PHYSICS 442 – WINTER 2021
Prof. André de Gouvêa

Homework 3 - **due Tuesday, February 16**

1. **Understanding Atmospheric Neutrino Oscillations** – In order to understand the effect of neutrino oscillations, considering two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations only, on the atmospheric muon-neutrino data, proceed as follows:

   (a) Numerically calculate and draw histograms of the average muon-neutrino survival probability in ten equal-size bins of $\cos\theta_z$, where θ_z is the angle between the neutrino direction and the vertical-axis at the detector's location ($\theta_z = 0$ for neutrinos coming straight from above, and $\theta_z = \pi$ for neutrinos coming from below). Make one histogram for $E_\nu = 0.2$ GeV, 2 GeV, and 20 GeV combined with $\Delta m^2 = 2.5 \times 10^{-4}$ eV^2, 2.5×10^{-3} eV^2, and 2.5×10^{-2} eV^2, for a grand total of nine plots. Assume throughout that the mixing is maximal, i.e., $\sin^2 2\theta = 1$, and that neutrinos are produced 20 km above the surface of the Earth.

   (b) Read "Super-Kamiokande Atmospheric Neutrino Results" by T. Toshito, hep-ex/0105023. It contains a summary of (quite old) atmospheric neutrino data. More recent data are more impressive but qualitatively the same. The historic talk by T. Kajita, hep-ex/981001, presented at the Neutrino 1998 Conference, may also prove helpful in understanding some of the Super-Kamiokande terminology. Look at Figure 1 and compare with the results you got in part (a). Can you qualitatively verify that $\Delta m^2 \sim 2.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta \sim 1$ is a good fit to the data (200 MeV is characteristic of sub-GeV events, 2 GeV is typical of multi-GeV events, and 20 GeV is typical of upward stopping muons. The fourth category, upward-through-going muons, has an average energy above 100 GeV)? In particular, explain why there is almost no depletion for $\cos\theta_z > 0.2$ in the multi-GeV data, but some depletion in the sub-GeV data.

2. **Day-Night Effect** – Solar neutrino oscillations can also be modified by the fact that, during the night, the neutrinos have to cross some significant amount of the Earth in order to reach the detectors. Hence, the oscillation probability is different for neutrinos arriving during the day and the night.

   To understand this effect, assume that solar neutrinos arrive at the surface of the Earth in the $|\nu_2\rangle$ state (a mass eigenstate). This is true of ^{8}B solar neutrinos as long as few $\times$ 10^{-9} eV$^2 < \Delta m^2 <$ few $\times$ 10^{-5} eV^2 and $\sin^2\theta$ is not too small ($\sin^2\theta > 0.1$ is safe). Even for the "correct" value of $\Delta m^2_{12} \sim 8 \times 10^{-5}$ eV^2, the approximation is pretty good (at the several percent level). Throughout, we assume there are only two flavors.

   (a) Compute the probability that $|\nu_2\rangle$ is detected as an electron-type neutrino. Define the mixing angle such that $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$, as we have been doing in class. The answer to this is very simple.

   (b) Assume that the solar neutrino, originally in the $|\nu_2\rangle$ state, propagates a distance L through the Earth before reaching the detector. Assume that the electron number density N_e in the neutrino's path is constant. Compute the probability that this neutrino is detected as an electron-type neutrino.

   (c) Assume $L = 3000$ km, $\sqrt{2}G_F N_e = 1.5 \times 10^{-7}$ eV^2/MeV, $\Delta m^2 = 10^{-5}$ eV^2, $\sin^2\theta = 0.3$, and $E_\nu = 8$ MeV, and compute $P_{ee}^{\rm night} - P_{ee}^{\rm day}$ using the answers you got for (a) and (b) above.

   Useful Formulae: In matter of constant density, the oscillation frequency is

$$\Delta_M = \sqrt{\Delta^2 \sin^2 2\theta + (\Delta\cos 2\theta - \sqrt{2}G_F N_e)^2}, \tag{1}$$

where $\Delta \equiv \Delta m^2/(2E_\nu)$. The matter mixing angle obeys

$$\Delta_M \sin 2\theta_M = \Delta \sin 2\theta \quad \text{and} \quad \Delta_M \cos 2\theta_M = \Delta \cos 2\theta - \sqrt{2} G_F N_e. \tag{2}$$

The second expression is easy to derive (try it!) and may help you simplify your answer in (b).