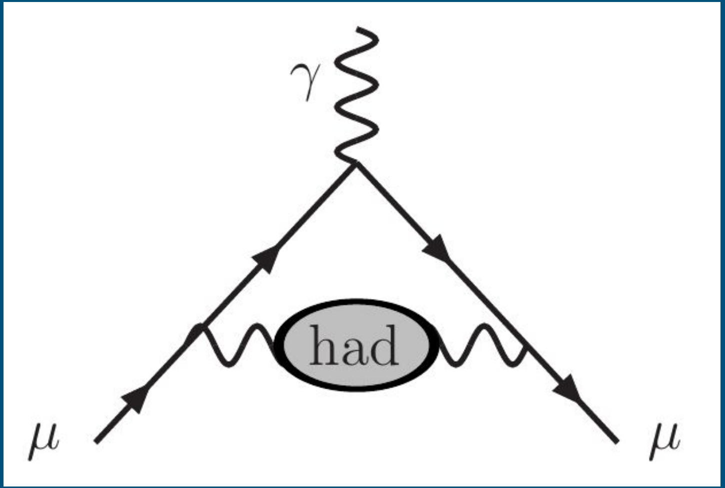
3-loop LL approx → $0.4 \pm 0.2 \times 10^{-11}$

$153.2(1.0)(1.5) \times 10^{-11}$

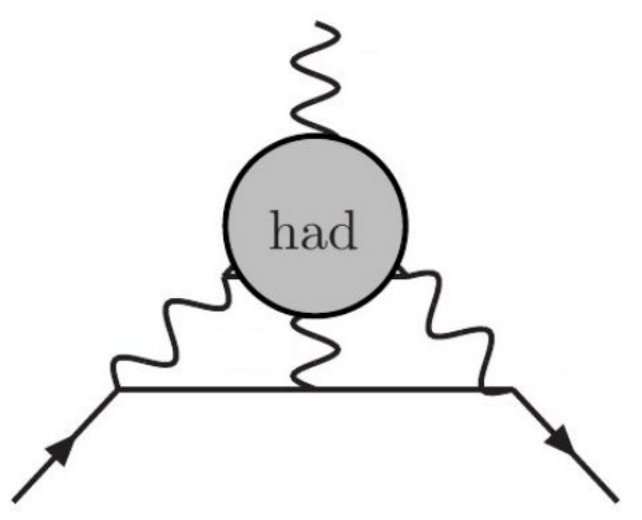
‘20: $153.6(1.0)$

Hadronic contributions

vacuum polarization



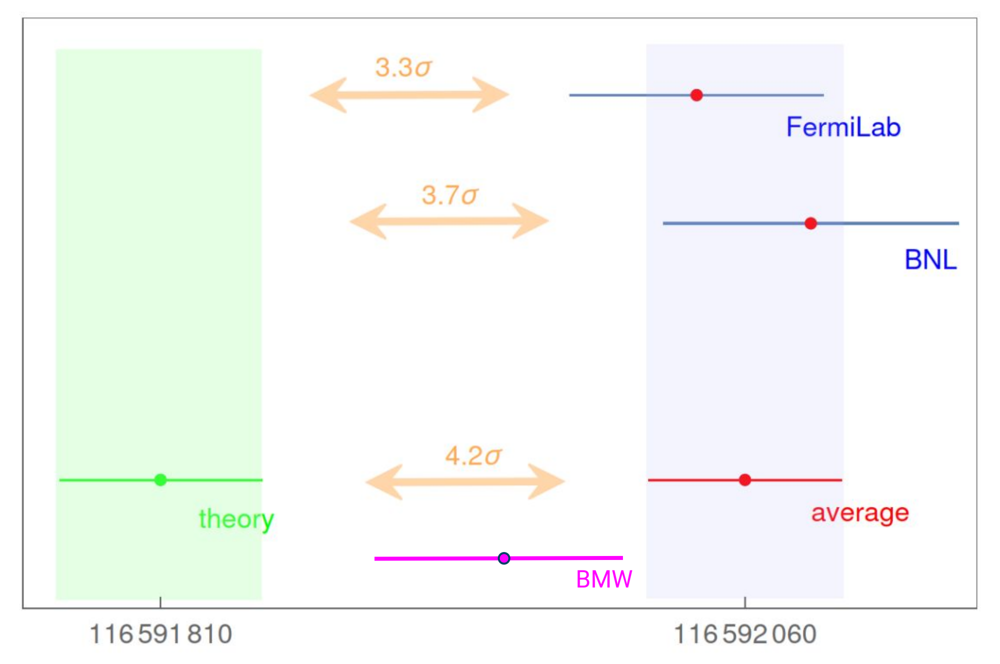
light-by-light scattering



$\Delta a_\mu(\times 10^{11}) =$ 6845(40) +92(18)

problem with sign - two independent groups had "minus"

Summary of theory vs experiments



$$\times 10^{-11}$$

$$a_\mu(\text{FNAL}) = 116592040(54)$$

$$a_\mu(\text{BNL}) = 116592089(63)$$

$$a_\mu(\text{comb.}) = 116592061(41)$$

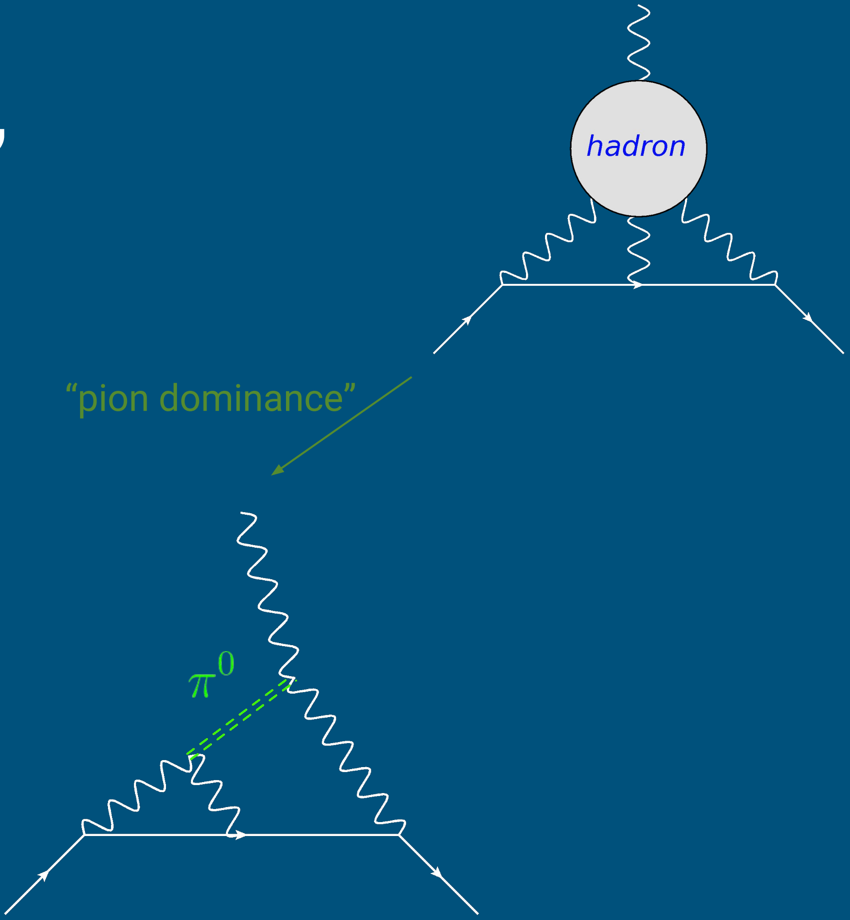
$$a_\mu(\text{theory}) = 116591810(43)$$

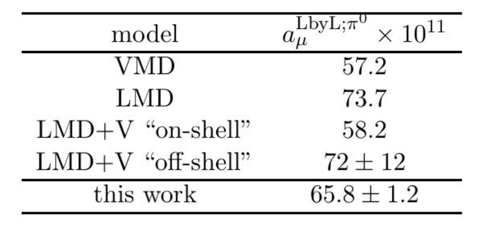
New lattice data on LO HVP: 6931(40) → 7075(55)

n.b. $\Delta a_\mu = 250(60) \approx 1.5 \times a_\mu^{EW}$

Our group “contributions”

- pion decay: KK, Moussallam ‘09
- resonances: KK, Novotný ‘11 + Kadavý *in prep.*
- radiative corrections: Husek, KK, Novotný, Sanchez-Puertas, Vaško
- rational approach: Masjuan, Sanchez-Puertas ‘17
- experiment proposal: KK, Kovaľ



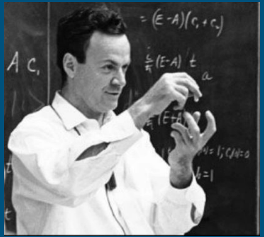


model	$a_\mu^{\mathrm{LbyL};\pi^0} \times 10^{11}$
VMD	57.2
LMD	73.7
LMD+V “on-shell”	58.2
LMD+V “off-shell”	72 ± 12
this work	65.8 ± 1.2

from [KK,Novotny ‘11]

The most precise physical measurement?

- Yes, if physical means in some sense fundamental
- Yes, if we are talking about electron *g-2*
- Yes, if no “null results” are considered
    - [pdg] photon mass < 1x10^{-18} eV
    - [MICROSCOPE] weak equivalence principle ~ 10^{-15}
    - ratio of electron and proton charge
    - ….



Feynman in 1985: “the equivalent of measuring the distance from Los Angeles to New York to within the width of a human hair.”

today: measuring the distance from the Earth to the Moon with the same precision

Conclusion

- electron *g-2* is the most precise fundamental physical measurement
- muon *g-2* is a possible window to new physics
- both played important role in particle physics development (past 74 years)
- NP hint? -> too early
- only one hint? -> interesting anomalies in B physics
- only one (and half) experiment on *g-2*? -> J-Parc proposal
  - different method: well controlled muon beam
  - 2020-2023 construction

THANK YOU!